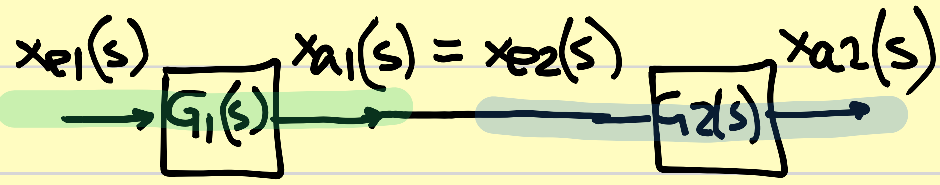
Verbindungsmöglichkeiten von RK-Gliedern

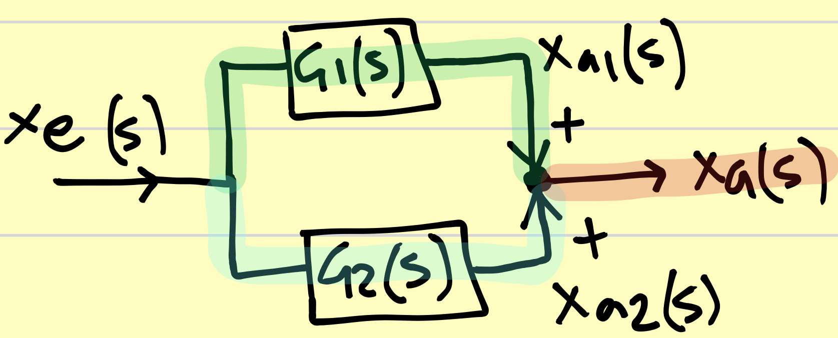
a) REIHENSCHALTUNG



$$\left.\begin{aligned} x_{a1}(s) &= G_1(s) \cdot x_{e1}(s) \\ x_{a2}(s) &= G_2(s) \cdot x_{e2}(s) \\ x_{a1}(s) &= x_{e2}(s) \end{aligned}\right\} \quad x_{a2}(s) = G_2(s) \cdot \underbrace{G_1(s) \cdot x_{e1}(s)}_{x_{a1}(s) = x_{e2}(s)}$$

$$\boxed{G_{REIHENSCHALTUNG} = \frac{x_{a2}(s)}{x_{e1}(s)} = G_1(s) \cdot G_2(s)}$$

b) PARALLELSCHALTUNG



$$\left.\begin{aligned} x_a(s) &= x_{a1}(s) + x_{a2}(s) \\ x_{a1}(s) &= G_1(s) \cdot x_e(s) \\ x_{a2}(s) &= G_2(s) \cdot x_e(s) \end{aligned}\right\} \quad x_a(s) = x_e(s) \cdot G_1(s) + x_e(s) \cdot G_2(s)$$

$$\boxed{G_{PARALLELSCHALTUNG} = \frac{x_a(s)}{x_e(s)} = G_1(s) + G_2(s)}$$

Übung. Bitte berechnen Sie $u_a(t)$, wenn die Eingangsfunktion $u_e(t)=k$, und die Reihengeschaltete Systeme 1 & 2 folgende Übertragungsfunktionen haben: $G_1(s)=\frac{s+3}{s+1}$ $G_2(s)=\frac{1}{s+2}$

$$x_e(s) \longrightarrow \boxed{G_1(s)} \longrightarrow \boxed{G_2(s)} \longrightarrow x_a(s)$$

$$\mathcal{L}(k(t)) = \mathcal{L}(k) = \frac{k}{s}$$

$$\left.\begin{aligned} G(s) &= \frac{x_a(s)}{x_e(s)} = \frac{s+3}{s+1}\cdot\frac{1}{s+2} \\ x_e(s) &= \frac{k}{s} \end{aligned}\right\} x_a(s) = \frac{k}{s}\cdot\frac{s+3}{s+1}\cdot\frac{1}{s+2} =$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$k(s+3) = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

$$s^* = 0 \longrightarrow 3k = 2A \longrightarrow A = \frac{3k}{2}$$

$$s^* = -1 \longrightarrow 2k = -B \longrightarrow B = -2k$$

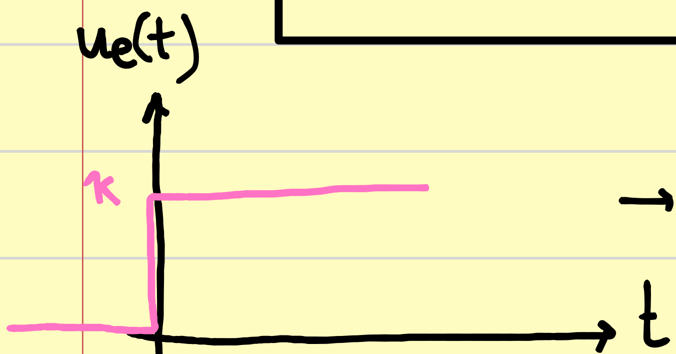
$$s^* = -2 \longrightarrow k = 2C \longrightarrow C = \frac{k}{2}$$

$$u_a(s) = \frac{3k}{2}\cdot\frac{1}{s} + \frac{-2k}{s+1} + \frac{k}{2}\cdot\frac{1}{s+2}$$

