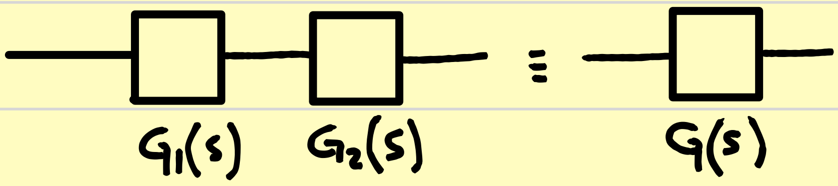
20231030_Regelungstechnik_BEL4

Grundkonzepte.

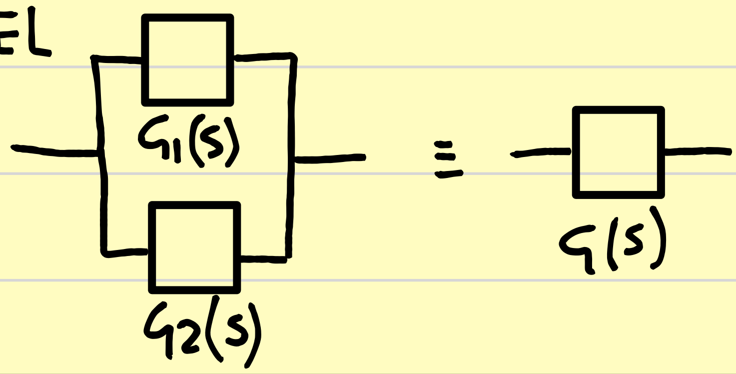
① Verbindungen von Regelkreisglieder

REIHE



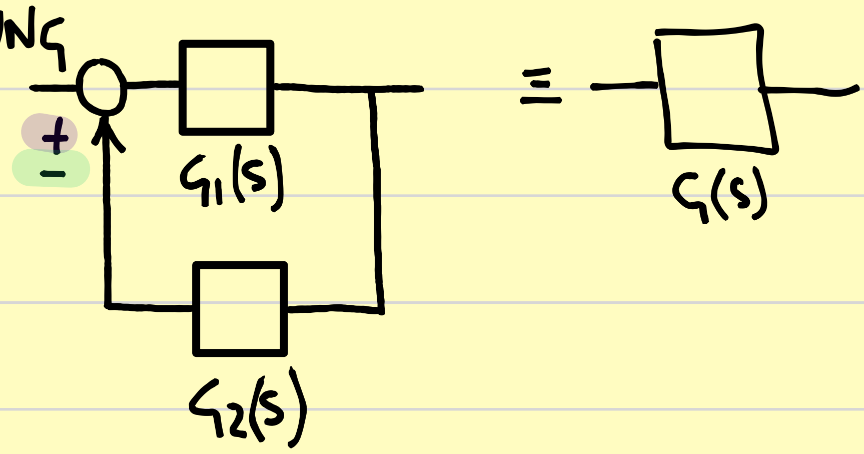
$G(s) = G_1(s) \cdot G_2(s)$

PARALLEL



$G(s) = G_1(s) + G_2(s)$

RÜCKFÜHRUNG



$$G(s) = \frac{G_1(s)}{1 \mp G_1(s) G_2(s)}$$

② LAPLACE TRANSFORM

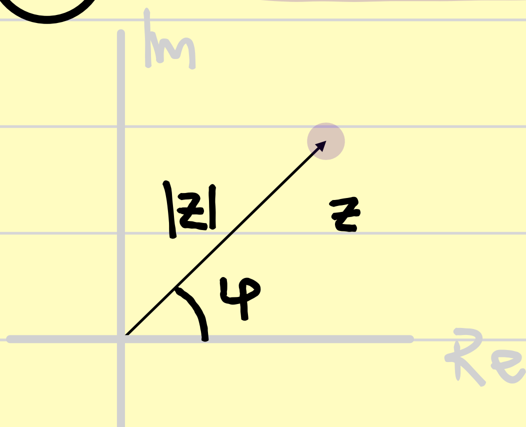
$$\boxed{\mathcal{L}[u(t)] = \int_0^\infty e^{-st}\, u(t)\, dt}$$

$$u(t) = \mathcal{L}^{-1}\left[G(s) = A \cdot \frac{1}{s+B}\right] = A \cdot e^{-Bt}$$

$\rightarrow$ $u(t) = K \rightarrow \mathcal{L}(u(t)) = \frac{K}{s}$ (Eingangsfunktion!)

