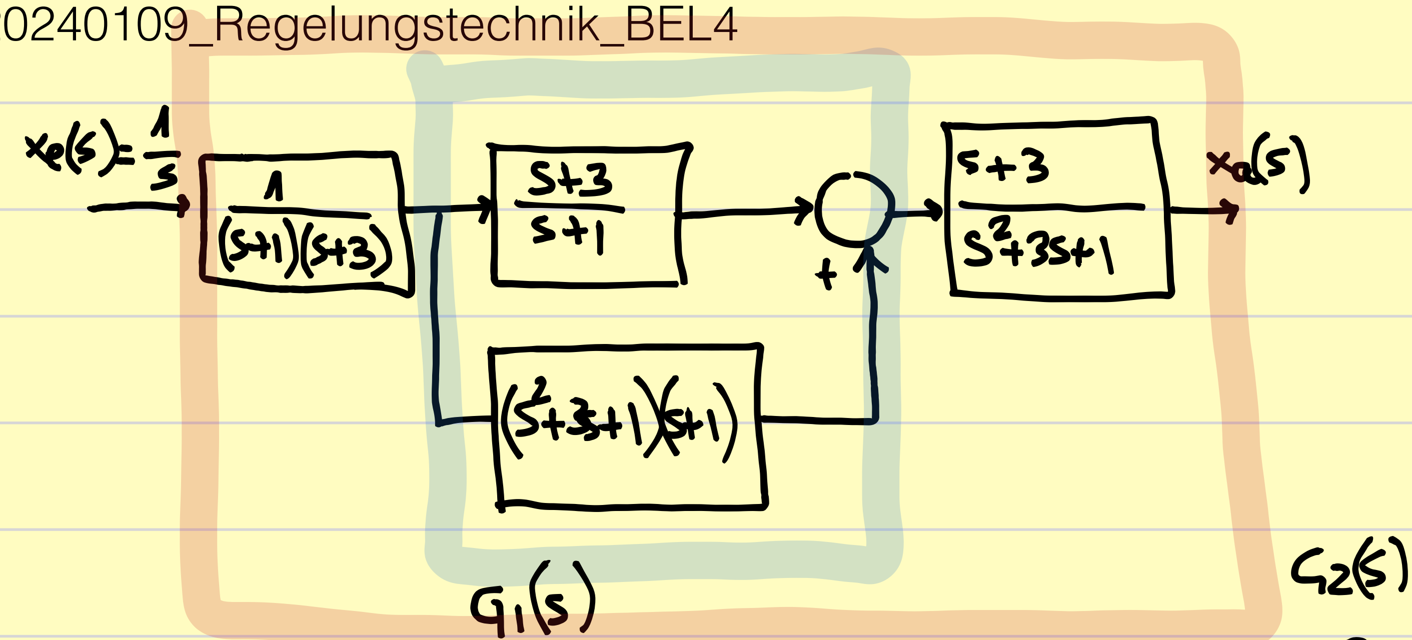
20240109_Regelungstechnik_BEL4

1)



$$G_1(s) = \frac{s+3}{s+1} + (s^2+3s+1)(s+1) = \frac{(s+3)+(s^2+3s+1)(s+1)^2}{s+1}$$

$$G_2(s) = \frac{1}{(s+1)\cancel{(s+3)}} \cdot G_1(s) \cdot \frac{\cancel{s+3}}{s^2+3s+1} = \frac{(s+3)+(s^2+3s+1)(s+1)^2}{(s+1)(s+1)(s^2+3s+1)}$$

$$\frac{x_a(s)}{x_e(s)} = \frac{s+3+(s^2+3s+1)(s^2+2s+1)}{(s+1)^2\left(s-\frac{-3+\sqrt{5}}{2}\right)\left(s-\frac{-3-\sqrt{5}}{2}\right)}$$

$$s^2+3s+1=0 \longrightarrow s^* = \frac{-3\pm\sqrt{9-4}}{2} = \begin{cases} \nearrow \frac{-3+\sqrt{5}}{2} \\ \searrow \frac{-3-\sqrt{5}}{2} \end{cases}$$

$$x_a(s) = \frac{1}{s} \cdot \frac{s+3+(s^2+3s+1)(s^2+2s+1)}{(s+1)^2\left(s-\frac{-3+\sqrt{5}}{2}\right)\left(s-\frac{-3-\sqrt{5}}{2}\right)}$$

