20250519_BU1_LM1_Mathe1

2D. Dynamisches System R&J

$$\left.\begin{array}{l}\frac{dR(t)}{dt} = a\,R(t) + b\,J(t) \\ \frac{dJ(t)}{dt} = c\,R(t) + d\,J(t)\end{array}\right\} \quad \frac{d\,X(t)}{dt} = \overset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \cdot X(t); \quad X(t) = \begin{bmatrix} R(t) \\ J(t) \end{bmatrix}$$

$$\det[A - \lambda I] = 0 \rightarrow \lambda = \frac{a+d \pm \sqrt{(a+d)^2 - 4(ad-cb)}}{2}$$

Reines Liebe-Hass. Oszillator:

$a = d = 0$ : keine Eigenverstärkung bei R oder J.

$b = 1$ : Julias Zuneigung regt Romeo an

$c = -1$ : Romeos Zuneigung weckt bei Julia Ablehnung

$$\left.\begin{array}{l}\frac{dR(t)}{dt} = J(t) \\ \frac{dJ(t)}{dt} = -R(t)\end{array}\right\} \frac{d\,X(t)}{dt} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} X(t)$$

$$\boxed{i = \sqrt{-1}}$$

$$\det[A - \lambda I] = 0 \rightarrow \det\begin{bmatrix} 0-\lambda & 1 \\ -1 & 0-\lambda \end{bmatrix} = 0 \rightarrow \lambda^2 + 1 = 0 \rightarrow \lambda = \pm i$$

$$\boxed{\lambda = i} \rightarrow A \cdot \vec{v}_1 = i \cdot \vec{v}_1 \rightarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} i v_{11} \\ i v_{12} \end{bmatrix} \rightarrow$$

$$\rightarrow \begin{array}{l} v_{12} = i v_{11} \\ -v_{11} = i v_{12} \end{array} \rightarrow v_{11} = 1 \rightarrow v_{12} = i \rightarrow \vec{v}_1 = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\boxed{\lambda = -i} \rightarrow A \cdot \vec{v}_2 = -i \cdot \vec{v}_2 \rightarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = -i \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} \rightarrow$$

$\rightarrow \quad v_{22} = -i v_{21} \quad \rightarrow \quad v_{21} = 1 \rightarrow v_{22} = -i$

$-v_{21} = -i v_{22}$

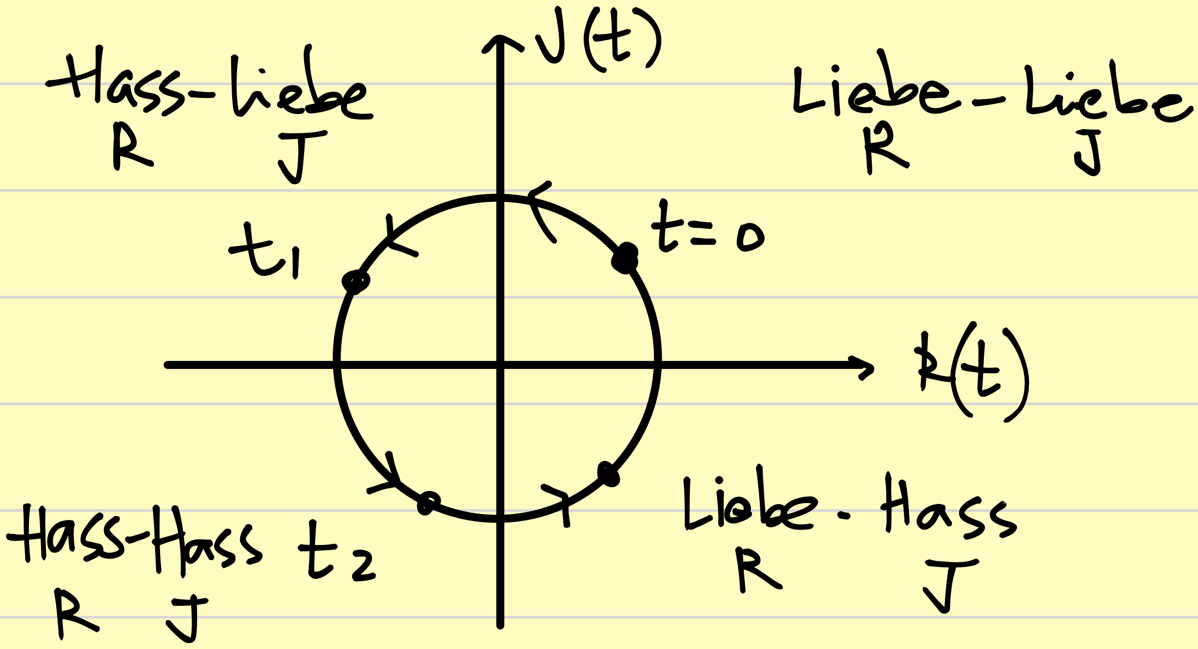
$\rightarrow \vec{v_2} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$$\vec{x}(t) = \begin{bmatrix} R(t) \\ J(t) \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ i \end{bmatrix} e^{+it} + c_2 \begin{bmatrix} 1 \\ -i \end{bmatrix} e^{-it}$$

