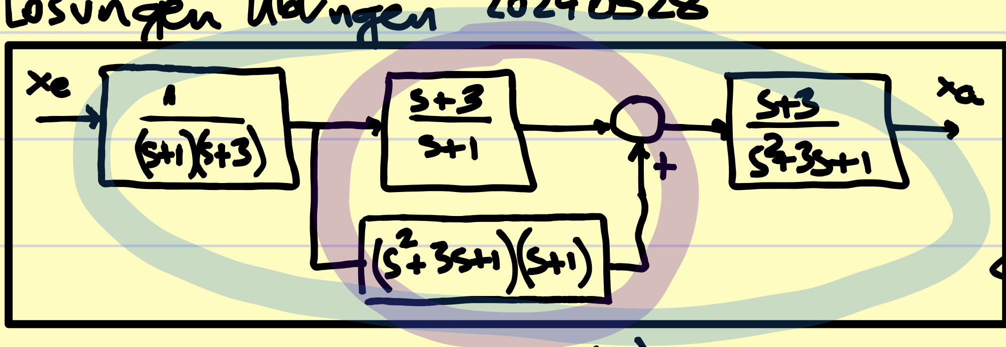
Lösungen Übungen 20240528

1) 

x_a ? $\quad x_e(t) = 1$

$$G_1(s) = \frac{s+3}{s+1} + (s^2+3s+1)(s+1) = \frac{(s+3)+(s^2+3s+1)(s+1)^2}{(s+1)}$$

$$G_2(s) = \frac{1}{(s+1)\cancel{(s+3)}} \cdot G_1(s) \cdot \frac{\cancel{s+3}}{s^2+3s+1} = \frac{(s+3)+(s^2+3s+1)(s+1)^2}{(s+1)^2(s^2+3s+1)}$$

$$s^2+3s+1 = 0 \longrightarrow \overset{*}{s} = \frac{-3 \pm \sqrt{9-4}}{2} = \begin{cases} \frac{-3+\sqrt{5}}{2} \\ \frac{-3-\sqrt{5}}{2} \end{cases}$$

$$G_2(s) = \frac{X_a(s)}{X_e(s)} = \frac{(s+3)+(s^2+3s+1)(s+1)^2}{(s+1)^2 \cdot \left(s - \frac{-3+\sqrt{5}}{2}\right) \cdot \left(s - \frac{-3-\sqrt{5}}{2}\right)} \quad ; \quad X_e(s) = \frac{1}{s}$$

$$X_a(s) = \frac{(s+3)+(s^2+3s+1)(s+1)^2}{s(s+1)^2 \cdot \left(s - \frac{-3+\sqrt{5}}{2}\right)\left(s - \frac{-3-\sqrt{5}}{2}\right)} = \frac{A}{s} + \frac{B}{(s+1)^2} + \frac{C}{\left(s - \frac{-3+\sqrt{5}}{2}\right)} + \frac{D}{\left(s - \frac{-3-\sqrt{5}}{2}\right)}$$

$$(s+3)+(s^2+3s+1)(s+1)^2 = A \cdot (s+1)^2 \cdot \left(s - \frac{-3+\sqrt{5}}{2}\right) \cdot \left(s - \frac{-3-\sqrt{5}}{2}\right) +$$
$$+ B \cdot s \cdot \left(s - \frac{-3+\sqrt{5}}{2}\right) \cdot \left(s - \frac{-3-\sqrt{5}}{2}\right) +$$
$$+ C \cdot s \cdot (s+1)^2 \cdot \left(s - \frac{-3-\sqrt{5}}{2}\right) +$$
$$+ D \cdot s \cdot (s+1)^2 \cdot \left(s - \frac{-3+\sqrt{5}}{2}\right)$$

$$\overset{*}{s} = 0 \longrightarrow 3 + 1 \cdot 1 = A \cdot 1 \cdot \left(-\frac{-3+\sqrt{5}}{2}\right)\left(-\frac{-3-\sqrt{5}}{2}\right) \longrightarrow A = 4$$