$$= \frac{A}{s} + \frac{B}{(s+1)^2} + \frac{C}{s-\frac{-3+\sqrt{5}}{2}} + \frac{D}{s-\frac{-3-\sqrt{5}}{2}}$$

$$s+3+(s^2+3s+1)(s^2+2s+1) = A(s+1)^2\left(s-\frac{-3+\sqrt{5}}{2}\right)\left(s-\frac{-3-\sqrt{5}}{2}\right)$$
$$+ B\,s\left(s-\frac{-3+\sqrt{5}}{2}\right)\left(s-\frac{-3-\sqrt{5}}{2}\right)$$
$$+ C\,s(s+1)^2\left(s-\frac{-3-\sqrt{5}}{2}\right) +$$

$$+ D\, s(s+1)^2\left(s - \frac{-3+\sqrt{5}}{2}\right)$$

$$s=0 \rightarrow 3+1.1 = A \cdot 1 \cdot \left(-\frac{-3+\sqrt{5}}{2}\right)\left(-\frac{-3-\sqrt{5}}{2}\right)$$

$$4 = A \cdot 1 \rightarrow \boxed{A = 4}$$

$$s=-1 \rightarrow -1+3 = B \cdot (-1) \cdot \left(-1 - \frac{-3+\sqrt{5}}{2}\right)\left(-1 - \frac{-3-\sqrt{5}}{2}\right)$$

$$\rightarrow B = b_0$$

$$s = \frac{-3+\sqrt{5}}{2} \rightarrow \frac{-3+\sqrt{5}}{2} + 3 = C \cdot \frac{-3+\sqrt{5}}{2} \cdot \left(\frac{-3+\sqrt{5}}{2} + 1\right)^2 \cdot 1$$

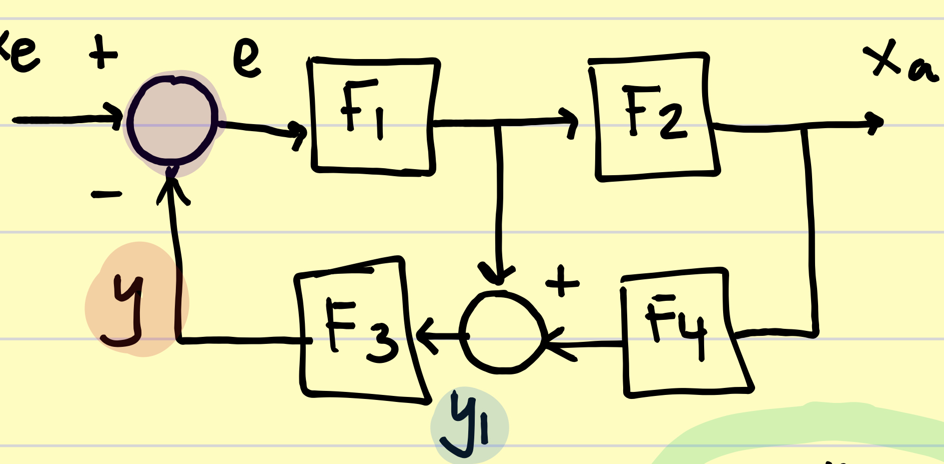
$$\rightarrow C = c_0$$

$$s = \frac{-3-\sqrt{5}}{2} \rightarrow \frac{-3-\sqrt{5}}{2} + 3 = D \cdot \frac{-3-\sqrt{5}}{2} \cdot \left(\frac{-3-\sqrt{5}}{2} + 1\right)^2 \cdot 1$$

$$\rightarrow D = d_0$$

$$x_a(s) = \frac{4}{s} + \frac{b_0}{(s+1)^2} + \frac{c_0}{\left(s - \frac{-3+\sqrt{5}}{2}\right)} + \frac{d_0}{\left(s - \frac{-3-\sqrt{5}}{2}\right)} \rightarrow x_a(t) = \dots$$

