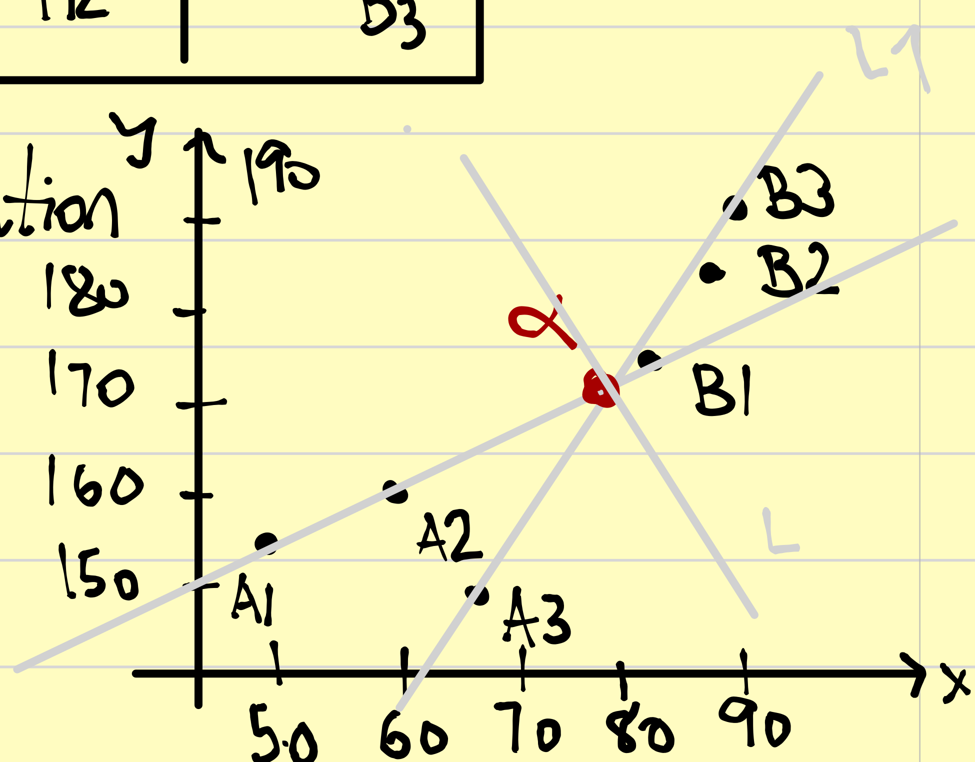
Support Vector Machines (SVM)

The goal is, given a set of labelled data, find the best separating line (2D), plane (3D), hyperplane (3+ D).
We need labelled Data
Because we are measuring distances, we need to check if normalization is needed.

Example:	x (Weight)	y (Height)	Klasse
	50	155	A_1
	60	160	A_2
2 Dimensions (x,y)	68	150	A_3
2 Groups (A,B)	85	175	B_1
	90	182	B_2
	92	192	B_3

Step 1. Graphical Representation



Step 2. The separation line has the form $y = mx + b$.

Step 3. Extreme separation lines.

$L1: A3 / B3 \quad \dfrac{y-150}{x-68} = \dfrac{192-150}{92-68} \rightarrow y = 150 + 1'75(x-68)$

$$\boxed{\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}}$$

Line goes through $[x_1, y_1]$ $[x_2, y_2]$

$L2: A2/B1. \quad \dfrac{y-160}{x-60} = \dfrac{175-160}{85-60} \rightarrow y = 160 + 0'33(x-60)$

$$\left.\begin{array}{l} y\alpha = 150 + 1'75(x\alpha - 68) \\ y\alpha = 160 + 0'33(x\alpha - 60) \end{array}\right| \rightarrow 0 = -10 + (1'75 - 0'33)x\alpha - 1'75.68 + 0'33.60$$

