20251212_Mathematik1_B

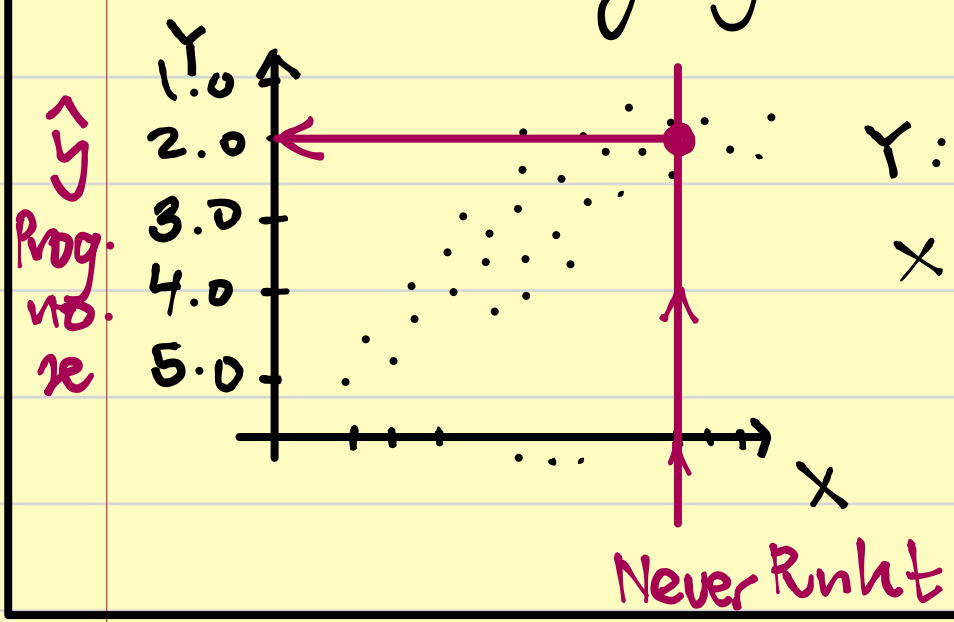
Regressionsalgorithmen

Zukunftsprognose anhand Daten. ≡ Regression.

- Linear ✓
- Polynomisch ✓
- Logistische

Hypothese. wir haben Daten (i.e. Management-system).

Ziel. Zukünftiges Verhalten hervorzusagen.



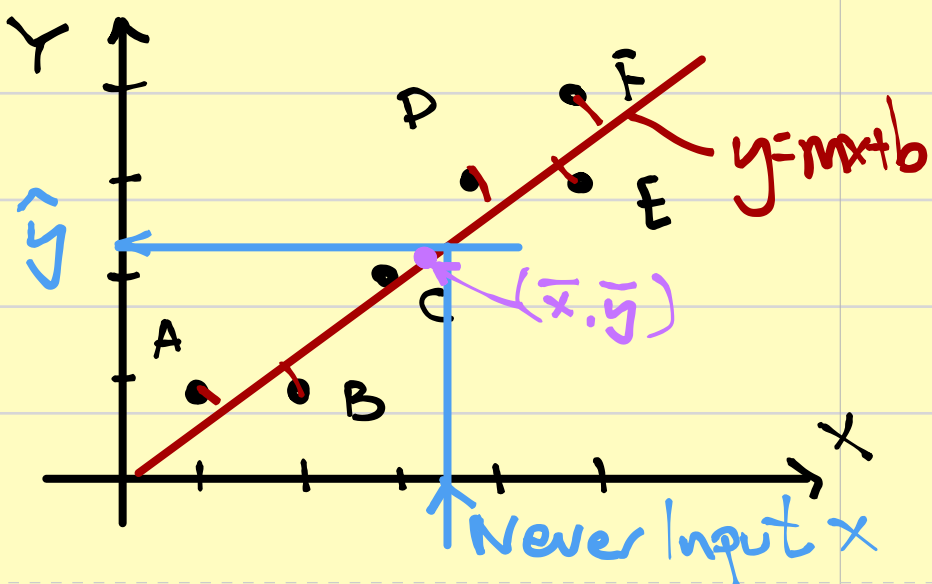
Y: Note

X: Std gelernt

Lineare Regression

Daten: X [1, 2, 3, 4, 5, 5]

Y [1, 1, 2, 3, 3, 4]



A B C D E F

Welche Linie beschreibt die Daten am besten?

· Die Linie die einen geringsten Abstand zu den Punkten hat.

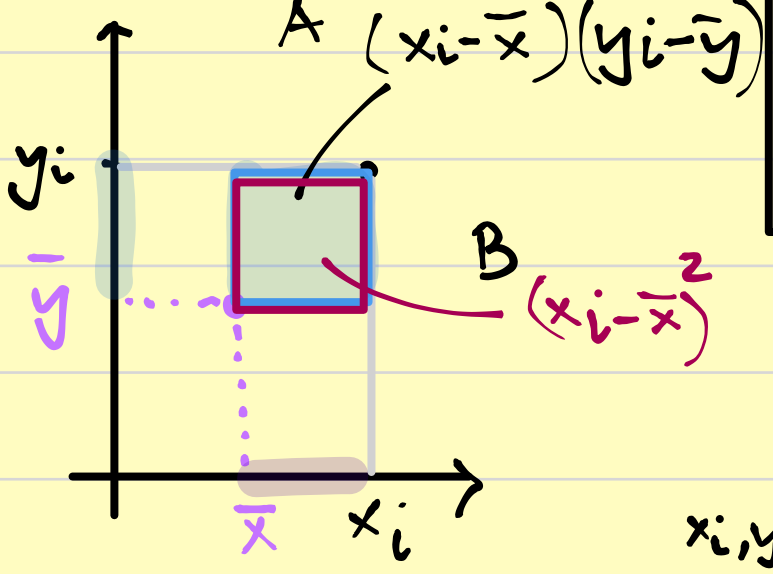
① Schritt 1. Die gesuchte lineare Regression geht durch den Mittelwert.