2.1)



$$e \cdot F_1 F_2 = x_a \rightarrow e = \frac{x_a}{F_1 F_2}$$

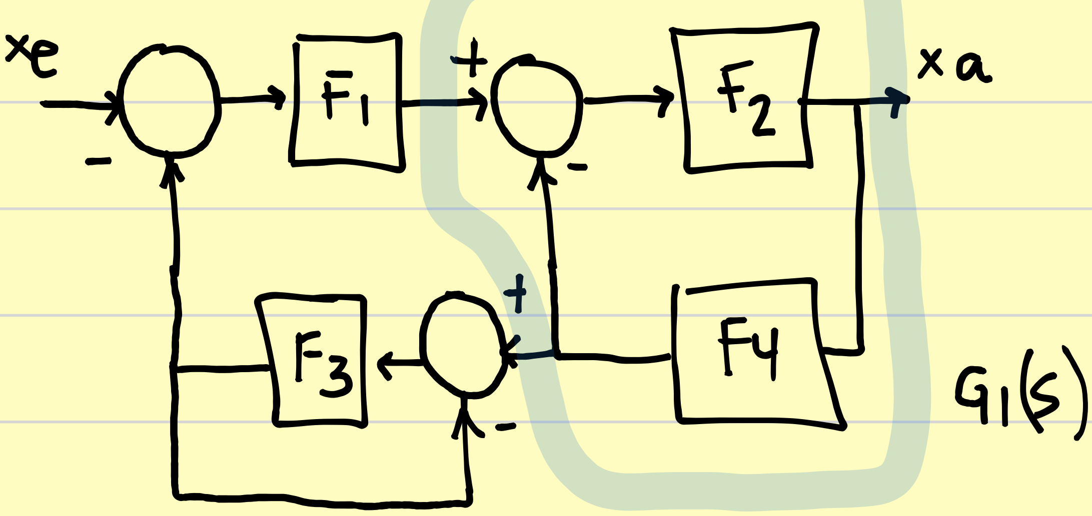
$$e = x_e - y = x_e - y_1 F_3 = x_e - \left[e F_1 + x_a F_4\right] F_3$$

$$\frac{x_a}{F_1F_2} = x_e - \left[\frac{x_a}{F_1F_2}\cdot F_1 + x_a F_4\right]F_3$$

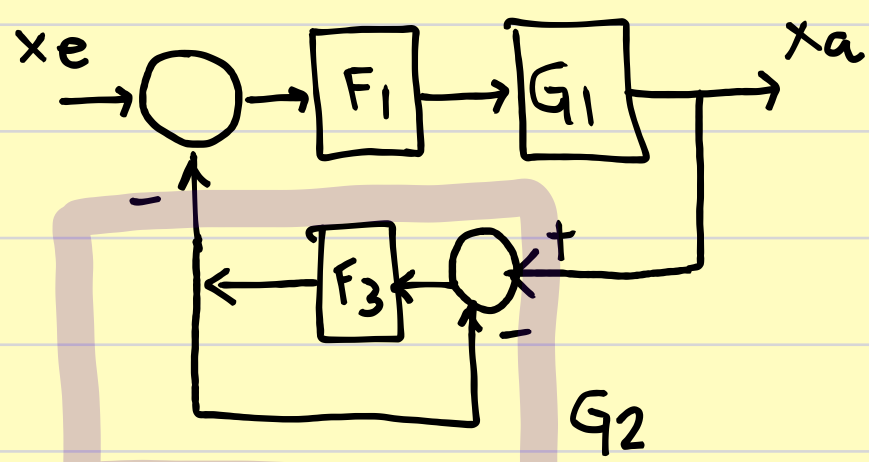
$$\frac{x_a}{F_1F_2} + \frac{x_a F_3}{F_2} + x_a F_4 F_3 = x_e$$

$$x_a\left[\frac{1}{F_1F_2} + \frac{F_3}{F_2} + F_4F_3\right] = x_e \rightarrow \boxed{\frac{x_a}{x_e} = \frac{1}{\frac{1}{F_1F_2} + \frac{F_3}{F_2} + F_4F_3}}$$

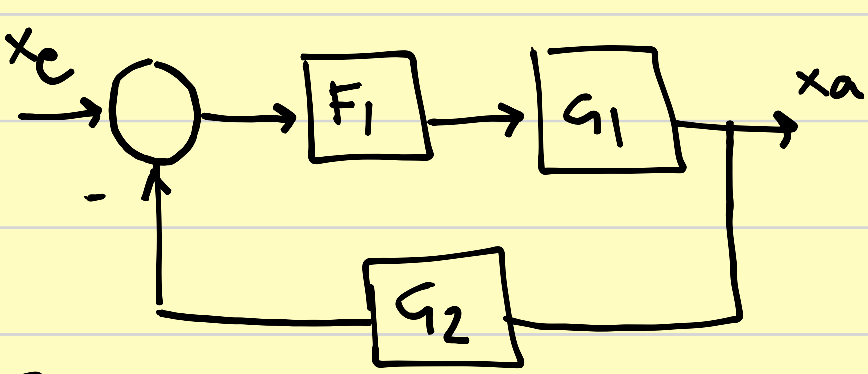
2.2.