$$e^{it} = \cos(t) + i\sin(t)$$
$$e^{-it} = \cos(t) - i\sin(t)$$

$$\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ i \end{bmatrix} \big(\cos(t) + i\sin(t)\big) + c_2 \begin{bmatrix} 1 \\ -i \end{bmatrix} \big(\cos(t) - i\sin(t)\big)$$

$$\vec{x}(t) = \begin{bmatrix} R(t) \\ J(t) \end{bmatrix} = \begin{bmatrix} c_1 \cos(t) + c_2 \sin(t) \\ -c_1 \sin(t) + c_2 \cos(t) \end{bmatrix}$$

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10. Dynamisches System.

Die Ansteckungsrate von einer sex. Krankheit ist gegeben durch $\beta = 3$ neu Infizierte pro 100 Individuen. Die Genessungsrate $\gamma = 2$ Menschen pro 100.

Bitte stellen Sie den Verlauf der Krankheit, wenn es 100 Menschen gibt und bei $t = 0$, 10 sind krank.

$$\frac{di(t)}{dt} = \text{Ansteckungsrate} - \text{Genesungsrate}$$

$$= 3\,i(1-i) - 2i = 3i - 2i - 3i^2 = i - 3i^2 =$$

$$= i(1-3i)$$

$$\int \frac{di(t)}{i(1-3i)} = \int dt \qquad (*)$$

$$\frac{1}{i(1-3i)} = \frac{1}{i \cdot 3\left(\frac{1}{3} - i\right)} = \frac{1}{3} \cdot \frac{1}{i\left(\frac{1}{3} - i\right)} = \frac{A}{i} + \frac{B}{\frac{1}{3} - i}$$

$$\frac{1}{3} = A\left(\frac{1}{3} - i\right) + B \cdot i$$

$$i = \frac{1}{3} \longrightarrow \frac{1}{3} = \cancel{A\left(\frac{1}{3} - \frac{1}{3}\right)} + B \cdot \frac{1}{3} \longrightarrow B = 1$$

$$i = 0 \longrightarrow \frac{1}{3} = A\left(\frac{1}{3} - 0\right) + B \cdot 0 \longrightarrow A = 1$$

$$\frac{1}{i(1-3i)} = \frac{1}{i} + \frac{1}{\frac{1}{3} - i} \qquad (*)$$

$$\int \frac{di(t)}{i(1-3i)} = \int dt \longrightarrow \int \frac{di(t)}{i} + \int \frac{di(t)}{\frac{1}{3} - i(t)} = \int dt$$

$$\ln|i| - \ln\left|\frac{1}{3} - i\right| = t + C$$

Konstante

Variable (Zeit)