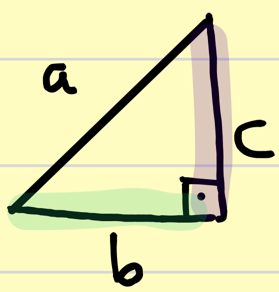
$$h(t) = \mathcal{L}^{-1}\left[G(s) = A\right] = A\,\delta(t) \qquad \delta(t) \doteq \text{DIRAC FUNC.}$$

③ KOMPLEXE ZAHLEN

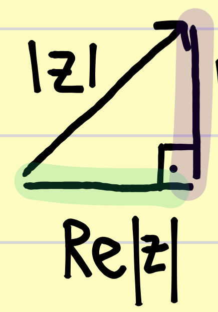


$$z = \underbrace{|z|}_{\text{MODUL}}\, e^{\overbrace{j\varphi}^{\text{PHASE}}} = |z|(\cos\varphi + j\sin\varphi)$$

PYTHA...
GORAS



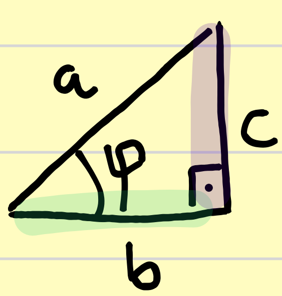
$$a^2 = b^2 + c^2$$



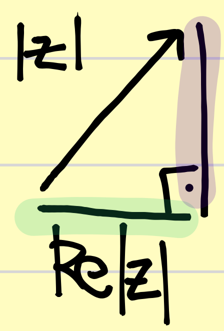
$$\text{Im}|z| \doteq |z|^2 = \text{Re}|z|^2 + \text{Im}|z|^2$$

$$\rightarrow \boxed{|z| = \sqrt{\text{Re}|z|^2 + \text{Im}|z|^2}}$$

TANGENTE



$$\operatorname{tg}\varphi = \frac{c}{b}$$

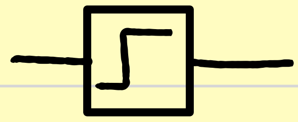


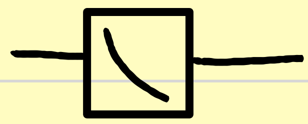
$$\text{Im}|z| \doteq \operatorname{tg}\varphi = \frac{\text{Im}|z|}{\text{Re}|z|}$$

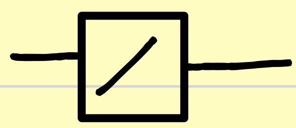
$$\rightarrow \boxed{\varphi = \operatorname{atan}\left(\frac{\text{Im}|z|}{\text{Re}|z|}\right)}$$

$$z = \frac{1}{b+jc} \rightarrow z = \frac{1}{b+jc} \cdot \frac{b-jc}{b-jc} = \frac{b-jc}{b^2+c^2} \doteq \frac{1}{b^2+c^2}(b-jc)$$

4 Klassifizierung von Regelkreisgliedern

P. $G(s) = K$ 

I. $G(s) = \frac{K}{s}$ 

D. $G(s) = Ks$ 

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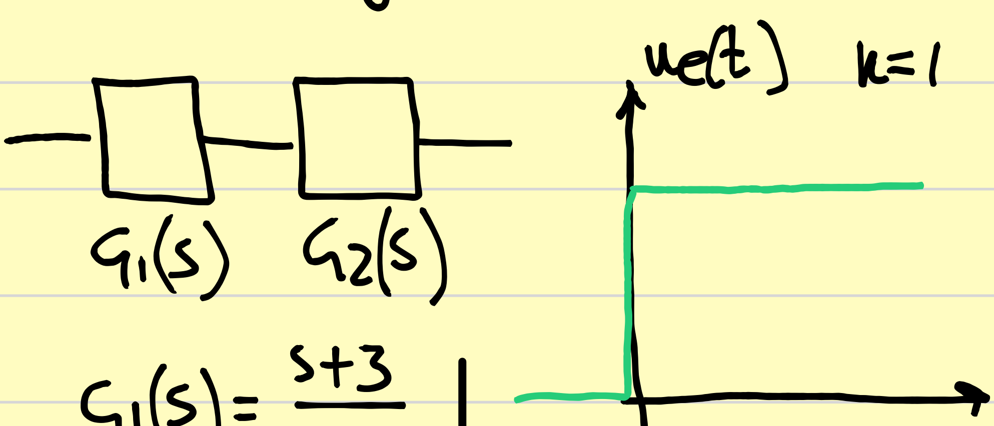
siehe Vorlesung 20231017