$x\alpha = 76'9. \rightarrow y\alpha = 165'58 \qquad \alpha[76'9, 165'58]$

$$d_{L,A2} = \dfrac{|m.60 + 160 + b|}{\sqrt{m^2+1}}$$

$$d_{L,B1} = \dfrac{|m.85 + 175 + b|}{\sqrt{m^2+1}}$$

$60 + 160 = 85m + 175$

$\rightarrow m = -0'6$

$d_{L,POINT} = \dfrac{|mx_0 + y_0 + b|}{\sqrt{m^2+1}}$

$y = mx + b$

POINT$[x_0 y_0]$

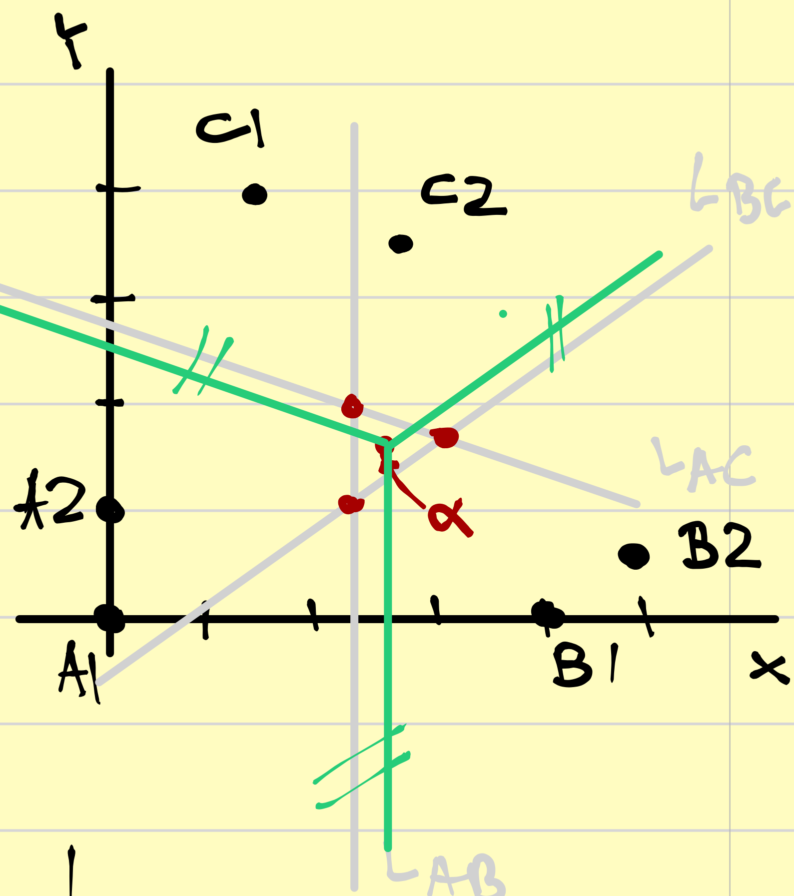
$y = -0'6x + b \rightarrow 165'58 = -0'6.76'9 + b \rightarrow b = 211'72$

$$\boxed{y = -0'6.x + 211'72}$$

$$d = \dfrac{|60.(-0'6) + 160 + 211'72|}{\sqrt{-0'6^2 + 1}} = 287'88$$

Example for 2 Dimensions and 3 Groups.

Customers	X	Y
A	0	0
	0	1
B	4	0
	5	0'5
C	2	4
	3	3'5



L_{AB}: $L_1: A2/B1$: $\frac{y-1}{x-0} = \frac{0-1}{4-0} \rightarrow y = 1 - \frac{1}{4}x$

$L_2: A1/B2$: $\frac{y-0}{x-0} = \frac{0'5-0}{5-0} \rightarrow y = 0'1x$

$\blacktriangleright$ $y\alpha_1 = 1 - \frac{1}{4}x\alpha_1$ | $y\alpha_1 = 0'1x\alpha_1$ $\blacktriangleright$ $x\alpha_1 = 2'857$ | $y\alpha_1 = 0'286$

$d_{L_{AB},A2} = \frac{|m \cdot 0 + 1 + b|}{\sqrt{m^2+1}}$

$d_{L_{AB},B1} = \frac{|m \cdot 4 + 0 + b|}{\sqrt{m^2+1}}$

$\blacktriangleright$ $\cancel{b}+1 = m \cdot 4 + \cancel{b} \longrightarrow m = \frac{1}{4}$

$y = \frac{1}{4}x + b \longrightarrow 0'286 = \frac{1}{4} \cdot 2'857 + b \longrightarrow b = -0'428$

$$\boxed{L_{AB}: \; y = 0'25x - 0'428 \; ; \; \alpha_1 = [2'857, 0'286]}$$

L_{AC}: $L_1: A2/C2$: $\frac{y-1}{x-0} = \frac{3'5-1}{3-0} \rightarrow y = 1 + 0'83x$

