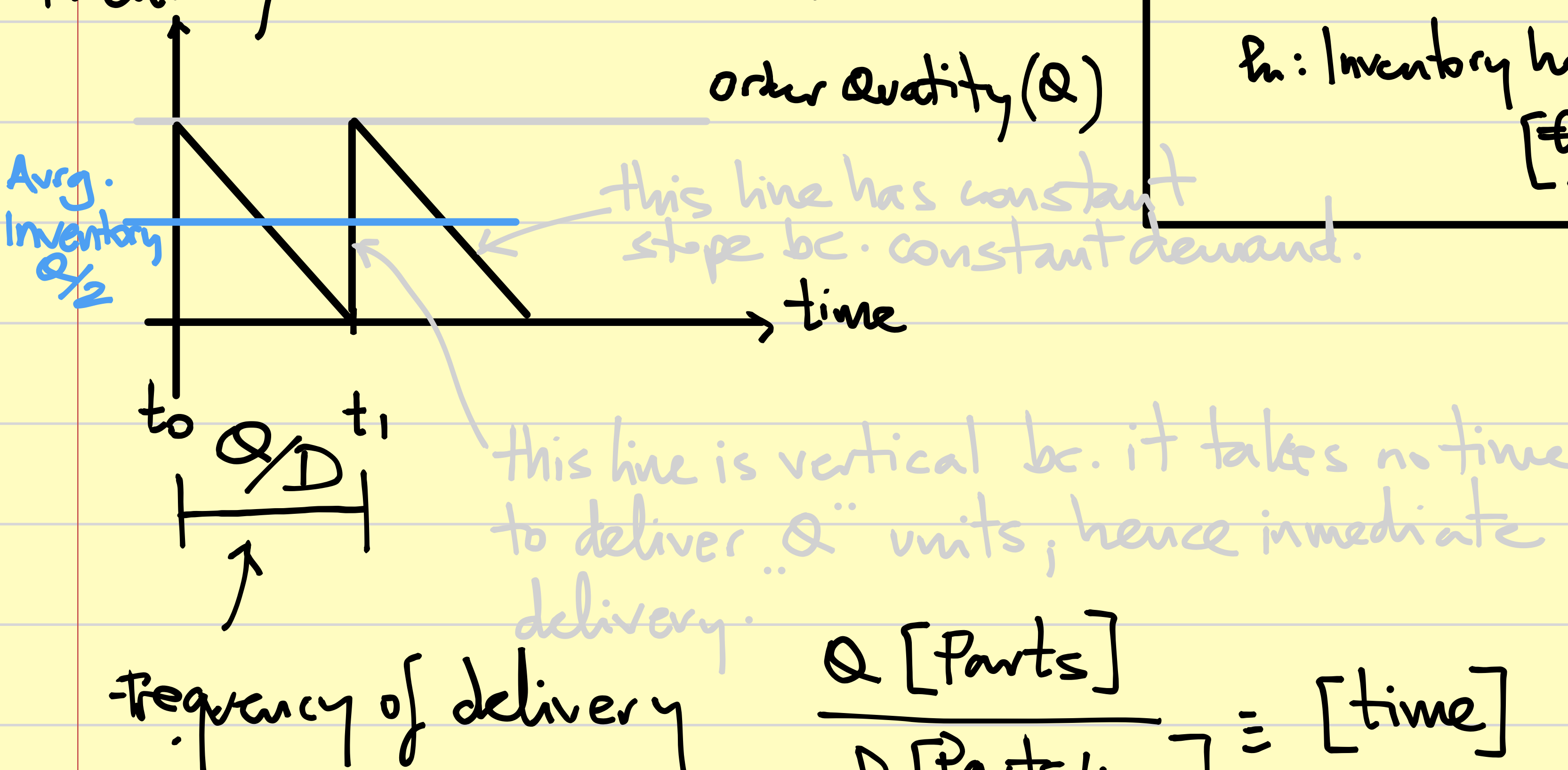
20240930_Supplier_Management_NBW7

Economic Order Quantity Model:

Parameters:
- D: Demand [Parts/time]
- c: Cost per Unit [€/part]
- A: Setup Cost [€]
- h: Inventory holding cost [€/part.time]

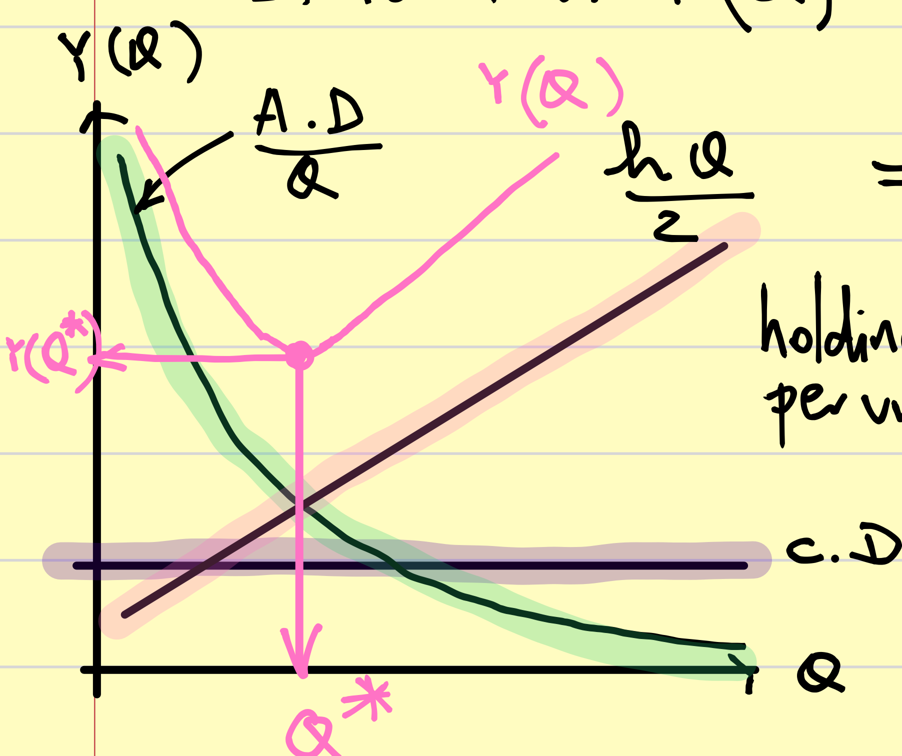
$\boxed{\text{EOQ I}}$

Hypothesis: immediate delivery and production
constant demand.



$$\frac{Q\,[\text{Parts}]}{D\,[\text{Parts/time}]} = [\text{time}]$$

Cost Function $Y(Q) =$ cost of inventory + setup cost + Production cost =



$$= \underset{\substack{\uparrow\\ \text{holding cost}\\ \text{per unit}}}{h} \cdot \underset{\substack{\uparrow\\ \text{Average}\\ \text{Inventory}}}{\frac{Q}{2}} + \underset{\substack{\uparrow\\ \text{Setup}\\ \text{Cost}}}{A} \cdot \underset{\substack{\uparrow\\ \text{Frequency}\\ \text{of setup}}}{\frac{1}{Q/D}} + \underset{\substack{\uparrow\\ \text{Cost}\\ \text{per Part}}}{c} \cdot \underset{\substack{\uparrow\\ \text{Demand}}}{D}$$