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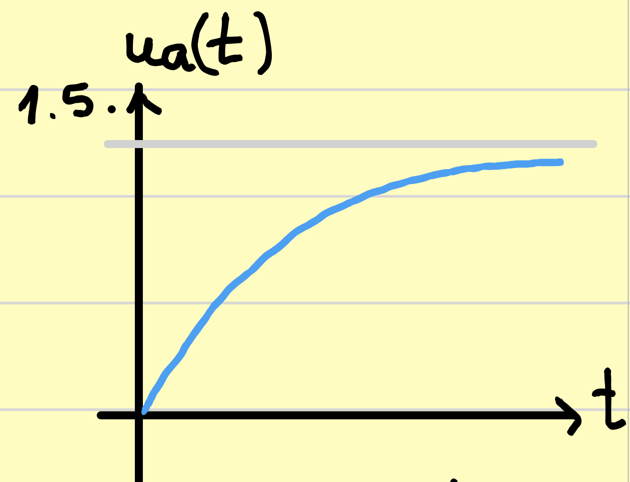


$G_1(s) = \frac{s+3}{s+1}$

$G_2(s) = \frac{1}{s+2}$

$u_e(t) = K$

$\rightarrow \dots \rightarrow u_a(t) = \frac{3k}{2} - 2ke^{-t} + \frac{k}{2}e^{-2t}$ ✓

$$G(s) = G_1(s) \cdot G_2(s) = \frac{s+3}{(s+1)(s+2)}$$

BODE DIAGRAMM ...

Schritt 1. G(s) IN DEM FREQUENZGANG

$$G(jw) = \frac{jw+3}{(jw+1)(jw+2)} = \frac{3+jw}{-w^2+3jw+2} =$$

$$= \frac{3+jw}{(2-w^2)+3jw} \cdot \frac{(2-w^2)-3jw}{(2-w^2)-3jw} =$$

$j^2=-1$

$$= \frac{6-3w^2-9jw+2jw-jw^3+3w^2}{(2-w^2)^2+9w^2} =$$

$$= \frac{6-7jw-jw^3}{4-4w^2+w^4+9w^2} =$$

$$\boxed{G(jw) = \frac{1}{w^4+5w^2+4} \cdot \left[6 - jw(7-w^2)\right]}$$

$|z| = \sqrt{Re|z|^2 + Im|z|^2}$

$$Re(G(jw)) = \frac{6}{w^4+5w^2+4} \; ; \; Im(G(jw)) = \frac{-w(7-w^2)}{w^4+5w^2+4}$$

$$|G(jw)| = \sqrt{\frac{6^2}{(w^4+5w^2+4)^2} - \frac{(w(7-w^2))^2}{(w^4+5w^2+4)^2}} =$$

$$= \frac{1}{w^4+5w^2+4}\sqrt{6^2 - w^2(49-14w^2+w^4)}$$

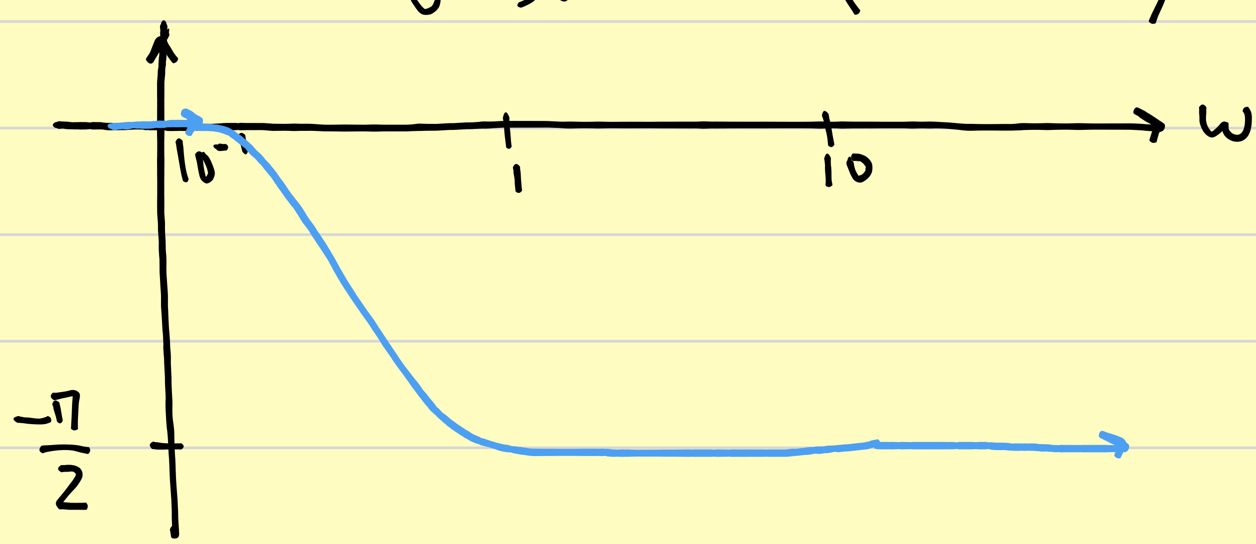
$$= \frac{1}{w^4+5w^2+4}\sqrt{-w^6+14w^4-49w^2+36}$$