$L_2: A1/C1$: $\frac{y-0}{x-0} = \frac{2-0}{4-0} \rightarrow y = 0'5x$ $\blacktriangleright$

$$y_{\alpha_2} = 1 + 0'83\, x_{\alpha_2}$$
$$y_{\alpha_2} = 0'5\, x_{\alpha_2}$$
$$\rightarrow \quad x_{\alpha_2} = 3 \quad y_{\alpha_2} = 1'5$$

$$d_{L_{AC}, C_1} = \frac{|m.2 + 4 + b|}{\sqrt{m^2+1}}$$
$$d_{L_{AC}, A_2} = \frac{|m.0 + 1 + b|}{\sqrt{m^2+1}}$$
$$\rightarrow \quad m.2 + 4 + \cancel{b} = \cancel{b} + 1 \qquad m = \frac{-3}{2} = -1'5$$

$$y = -1'5x + b \rightarrow 1'5 = -1'5 \cdot 3 + b \rightarrow b = 6$$

$$\boxed{L_{AC}: \; y = -1'5x + 6 \qquad \alpha_2 [3, 1'5]}$$

L_{BC} :

$$L_1 : B1/C2 : \quad \frac{y-0}{x-4} = \frac{3'5-0}{3-4} \rightarrow y = 3'5(x-4)$$

$$L2 : B2/C1 : \quad \frac{y-0'5}{x-5} = \frac{4-0'5}{2-5} \rightarrow y = 0'5 - 1'67(x-5)$$

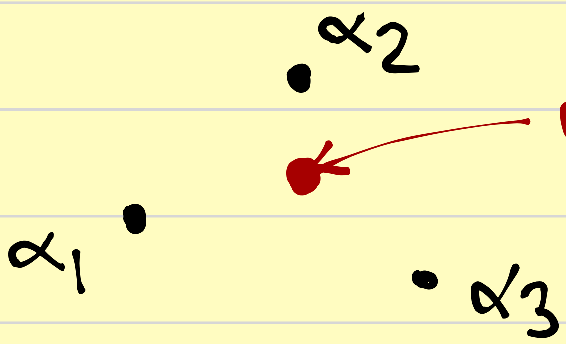
$$y_{\alpha_3} = 3'5(x_{\alpha_3} - 4)$$
$$y_{\alpha_3} = 0'5 - 1'67(x_{\alpha_3} - 5)$$
$$\rightarrow \quad 0 = -3'5\, x_{\alpha_3} + 14 - 0'5 + 1'67\, x_{\alpha_3} - 8'35$$

$$x_{\alpha_3} = 2'814 \qquad y_{\alpha_3} = 4'15$$

$$d_{L_{BC}, C_2} = \frac{|m.3 + 3'5 + b|}{\sqrt{m^2+1}}$$
$$d_{L_{BC}, B_2} = \frac{|m.5 + 0'5 + b|}{\sqrt{m^2+1}}$$
$$\rightarrow \quad 3m + 3'5 + \cancel{b} = 5m + 0'5 + \cancel{b} \qquad m = 1'5$$

$$y = 1'5x + b \rightarrow 4'15 = 1'5 \cdot 2'814 + b \rightarrow b = -0'071$$

$$\boxed{L_{BC}: \quad y = 1'5 \cdot x - 0'071 \qquad \alpha_3\left[2'814, 4'15\right]}$$



$$\alpha\left[\frac{x_{\alpha_1}+x_{\alpha_2}+x_{\alpha_3}}{3}, \frac{y_{\alpha_1}+y_{\alpha_2}+y_{\alpha_3}}{3}\right]$$

$$\alpha\left[2'89, 1'978\right]$$

$L_{AB}: \quad y = 0'25x - 0'428$

$/\!/\alpha \quad y = 0'25x + (1'978 - 0'25 \cdot 2'89)$

$$\boxed{L'_{AB} \equiv y = 0'25x + 1'256}$$

The parallel line to $y = mx + b$ going through $[x_1, y_1]$

$$y = mx + (y_1 - mx_1)$$

$L_{AC}: \quad y = -1'5x + 6$

$/\!/\alpha \quad y = -1'5x + (1'978 + 1'5 \; 2'89)$

$$\boxed{L'_{AC} \equiv y = -1'5x + 6'313}$$

$L_{BC}: \quad y = -x + 4'5$

$/\!/\alpha \quad y = -x + (1'978 + 1 \cdot 2'89)$

$$\boxed{L'_{BC} \equiv y = -x + 4'868}$$