$$\boxed{G_1(s) = \frac{F_2}{1+F_2F_4}} \equiv$$

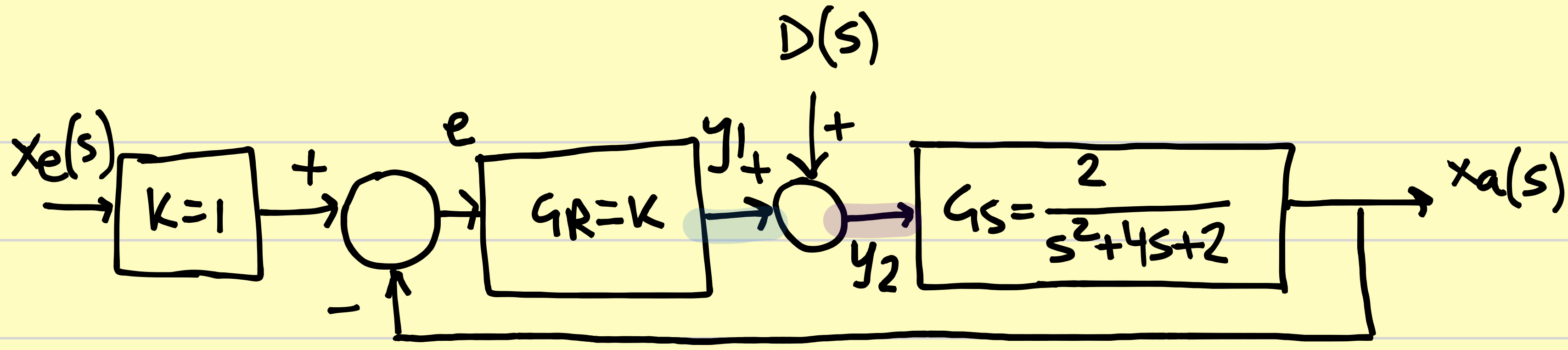


$$\boxed{G_2(s) = \frac{F_3}{1+F_3}} \equiv$$



$$G(s) = \frac{F_1G_1}{1+F_1G_1G_2} = \frac{F_1\cdot\frac{F_2}{1+F_2F_4}}{1+F_1\cdot\frac{F_2}{1+F_2F_4}\cdot\frac{F_3}{1+F_3}}$$

3.



a) Annahme: $xe(s) = \frac{1}{s}$; $D(s) = \frac{1}{s}$

$$e = x_e \cdot K - x_a \rightarrow x_a = x_e - e$$

$$x_a = \Big(\underbrace{\underbrace{e \cdot G_R}_{y_1} + D(s)}_{y_2}\Big) \cdot G_s = \left[(x_e - x_a) \cdot K + \frac{1}{s}\right] \cdot \frac{2}{(s^2+4s+2)}$$

$$x_a = \left[x_e \cdot K \cdot \frac{2}{(s^2+4s+2)} - x_a \cdot K \cdot \frac{2}{(s^2+4s+2)} + \frac{2}{s(s^2+4s+2)}\right]$$

$$x_a + x_a \cdot K \cdot \frac{2}{(s^2+4s+2)} = x_e \cdot K \cdot \frac{2}{s^2+4s+2} + \frac{2}{s(s^2+4s+2)}$$

$$x_a\left(1 + \frac{2K}{(s^2+4s+2)}\right) = x_e \cdot \frac{2K}{s^2+4s+2} + \frac{2}{(s^2+4s+2)} \cdot D(s)$$

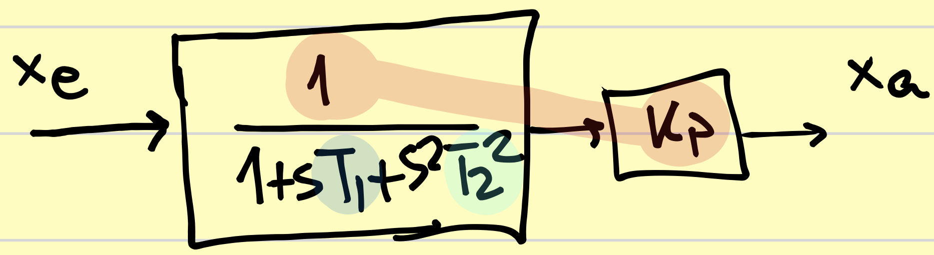
$D(s) = \varepsilon \rightarrow 0$ ⇒ $D(s)$ ist zu vernachlässigen.