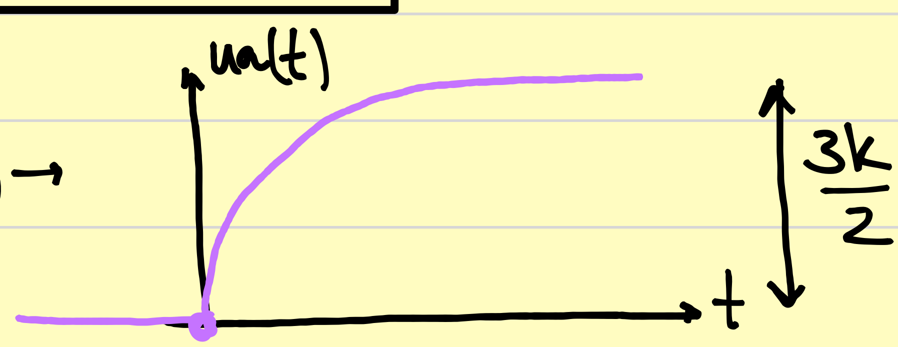
$$\mathcal{L}^{-1}(u(t)) \equiv \boxed{u_a(t) = \frac{3k}{2} - 2k\,e^{-t} + \frac{k}{2}e^{-2t}}$$

$$u_a(t=0) = \frac{3k}{2} - 2k + \frac{k}{2} = 0$$

$$u_a(t\to\infty) = \frac{3k}{2}$$

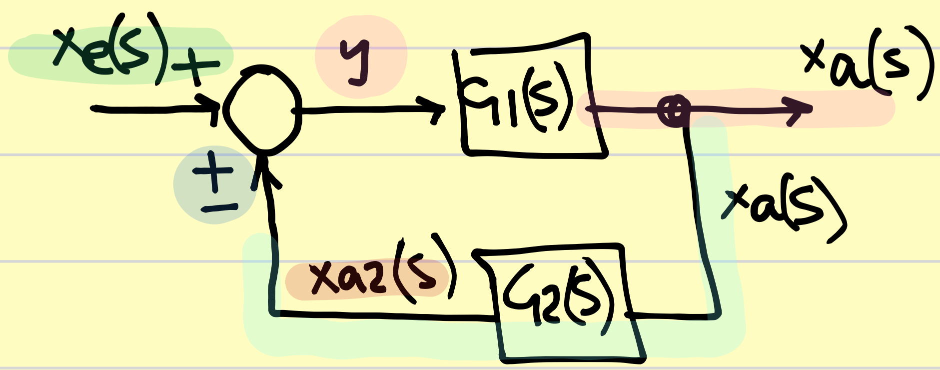


$\rightarrow G(s) \rightarrow$



Übung: bitte die obere Aufgabe mit einer Parallelschaltung der Glieder lösen.

c) RÜCKFÜHRUNGSCHALTUNG



$$\left.\begin{aligned} xa(s) &= G_1(s)\cdot \underbrace{\left[xe(s) \pm xa_2(s)\right]}_{y} \\ xa_2(s) &= G_2(s)\cdot xa(s) \end{aligned}\right\}$$

$$xa(s) = G_1(s)\cdot\left[xe(s) \pm G_2(s)\cdot xa(s)\right] \rightarrow$$

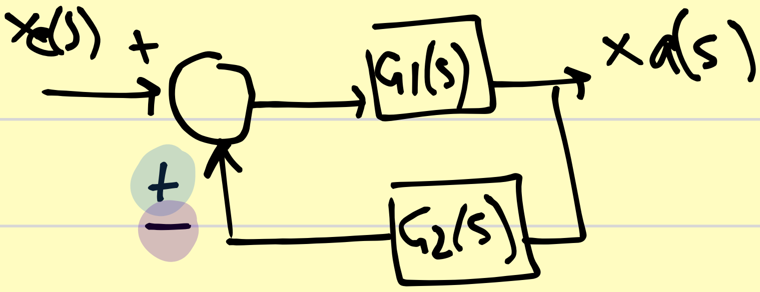
$$\rightarrow\ xa(s) = G_1(s)\cdot xe(s) \pm G_1(s)G_2(s)\cdot xa(s) \rightarrow$$

$$\rightarrow\ xa(s) \mp G_1(s)G_2(s)\, xa(s) = G_1(s)\cdot xe(s) \rightarrow$$

$$\rightarrow\ xa(s)\left[1 \mp G_1(s)G_2(s)\right] = G_1(s)\, xe(s) \rightarrow$$

$$\boxed{G(s)_{\text{RÜCKFÜHRUNGSSCHALTUNG}} = \frac{xa(s)}{xe(s)} = \frac{G_1(s)}{1 \mp G_1(s)G_2(s)}}$$