$$|G(w=0)| = 1'5 \longrightarrow |G(w=0)|_{dB} = 20 \log 1'5$$

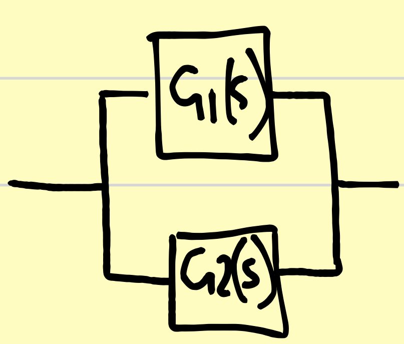
$$|G(w=\infty)| \simeq -\frac{1}{w} \longrightarrow -\infty$$

AMPLITUDE

$$\varphi = \operatorname{atan}\left(\frac{\operatorname{Im}(G(jw))}{\operatorname{Re}(G(jw))}\right) = \operatorname{atan}\left(\frac{-w(7-w^2)}{6}\right)$$



Übung. bitte die obere Aufgabe mit Parallelschaltung der Glieder lösen ...



$G_1(s) = \frac{s+3}{s+1}$; $G_2(s) = \frac{1}{s+2}$; $u_e(t) = K$

$$G(s) = G_1(s) + G_2(s) = \frac{s+3}{s+1} + \frac{1}{s+2} = \frac{(s+3)(s+2)+s+1}{(s+1)(s+2)}$$

$$u_a(s) = u_e(s) \cdot G(s) = \frac{K}{s} \cdot \frac{(s+3)(s+2)+s+1}{(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

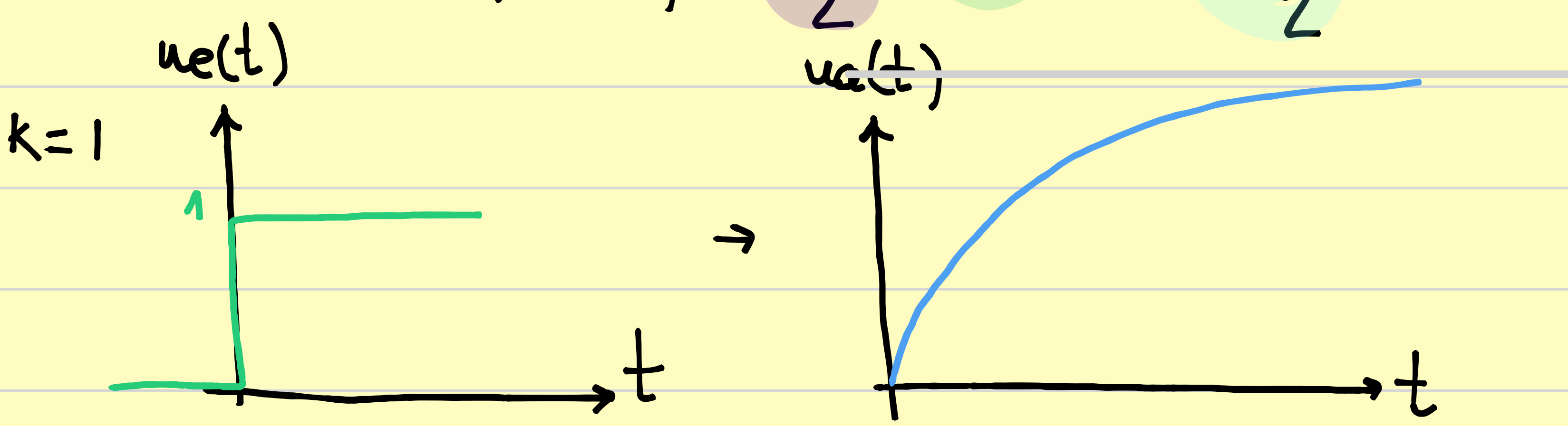
$$k \cdot \left[(s+3)(s+2)+s+1\right] = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

$$s=0 \rightarrow k \cdot 7 = A \cdot 2 \rightarrow A = \frac{7k}{2}$$

$$s=-1 \rightarrow k \cdot 2 = B \cdot (-1) \rightarrow B = -2k$$

$$s=-2 \rightarrow k \cdot (-1) = C \cdot 2 \rightarrow C = \frac{-k}{2}$$

$$u_a(t) = \mathcal{L}^{-1}(u_a(s)) = \frac{7k}{2} - 2k\,e^{-t} - \frac{k}{2}\,e^{-2t}$$