$$\frac{i}{\frac{1}{3} - i} = e^{t} + C$$

$$t=0 \rightarrow i(0)=10/100$$

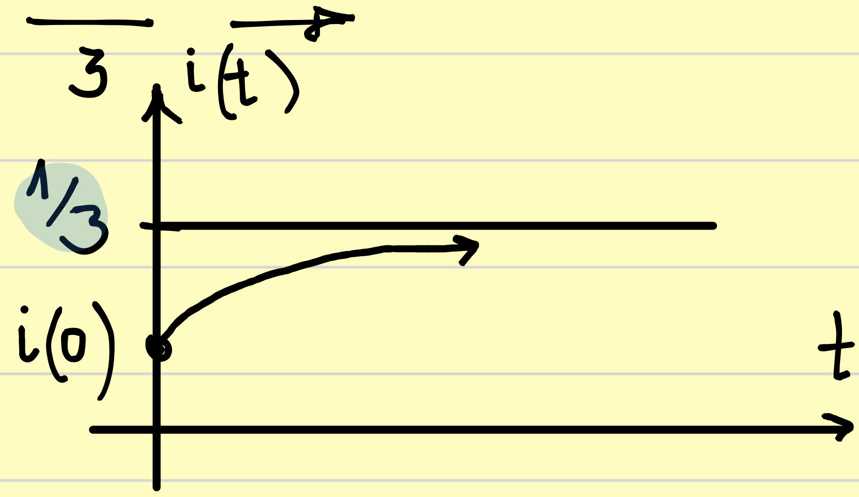
$$\frac{i(0)}{\frac{1}{3}-i(0)} = e^{0} + C \rightarrow \frac{0'1}{\frac{1}{3}-0'1} = C \rightarrow C = 0'428$$

$$\frac{i}{\frac{1}{3}-i} = e^{t} - 0'428 \rightarrow i = \left(\frac{1}{3}-i\right)\left(e^{t}-0'428\right) \rightarrow$$

$$i = \frac{1}{3}e^{t} - \frac{1}{3}\cdot 0'428 - i\,e^{t} + i\cdot 0'428 \rightarrow$$

$$i\left[1+e^{t}-0'428\right] = \frac{1}{3}e^{t} - \frac{0'428}{3} \rightarrow$$

$$\rightarrow \boxed{i(t) = \frac{\frac{1}{3}e^{t} - 0'143}{e^{t} - 0'572}}$$

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$$\beta = 2 \quad ; \quad \gamma = 3 \quad ; \quad i(0) = \frac{10}{100}$$

$$\frac{di(t)}{dt} = \beta\, i(1-i) - \gamma\, i = 2i(1-i) - 3i = -i - 2i^{2}$$
$$= -i\left[1+2i\right]$$

$$\int \frac{di(t)}{-i(1+2i)} = \int dt$$

$$\frac{-1}{i(1+2i)} = \frac{-1}{i\cdot 2\left(i+\frac{1}{2}\right)} = \frac{A}{i} + \frac{B}{i+\frac{1}{2}}$$

$$-\frac{1}{2} = A\left(i+\frac{1}{2}\right) + B \cdot i$$

$$i = \frac{-1}{2} \rightarrow \frac{-1}{2} = A\left(\frac{-1}{2} + \frac{1}{2}\right) + B \cdot \frac{-1}{2} \rightarrow B = 1.$$

$$i = 0 \rightarrow \frac{-1}{2} = A\left(0 + \frac{1}{2}\right) + B \cdot 0 \rightarrow A = -1$$

$$\int \frac{di(t)}{-i(1+2i)} = \int \frac{-di(t)}{i} + \int \frac{1 \cdot dt(t)}{i+\frac{1}{2}} = \int dt$$

$$-\ln|i| + \ln\left|i + \frac{1}{2}\right| = t + c$$

$$\frac{i+\frac{1}{2}}{i} = e^{t} + c$$

$$t = 0 \rightarrow i(0) = 0'1 \rightarrow \frac{0'1 + \frac{1}{2}}{0'1} = 1 + c \rightarrow c = 5$$

$$i + \frac{1}{2} = i \cdot \left[e^{t} + 5\right] \rightarrow \frac{1}{2} = i\left[e^{t} + 5 - 1\right] \rightarrow$$

$$\boxed{i(t) = \frac{1}{2\left[e^{t} + 5 - 1\right]}}$$


$$i(t=10) = \frac{1}{2\left[e^{10} + 5 - 1\right]} = 0'0000227$$