$$\frac{x_a}{x_e} = \frac{\frac{2K}{s^2+4s+2}}{1 + \frac{2K}{s^2+4s+2}} = \frac{2K}{s^2+4s+2+2K} \quad (*)$$

$$s^* = \frac{-4 \pm \sqrt{16-4(2+2k)}}{2} = -2 \pm \sqrt{4-2-2k} = -2 \pm \sqrt{2-2k}$$

$$2-2k < 4 \rightarrow -2k < 2 \rightarrow -k < 1 \rightarrow \boxed{k > -1}$$

b) Vorlesung 2023|130



$$\text{Überschwingung} = |G(j\omega)|_{\omega=\omega_E} = \frac{K_p \cdot T_2}{T_1}$$

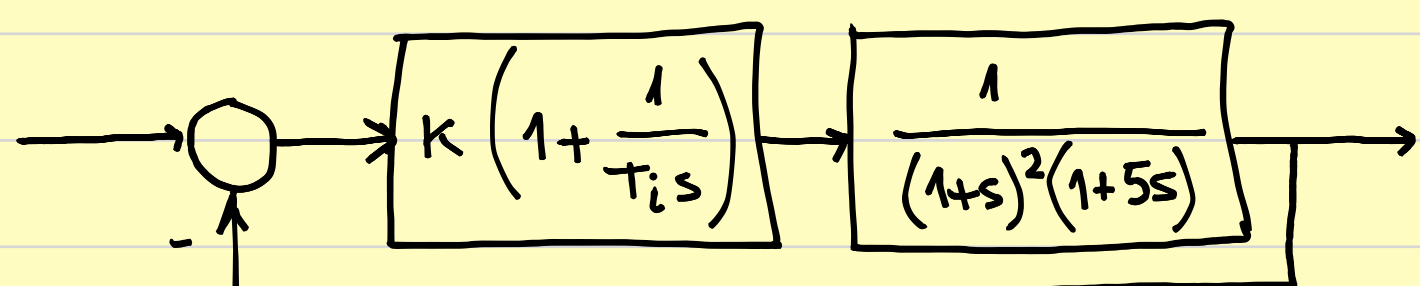
$$(*) \quad \frac{x_a}{x_e} \overset{(*)}{=} \frac{2k}{s^2+4s+2+2k} = \frac{2k}{2+2k} \cdot \frac{1}{1+\frac{4s}{2+2k}+\frac{1 \cdot s^2}{2+2k}}$$

$$\text{Überschwingung} = \frac{2k \cdot \sqrt{\frac{1}{2+2k}}}{\frac{4}{2+2k}} \leq 1 \rightarrow 2k \cdot \sqrt{\frac{1}{2+2k}} \leq \frac{4}{2+2k}$$

$$2k\sqrt{2+2k} \leq 4 \rightarrow k\sqrt{2+2k} \leq 2 \rightarrow \boxed{k \leq 1}$$

(*) korrigiert ab dem Punkt

4.



$$G(s) = \frac{K\left(1+\frac{1}{T_i s}\right)\left(\frac{1}{(1+s^2)(1+5s)}\right)}{1+K\left(1+\frac{1}{T_i s}\right)\left(\frac{1}{(1+s^2)(1+5s)}\right)} =$$

$$= \frac{k\left(\frac{1+T_i s}{T_i s}\right)\left(\frac{1}{(1+s^2)(1+5s)}\right)}{1+\left(\frac{1+T_i s}{T_i s}\right)\left(\frac{1}{(1+s^2)(1+5s)}\right)k} =$$