$\frac{7 \cdot 1}{2} = 3{,}5$

BODE DIAGRAMM

$$G(s) = \frac{(s+3)(s+2)+s+1}{(s+1)(s+2)} = \frac{s^2+6s+7}{s^2+3s+2}$$

$$G(jw) = \frac{(-w^2+7)+6jw}{(-w^2+2)+3jw} \cdot \frac{(-w^2+2)-3jw}{(-w^2+2)-3jw} =$$

$$= \frac{(-w^2+7)(-w^2+2)+18w^2+j\left[6w(-w^2+2)-3(-w^2+7)\right]}{(-w^2+2)^2+9w^2}$$

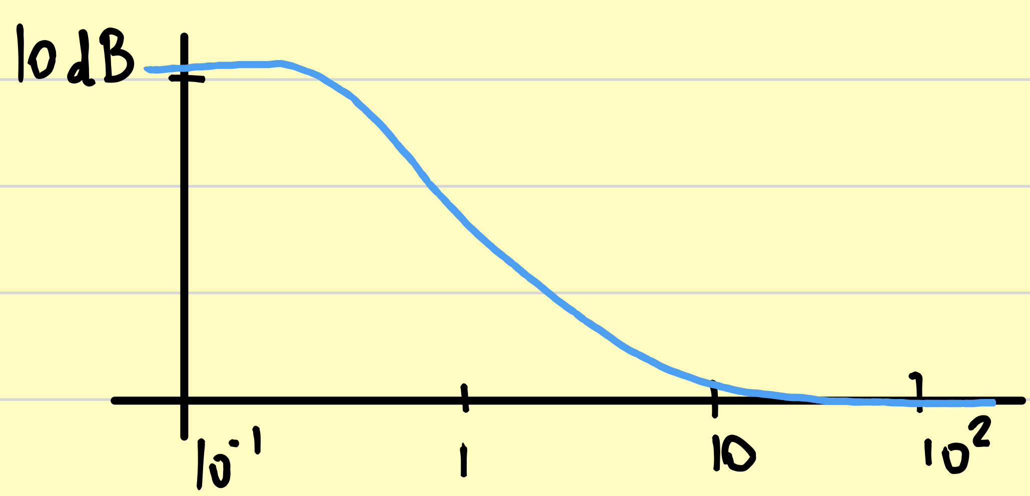
$$= \frac{w^4-9w^2+14+18w^2+j\left[-6w^2+12+3w^2-21\right]w}{w^4-4w^2+4+9w^2} =$$

Re(G(jw)) Im(G(jw))

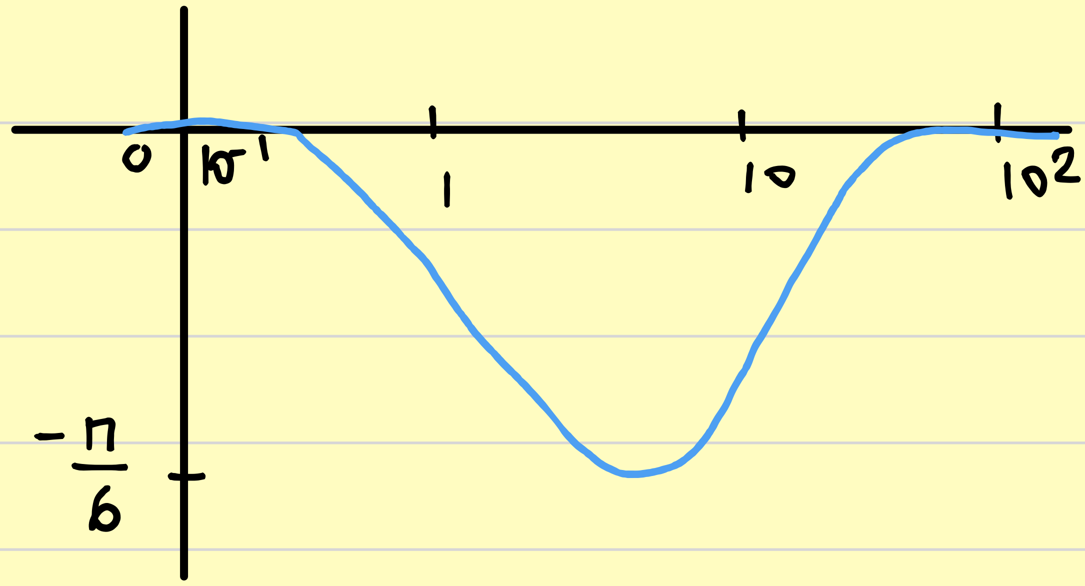
$$G(jw) = \frac{w^4+9w^2+14}{w^4+5w^2+4} + j\,\frac{-3w^3-9w}{w^4+5w^2+4}$$

$$|G(jw)| = \sqrt{Re(G(jw))^2+Im(G(jw))^2} =$$

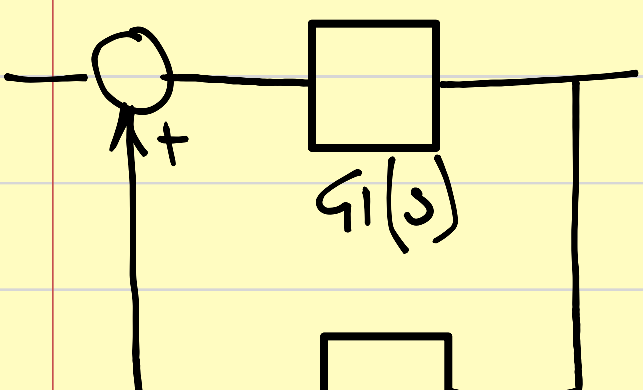
$$= \frac{1}{w^4+5w^2+4}\sqrt{(w^4+9w^2+14)^2-(3w^3+9w)^2} =$$



$$\varphi = \text{atan}\left(\frac{Im\,G(jw)}{Re\,G(jw)}\right) = \text{atan}\left(\frac{-3w^3-9w}{w^4+9w^2+14}\right)$$



Übung. die oberen Gliedern in Rückkopplungsschaltung setzen und $u_a(t)$ bzw. Bode-Diagramm ermitteln. (Gegenkopplung)



$$G(s) = \frac{G_1(s)}{1 - G_1(s)G_2(s)} = \frac{\frac{s+3}{s+1}}{1 - \frac{s+3}{s+1}\cdot\frac{1}{s+2}}$$

$$u_a(s) = u_e(s)\cdot G(s) = \frac{\frac{K}{s}\cdot\frac{s+3}{\cancel{s+1}}}{\frac{(s+1)(s+2)-(s+3)}{\cancel{(s+1)}(s+2)}} = \frac{\frac{K}{s}\cdot(s+3)(s+2)}{(s+1)(s+2)-(s+3)} =$$

$$= \frac{K\cdot(s+3)(s+2)}{s\left[s^2+3s+2-s-3\right]} = \frac{K(s+3)(s+2)}{(s^2+2s-1)s}$$

$$s=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \rightarrow s=\frac{-2\pm\sqrt{4+4}}{2}=\begin{cases}-1+2\sqrt{2}=s_1\\ -1-2\sqrt{2}=s_2\end{cases}$$

$s_1=1'8$, $s_2=-3'8$

$$u_a(s)=\frac{K(s+3)(s+2)}{(s^2+2s-1)s}=\frac{K(s+3)(s+2)}{(s-s_1)(s-s_2)s}=\frac{k(s+3)(s+2)}{(s-1'8)(s+3'8)s}$$

$$=\frac{A}{s}+\frac{B}{(s-1'8)}+\frac{C}{(s+3'8)}$$

$$K(s+3)(s+2)=A(s-1'8)(s+3'8)+Bs(s+3'8)+Cs(s-1'8)$$

$s=0 \rightarrow K\cdot 6=A(-6'84) \rightarrow A=-0'87K$

$s=1'8 \rightarrow K\cdot 18'24=B\cdot 16'08 \rightarrow B=1'81K$

$s=-3'8 \rightarrow K\cdot 1'44=C\cdot(-5'6) \rightarrow C=-0'26K$

$$u_a(t)=-0'87K+1'81Ke^{1'8t}-0'26ke^{-3'8t}$$

$$G(s)=\frac{\frac{s+3}{s+1}}{1-\frac{s+3}{s+1}\cdot\frac{1}{s+2}}=\frac{\frac{s+3}{s+1}}{(s+1)(s+2)-(s+3)}=\frac{(s+3)}{(s+1)\cdot(s^2+2s-1)}=$$

$$=\frac{s+3}{(s+1)(s-1'8)(s+3'8)}$$

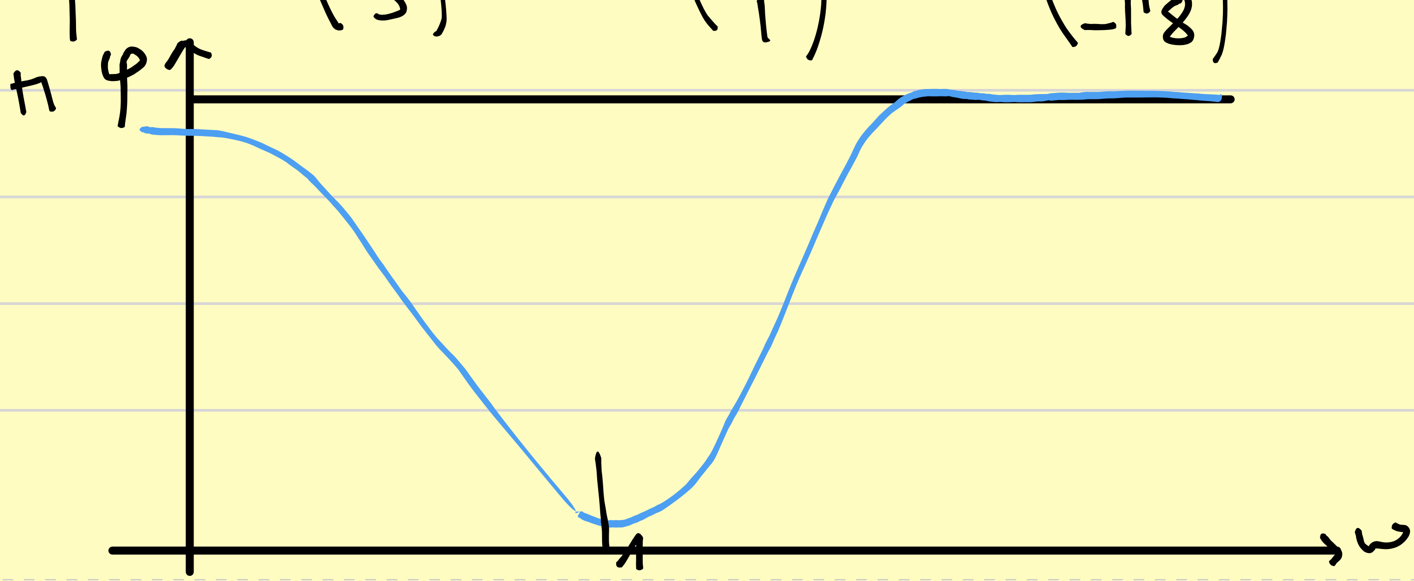
$$G(jw) = \frac{jw+3}{(jw+1)(jw-1'8)(jw+3'8)}$$

$$|G(jw)| = \frac{|jw+3|}{|jw+1|\,|jw-1'8|\,|jw+3'8|} =$$

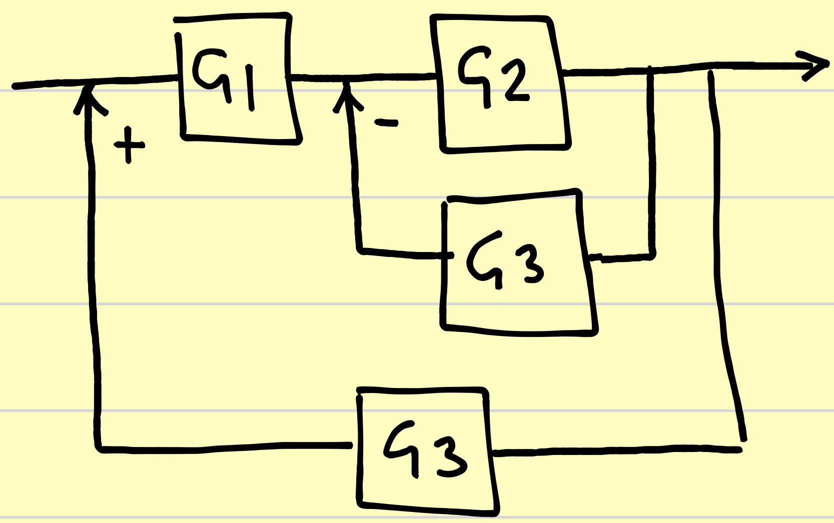
$$= \frac{\sqrt{9+w^2}}{\sqrt{1+w^2}\,\sqrt{1'8^2+w^2}\,\sqrt{3'8^2+w^2}}$$

$$|G(jw)|_{dB} = 20 \cdot \frac{1}{2}\left[\log(9+w^2) - \log(1+w^2) - \log(1'8^2+w^2) - \log(3'8^2+w^2)\right]$$