$Q^* = \text{EOQ I}$ $\quad Y(Q^*) = \text{Cost by EOQ I}$

$$\left.\frac{dY(Q)}{dQ}\right|_{Q=Q^*} = 0 \qquad \text{[this way we find } Q^*\text{]}$$

$$\frac{d}{dQ}\left[\frac{hQ}{2}+\frac{AD}{Q}+cD\right]_{Q=Q^*_I}=0 \rightarrow \frac{h}{2}-\frac{AD}{Q^{*2}_I}=0 \rightarrow$$

$$\boxed{Q^*_I=\sqrt{\frac{2AD}{h}}} \rightarrow Y(Q^*_I)=\frac{hQ^*_I}{2}+\frac{AD}{Q^*_I}+\cancel{cD}$$

cD=0 as aproximation

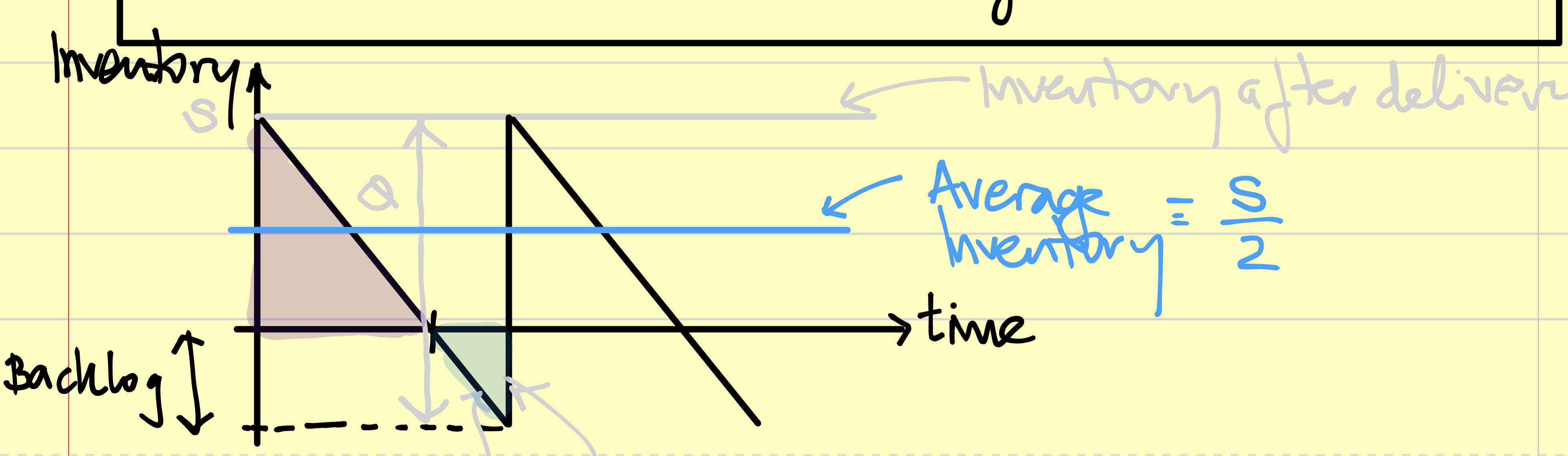
$$Y(Q^*_I)=\frac{h\cdot\sqrt{\frac{2AD}{h}}}{2}+\frac{AD}{\sqrt{\frac{2AD}{h}}}=\boxed{\sqrt{2ADh}=Y(Q^*_I)}$$

EOQ II. Supplier Model

Hypothesis:
- we accept "stock out" in which supply has a backlog of orders.
- delivery & production are inmediate
- demand is constant.

Parameters:
- p . Cost per not supplied orders [€/unit]
- s . Inventory after delivery [Units]

$Q-s$ = Backlog



t_0 t_1 t_2

This line is vertical bc. production & supply are immediate

this line has constant slope bc. demand is constant

$$t_1 = \frac{S}{P} \qquad t_2 = \frac{Q}{D}$$

Cost function $\equiv Y(Q,S) =$ Inventory holding cost + Backlog Cost + Setup Cost + Production Cost

$$= \underbrace{h}_{\text{holding cost per unit}} \cdot \underbrace{\frac{S}{2}}_{\text{avrage inventory}} \cdot \underbrace{\frac{S}{D}}_{\text{time to consume inventory}} \cdot \underbrace{\frac{D}{Q}}_{\text{frequency (how often)}} + \underbrace{p}_{\text{backlog cost/unit}} \cdot \underbrace{\frac{Q-S}{2}}_{\text{average backlog}} \cdot \underbrace{\frac{D}{Q}}_{\text{frequency}} +$$

$$+ \underbrace{\frac{AD}{Q}}_{\text{setup cost}} + \underbrace{c.D}_{\text{Production cost}}$$

