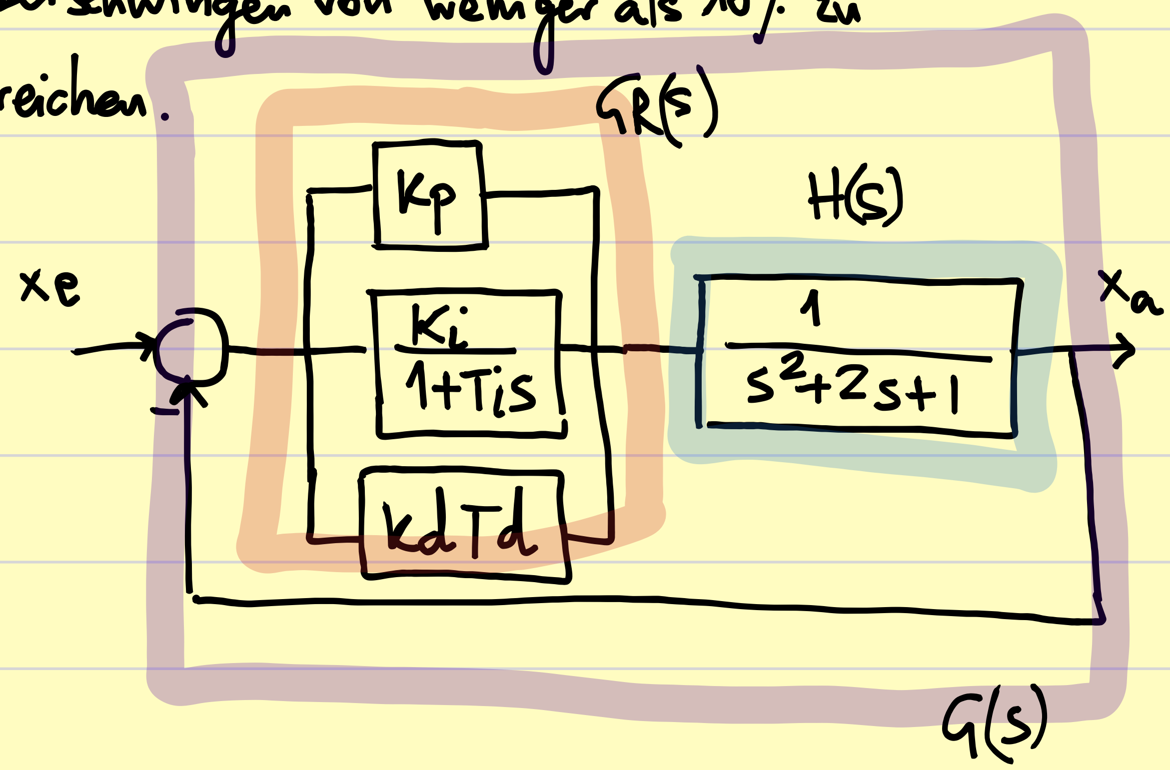
20240611_Regelungstechnik_BEL4

1. Entwerfen Sie einen PID-Regler für ein System mit Übertragungsfunktion $H(s)=\frac{1}{s^2+2s+1}$. Bestimmen Sie die k_p, k_i, und k_d um ein Überschwingen von weniger als 10% zu erreichen.



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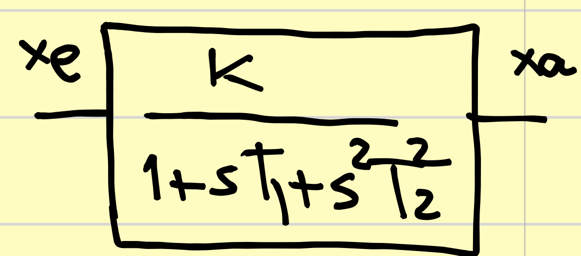

$$\text{Überschw} = \frac{KT_2}{T_1}$$

$$\left.\begin{aligned} G(s) &= \frac{G_R(s)\cdot H(s)}{1+G_R(s)\cdot H(s)} \\ G_R(s) &= K_p + \frac{K_i}{1+T_i s} + K_d T_d \end{aligned}\right\} \rightarrow$$

$$\rightarrow G(s) = \frac{\left[K_p + \frac{K_i}{1+T_i s} + K_d T_d\right]\cdot \frac{1}{s^2+2s+1}}{1+\left[K_p + \frac{K_i}{1+T_i s} + K_d T_d\right]\cdot \frac{1}{s^2+2s+1}} =$$

$$= \frac{K_p + \frac{K_i}{1+T_i s} + K_d T_d}{s^2+2s+1 + K_p + \frac{K_i}{1+T_i s} + K_d T_d} =$$

$$= \frac{(1+\cancel{T_i s})K_p + K_i + K_d T_d (1+\cancel{T_i s})}{(1+\cancel{T_i s})(s^2+2s+1) + (1+\cancel{T_i s})K_p + K_i + K_d T_d}$$

