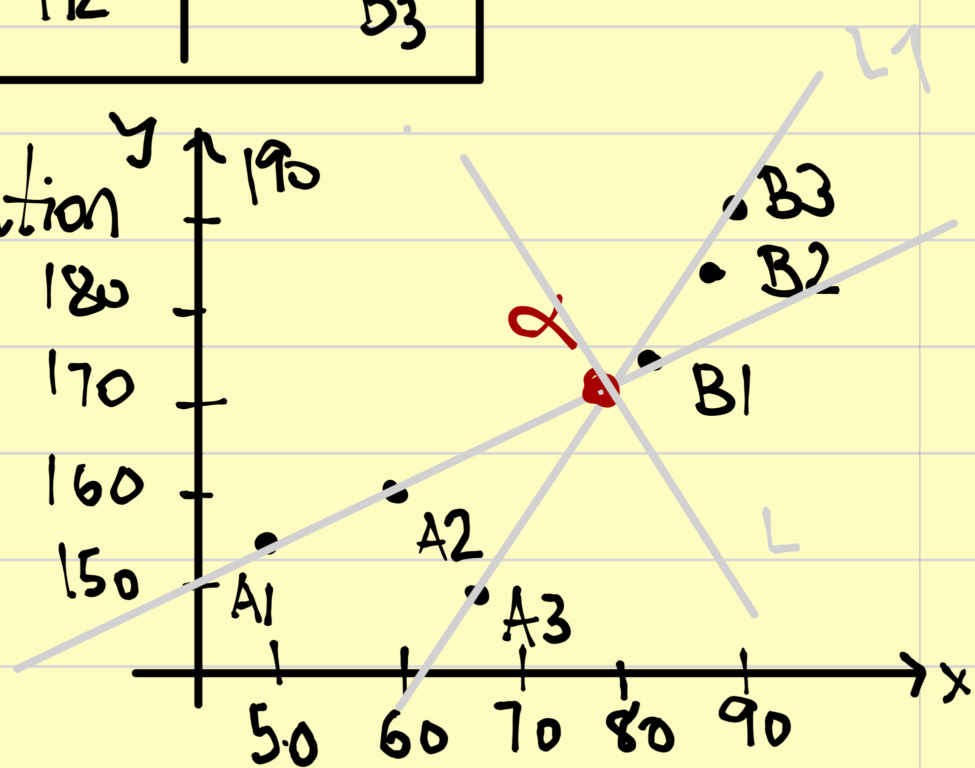
Support Vector Machines (SVM)

The goal is, given a set of labelled data, find the best separating line (2D), plane (3D), hyperplane (3+ D).
We need labelled Data
Because we are measuring distances, we need to check if normalization is needed.

Example:	x (Weight)	y (Height)	Klasse
	50	155	A_1
	60	160	A_2
2 Dimensions (x,y)	68	150	A_3
2 Groups (A,B)	85	175	B_1
	90	182	B_2
	92	192	B_3

Step 1. Graphical Representation



Step 2. The separation line has the form $y = mx + b$.

Step 3. Extreme separation lines.

$L1: A3 / B3 \quad \frac{y-150}{x-68} = \frac{192-150}{92-68} \rightarrow y = 150 + 1'75(x-68)$

$$\boxed{\begin{array}{l} \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \\ \text{Line goes through } [x_1, y_1] \\ \qquad [x_2, y_2] \end{array}}$$

$L2: A2/B1. \quad \frac{y-160}{x-60} = \frac{175-160}{85-60} \rightarrow y = 160 + 0'33(x-60)$

$y\alpha = 150 + 1'75(x\alpha - 68)$

$y\alpha = 160 + 0'33(x\alpha - 60)$

$\rightarrow 0 = -10 + (1'75 - 0'33)x\alpha - 68 + 60 \rightarrow$

$x\alpha = 12'67 \rightarrow y\alpha = 53'18 \qquad \alpha[12'67, 53'18]$

$d_{L,A2} = \frac{|m.60 + 160 + b|}{\sqrt{m^2+1}}$

$$\boxed{\begin{array}{l} d_{L,\text{POINT}} = \frac{|mx_0 + y_0 + b|}{\sqrt{m^2+1}} \\ y = mx + b \\ \text{POINT}[x_0 y_0] \end{array}}$$

$d_{L,B_1} = \frac{|m.85 + 175 + b|}{\sqrt{m^2+1}}$

$\rightarrow 60 + 160 = 85m + 175$

$\rightarrow m = -0'6$

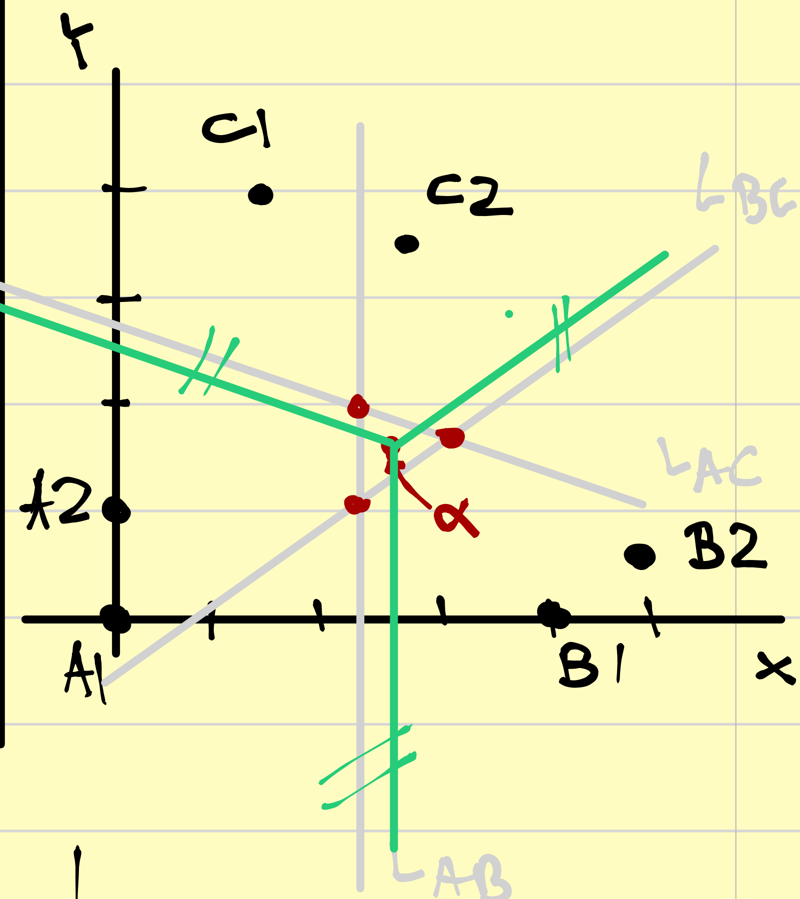
$y = -0'6x + b \rightarrow 53'18 = -0'6.12'67 + b \rightarrow b = 60'78$

$$\boxed{y = -0'6.x + 60'78}$$

$d = \frac{|60.(-0'6) + 160 + 60'78|}{\sqrt{0'6^2+1}} = 158'45$

Example for 2 Dimensions and 3 Groups.

Customers	X	Y
A	0	0
	0	1
B	4	0
	5	0'5
C	2	4
	3	3'5



L_{AB}: $L_1: A2/B1$: $\frac{y-1}{x-0} = \frac{0-1}{4-0} \longrightarrow y = 1 - \frac{1}{4}x$

$L_2: A1/B2$: $\frac{y-0}{x-0} = \frac{0'5-0}{5-0} \longrightarrow y = 0'1x$

$\blacktriangleright\ y_{\alpha_1} = 1 - \frac{1}{4}x_{\alpha_1}$ $\blacktriangleright\ x_{\alpha_1} = 2'857$

$y_{\alpha_1} = 0'1x_{\alpha_1}$; $y_{\alpha_1} = 0'286$

$$d_{L_{AB},A2} = \frac{|m\cdot 0 + 1 + b|}{\sqrt{m^2+1}}$$

$$d_{L_{AB},B1} = \frac{|m\cdot 4 + 0 + b|}{\sqrt{m^2+1}}$$

$\blacktriangleright\ \cancel{b}+1 = m\cdot 4 + \cancel{b} \longrightarrow m = \frac{1}{4}$

$y = \frac{1}{4}x + b \longrightarrow 0'286 = \frac{1}{4}\cdot 2'857 + b \longrightarrow b = -0'428$

$$\boxed{L_{AB}:\ y = 0'25x - 0'428\ ;\ \alpha_1 = [2'857, 0'286]}$$

L_{AC}: $L_1: A2/C2$: $\frac{y-1}{x-0} = \frac{3'5-1}{3-0} \longrightarrow y = 1 + 0'83x$

$L_2: A1/C1$: $\frac{y-0}{x-0} = \frac{2-0}{4-0} \longrightarrow y = 0'5x$ $\blacktriangleright$

$$y_{\alpha_2} = 1 + 0'83\, x_{\alpha_2}$$
$$y_{\alpha_2} = 0'5\, x_{\alpha_2}$$
$$\rightarrow x_{\alpha_2} = 3 \qquad y_{\alpha_2} = 1'5$$

$$d_{L_{AC}, C_1} = \frac{|m.2+4+b|}{\sqrt{m^2+1}}$$
$$d_{L_{AC}, A_2} = \frac{|m.0+1+b|}{\sqrt{m^2+1}}$$
$$\rightarrow m.2+4+\cancel{b} = \cancel{b}+1 \qquad m = \frac{-3}{2} = -1'5$$

$$y = -1'5x + b \rightarrow 1'5 = -1'5 \cdot 3 + b \rightarrow b = 6$$

$$\boxed{L_{AC}: \ y = -1'5x + 6 \qquad \alpha_2 [3, 1'5]}$$

L_{BC} :

$$L_1: B1/C2: \ \frac{y-0}{x-4} = \frac{3'5-0}{3-4} \rightarrow y = 3'5(x-4)$$

$$L2: B2/C1: \ \frac{y-0'5}{x-5} = \frac{4-0'5}{2-5} \rightarrow y = 0'5 - 1'67(x-5)$$

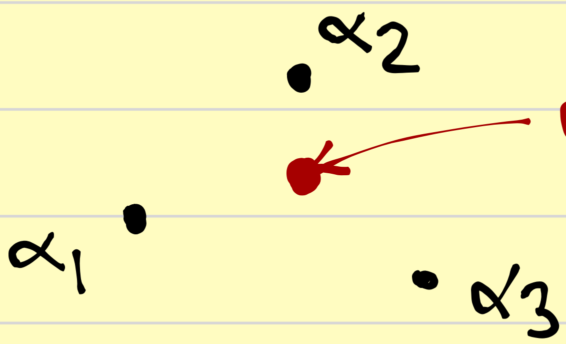
$$y_{\alpha_3} = 3'5(x_{\alpha_3} - 4)$$
$$y_{\alpha_3} = 0'5 - 1'67(x_{\alpha_3} - 5)$$
$$\rightarrow 0 = -3'5\, x_{\alpha_3} + 14 - 0'5 + 1'67\, x_{\alpha_3} - 8'35$$

$$x_{\alpha_3} = 2'814 \qquad y_{\alpha_3} = 4'15$$

$$d_{L_{BC}, C_2} = \frac{|m.3+3'5+b|}{\sqrt{m^2+1}}$$
$$d_{L_{BC}, B_2} = \frac{|m.5+0'5+b|}{\sqrt{m^2+1}}$$
$$\rightarrow 3m + 3'5 + \cancel{b} = 5m + 0'5 + \cancel{b} \qquad m = 1'5$$

$$y = 1'5x + b \rightarrow 4'15 = 1'5 \cdot 2'814 + b \rightarrow b = -0'071$$

$$\boxed{L_{BC}:\ y = 1'5\cdot x - 0'071 \qquad \alpha_3\left[2'814,\ 4'15\right]}$$



$$\alpha\left[\frac{x_{\alpha_1}+x_{\alpha_2}+x_{\alpha_3}}{3},\ \frac{y_{\alpha_1}+y_{\alpha_2}+y_{\alpha_3}}{3}\right]$$

$$\alpha\left[2'89,\ 1'978\right]$$

$L_{AB}:\ y = 0'25x - 0'428$

$/\!/\alpha \quad y = 0'25x + (1'978 - 0'25\cdot 2'89)$

$$\boxed{L'_{AB} \equiv y = 0'25x + 1'256}$$

$$\boxed{\begin{array}{l}\text{The parallel line to } y=mx+b \text{ going through } [x_1, y_1]\\ y = mx + (y_1 - mx_1)\end{array}}$$

$L_{AC}:\ y = -1'5x + 6$

$/\!/\alpha \quad y = -1'5x + (1'978 + 1'5\ 2'89)$

$$\boxed{L'_{AC} \equiv y = -1'5x + 6'313}$$

$L_{BC}:\ y = -x + 4'5$

$/\!/\alpha \quad y = -x + (1'978 + 1\cdot 2'89)$

$$\boxed{L'_{BC} \equiv y = -x + 4'868}$$