We ask ourselves the question, which is the economic order quantity Q^*_{II} and the optimum inventory after delivery S^*_{II}. For that we equal zero both first partial derivatives of the function.

$$\left.\frac{\partial Y(Q,S)}{\partial Q}\right|_{Q^*,S^*} = 0$$

$$\left.\frac{\partial Y(Q,S)}{\partial S}\right|_{Q^*,S^*} = 0$$

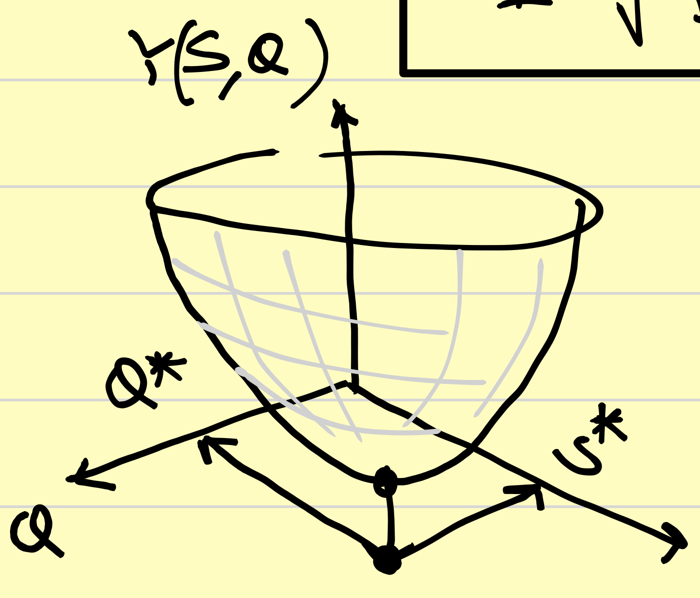
$$Y(Q,S) = \frac{hS^2}{2Q} + \frac{p(Q-S)^2}{2Q} + \frac{AD}{Q} + cD$$

$$\left.\frac{\partial Y(Q,S)}{\partial Q}\right|_{Q^*,S^*} = \frac{-hS^{*2}}{Q^{*2}} + \frac{p(Q^*-S^*)}{Q^*} - \frac{p(Q^*-S^*)^2}{2Q^{*2}} - \frac{AD}{Q^{*2}} = 0$$

$$\left.\frac{\partial Y(Q,S)}{\partial S}\right|_{Q^*,S^*} = \frac{hS^*}{Q^*} - \frac{p(Q^*-S^*)}{Q^*} = 0$$

