BD VON REIHEN-GESCHALTETEN GLIEDERN

Sind „n" Glieder mit Frequenzgängen $G_1(j\omega), \dots, G_n(j\omega)$ in Reihe geschaltet, so ist der Gesamtfrequenzgang gleich dem PRODUKT der einzelnen Frequenzgängen.

$$- \boxed{G_1} - \boxed{G_2} - \cdots - \boxed{G_n} -$$

$$G(s) = \prod_{i=1}^{n} G_i(s)$$

$$G(j\omega) = \prod_{i=1}^{n} G_i(j\omega) \quad (*)$$

Zur Darstellung in BD wird $G(j\omega)$ zerlegt: $G(j\omega) = |G(j\omega)| e^{j\varphi(\omega)}$

Angewandt $(*)$ wird dann:

$$G(j\omega) = |G_1(j\omega)| e^{j\varphi_1(\omega)} \cdot |G_2(j\omega)| e^{j\varphi_2(\omega)} \cdots\cdots |G_n(j\omega)| e^{j\varphi_n(\omega)}$$

$$= |G_1(j\omega)| |G_2(j\omega)| \dots |G_n(j\omega)| e^{j(\varphi_1 + \varphi_2 + \dots + \varphi_n)}$$

$$|G(j\omega)| = |G_1(j\omega)| |G_2(j\omega)| \cdots |G_n(j\omega)|$$

$$\varphi(\omega) = \varphi_1(\omega) + \varphi_2(\omega) + \dots + \varphi_n(\omega)$$

Infolge der logaritmischen Darstellung des Amplitudganges

$$\log|G(j\omega)| = \log|G_1(j\omega)| + \log|G_2(j\omega)| + \dots + \log|G_n(j\omega)|$$

$$\boxed{\begin{aligned} |G(j\omega)|_{dB} &= \sum_{i=1}^{n} |G_i(j\omega)|_{dB} \\ \varphi(\omega) &= \sum_{i=1}^{n} \varphi_i(\omega) \end{aligned}}$$

Beispiel. BD der folgenden Gliedern darstellen.

$$-\boxed{G_1}-\boxed{G_2}-\boxed{G_3}- \qquad G_1=\frac{K_{P1}}{1+sT_1};\ G_2=\frac{K_{P2}}{1+sT_2};\ G_3=K_{P3}(1+sT_V)$$

$K_{P1}=2;\ T_1=5s;\ K_{P2}=4;\ T_2=1s;\ K_{P3}=8;\ T_V=0{,}25s$

$$G(s)=G_1\cdot G_2\cdot G_3=\frac{64(1+0{,}25s)}{(1+5s)(1+s)}=\frac{A}{1+5s}+\frac{B}{1+s}$$