$T_i = 0$

$$= \frac{K_p + K_i + K_d T_d}{s^2+2s+1+K_p+K_i+K_d T_d} = \frac{K}{1+sT_1+s^2T_2^2}$$

$$= \frac{K_p+K_i+K_d T_d}{1+K_p+K_i+K_d T_d} \cdot \frac{1}{1 + \frac{2s}{1+K_p+K_i+K_d T_d} + \frac{s^2}{1+K_p+K_i+K_d T_d}}$$

$$K = \frac{K_p+K_i+K_d T_d}{1+K_p+K_i+K_d T_d}; \quad T_1 = \frac{2}{1+K_p+K_i+K_d T_d}; \quad T_2 = \sqrt{\frac{1}{1+K_p+K_i+K_d T_d}}$$

$$\ddot{U}bs = \frac{K T_2}{T_1} = \frac{\frac{K_p+K_i+K_d T_d}{\cancel{1+K_p+K_i+K_d T_d}} \cdot \sqrt{\frac{1}{1+K_p+K_i+K_d T_d}}}{\frac{2}{\cancel{1+K_p+K_i+K_d T_d}}} = 1'1$$

10% Überschw.

$$\left.\begin{array}{l} K_p+K_i+K_d T_d = 2'2\sqrt{K_p+K_i+K_d T_d} \\ \\ T_d = 0 \;;\; K_p = K_i = K \end{array}\right\} \; 2K = 2'2\sqrt{2K}$$

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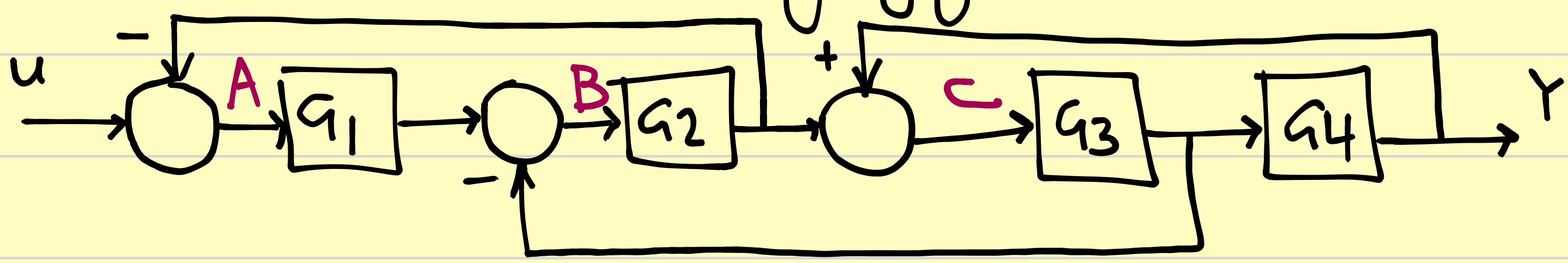
2. Blockschaltbild. Bitte Übertragungsfunktion ermitteln.



$$\left.\begin{aligned}
Y &= C \cdot G_3 \cdot G_4 && (1)\\
C &= B \cdot G_2 + Y && (2)\\
B &= A \cdot G_1 - C \cdot G_3 && (3)\\
A &= U - B \cdot G_2 && (4)
\end{aligned}\right\}$$

$(3)+(4) \longrightarrow \quad B = \left[U - B \cdot G_2\right] \cdot G_1 - C \cdot G_3$

$$B + BG_1G_2 = UG_1 - CG_3$$

$$B(1+G_1G_2) = UG_1 - CG_3$$

$$B = \frac{G_1}{1+G_1G_2}U - \frac{G_3}{1+G_1G_2}C \qquad (5)$$

$(2)+(5) \longrightarrow \quad C = \left[\frac{G_1}{1+G_1G_2}U - \frac{G_3}{1+G_1G_2}C\right] \cdot G_2 + Y$

$$C + \frac{G_2G_3}{1+G_1G_2}C = \frac{G_1G_2}{1+G_1G_2}U + Y$$

$$C\left[\frac{1+G_1G_2+G_2G_3}{1+G_1G_2}\right] = \frac{G_1G_2}{1+G_1G_2}U + Y$$