$\rightarrow \dots \rightarrow$

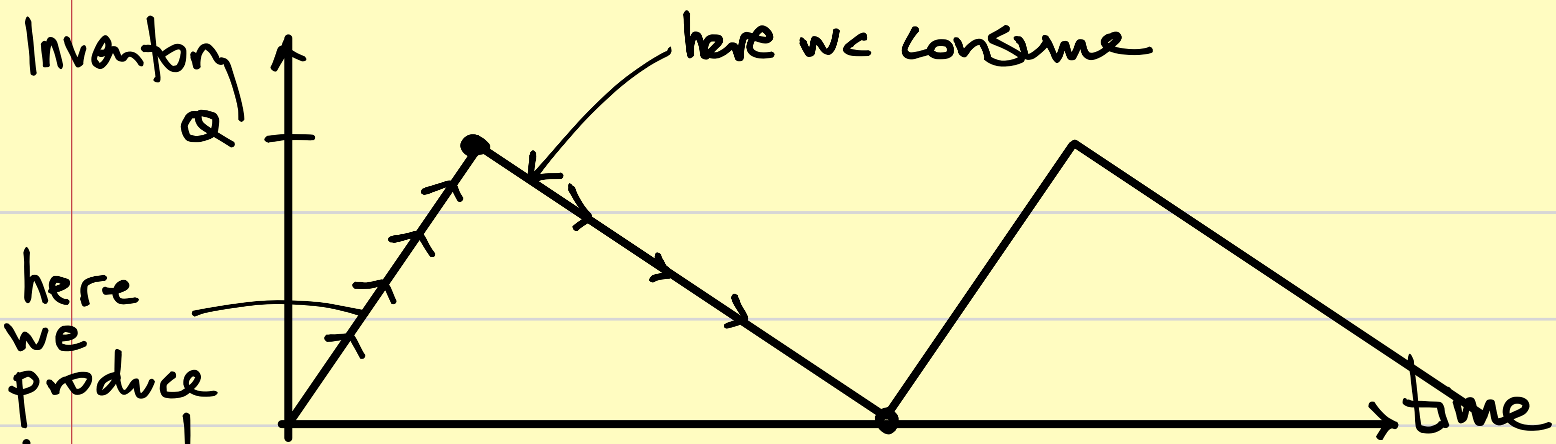
$$\boxed{\begin{aligned} S^*_{II} &= \sqrt{\frac{2AD}{h}} \cdot \sqrt{\frac{p}{p+h}} = Q^*_I \sqrt{\frac{p}{p+h}} \leq Q^*_I \\ Q^*_{II} &= \sqrt{\frac{2AD}{h}} \sqrt{\frac{p+h}{p}} = Q^*_I \sqrt{\frac{p+h}{p}} \geq Q^*_I \end{aligned}}$$



EOQ III (Manufacturing Model) - without Backlog

Hypothesis:
- constant demand
- no backlog
- production & delivery are NOT immediate

$\boxed{\text{Parameter: } K \equiv \text{production rate}}$



Line not vertical bc. not immediate

$$r(Q) = \text{holding cost of inventory} + \text{setup cost} + \text{Production cost}$$

$$= h \cdot \frac{Q}{2}\left[1 - \frac{D}{K}\right] + \frac{AD}{Q} + c \cdot D$$

Inventory holding cost

Average inventory

$$\left.\frac{dr}{dQ}\right|_{Q=Q^*_{III}} = 0 \rightarrow \frac{hQ^*_{III}}{2}\left[1 - \frac{D}{K}\right] - \frac{AD}{Q^{*2}_{III}} = 0 \longrightarrow$$

$$\boxed{Q^*_{III} = \sqrt{\frac{2AD}{h\left(1 - \frac{D}{K}\right)}}}$$

Note that if the production rate is infinite $K >> D$, then $1 - \frac{D}{K} \simeq 1 \rightarrow Q^*_{III} = Q^*_{I}$

$$r(Q^*_{III}) = \frac{h \cdot Q^*_{III}}{2}\left(1 - \frac{D}{K}\right) + \frac{AD}{Q^*_{III}} + \cancel{cD}^{0} =$$

$$= \frac{h}{2}\left(1 - \frac{D}{K}\right) \cdot \sqrt{\frac{2AD}{h\left(1 - \frac{D}{K}\right)}} + AD \cdot \frac{1}{\sqrt{\frac{2AD}{h\left(1 - \frac{D}{K}\right)}}} =$$

$$= \frac{h}{2}\sqrt{\frac{2AD\left(1 - \frac{D}{K}\right)^2}{h\left(1 - \frac{D}{K}\right)}} + \sqrt{\frac{A^2D^2\, h\left(1 - \frac{D}{K}\right)}{2AD}} =$$

$$\boxed{Y(Q^*_{III}) = \sqrt{2ADh\left(1-\frac{D}{K}\right)}}$$

again if production rate is much faster than demand, then $K \gg D \rightarrow 1-\frac{D}{K} \simeq 1 \rightarrow Y(Q^*_{III}) = Y(Q^*_{I})$

Conclusion: the more flexible the production (the bigger K is), the better we can adapt to demand changes!