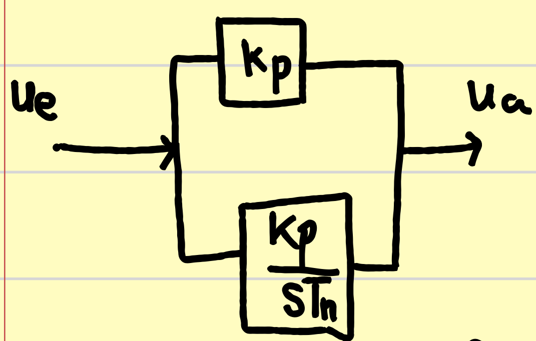
... BODE DIAGRAMME ...

5. BODE DIAGRAMM PI - Regelkreisglied

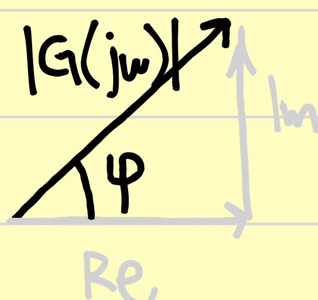
PI-Glied = Parallelschaltung von P und I Glied.



$$G(s)=\frac{u_a(s)}{u_e(s)}=K_p+\frac{K_p}{sT_n}=K_p\left(1+\frac{1}{sT_n}\right)$$

$$s=j\omega \rightarrow G(j\omega)=K_p\left(1-j\frac{1}{\omega T_n}\right)$$

$$Re(G(j\omega))=K_p \quad ; \quad Im(G(j\omega))=\frac{-K_p}{\omega T_n}$$



$$|G(j\omega)|=\sqrt{|Re(G(j\omega))|^2+|Im(G(j\omega))|^2}=$$

$$=\sqrt{K_p^2+K_p^2\left(\frac{1}{\omega T_n}\right)^2}=K_p\sqrt{1+\left(\frac{1}{\omega T_n}\right)^2} \quad \odot$$

$$\log|G(j\omega)|=\log\left[K_p\cdot\left(1+\left(\frac{1}{\omega T_n}\right)^2\right)^{1/2}\right]\overset{*}{=}$$

$$\boxed{\log(a\cdot b)=\log a+\log b}\ *$$

$$=\log K_p+\log\left[\left(1+\left(\frac{1}{\omega T_n}\right)^2\right)^{1/2}\right]\overset{**}{=}$$

$$\boxed{\log a^b=b\log a}\ **$$

$$=\log K_p+\frac{1}{2}\log\left[1+\left(\frac{1}{\omega T_n}\right)^2\right]$$

***

$$\tan\varphi(\omega) = \frac{\mathrm{Im}(G(j\omega))}{\mathrm{Re}(G(j\omega))} = \frac{\frac{-k_p}{\omega T_n}}{k_p} = \frac{-1}{\omega T_n}$$

$$\omega \to 0 \;\rightarrow\; \frac{1}{\omega T_n} \gg 1 \;\xrightarrow[***]{}\; \log|G(j\omega)| =$$

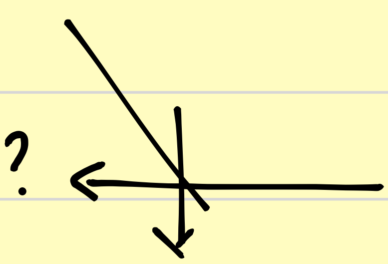
$$\cong \log k_p + \frac{1}{2}\log\left[\left(\frac{1}{\omega T_n}\right)^2\right] \underset{**}{=}$$

$$= \log k_p + \frac{1}{2}\cdot 2\log\left[(\omega T_n)^{-1}\right] =$$

$$= \log k_p - \log(\omega T_n)$$

GERADE MIT NEGATIVEN STEIGUNG

$$\omega \to \infty \;\rightarrow\; \frac{1}{\omega T_n} \ll 1 \;\xrightarrow[***]{}\; \log|G(j\omega)| = \log k_p$$