$$s^* = -1 \rightarrow -1+3 = B(-1)\cdot\left(-1-\frac{-3+\sqrt{5}}{2}\right)\left(-1-\frac{-3-\sqrt{5}}{2}\right) \rightarrow \cdots \rightarrow B = b_0$$

$$s^* = \frac{-3+\sqrt{5}}{2} \rightarrow \frac{-3+\sqrt{5}}{2}+3 = C\cdot\frac{-3+\sqrt{5}}{2}\cdot\left(\frac{-3+\sqrt{5}}{2}+1\right)^2\cdot 1 \rightarrow \cdots \rightarrow C = c_0$$

$$s^* = \frac{-3-\sqrt{5}}{2} \rightarrow \frac{-3-\sqrt{5}}{2}+3 = D\cdot\frac{-3-\sqrt{5}}{2}\left(\frac{-3-\sqrt{5}}{2}+1\right)^2\cdot 1 \rightarrow \cdots \rightarrow D = d_0$$

$$x_a(s) = \frac{4}{s} + \frac{b_0}{(s+1)^2} + \frac{c_0}{s-\frac{-3+\sqrt{5}}{2}} + \frac{d_0}{s-\frac{-3-\sqrt{5}}{2}} \rightarrow x_a(t) = \ldots$$

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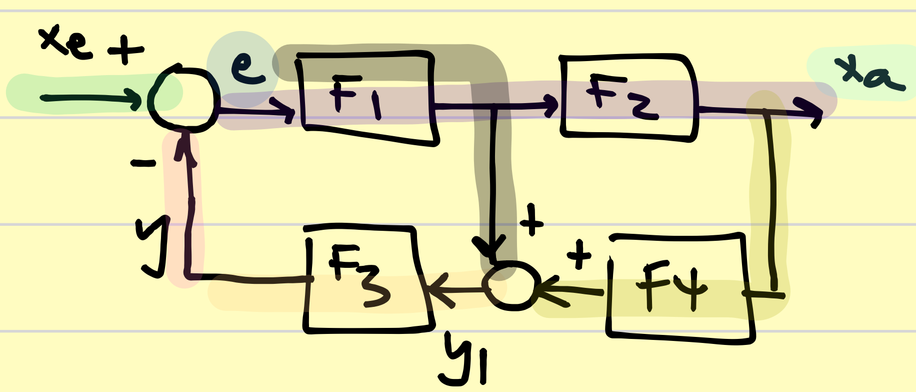
2) 2.1)



$$\left.\begin{aligned} e &= x_e - y \\ x_a &= e\cdot F_1\cdot F_2 \\ y &= y_1\cdot F_3 \end{aligned}\right\} \quad \left.\begin{aligned} e &= x_e - y_1\cdot F_3 \\ \\ y_1 &= e\cdot F_1 + x_a F_4 \end{aligned}\right\}$$

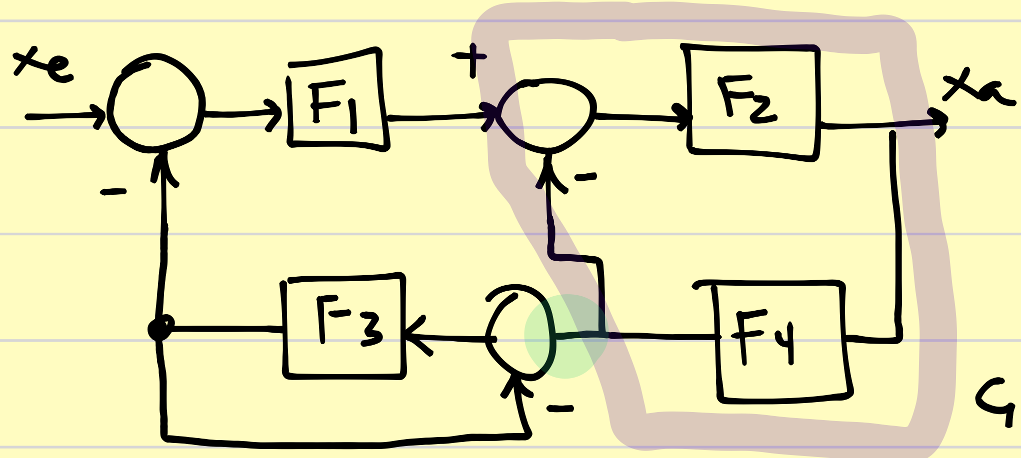
$$\left.\begin{aligned} e &= x_e - (e F_1 + x_a F_4) F_3 \\ e &= \frac{x_a}{F_1 F_2} \end{aligned}\right\} \frac{x_a}{F_1 F_2} = x_e - \left(\frac{x_a}{\cancel{F_1} F_2}\cdot \cancel{F_1} + x_a F_4\right) F_3$$