$$\bar{y} = m\bar{x} + b$$

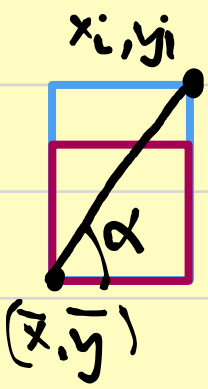
$$\bar{x} = \frac{1+2+3+4+5+5}{6} = 3'33 \qquad \bar{y} = \frac{1+1+2+3+3+4}{6} = 2'33$$

② Schritt 2.

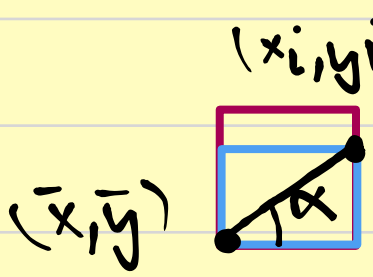
$$b = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$



$\frac{A}{B} = 1 \rightarrow \alpha = 45^\circ \ (tg\alpha = 1)$



$\frac{A}{B} > 1 \rightarrow \alpha > 45^\circ \ (tg\alpha > 1)$



$\frac{A}{B} < 1 \rightarrow \alpha < 45^\circ \ (tg\alpha < 1)$

$$b = \frac{(1-3{,}33)(1-2{,}33)+(2-3{,}33)(1-2{,}33)+(3-3{,}33)(2-2{,}33)+}{+(4-3{,}33)(3-2{,}33)+(5-3{,}33)(3-2{,}33)+(5-3{,}33)(4-2{,}33)} = \checkmark$$
$$(1-3{,}33)^2+(2-3{,}33)^2+(3-3{,}33)^2+(4-3{,}33)^2+2(5-3{,}33)^2$$

③ Schritt 3. $\bar{y} = m\bar{x} + b \rightarrow m = \frac{\bar{y}-b}{\bar{x}}$

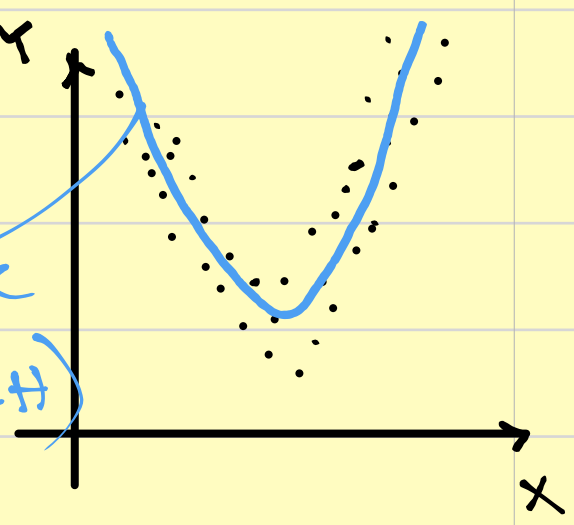
$2{,}33 = m \cdot 3{,}33 + b \rightarrow m = \frac{2{,}33-b}{3{,}33}$ ✓

$y = mx + b$

Was ist y_0, wenn $x = x_0$? $\rightarrow$ $y_0 = mx_0 + b$

Polynomische Regression

	A	B	C		
x	1	2	3	4	5
y	2,5	5,8	11,9	21,4	31,2



① Schritt 1. Hipothese Regression 2. Grades

$y = ax^2 + bx + c$

1. $[1, 2{,}5] \rightarrow 2{,}5 = a \cdot 1^2 + b \cdot 1 + c$ (1)
2. $[2, 5{,}8] \rightarrow 5{,}8 = a \cdot 2^2 + b \cdot 2 + c$ (2)
3. $[3, 11{,}9] \rightarrow 11{,}9 = a \cdot 3^2 + b \cdot 3 + c$ (3)

$(2)-(1): 5'8-2'5 = a(4-1)+b(2-1) \quad (4)$

$(3)-(2): 11'9-5'8 = a(9-4)+b(3-2) \quad (5)$

$(5)-(4): 11'9-2.5'8+2'5 = 2a \rightarrow a = 1'4$

$(4)+a: 5'8-2'5 = 1'4.3+b \rightarrow b = -0'9$

$(1)+a+b: 2'5 = 1'4+(-0'9)+c \rightarrow c = 2$

$$\hat{y} = 1'4x^2 - 0'9x + 2$$

x	y	$\hat{y}$	Error $(\hat{y}-y)$
1	2'5	2'5	0
2	5'8	5'7	-0'1
3	11'9	11'9	0
4	21'4	21'4	0
5	31'2	31'5	0'3