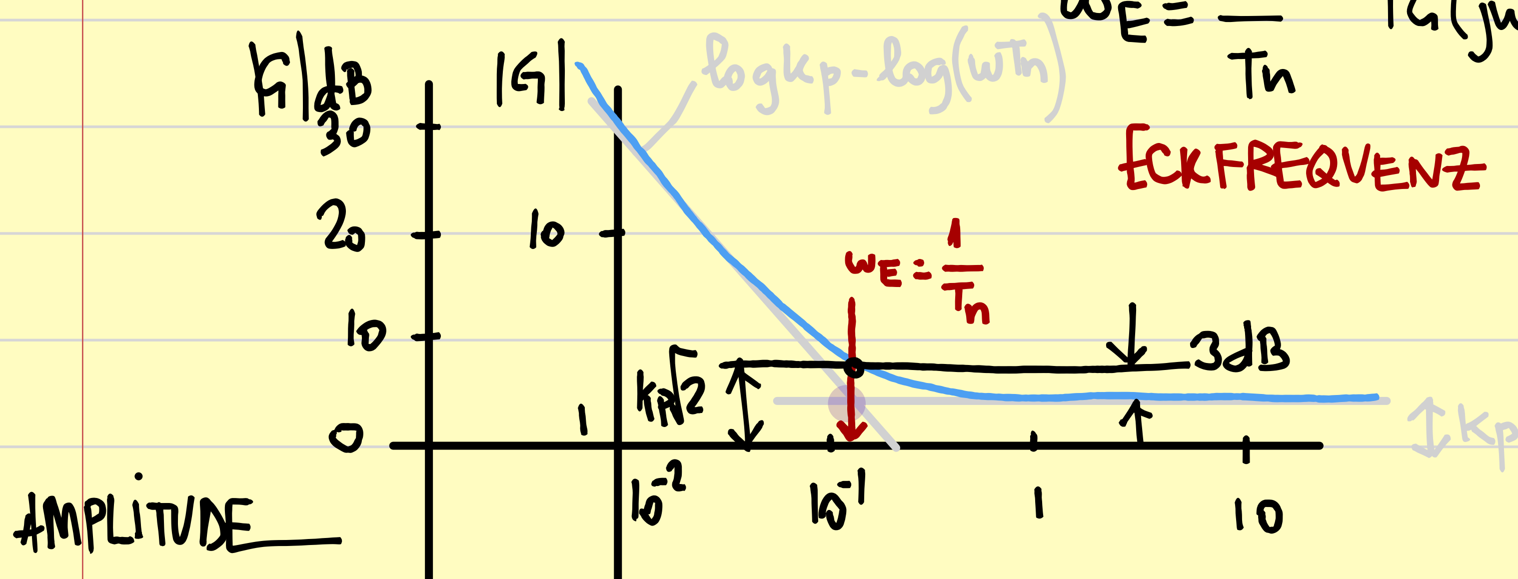


$$\omega_E = \frac{1}{T_n} \rightarrow \log k_p - \log(\omega_E T_n) = \log k_p \;\checkmark$$

$$|G(j\omega)| = k_p\sqrt{1+\left(\frac{1}{\omega T_n}\right)^2} \underset{\omega_E = \frac{1}{T_n}}{=} k_p\sqrt{1+1} = \sqrt{2}\,k_p$$

