

Übung. Gegeben ist die Position von 6 Werke mit  $(x, y)$  Koordinaten. 1) Bitte mit k.M.C. die Gruppen in 2 Kategorien teilen 2) Mit KNN entscheiden zu welcher Gruppe Werk  $w_x$  gehört?

- $w_1[0,0]$   $w_2[0,2]$   $w_3[1,1]$   $w_4[3,2]$   $w_5[4,1]$   $w_6[4,3]$
- $w_x[2,5]$

1) k.M.C. Gruppe 1  $[w_1, w_2]$  Gruppe 2  $[w_3, w_4, w_5, w_6]$

• Zentroide  $Z_1 = \left[ \frac{0+0}{2}, \frac{0+2}{2} \right] = [0, 1]$ ;  $Z_2 = \left[ \frac{1+3+4+4}{4}, \frac{1+2+1+3}{4} \right] = [3, 1.75]$

•  $d_{w_1, Z_1} = \sqrt{(0-0)^2 + (0-1)^2} = 1$ ;  $d_{w_1, Z_2} = \sqrt{(0-3)^2 + (0-1.75)^2} = 3.47$

$d_{w_2, Z_1} = \sqrt{(0-0)^2 + (2-1)^2} = 1$ ;  $d_{w_2, Z_2} = \sqrt{(0-3)^2 + (2-1.75)^2} = 3.01$

x  $d_{w_3, Z_1} = \sqrt{(1-0)^2 + (1-1)^2} = 1$ ;  $d_{w_3, Z_2} = \sqrt{(1-3)^2 + (1-1.75)^2} = 2.17$

$d_{w_4, Z_1} = \sqrt{(3-0)^2 + (2-1)^2} = 3.16$ ;  $d_{w_4, Z_2} = \sqrt{(3-3)^2 + (2-1.75)^2} = 0.25$

$d_{w_5, Z_1} = \sqrt{(4-0)^2 + (1-1)^2} = 4$ ;  $d_{w_5, Z_2} = \sqrt{(4-3)^2 + (1-1.75)^2} = 1.25$

$d_{w_6, Z_1} = \sqrt{(4-0)^2 + (3-1)^2} = 4.47$ ;  $d_{w_6, Z_2} = \sqrt{(4-3)^2 + (3-1.75)^2} = 1.6$

• Neue Gruppen:  $G_1[w_1, w_2, w_3]$   $G_2[w_4, w_5, w_6]$

$$z_1^* = \left[ \frac{0+0+1}{3}, \frac{0+1+1}{3} \right] = \left[ \frac{1}{3}, 1 \right] \quad z_2^* = \left[ \frac{3+4+4}{3}, \frac{2+1+3}{3} \right] = \left[ 3.67, 2 \right]$$

$$dw_1, z_1^* = \sqrt{\left(0 - \frac{1}{3}\right)^2 + (0-1)^2} = 1.05; \quad dw_1, z_2^* = \sqrt{\left(0 - 3.67\right)^2 + (0-2)^2} = 4.18$$

$$dw_2, z_1^* = \sqrt{\left(0 - \frac{1}{3}\right)^2 + (2-1)^2} = 1.05; \quad dw_2, z_2^* = \sqrt{\left(0 - 3.67\right)^2 + (2-2)^2} = 3.67$$

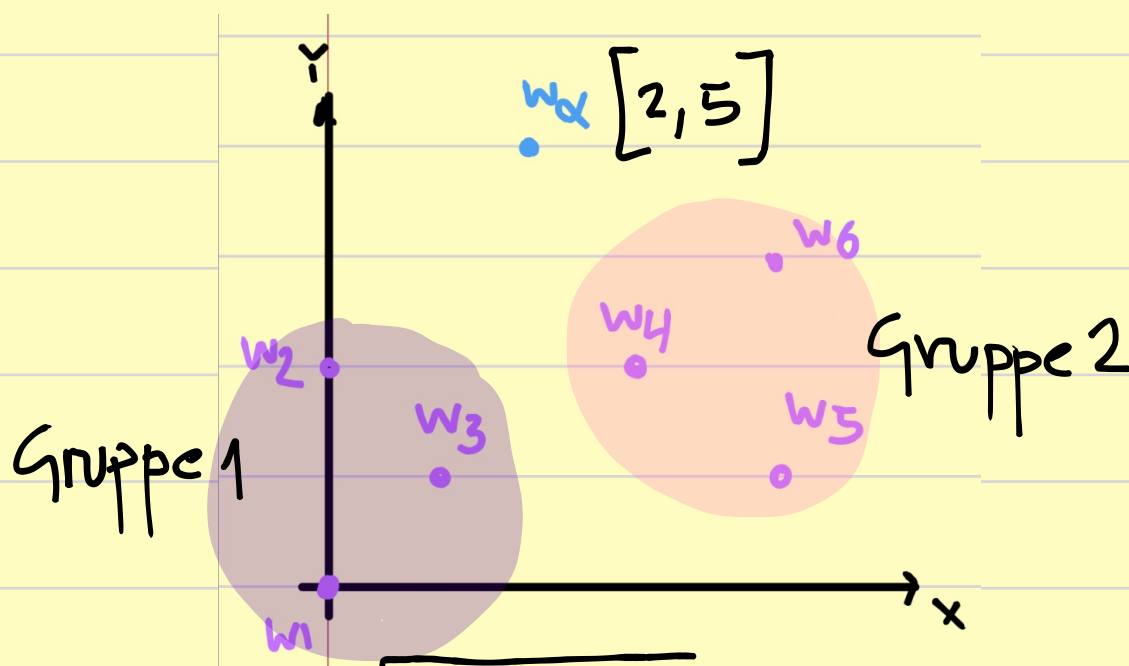
$$dw_3, z_1^* = \sqrt{\left(1 - \frac{1}{3}\right)^2 + (1-1)^2} = 1.05; \quad dw_3, z_2^* = \sqrt{\left(1 - 3.67\right)^2 + (1-2)^2} = 2.85$$

$$dw_4, z_1^* = \sqrt{\left(3 - \frac{1}{3}\right)^2 + (2-1)^2} = 2.84; \quad dw_4, z_2^* = \sqrt{\left(3 - 3.67\right)^2 + (2-2)^2} = 0.67$$

$$dw_5, z_1^* = \sqrt{\left(4 - \frac{1}{3}\right)^2 + (1-1)^2} = 3.67; \quad dw_5, z_2^* = \sqrt{\left(4 - 3.67\right)^2 + (1-2)^2} = 1.05$$

$$dw_6, z_1^* = \sqrt{\left(4 - \frac{1}{3}\right)^2 + (3-1)^2} = 4.18; \quad dw_6, z_2^* = \sqrt{\left(4 - 3.67\right)^2 + (3-2)^2} = 1.05$$

Die Gruppen sind korrekt.  $G_1[w_1, w_2, w_3]$   $G_2[w_4, w_5, w_6]$



KNN.

$$d_{\alpha, w_1} = \sqrt{(2-0)^2 + (5-0)^2} = 5.38 \quad G_1$$

$$d_{\alpha, w_2} = \sqrt{(2-0)^2 + (5-2)^2} = 3.68 \quad G_1$$

$$d_{\alpha, w_3} = \sqrt{(2-1)^2 + (5-1)^2} = 4.12 \quad G_1$$

$$d_{\alpha, w_4} = \sqrt{(2-3)^2 + (5-2)^2} = 3'16 \quad G_2$$

$$d_{\alpha, w_5} = \sqrt{(2-4)^2 + (5-1)^2} = 4'47 \quad G_2$$

$$d_{\alpha, w_6} = \sqrt{(2-4)^2 + (5-4)^2} = 2'23 \quad G_2$$

$$\bullet d_{\alpha}, [w_6 < w_4 < w_2 < w_3 < w_5 < w_1]$$

$$K=1: p(\alpha \in G_1) = \frac{0}{1} = 0; p(\alpha \in G_2) = \frac{1}{1} = 1$$

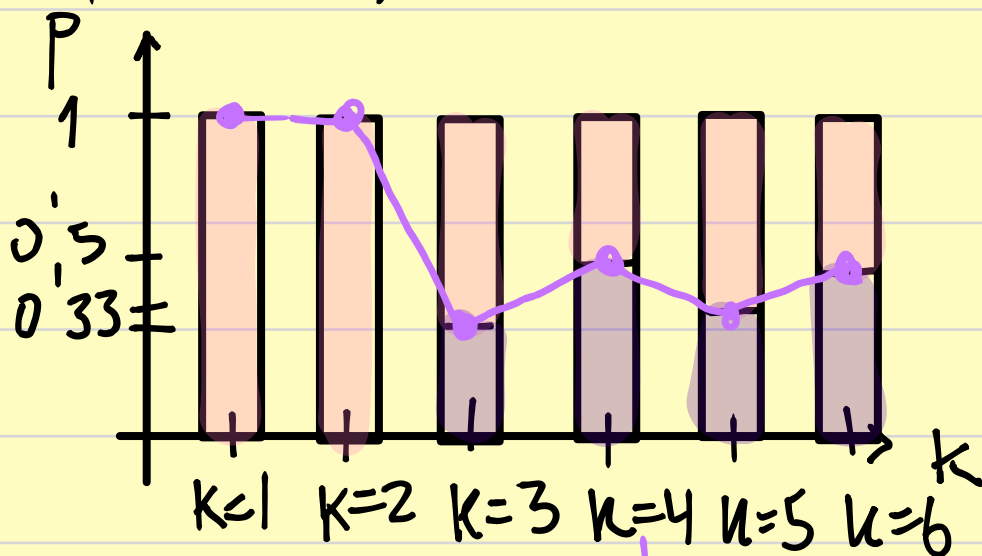
$$K=2: p(\alpha \in G_1) = \frac{0}{2} = 0; p(\alpha \in G_2) = \frac{2}{2} = 1$$

$$K=3: p(\alpha \in G_1) = \frac{1}{3} = 0'33; p(\alpha \in G_2) = \frac{2}{3} = 0'67$$

$$K=4: p(\alpha \in G_1) = \frac{2}{4} = 0'5; p(\alpha \in G_2) = \frac{2}{4} = 0'5$$

$$K=5: p(\alpha \in G_1) = \frac{2}{5} = 0'4; p(\alpha \in G_2) = \frac{3}{5} = 0'6$$

$$K=6: p(\alpha \in G_1) = \frac{3}{6} = 0'5; p(\alpha \in G_2) = \frac{3}{6} = 0'5$$



• Bei  $K=3$  erreichen wir die beste Erklärung der Daten.  
Somit gehört  $w_{\alpha}$  zur Gruppe 2.