$$= \frac{K\,(1+T_i s)}{T_i s\,(1+s^2)(1+5s)+(1+T_i s)K} =$$

$$= \frac{K(1+T_i s)}{T_i s\,(1+5s+s^2+5s^3)+(1+T_i s)K} =$$

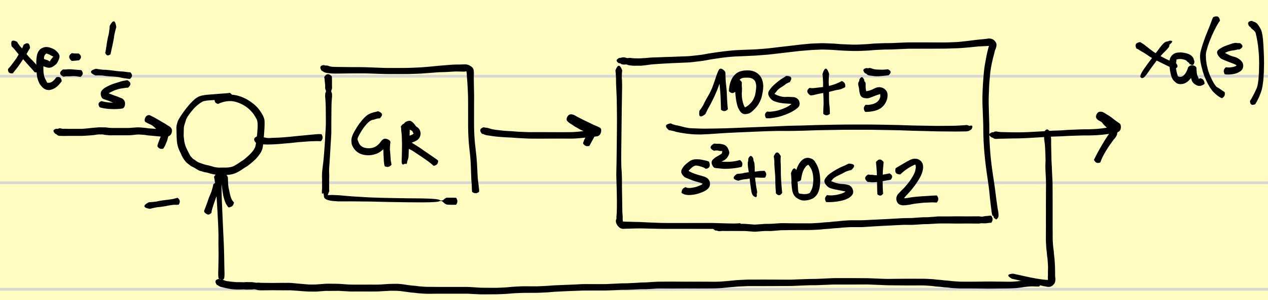
$$= \frac{K(1+T_i s)}{5T_i s^4+T_i s^3+5T_i s^2+(K+1)T_i s+K}$$

$K = T_i = 1$

$$= \frac{(1+s)}{5s^4+s^3+5s^2+2s+1}$$

$s^* = -0'15 \pm 0'86j$; $s^* = 0'05 \pm 0'5j$ NICHT STABIL

5.



$$G(s) = \frac{\dfrac{G_R\cdot(10s+5)}{s^2+10s+2}}{1+\dfrac{G_R(10s+5)}{s^2+10s+2}} = \frac{G_R(10s+5)}{s^2+10s+2+G_R(10s+5)}$$

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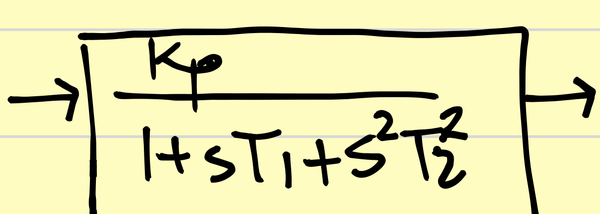


$W_E = \frac{1}{T_2}$

$Üb. = \frac{K_p T_2}{T_1}$

$$G(s)=\frac{G_R(10s+5)}{\left[G_R(10s+5)+2\right]\left[1+\frac{10s}{2+G_R(10s+5)}+\frac{1\cdot s^2}{2+G_R(10s+5)}\right]}$$

$$T_2=\sqrt{\frac{1}{2+G_R(10s+5)}}\qquad T_1=\frac{10}{2+G_R(10s+5)}\qquad K_p=\frac{G_R(10s+5)}{2+G_R(10s+5)}$$

$$w_E=\frac{1}{T_2}=\sqrt{2+G_R(10s+5)}$$

$$\text{Überschw}=\frac{K_p\sqrt{\frac{1}{2+G_R(10s+5)}}}{\frac{10}{2+G_R(10s+5)}}=1'1$$

G_R ?

$$G_R=\frac{K}{10s+5}$$

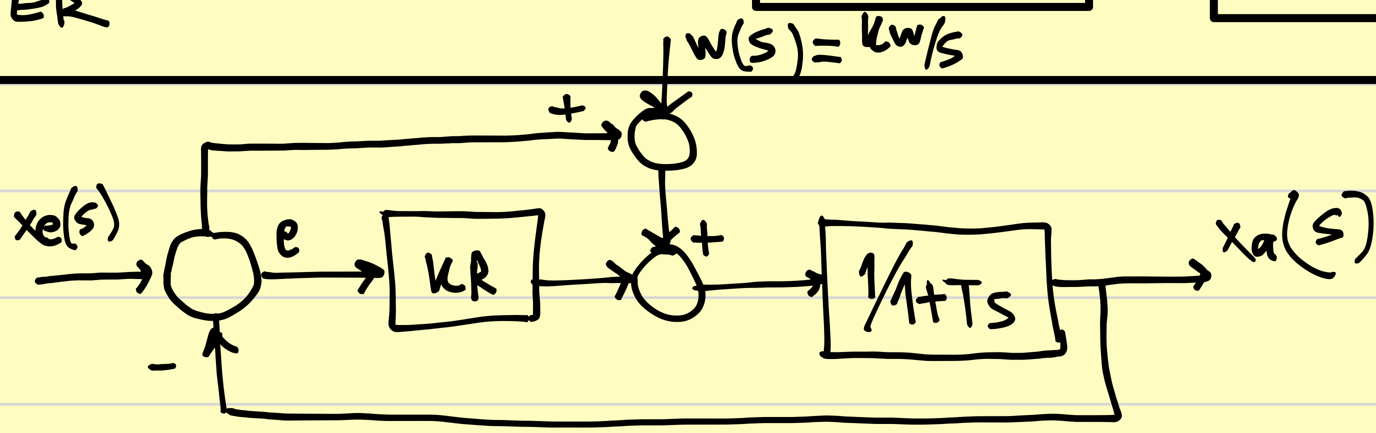
$$\text{Überschw.}=\overset{K_p}{\frac{K}{2+K}}\cdot\frac{\sqrt{\frac{1}{2+K}}}{\frac{10}{2+K}}=\frac{K}{10\sqrt{2+K}}=1'1$$

