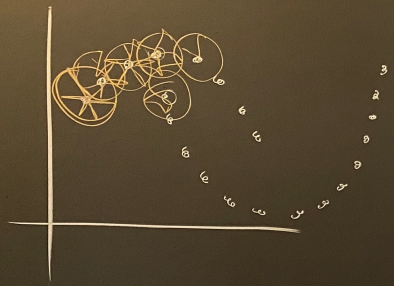
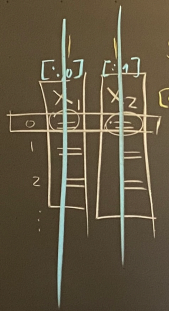
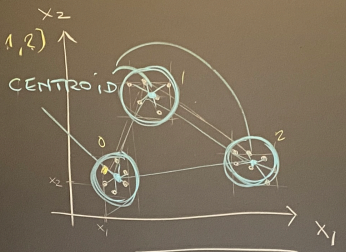
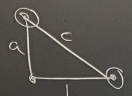




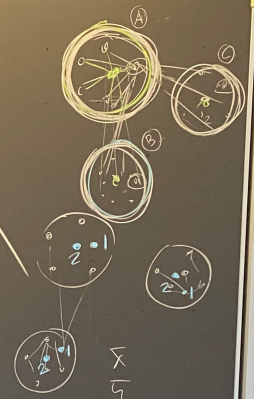
y_true K-MEANS CLUSTERING



$$d_{ij} = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2}$$



$$c^2 = a^2 + b^2$$



$\bar{x}$

$\bar{y}$

Untitled5

June 14, 2023

[1]:
```
# k -- Means Clustering
```

[1]:
```
%matplotlib inline
import matplotlib.pyplot as plt
import seaborn as sns
import numpy as np
```

[2]:
```
'''Der k-means Algorithmus sucht nach einer vorgegebenen Anzahl von Clustern
innerhalb eines unbeschriteten mehrdimensionalen Datensatzes.Er erreicht
durch dies eine "einfache" Vorstellung davon, wie die optimale Clusterbildung
aussieht.Das Clusterzentrum ist das arythmetische Mittel Aller Punkte die
zum Cluster gehören.Jeder Punkt liegt näher an seinem eigenen Clusterzentrum
als an anderen Clusterzentren.Diese Beiden Annahmen sind die Grundlage des
k-means Clusters Modells.'''
```

[2]:
```
'Der k-means Algorithmus sucht nach einer vorgegebenen Anzahl von
Clustern\ninnerhalb eines unbeschriteten mehrdimensionalen Datensatzes.Er
erreicht\ndurch dies eine "einfache" Vorstellung davon, wie die optimale
Clusterbildung\naussieht.Das Clusterzentrum ist das arythmetische Mittel Aller
Punkte die\nzum Cluster gehören.Jeder Punkt liegt näher an seinem eigenen
Clusterzentrum\nals an anderen Clusterzentren.Diese Beiden Annahmen sind die
Grundlage des\nk-means Clusters Modells.'
```

[3]:
```
# Beispiel 1 - "Blobs 3 Gruppen"
```

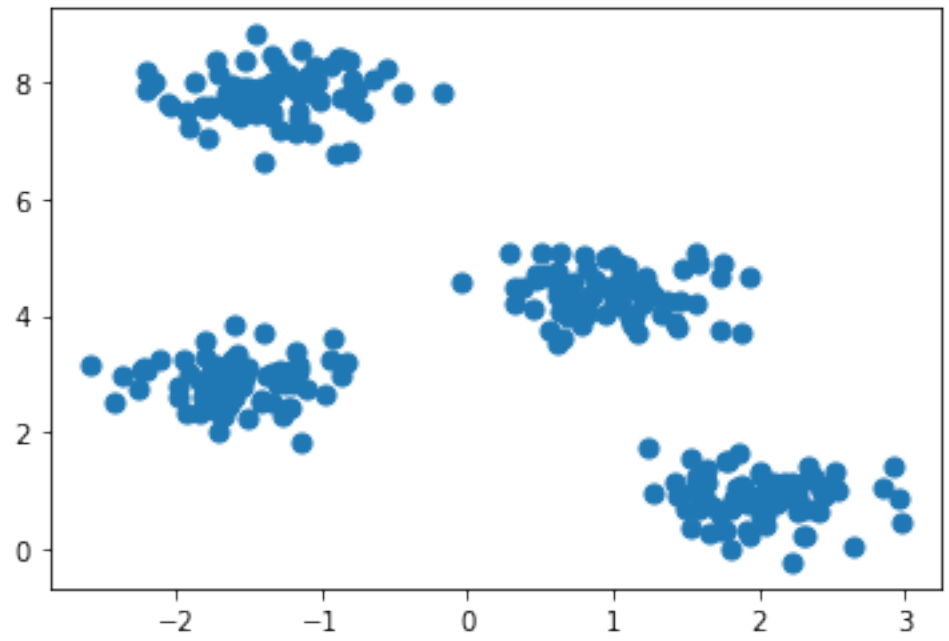
[4]:
```
from sklearn.datasets import make_blobs

X, y_true = make_blobs(n_samples = 300, # anzahl punkte
                       centers = 4, # anzahl gruppen
                       cluster_std = 0.4, # abstand zw. den gruppen
                       random_state = 0)
```