Negatives Vorzeichen = Mitkopplung / Positives Vorzeichen = Gegenkopplung



Übung. die oberen Gliedern in Gegenkopplungsschaltung setzen und $u_a(t)$ berechnen.

$u_e(t) = k \quad G_1(s) = \frac{s+3}{s+1} \quad G_2(s) = \frac{1}{s+2}$

$$G(s) = \frac{u_a(s)}{u_e(s)} = \frac{G_1(s)}{1 - G_1(s) G_2(s)} = \frac{\frac{s+3}{s+1}}{1 - \frac{s+3}{s+1} \cdot \frac{1}{s+2}} =$$

$$\mathcal{L}^{-1}(k) = \frac{k}{s}$$

$$= \frac{\frac{s+3}{s+1}}{\frac{(s+1)(s+2) - (s+3)}{(s+1)(s+2)}} = \frac{(s+3)(s+2)}{(s+1)(s+2) - (s+3)} =$$

$$u_a(s) = \frac{k}{s} \cdot \frac{(s+3)(s+2)}{(s+1)(s+2) - (s+3)} \underset{(*)}{=} \frac{k}{s} \cdot \frac{(s+3)(s+2)}{s^2 + 2s - 1}$$

$$\boxed{(s+1)(s+2) = s^2 + 3s + 2}$$

(*)

$$s^2 + 2s - 1 = 0 \rightarrow s = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2} = \begin{cases} 0'414 \\ -2'414 \end{cases}$$

$$u_a(s) = \frac{k}{s} \cdot \frac{(s+3)(s+2)}{(s - 0'414)(s + 2'414)} = \frac{A}{s} + \frac{B}{s - 0'414} + \frac{C}{s + 2'414}$$

$$k(s+3)(s+2) = A(s-0'414)(s+2'414) + Bs(s+2'414) + Cs(s-0'414)$$

$$s^* = 0 \longrightarrow 6k = A(-0'414)(2'414) \rightarrow A = -6k$$

$$s^* = 0'414 \rightarrow k(0'414+3)(0'414+2) = B \cdot 0'414 \cdot (0'414+2'414)$$

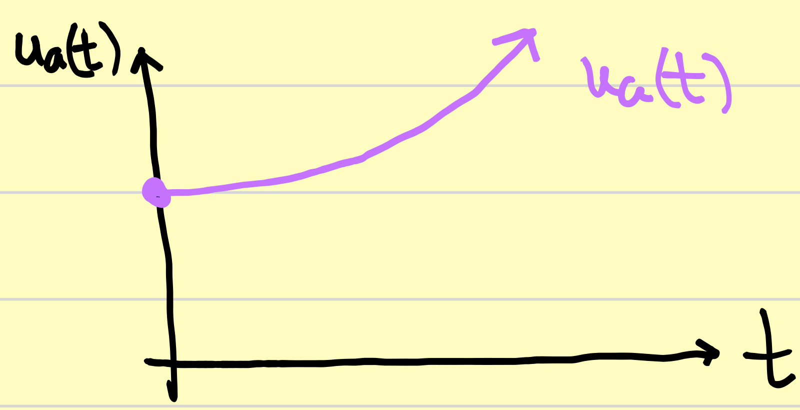
$$k \cdot 8'241 = B \cdot 1'171 \rightarrow B = 7'037k$$

$$s^* = -2'414 \longrightarrow k(-2'414+3)(-2'414+2) = C \cdot (-2'414)(-2'414-0'414)$$

$$k(-0'243) = C \cdot 6'827 \rightarrow C = -0'036k$$

$$u_a(s) = \frac{-6k}{s} + \frac{7'037k}{s-0'414} + \frac{-0'036k}{s+2'414}$$

$$\mathcal{L}^{-1} \quad u_a(t) = -6k + 7'037k\, e^{0'414t} - 0'036k\, e^{-2'414t}$$



$$u_a(t=0) = -6k + 7'037k - 0'036k$$
$$= k$$