$$X_a = F_1 F_2 x_e - x_a(1+F_4) F_1 F_2 F_3$$

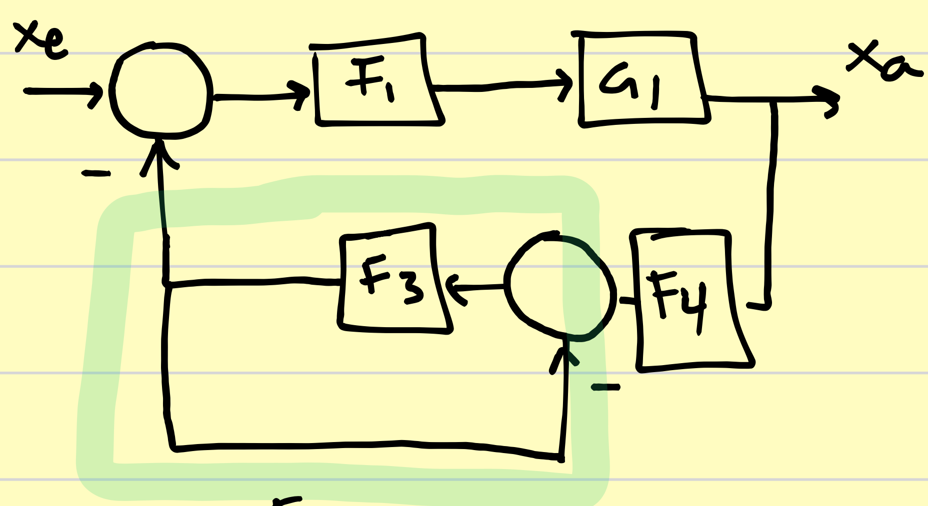
$$x_a\left(1+\left(1+F_4\right)F_1F_2F_3\right) = F_1F_2\,x_e$$

$$\boxed{\frac{x_a}{x_e} = \frac{F_1\,F_2}{1+F_1F_2F_3\left(1+F_4\right)}}$$

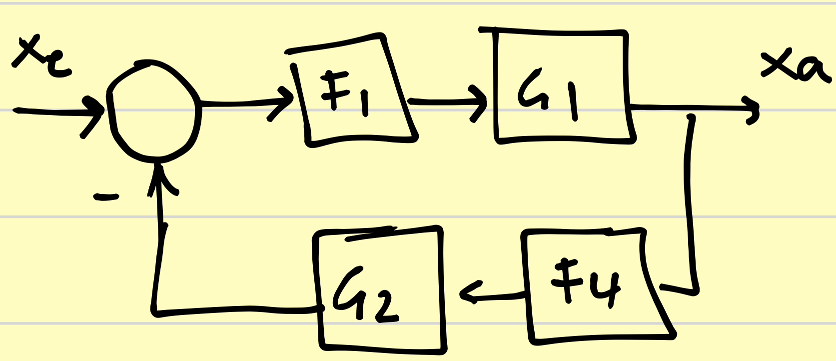
2.2)



$$G_1(s) = \frac{F_2}{1+F_2F_4}$$



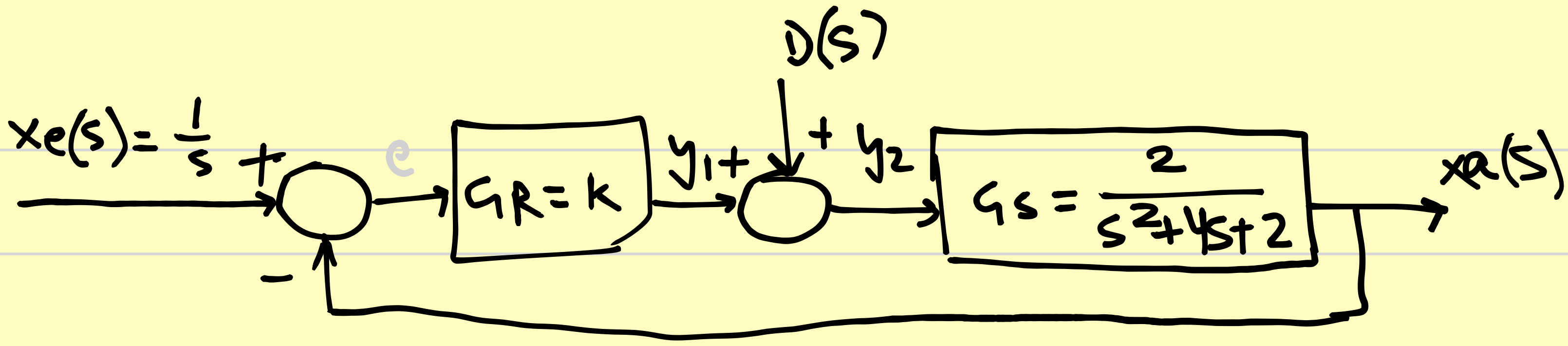
$$G_2(s) = \frac{F_3}{1+F_3}$$



$$G(s) = \frac{F_1G_1}{1+F_1G_1G_2F_4} = \frac{F_1\dfrac{F_2}{1+F_2F_4}}{1+F_1\dfrac{F_2}{1+F_2F_4}\cdot\dfrac{F_3}{1+F_3}\cdot F_4} =$$

$$\boxed{G(s) = \frac{F_1\,F_2}{1+F_2F_4+F_1F_2F_1F_4\cdot\dfrac{1}{1+F_3}}}$$

3.



a) Annahme: $xe(s) = D(s) = \frac{1}{s}$

$$e = xe - xa \rightarrow xa = \left(e \cdot G_R + D(s)\right) \cdot G(s) =$$

$$= \left[(xe - xa) \cdot K + \frac{1}{s}\right] \cdot \frac{2}{s^2+4s+2}$$

$$xa = \left[xe \cdot k \cdot \frac{2}{s^2+4s+2} - xa\,k \cdot \frac{2}{s^2+4s+2} + \frac{2}{s(s^2+4s+2)}\right]$$

$$xa\left[1 + \frac{2k}{s^2+4s+2}\right] = xe \cdot \frac{2k}{s^2+4s+2} + D(s) \cdot \frac{2}{s^2+4s+2}$$

$D(s) = \varepsilon \ll \rightarrow D(s)$ ist vernachlässigbar (Annahme)

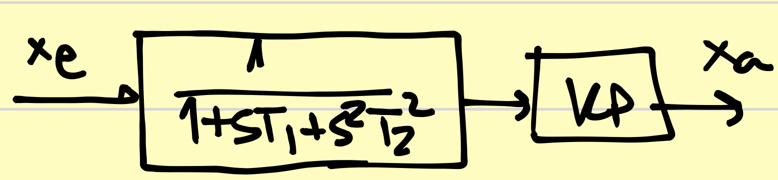
$$\frac{xa}{xe} = \frac{\frac{2k}{s^2+4s+2}}{1 + \frac{2k}{s^2+4s+2}} = \frac{2k}{s^2+4s+2+2k}$$

STABILITÄT

$$s^* = \frac{-4 \pm \sqrt{16 - 4(2+2k)}}{2} = -2 \pm \sqrt{2-2k} < 0$$

$$2 - 2k < 4 \rightarrow \boxed{k > -1}$$ ✓

b) Theorievorlesung



$$\text{Überschwingung} = \frac{kp \cdot T_2}{T_1}$$

$$\frac{xa}{xe} = \frac{2k}{s^2+4s+2+2k} = \frac{2k}{2+2k} \cdot \frac{1}{1 + \frac{4s}{2+2k} + \frac{1}{2+2k}s^2}$$

$$T_1=\frac{4}{2+2k}\,;\;T_2=\sqrt{\frac{1}{2+2k}}$$

$$\text{Überschw.}=\frac{2k\cdot\sqrt{\frac{1}{2+2k}}}{\frac{4}{2+2k}}\leq 1 \rightarrow 2k\sqrt{\frac{1}{2+2k}}<\frac{4}{\sqrt{2+2k}}$$

$$\rightarrow\ldots\rightarrow \boxed{k\leq 1}$$

$$\boxed{\text{Lösung: } -1\leq k\leq 1}$$

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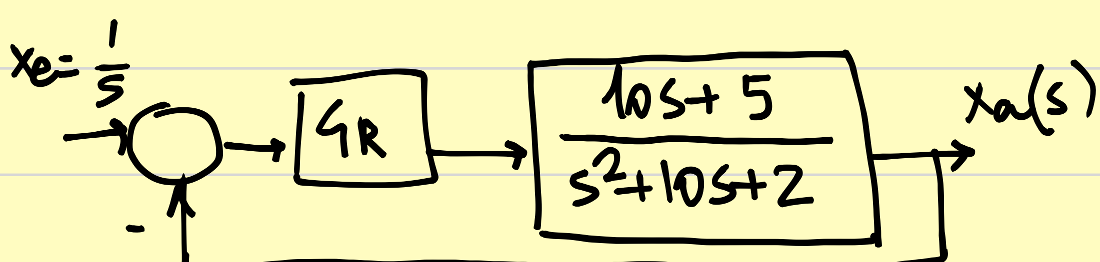
4. ignorieren

Gegeben ist die Strecke $G_S(s)=\frac{10s+5}{s^2+10s+5}$. Entwerfen Sie einen Regler, so dass das System bei einem Einheitssprung eine Anstiegszeit $t_R=1$ und eine Überschw. von 10%.

5.



$$G(s)=\frac{G_R\cdot\frac{10s+5}{s^2+10s+2}}{1+\frac{G_R(10s+5)}{s^2+10s+2}}=\frac{G_R(10s+5)}{s^2+10s+2+G_R(10s+5)}$$

Theorievorlesung: $\rightarrow\boxed{\frac{K_P}{1+sT_1+s^2T_2^2}}\rightarrow$ $\quad\omega_E=\frac{1}{T_2}$, $\quad \text{Üb}=\frac{K_P T_2}{T_1}$

$$G(s)=\frac{G_R(10s+5)}{[G_R(10s+5)+2]}\cdot\frac{1}{1+\frac{10s}{[G_R(10s+5)+2]}+\frac{s^2}{[G_R(10s+5)+2]}}$$

$$T_2=\sqrt{\frac{1}{2+G_R(10s+5)}}\qquad T_1=\frac{10}{2+G_R(10s+5)}\qquad K_P=\frac{G_R(10s+5)}{2+G_R(10s+5)}$$

$$w_E = \frac{1}{T_2} = \sqrt{2 + G_R(10s+5)}$$

10 % Überschwingung

$$\text{Überschw.} = \frac{K_p \cdot \sqrt{\frac{1}{2+G_R(10s+5)}}}{\frac{10}{2+G_R(10s+5)}} = 1{,}1 \qquad G_R\,?$$

$$G_R = \frac{K}{10s+5} \rightarrow \text{überschw.} = \frac{K_p \sqrt{\frac{1}{2+K}}}{\frac{10}{2+K}}$$

$$K_p = \frac{K}{2+K} \longrightarrow \text{überschw} = \frac{K}{10\sqrt{2+K}} = 1{,}1 \rightarrow \dots \rightarrow K = 122$$

$$\boxed{G_R = \frac{122}{10s+5}}$$