$$|G(j\omega)|_{dB} = 20\log(k_p\sqrt{2}) = 20\log k_p + 3\,dB$$

ECKFREQUENZ



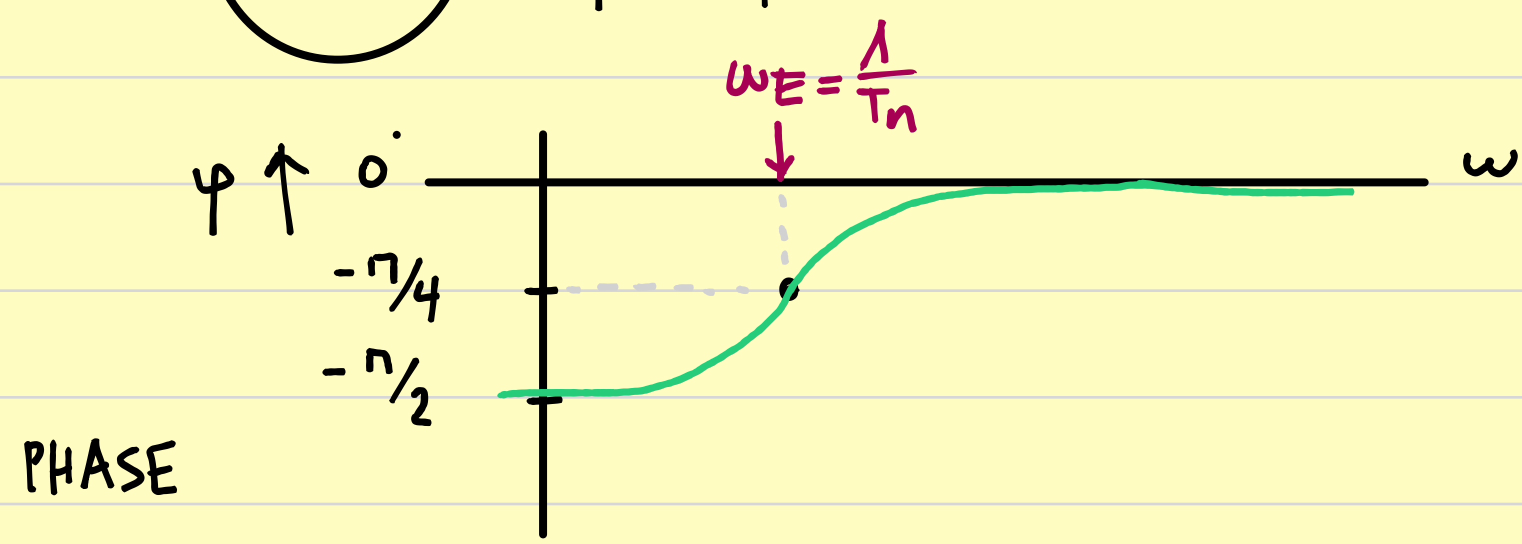
$k_p = 2$
$T_n = 5s$

$$\tan \varphi(\omega) = \frac{-1}{\omega T_n} \rightarrow \varphi = \operatorname{atan}\left(\frac{-1}{\omega T_n}\right)$$

$$\omega = 0 \rightarrow \varphi = -\frac{\pi}{2} \quad (-90^\circ)$$

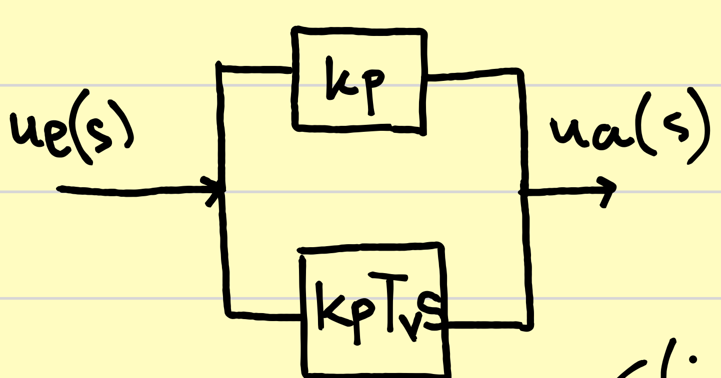
$$\omega = \infty \rightarrow \varphi = 0^\circ \quad (0^\circ)$$

$$\omega_E = \frac{1}{T_n} \rightarrow \varphi = -\frac{\pi}{4} \quad (-45^\circ)$$



6. BODE-DIAGRAMM EINES PD-GLIEDES

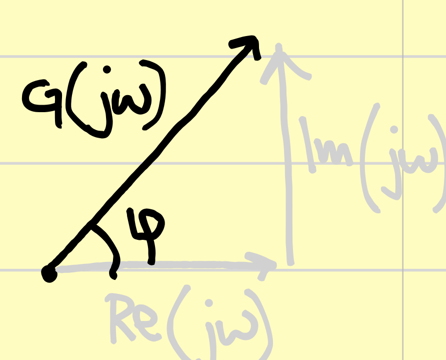
PD-GLIED ≡ PARALLELSCHALTUNG P + D GLIED