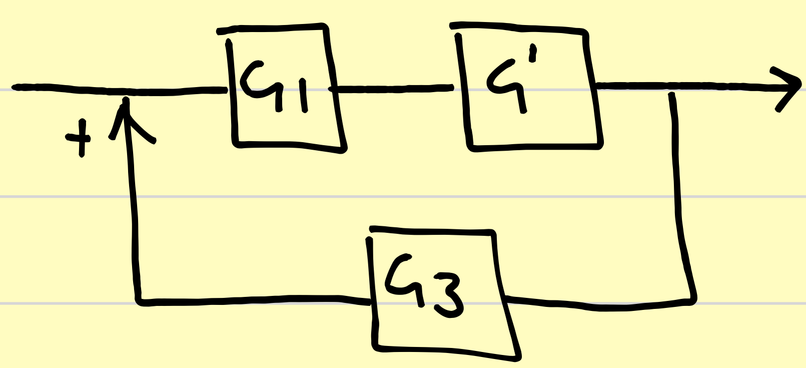
$$\varphi = \operatorname{atan}\left(\frac{w}{3}\right) - \operatorname{atan}\left(\frac{w}{1}\right) - \operatorname{atan}\left(\frac{w}{-1'8}\right) - \operatorname{atan}\left(\frac{w}{3'8}\right)$$



Übung: bitte ermitteln Sie die $u_a(t)$ und das Bode-Diagramm wenn $G_1 = \frac{1}{1+2s}$, $G_2 = \frac{1}{1+s}$, $G_3 = \frac{1}{2+s}$ in folgender Konfiguration:

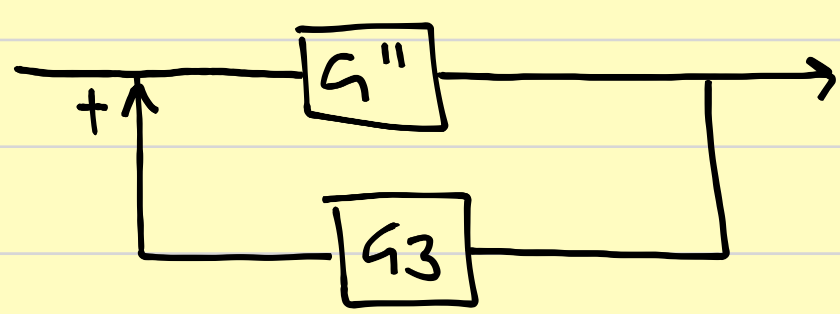


$\equiv$



$$G' = \frac{G_2}{1+G_2G_3}$$

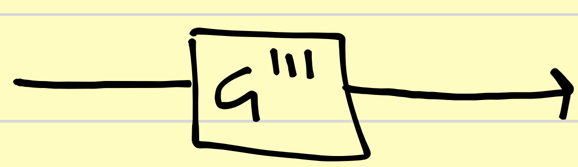
$\equiv$



$$G'' = \frac{G_1G_2}{1+G_2G_3}$$

$$= G_1 \cdot G'$$

$\equiv$



$$G''' = \frac{\frac{G_1G_2}{1+G_2G_3}}{1-\frac{G_1G_2G_3}{1+G_2G_3}}$$

$$G''' = \frac{\dfrac{G_1 G_2}{\cancel{1+G_2G_3}}}{\dfrac{1+G_2G_3-G_1G_2G_3}{\cancel{1+G_2G_3}}} = \frac{G_1G_2}{1+G_2G_3-G_1G_2G_3}$$

$$G_1 = \frac{1}{1+2s} \qquad G_2 = \frac{1}{1+s} \qquad G_3 = \frac{1}{2+s}$$

$$G''' = \frac{\dfrac{1}{1+2s}\cdot\dfrac{1}{1+s}}{1+\dfrac{1}{1+s}\cdot\dfrac{1}{2+s}-\dfrac{1}{1+2s}\cdot\dfrac{1}{1+s}\cdot\dfrac{1}{2+s}} =$$

$$= \frac{\dfrac{1}{(1+2s)(1+s)}}{\dfrac{(1+2s)(1+s)(2+s)+(1+2s)-1}{(1+2s)(1+s)(2+s)}} =$$

$$= \frac{(2+s)}{(1+2s)(1+s)(2+s)+2s} = \dots$$