[5]:
```
plt.scatter(X[:,0], X[:,1], s = 50)

# X[:,0] X Koordinaten der Daten
# X[:,1] Y Koordinaten der Daten
```

```
plt.show()
```



[6]:
```
X
```

[6]:
```
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       [-1.09516280e+00,  7.82176310e+00],
       [ 1.53265678e+00,  1.56091593e+00],
       [ 2.97683420e+00,  4.73657331e-01],
       [-1.71444969e+00,  2.03730575e+00],
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       [-2.19609054e+00,  8.18108094e+00],
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       [-1.90130959e+00,  7.20835293e+00],
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       [ 1.86982913e+00,  1.09025625e+00],
```

```
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       [-9.40272527e-01,  3.25890304e+00],
       [-1.30067738e+00,  8.28869197e+00],
       [ 9.36889069e-01,  4.03839601e+00],
       [ 2.21488606e+00, -2.11373443e-01],
       [ 1.89548791e+00,  1.04568602e+00],
       [-1.16286376e+00,  7.35203055e+00],
       [-1.51946609e+00,  8.35459833e+00],
       [-1.26653736e+00,  7.92366308e+00],
       [ 2.00088764e+00,  1.35242020e+00],
       [ 2.36398374e+00,  1.22706532e+00],
       [ 6.18083454e-01,  4.45854833e+00],
       [-1.69766154e-01,  7.80589015e+00],
       [-1.70327307e+00,  2.80574006e+00]])
```

[7]:
```
y_true
```

[7]:
```
array([1, 3, 0, 3, 1, 1, 2, 0, 3, 3, 2, 3, 0, 3, 1, 0, 0, 1, 2, 2, 1, 1,
       0, 2, 2, 0, 1, 0, 2, 0, 3, 3, 0, 3, 3, 3, 3, 3, 2, 1, 0, 2, 0, 0,
       2, 2, 3, 2, 3, 1, 2, 1, 3, 1, 1, 2, 3, 2, 3, 1, 3, 0, 3, 2, 2, 2,
       3, 1, 3, 2, 0, 2, 3, 2, 2, 3, 2, 0, 1, 3, 1, 0, 1, 1, 3, 0, 1, 0,
       3, 3, 0, 1, 3, 2, 2, 0, 1, 1, 0, 2, 3, 1, 3, 1, 0, 1, 1, 0, 3, 0,
       2, 2, 1, 3, 1, 0, 3, 1, 1, 0, 2, 1, 2, 1, 1, 1, 1, 2, 1, 2, 3, 2,
       2, 1, 3, 2, 2, 3, 0, 3, 3, 2, 0, 2, 0, 2, 3, 0, 3, 3, 3, 0, 3, 0,
       1, 2, 3, 2, 1, 0, 3, 0, 0, 1, 0, 2, 2, 0, 1, 0, 0, 3, 1, 0, 2, 3,
       1, 1, 0, 2, 1, 0, 2, 2, 0, 0, 0, 0, 1, 3, 0, 2, 0, 0, 2, 2, 2, 0,
       2, 3, 0, 2, 1, 2, 0, 3, 2, 3, 0, 3, 0, 2, 0, 0, 3, 2, 2, 1, 1, 0,
       3, 1, 1, 2, 1, 2, 0, 3, 3, 0, 0, 3, 0, 1, 2, 0, 1, 2, 3, 2, 1, 0,
       1, 3, 3, 3, 3, 2, 2, 3, 0, 2, 1, 0, 2, 2, 2, 1, 1, 3, 0, 0, 2, 1,
       3, 2, 0, 3, 0, 1, 1, 2, 2, 0, 1, 1, 1, 0, 3, 3, 1, 1, 0, 1, 1, 1,
       3, 2, 3, 0, 1, 1, 3, 3, 3, 1, 1, 0, 3, 2])
```