$$C = \frac{G_1G_2}{1+G_1G_2+G_2G_3}U + \frac{1+G_1G_2}{1+G_1G_2+G_2G_3}Y \quad (6)$$

$$(6)+(1) \longrightarrow Y = \left(\frac{G_1G_2}{1+G_1G_2+G_2G_3}\cdot U + \frac{1+G_1G_2}{1+G_1G_2+G_2G_3}Y\right)G_3G_4$$

$$Y - \frac{1+G_1G_2}{1+G_1G_2+G_2G_3}\cdot G_3G_4\,Y = \frac{G_1G_2G_3G_4}{1+G_1G_2+G_2G_3}\cdot U$$

$$\boxed{G(s) = \frac{Y}{U} = \frac{G_1G_2G_3G_4}{1+G_1G_2+G_2G_3-G_3G_4-G_1G_2G_3G_4}}$$

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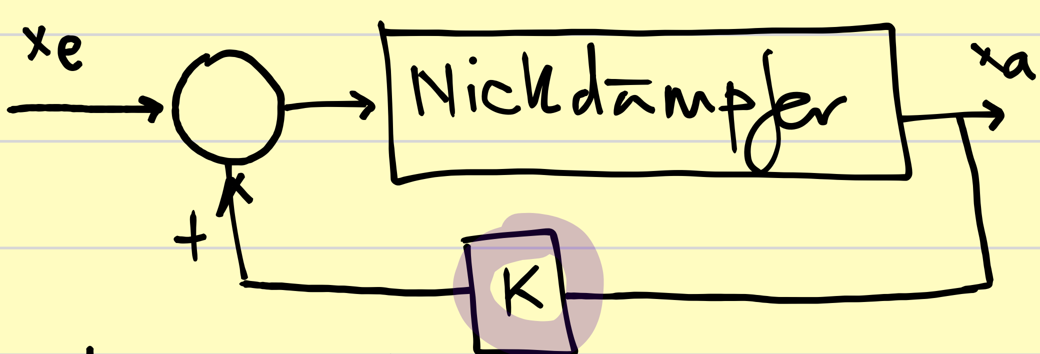
3. F·16



$$G(s) = \frac{-0'1137s - 0'0705}{s^2 + 1'5189s + 2'1303}$$

a) Berechnen Sie die Pol & Nullstellen, Eigenfrequenz & Dämpfung.

Zählerpolynom: $-0'1137s - 0'0705 = 0 \rightarrow s = -0'6201$

Nennerpolynom: $s^2 + 1'5189s + 2'1303 = 0$

$$s = \frac{-1'5189 \pm \sqrt{1'5189^2 - 4\cdot 2'1303}}{2} = \begin{cases} -0'7594 + j\cdot 1'2404 \\ -0'7594 - j\cdot 1'2404 \end{cases}$$

$$G(s)=\frac{-0'1137s-0'0705}{2'1303}\cdot\frac{1}{1+\frac{1'5189}{2'1303}s+\frac{1}{2'1303}s^2}$$

$$\omega_E=\frac{1}{T_2}=\frac{1}{\sqrt{\frac{1}{2'1303}}}=1'4596\,s^{-1}$$

$$D=\frac{1'5189}{2\cdot\omega_E}=0'52$$

b) Eigenfrequenz & Dämpfung mit Regelkreis

$$G(s)=\frac{k\cdot\frac{-0'1137s-0'0705}{s^2+1'5189s+2'1303}}{1-k\cdot\frac{-0'1137s-0'0705}{s^2+1'5189s+2'1303}}=$$

$$=k\cdot\frac{-0'1137s-0'0705}{s^2+(1'5189+0'1137k)+(2'1303+0'0705k)}=$$

$$=\frac{k}{2'1303+0'0705k}\cdot\frac{-0'1137s-0'0705}{1+\frac{1'5189+0'1137k}{2'1303+0'0705k}\cdot s+\frac{s^2}{2'1303+0'0705k}}$$

c) Vergleichen Sie das System ohne Regler und mit Regler (k=5)

OHNE REGLER $\quad w_E = 1'4596\ s^{-1} \qquad D = 0'52$

$\qquad\qquad\qquad\wedge \qquad\qquad\qquad\qquad \wedge$

MIT REGLER $K=5$ $\quad w_E = 1'5757\ s^{-1} \qquad D = 0'66$