$$G(s) = \frac{u_a(s)}{u_e(s)} = K_p + K_p T_v s$$

$$G(j\omega) = k_p(1 + j\omega T_v)$$

$$\operatorname{Re} G(j\omega) = k_p \; ; \; \operatorname{Im} G(j\omega) = k_p \omega T_v$$



$$|G(j\omega)| = \sqrt{K_p^2 + (K_p\omega T_v)^2} = K_p\sqrt{1+(\omega T_v)^2} = K_p\left(1+(\omega T_v)^2\right)^{1/2} \odot$$

$$\boxed{\log(ab) = \log a + \log b} \qquad \log|G(j\omega)| = \log K_p + \log\left[\left(1+(\omega T_v)^2\right)\right]^{1/2} =$$

$$* \qquad = \log K_p + \frac{1}{2}\log\left[1+(\omega T_v)^2\right]$$

$$\tan\varphi(\omega) = \frac{\mathrm{Im}(G(j\omega))}{\mathrm{Re}(G(j\omega))} = \omega T_v \rightarrow \varphi = \operatorname{atan}(\omega T_v)$$

$$\omega \to 0 \quad \rightarrow \log|G(j\omega)| \cong \log K_p$$

$$\omega \to \infty \quad \rightarrow \log|G(j\omega)| \overset{*}{\cong} \log K_p + \log(\omega T_v)$$