[8]:
```
# Lösung 1. Direkt

from sklearn.cluster import KMeans

kmeans = KMeans(n_clusters=4)
# hier sagen wir KMeans wie viele Clusters wir haben!

kmeans.fit(X) # lassen wir die Clusters berechnen
y_means = kmeans.predict(X) # hier geben wir Jedem Punkt einen neuen Label
```

[16]:
```
np.__version__
```

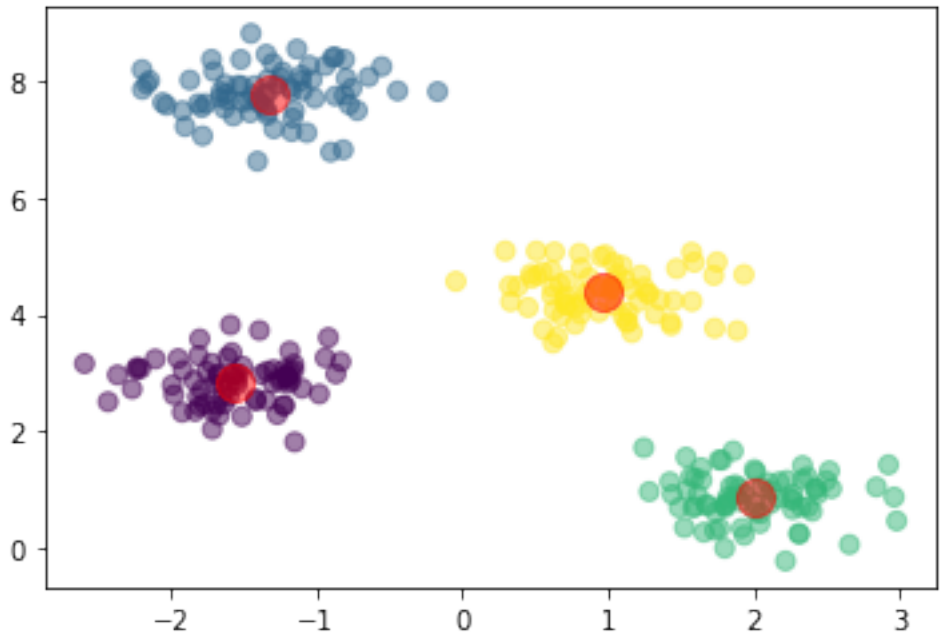
[16]:
```
'1.23.2'
```

[17]:
```
pip install --upgrade numpy==1.21.4
```

```
Collecting numpy==1.21.4
  Downloading numpy-1.21.4-cp39-cp39-macosx_11_0_arm64.whl (12.4 MB)
                              12.4/12.4 MB
14.0 MB/s eta 0:00:0000:0100:01
Installing collected packages: numpy
  Attempting uninstall: numpy
    Found existing installation: numpy 1.23.2
    Uninstalling numpy-1.23.2:
      Successfully uninstalled numpy-1.23.2
```

```
ERROR: pip's dependency resolver does not currently take into account all
the packages that are installed. This behaviour is the source of the following
dependency conflicts.
qiskit-machine-learning 0.5.0 requires dill<0.3.6,>=0.3.4, but you have dill
0.3.6 which is incompatible.
Successfully installed numpy-1.21.4
Note: you may need to restart the kernel to use updated packages.
```