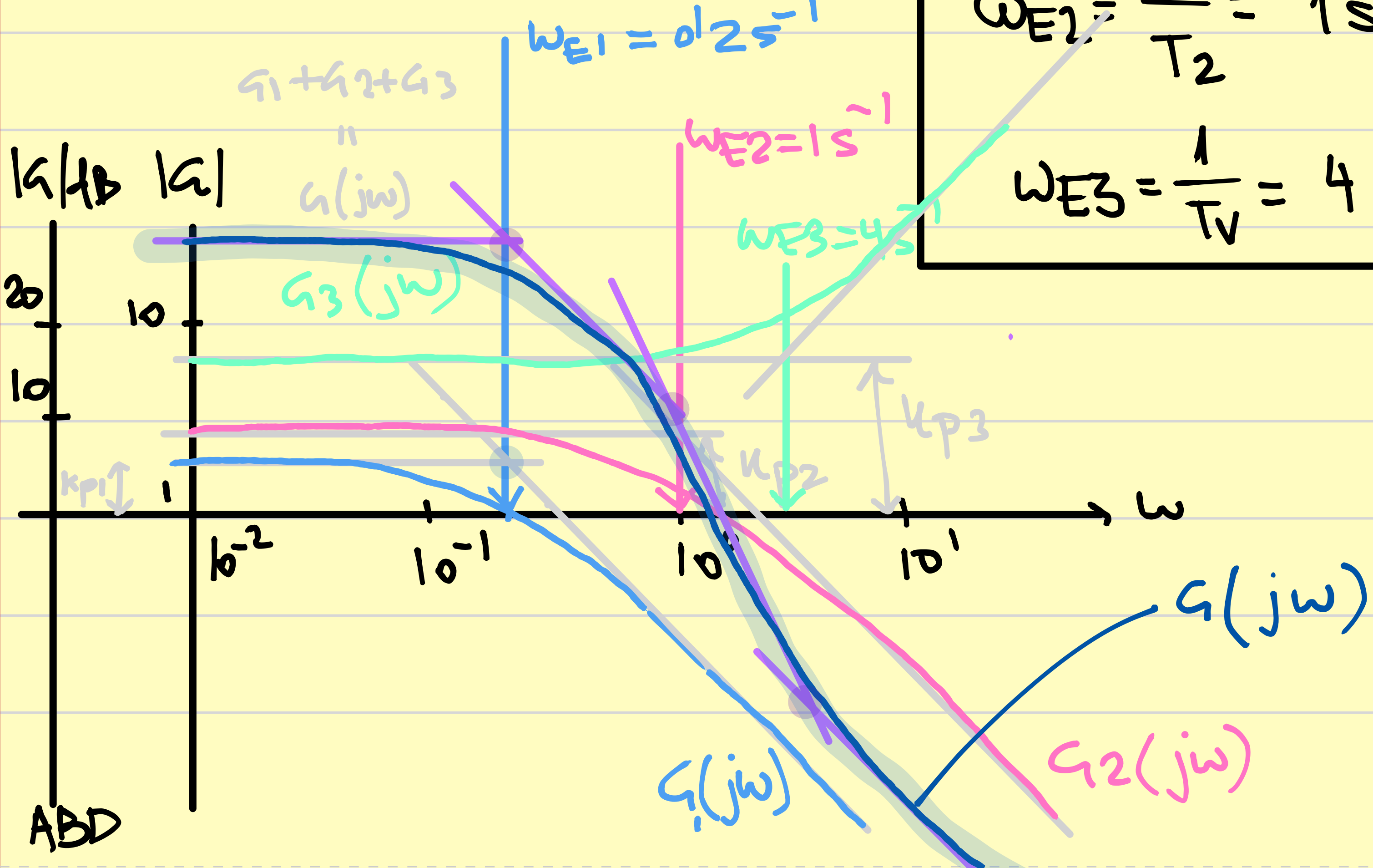
$$64(1+0{,}25s)=A(1+s)+B(1+5s)$$

$$\left.\begin{aligned}&s^*=-1 \rightarrow 48=-4B \rightarrow B=-12\\ &s^*=-0{,}2 \rightarrow 60{,}8=0{,}8A \rightarrow A=76\end{aligned}\right\} G(s)=\frac{76}{1+s}-\frac{12}{1+5s}$$

$$\boxed{G(t)=76e^{-t}-12e^{-5t}}$$

BD... Zunächst werden die Asymptoten der einzelnen Amplitudengänge gezeichnet

$$G_1=\frac{1}{s+T_1}=\frac{1}{T_1}\cdot\frac{1}{1+\frac{1}{T_1}s} \longrightarrow \boxed{\begin{aligned}\omega_{E1}&=\frac{1}{T_1}=0{,}2s^{-1}\\ \omega_{E2}&=\frac{1}{T_2}=1s^{-1}\\ \omega_{E3}&=\frac{1}{T_V}=4s^{-1}\end{aligned}}$$



$G_1 = \frac{2}{1+5s}$; $G_1(jw) = \frac{2}{1+5wj} \cdot \frac{1-5wj}{1-5wj} \rightarrow |G_1(jw)| = \frac{1}{1+(5w)^2}\sqrt{4-(10w)^2}$

$w \ll \rightarrow |G_1(jw)| = 2 = K_{p1}$

$w \gg \rightarrow |G_1(jw)| = -\infty$

$G_2 = \frac{4}{1+s}$; $G_2(jw) = \frac{4}{1+jw} \cdot \frac{1-jw}{1-jw} \rightarrow |G(jw)| = \frac{4}{1+w^2}\sqrt{1-w^2}$

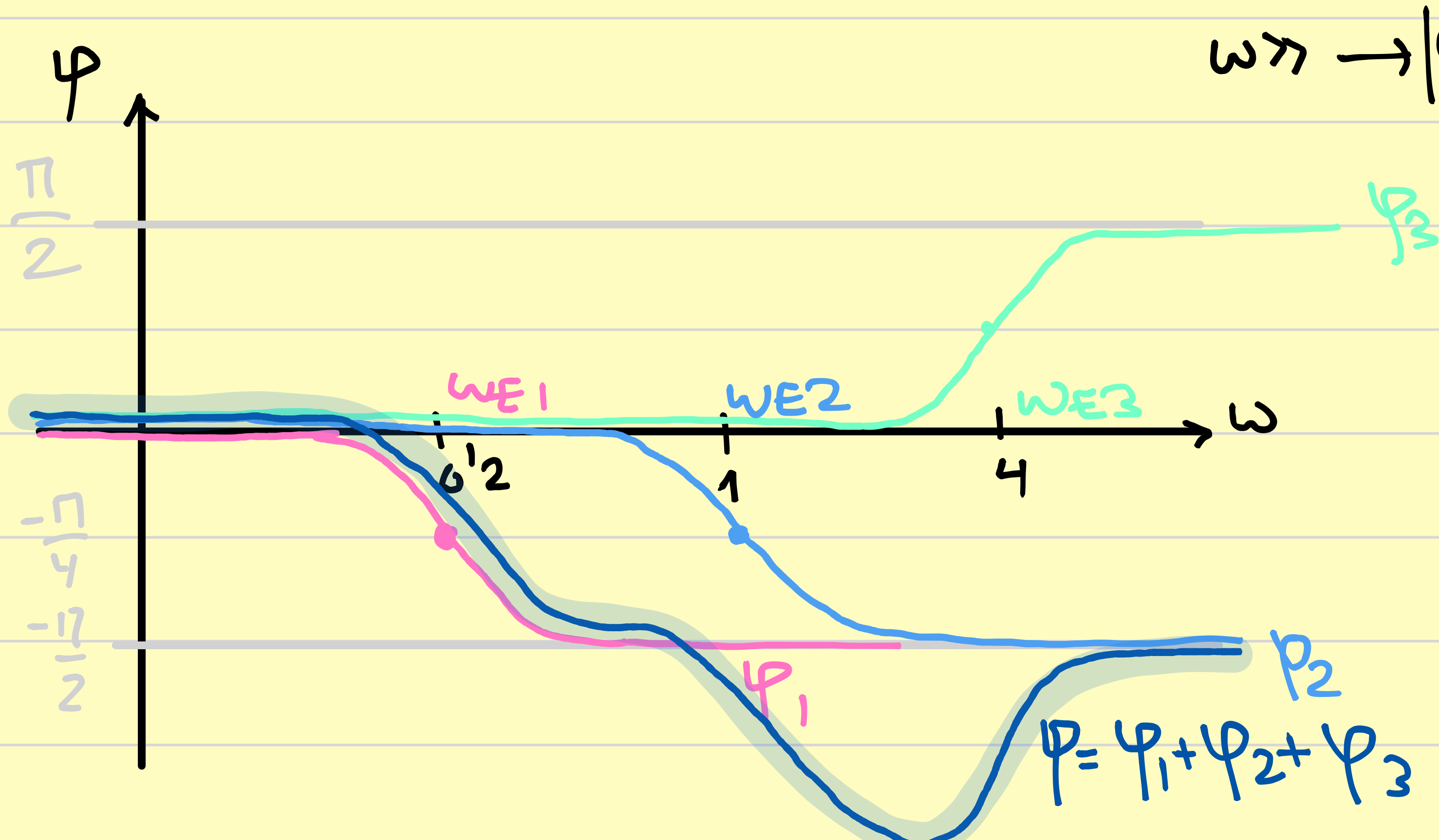
$w \ll \rightarrow |G_2(jw)| = 4 = K_{p2}$

$w \gg \rightarrow |G_2(jw)| = -\infty$

$G_3 = 8(1+0'25s) \rightarrow G_3(jw) = 8(1+0'25wj) \rightarrow |G(jw)| = 8\sqrt{1+\left(\frac{w}{4}\right)^2}$

$w \ll \rightarrow |G_3(jw)| = 8 = K_{p3}$

$w \gg \rightarrow |G_3(jw)| = \infty$



$G_1(jw) = \frac{2-10wj}{1+(5w)^2} \rightarrow \varphi_1 = \operatorname{atan}\left(\frac{-10w}{2}\right) = \operatorname{atan}(-5w)$

$w \ll \rightarrow \varphi_1 = 0$

$w \gg \rightarrow \varphi_1 = \frac{-\pi}{2}$

$G_2(jw) = \frac{4-4wj}{1+w^2} \rightarrow \varphi_2 = \operatorname{atan}(-w)$

$\omega \ll \rightarrow \varphi_2 = 0$

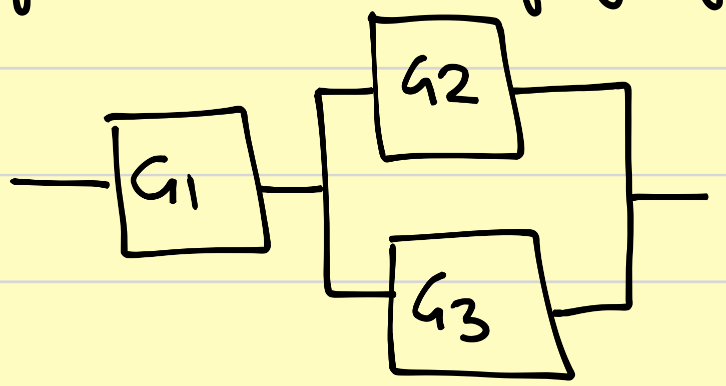
$\omega \gg \rightarrow \varphi_2 = -\frac{\pi}{2}$

$$G_3(j\omega) = 8(1+0'25\omega j) \rightarrow \varphi_3 = \operatorname{atan}(0'25\omega)$$

$\omega \ll \rightarrow \varphi_3 = 0$

$\omega \gg \rightarrow \varphi_3 = \frac{\pi}{2}$

Übung: Bitte BD von folgengen Rk·Gliedern darstellen.



$\boxed{G_1, G_2, G_3 \text{ wie oben}}$

$$G(s) = G_1 \cdot [G_2 + G_3] = \frac{K_{P1}}{1+T_1 s} \cdot \left[\frac{K_{P2}}{1+T_2 s} + K_{P3}(1+T_v s)\right] =$$

$$= \frac{2}{1+5s}\left[\frac{4}{1+s} + 8(1+0'25s)\right] = \frac{2}{1+5s}\left[\frac{4+8(1+s)(1+0'25s)}{1+s}\right]$$

$$= \frac{8+16[1+0'25s+s+0'25s^2]}{1+s} =$$

$$= \frac{24+20s+4s^2}{1+s} = 4 \cdot \frac{(s+3)(s+2)}{(s+1)}$$

$$s^* = \frac{-20 \pm \sqrt{20^2 - 4.4.24}}{2.4} = \frac{-20 \pm 4}{8} = \begin{cases} -3 \\ -2 \end{cases}$$

$$G(j\omega) = \frac{24 - 4\omega^2 + 20j\omega}{1+j\omega} \cdot \frac{1-j\omega}{1-j\omega} =$$

$$= \frac{24 - 4\omega^2 + 20\omega j - 24\omega j + 4\omega^3 j - 20\omega^2}{1+\omega^2}$$

$$= \frac{24 - 24\omega^2 + j\left[-4\omega + 4\omega^3\right]}{1+\omega^2} =$$

$$= \frac{24\left(1-\omega^2\right) - 4j\omega\left[1-\omega^2\right]}{1+\omega^2} =$$

$$= \frac{4\cdot\left(1-\omega^2\right)}{\left(1+\omega^2\right)}\left[6 - \omega j\right]$$

$$|G(j\omega)| = \frac{4\left(1-\omega^2\right)}{1+\omega^2}\sqrt{36+\omega^2}$$

$\omega \ll \quad |G(j\omega)| = 24$

$\omega \gg \quad |G(j\omega)| = +\infty$

$$\varphi = \operatorname{atan}\left[\frac{+\omega}{6}\right] \longrightarrow$$

$\omega \ll \quad \varphi = 0$

$\omega \gg \quad \varphi = \frac{+\pi}{2}$

$$G(s) = 4\,\frac{(s+3)(s+2)}{s+1} = \underbrace{\frac{4}{s+1}}_{G'_1}\cdot\underbrace{(s+3)}_{G'_2}\cdot\underbrace{(s+2)}_{G'_3}$$

$\omega'_{E1} = \frac{1}{1} = 1 \qquad \omega'_{E2} = \frac{1}{3} \qquad \omega'_{E3} = \frac{1}{2}$