$\rightarrow \dots \rightarrow k \cong 122$

WOLFRAM SOLVER

$$\boxed{k=122}\rightarrow\boxed{G_R=\frac{122}{10s+5}}$$

6.



$$e = x_e - x_a$$

$$x_a = \left[e\,K_R + e + w(s)\right] \cdot \frac{1}{1+Ts} \rightarrow$$

$$\rightarrow \quad e = \left[x_a \cdot (1+Ts) - w(s)\right] \frac{1}{1+K_R}$$

$$\left[x_a (1+Ts) - w(s)\right] \frac{1}{1+K_R} = x_e - x_a$$

$$x_a(1+Ts) + x_a - w(s) = x_e(1+K_R)$$

$$x_a(2+Ts) - \frac{K_W}{s} = x_e(1+K_R)$$

$$K_W = 0$$

$$FV: \quad \frac{x_a}{x_e} = \frac{1+K_R}{2+Ts} = \frac{1+K_R}{2} \cdot \frac{1}{1+\frac{T}{2}s}$$

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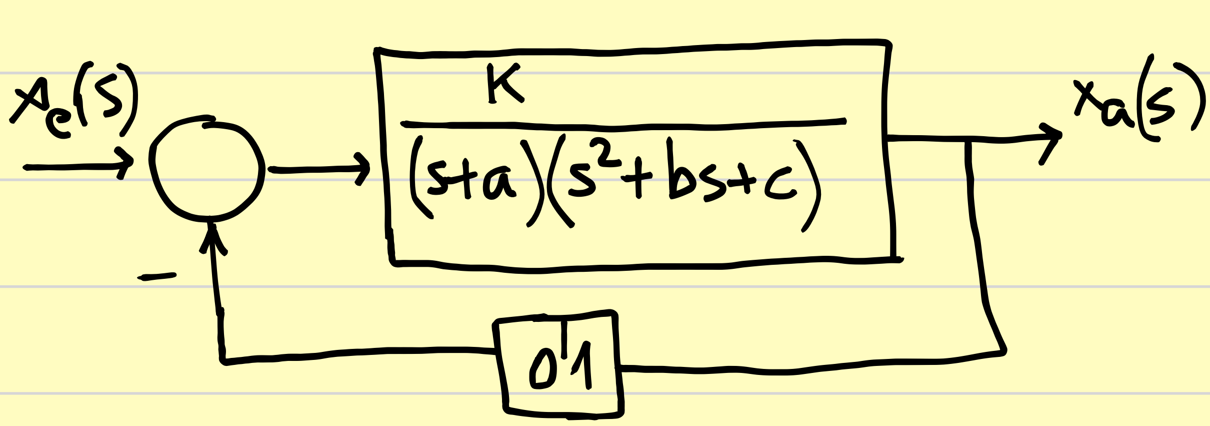
7. $$G(t) = 1'8\, e^{-2t} - 2\, e^{-0'9t}\left[0'9 \cos t - \sin t\right]$$



$$G(s) = \frac{\frac{K}{(s+a)(s^2+bs+c)}}{1 + \frac{0'1K}{(s+a)(s^2+bs+c)}} =$$

$$= \frac{K}{(s+a)(s^2+bs+c)+0\text{'}1K} = \frac{K}{s^3+(a+b)s^2+(ab+c)s+ac+0\text{'}1K}$$

$$G(s) = \frac{\sim}{s^3+3\text{'}8s^2+5\text{'}41s+3\text{'}62}$$