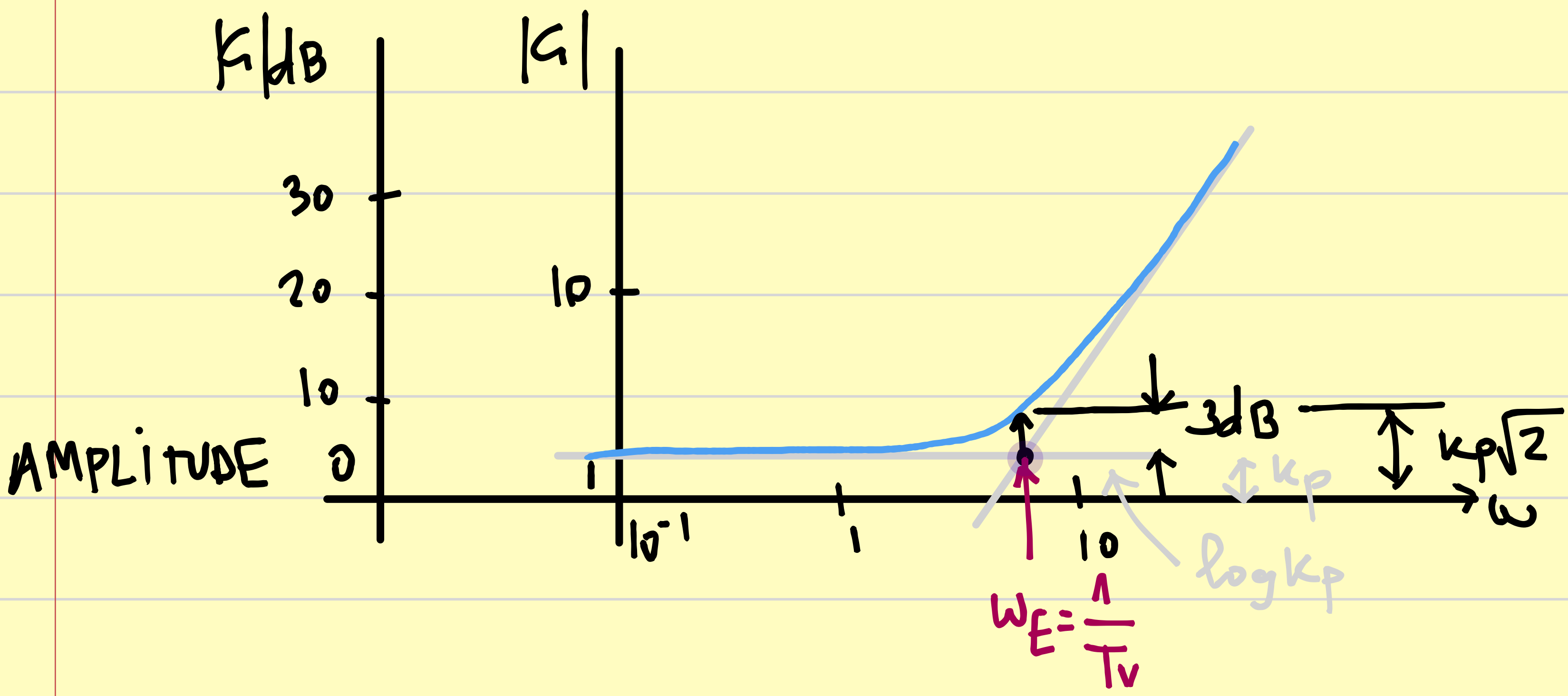
GERADE MIT POSITIVEN STEIGUNG

$$\omega_E \equiv \quad \log K_p = \log K_p + \log(\omega_E T_v) \rightarrow \omega_E = \frac{1}{T_v}$$

$$* \quad \boxed{\begin{aligned} \log|G(j\omega)| &= \log K_p + \frac{1}{2}\log\left[1+(\omega T_v)^2\right] \underset{\omega\to\infty}{=} \\ &\cong \log K_p + \frac{1}{2}\log\left[(\omega T_v)^2\right] = \\ &= \log K_p + \frac{1}{2}\cdot 2\log(\omega T_v) = \\ &= \log K_p + \log(\omega T_v) \end{aligned}}$$

$$\odot \quad |G(j\omega)|_{\omega=\omega_E} = K_p\sqrt{1+(\omega_E T_v)^2} = K_p\sqrt{2}$$