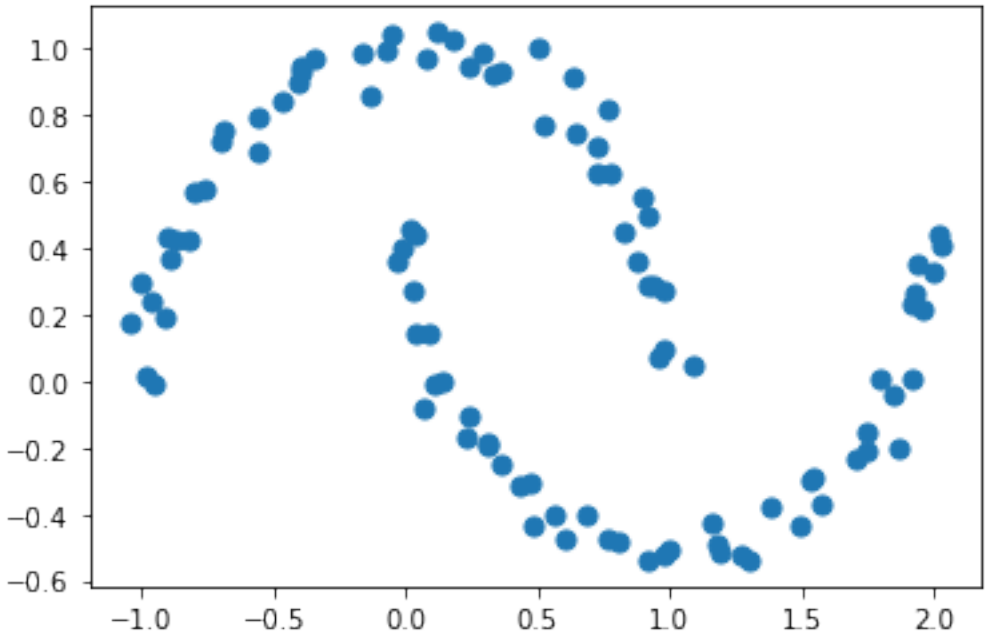
[12]:
```
plt.scatter(X[:,0], # alle X Koordinaten
            X[:,1], # alle Y Koordinaten
            c = y_means, # die neuen Labels als Farbe (c = color)
            s = 50, # s = size
            alpha = 0.5) # alpha = transparenz

centroids=kmeans.cluster_centers_
# centroids werden berechnet(roten punkten)

plt.scatter(centroids[:,0], centroids[:,1],
            c='red', s=200, alpha=0.5)
#centroids werden dargestellt
plt.show()
```



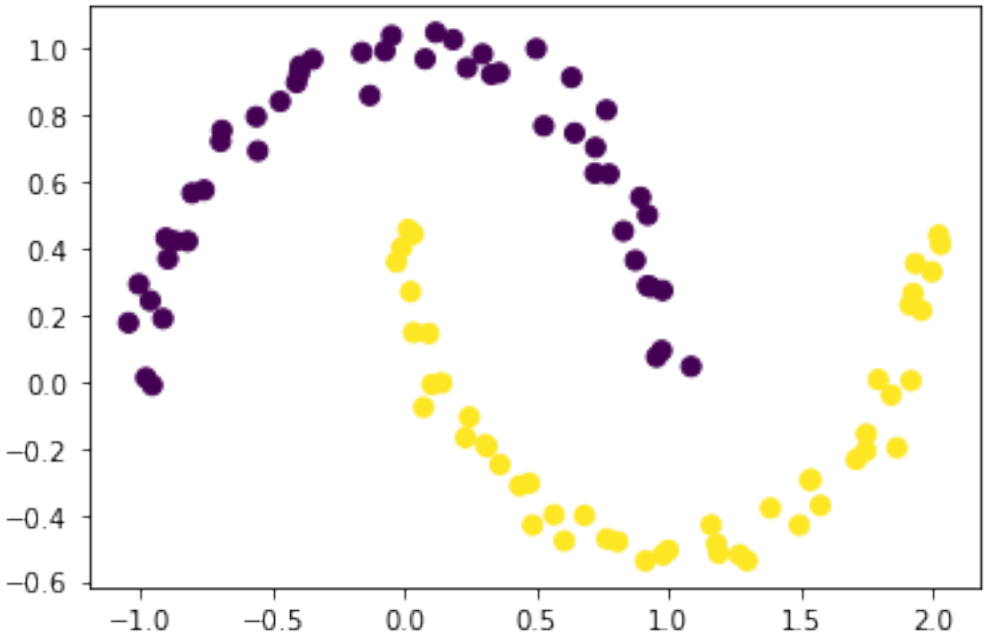
[13]:
```
# K means clustering funktioniert nur wenn:
# 1. ich kenne die Anzahl Clusters
# 2. die Clusters sind LINEAR trennbar.
```

[14]:
```
# Wenn nicht, brauche ich Spectral Clustering!
```

[15]:
```
# Beispiel 2. Moons Cluster

from sklearn.datasets import make_moons

X, y = make_moons(100, noise =0.05, random_state =0)

plt.scatter(X[:,0], X[:,1], s=50)
plt.show()
```



[16]:
```
from sklearn.cluster import SpectralClustering

model = SpectralClustering(n_clusters = 2,
                           affinity='nearest_neighbors',
                           assign_labels='kmeans')

labels = model.fit_predict(X)
# hier kriege ich den Vorschau der Labels
plt.scatter(X[:,0], X[:,1],
            c=labels, s=50)
plt.show()
```


1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

[ ]: