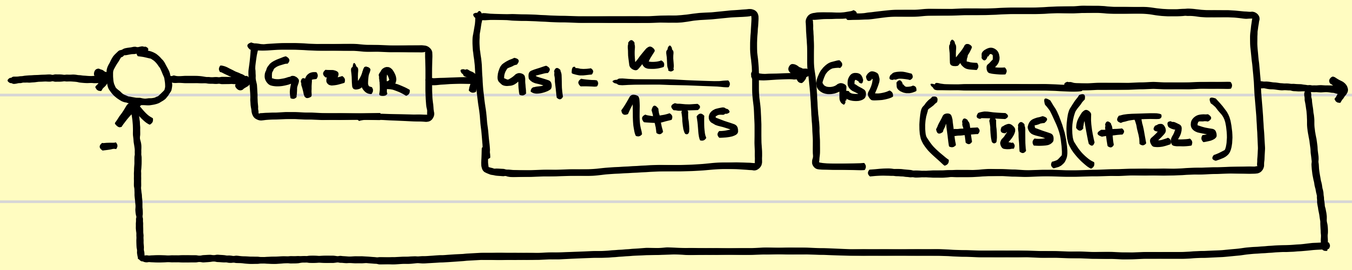
20240528_Regelungstechnik_BEL4

3.



$T_1, T_{21}, T_{22} \in \mathbb{R}^+$

$$G_g(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{\frac{K_1 K_2 K_R}{(1+T_1 s)(1+T_{21}s)(1+T_{22}s)}}{1+\frac{K_1 K_2 K_R}{(1+T_1 s)(1+T_{21}s)(1+T_{22}s)}} =$$

$$= \frac{K_1 K_2 K_R}{(T_1 T_{21} T_{22})s^3 + (T_1 T_{21} + T_1 T_{22} + T_{21} T_{22})s^2 + (T_1 + T_{21} + T_{22})s + 1 + K_1 K_2 K_R}$$

$$= \frac{K_1 K_2 K_R}{(T_1 T_{21} T_{22})} \cdot \frac{1}{s^3 + \frac{T_1 T_{21} + T_1 T_{22} + T_{21} T_{22}}{T_1 T_{21} T_{22}} s^2 + \frac{T_1 + T_{21} + T_{22}}{T_1 T_{21} T_{22}} s + \frac{1 + K_1 K_2 K_R}{T_1 T_{21} T_{22}}}$$

Annahme OHNE Verlust an Erklärbarkeit vom Model:

$T_1 = T_{21} = T_{22} = T \qquad K_1 K_2 K_R = K$

$$= \frac{K}{T^3} \cdot \frac{1}{s^3 + \frac{3}{T}s^2 + \frac{3}{T^2}s + \frac{1+K}{T^3}} = \frac{K}{T^3} \cdot \frac{1}{s^3 + Bs^2 + Cs + D}$$

CARDAN

$A = 1 \quad ; \quad B = \frac{3}{T} \quad ; \quad C = \frac{3}{T^2} \quad ; \quad D = \frac{1+K}{T^3}$

- $p = C - \frac{1}{3}B^2 = \frac{3}{T^2} - \frac{1}{3}\left(\frac{3}{T}\right)^2 = 0$
- $q = D + \frac{2}{27}B^3 - \frac{1}{3}BC = \dots = \frac{K}{T^3}$

- $s^* = y^* - \frac{B}{3} = y^* - \frac{1}{T}$

p=0

- $y^* = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \cancel{\frac{p^3}{27}}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \cancel{\frac{p^3}{27}}}} =$

$$= \sqrt[3]{-\frac{q}{2} - \frac{q}{2}} = \sqrt[3]{-q} = \sqrt[3]{\frac{-K}{T^3}}$$

STABILITÄTS-BEDINGUNG

$$s^* = \sqrt[3]{\frac{-K}{T^3}} - \frac{1}{T} = -\frac{\sqrt[3]{K+1}}{T} < 0$$

$\boxed{K > -1}$ STABILITÄT

ANNAHME:

$K = 1 \quad T = 1$

$$G_g(s) = \frac{1}{s^3 + 3s^2 + 3s + 2} = \dots = \frac{1}{(s+2)(s+0'5+j0'8)(s+0'5-j0'8)}$$

$$G_g(j\omega) = \frac{1}{(2+j\omega)(0'5+j(0'8+\omega))(0'5+j(\omega-0'8))} \longrightarrow \text{BODE}$$

$$G_g(s) = \frac{A}{(s+2)} + \frac{B}{(s+0'5+j0'8)} + \frac{C}{(s+0'5-j0'8)} \longrightarrow$$

Laplace inverse Zeitverhalten

$$1 = A(s+0'5+j0'8)(s+0'5-j0'8) +$$
$$B(s+2)(s+0'5-j0'8) +$$
$$C(s+2)(s+0'5+j0'8)$$

$$s^* = -2 \longrightarrow 1 = A(-2+0'5+j0'8)(-2+0'5-j0'8)$$
$$1 = A(-1'5+j0'8)(-1'5-j0'8)$$
$$1 = A(1'5^2+0'8^2) \longrightarrow A = 0'35$$

$$s^* = -0'5-j0'8 \longrightarrow 1 = B(-0'5-j0'8+2)(-0'5-j0'8+0'5-j0'8)$$
$$\longrightarrow 1 = B(+1'5-j0'8)(-2j0'8)$$
$$\longrightarrow 1 = B(2'4j - 1'28)$$
$$\longrightarrow B = \frac{1}{-1'28+j2'4} \cdot \frac{-1'28-j\cdot 2'4}{-1'28-j\cdot 2'4} =$$
$$= \frac{-1'28-j\cdot 2'4}{(1'28)^2+(2'4)^2} = \frac{-1'28}{(1'28)^2+(2'4)^2} - j\frac{2'4}{(1'28)^2+(2'4)^2}$$

$$s^* = -0'5+j0'8 \longrightarrow C = \cdots$$

$$xa(s) = \frac{1}{s} \cdot Gg(s) = \frac{1}{s} \cdot \frac{1}{(s+2)(s+0'5+j0'8)(s+0'5-j0'8)}$$

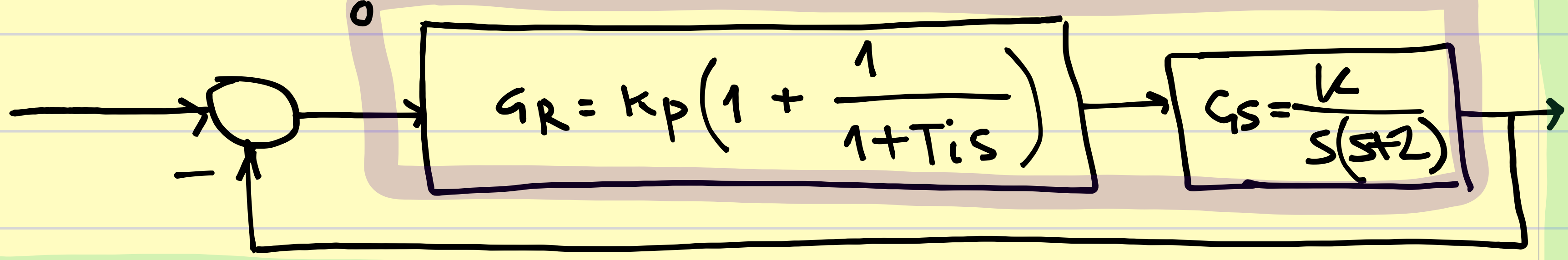
$$\rightarrow xa(s) = \frac{A'}{s} + \frac{B'}{s+2} + \frac{C'}{(s+0'5+j0'8)} + \frac{D'}{(s+0'5-j0'8)}$$

$$A', B', C', D' \longrightarrow xa(t) = \ldots$$

HINWEIS: $x_a(s) = \frac{a+bj}{1+Ts} \longrightarrow x_a(t) = e^{a+bjt} \mathcal{L}^{-1}\left(\frac{1}{1+Ts}\right) =$

$= \cos t + j\sin(t)$

4.



a) $G_0(s) = K \cdot K_P \cdot \frac{1}{s(s+2)} \cdot \left(1 + \frac{1}{1+T_i s}\right) =$

$= K \cdot K_P \cdot \frac{1}{s(s+2)} \cdot \frac{1+T_i s+1}{1+T_i s} =$

$= K \cdot K_P \cdot \frac{1}{s(s+2)} \cdot \frac{2+T_i s}{1+T_i s} =$

$= \frac{K \cdot K_P}{T_i} \cdot \frac{1}{s(s+2)} \cdot \frac{2+T_i s}{s+\frac{1}{T_i}} = G_0(s)$

STABILITÄTSBEDINGUNG: $\frac{1}{T_i} > 0 \longrightarrow T_i > 0$

b) $K = K_P = T_i = 1$

$$G_g(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{\frac{1}{(s+2)s} \cdot \frac{s+2}{s+1}}{1 + \frac{1}{s(s+2)} \cdot \frac{s+2}{s+1}} = \dots = \frac{1}{s^2+s+1}$$

$$s^* = \frac{-1 \pm \sqrt{-1-4}}{2} = \frac{-1 \pm \sqrt{5}\,j}{2}$$

$$G_g(s) = \frac{1}{\left(s + \frac{1-\sqrt{5}j}{2}\right)\left(s + \frac{1+\sqrt{5}j}{2}\right)}$$

$$a = \frac{1-\sqrt{5}j}{2} \qquad b = \frac{1+\sqrt{5}\,j}{2}$$

c) $$x_a(s) = \frac{1}{s} \cdot \frac{1}{s+a} \cdot \frac{1}{s+b} = \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b}$$

$$\begin{aligned} 1 = \; & A(s+a)(s+b) + \\ & B\,s(s+b) + \\ & C\,s(s+a) \end{aligned}$$

$$s^* = 0 \rightarrow 1 = A \cdot ab \rightarrow A = \frac{1}{ab} = \frac{1}{\left(\frac{1-\sqrt{5}j}{2}\right)\left(\frac{1+\sqrt{5}j}{2}\right)} = \frac{4}{6} = \frac{2}{3}$$

$$s^* = -a \rightarrow 1 = B(-a)(-a+b) \rightarrow B = \frac{1}{a(a-b)} \rightarrow$$

$$\rightarrow B = \frac{1}{\frac{1-\sqrt{5}j}{2} \cdot \left(\frac{1-\sqrt{5}j}{2} - \frac{1+\sqrt{5}j}{2}\right)} = \frac{1}{\frac{1-\sqrt{5}j}{2}\left(-\sqrt{5}j\right)} =$$

$$= \frac{1}{\frac{-\sqrt{5}j - 5}{2}} = \frac{-2}{5+\sqrt{5}j} \cdot \frac{5-\sqrt{5}j}{5-\sqrt{5}j}$$

$$= \frac{-10 + 2\sqrt{5}j}{30} = \frac{-1}{3} + \frac{\sqrt{5}}{15} j$$

$$s^* = -b \to \dots \to c = \frac{1}{3} + \frac{\sqrt{5}}{15} j$$

$$x_a(s) = \frac{2/3}{s} + \frac{-\frac{1}{3} + \frac{\sqrt{5}}{15} j}{s + \frac{1-\sqrt{5} j}{2}} + \frac{\frac{1}{3} + \frac{\sqrt{5}}{15} j}{s + \frac{1+\sqrt{5} j}{2}}$$

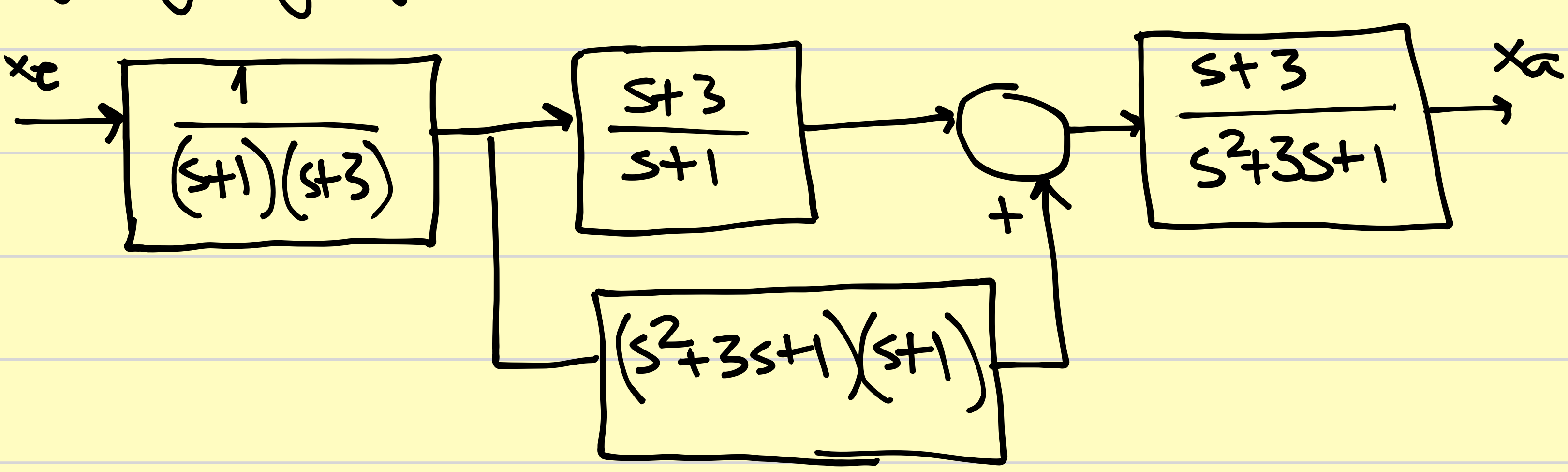
$$x_a(t) = \frac{2}{3} + \left(\frac{-1}{3} + \frac{\sqrt{5}}{15} j\right) e^{\frac{-t}{\frac{1-\sqrt{5} j}{2}}} + \left(\frac{1}{3} + \frac{\sqrt{5}}{15} j \cdot e^{\frac{-t}{\frac{1+\sqrt{5} j}{2}}}\right)$$

$\vdots$

$$x_a(t) = \frac{2}{3} + \left(\frac{-1}{3} + \frac{\sqrt{5}}{15} j\right)\left(\cos \frac{-2t}{1-\sqrt{5} j} + \sin \frac{-2t}{1-\sqrt{5} j}\right) + \left(\frac{1}{3} + \frac{\sqrt{5}}{15} j\right)\left(\cos \frac{-2t}{1+\sqrt{5} j} + \sin \frac{-2t}{1+\sqrt{5} j}\right)$$

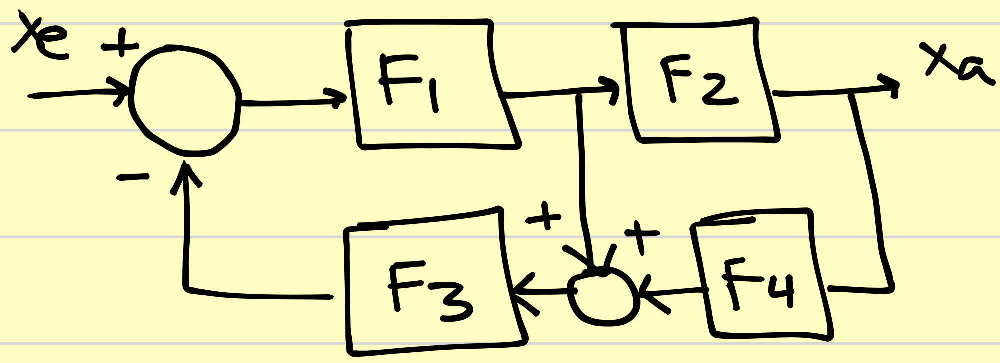
Übungen:

1) Ermitteln Sie das Ausgangssignal $x_a(t)$ bei einem Sprungeingangssignal $x_e(t) = 1$.

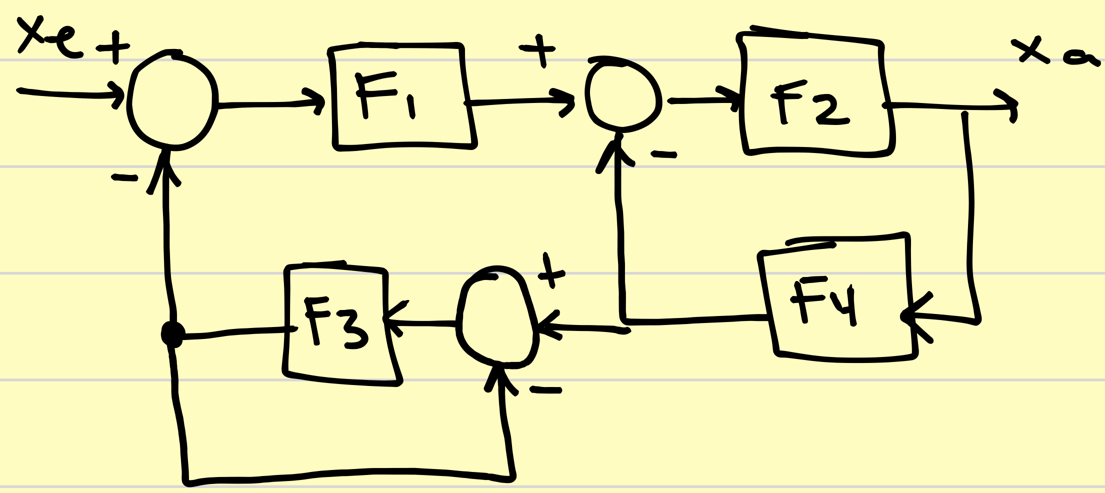


2) Von den Beiden Blockschaltbildern, bitte die Übertragungsfunktion jeweils angeben:

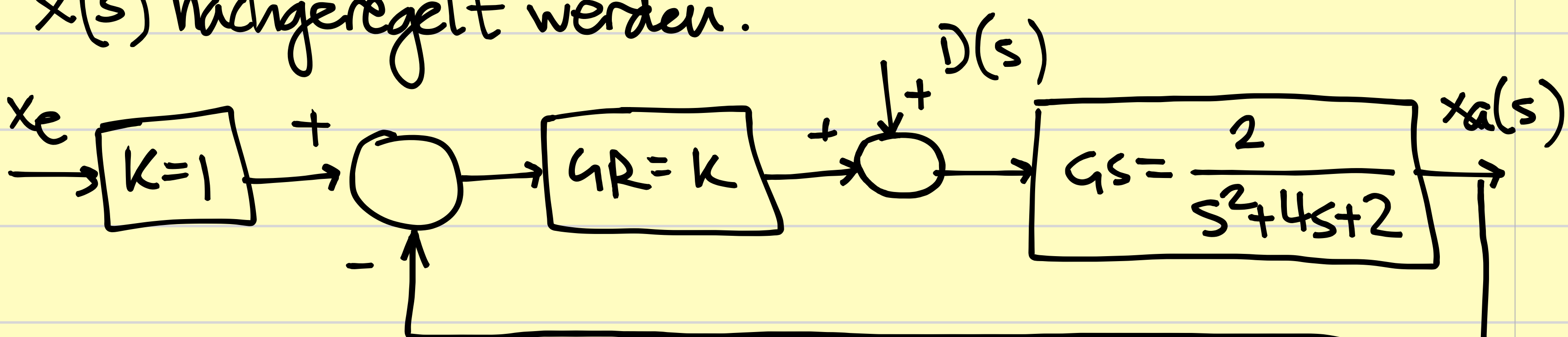
2.1)



2.2)

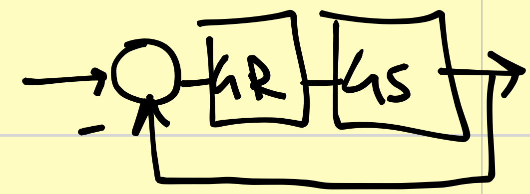


3) Ein Lase für Augenchirurgie wird über eine automatische Positionierung bestimmt.
Sollte sich das Auge bewegen D(s), so muss die Position x(s) nachgeregelt werden.



a) Ermitteln Sie den Bereich der Verstärkung (K), damit das System stabil ist.

b) Und damit es zu keiner Überschwingung kommen kann.

4) Für einen Standardregelkreis 

$$G_S = \frac{1}{(1+s)^2(1+5s)} \qquad G_R = K_t \frac{1}{1+T_i s}$$

Ermitteln Sie die Parameter K_t und T_i für einen stabilen Betrieb.