$$|G(j\omega)| \underset{\omega=\omega_E}{=} 20\cdot\log K_p + 3\,dB$$



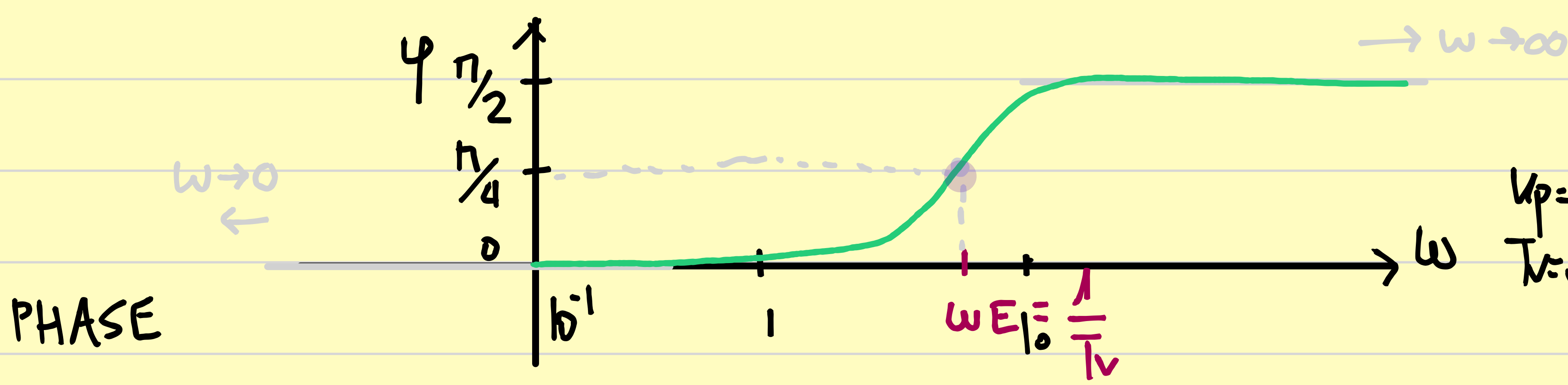
$K_p = 2$
$T_v = 0{,}2\,s$

$$\varphi = \operatorname{atan}(\omega T_v)$$

$$\omega = 0 \rightarrow \varphi = 0$$

$$\omega = \infty \rightarrow \varphi = \frac{\pi}{2} \quad (90^\circ)$$

$$\omega = \omega_E \rightarrow \varphi = \frac{\pi}{4} \quad (45^\circ)$$



$K_p = 1$
$T_v = 0{,}2\,s$

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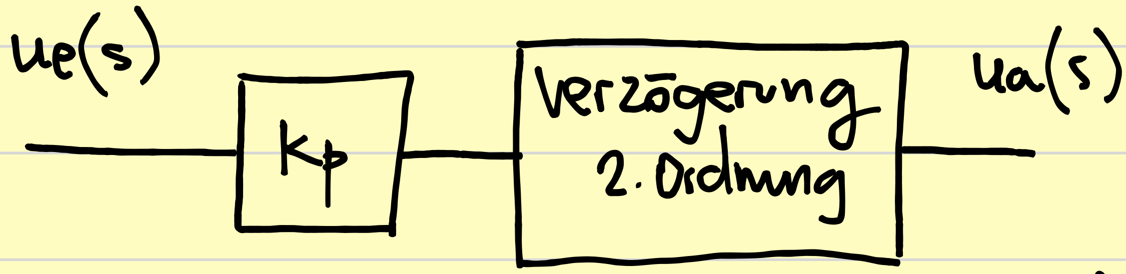
7. BODE DIAGRAMM VON EINEM PT_2-GLIED

Als PT_2-Glied bezeichnet man ein proportionales Übertragungsverhalten (P-Glied) mit einer Verzögerung 2. Ordnung.



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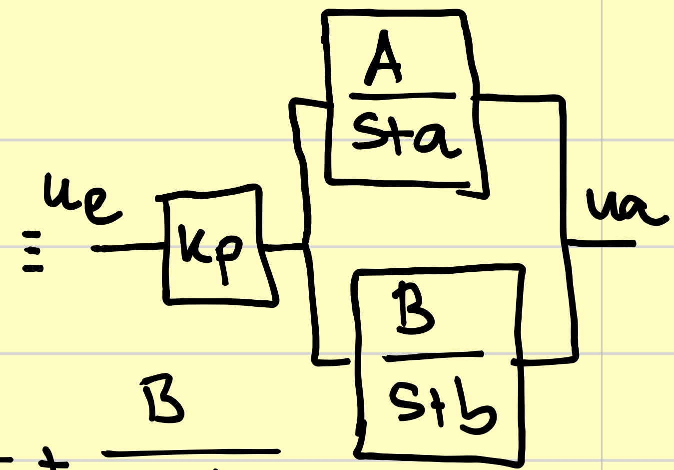


$$\frac{1}{1+sT_1+s^2T_2^2} = \frac{A}{s+a} + \frac{B}{s+b}$$

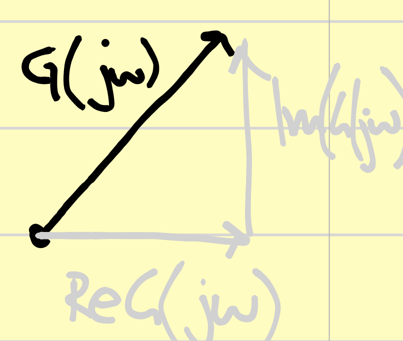
PT_2-Glied

Dämpfung = $D = \frac{T_1}{2T_2}$

$$G(s) = \frac{ua(s)}{ue(s)} = \frac{K_p}{1+sT_1+s^2T_2^2}$$

$$G(j\omega) = \frac{K_p}{1+j\omega T_1-(\omega T_2)^2} = \frac{K_p}{1-(\omega T_2)^2} + \frac{K_p}{\omega T_1} j$$

$$Re(G(j\omega)) = \frac{K_p}{1-(\omega T_2)^2} \; ; \; Im(G(j\omega)) = \frac{K_p}{\omega T_1}$$



$$|G(j\omega)| = \sqrt{Re(G(j\omega))^2 + Im(G(j\omega))^2} = \frac{K_p}{\sqrt{(1-(\omega T_2)^2)^2 + (\omega T_1)^2}}$$

$$\log|G(j\omega)| = \log K_p - \frac{1}{2}\log\left[\left[1-(\omega T_2)^2\right]^2 + (\omega T_1)^2\right]$$

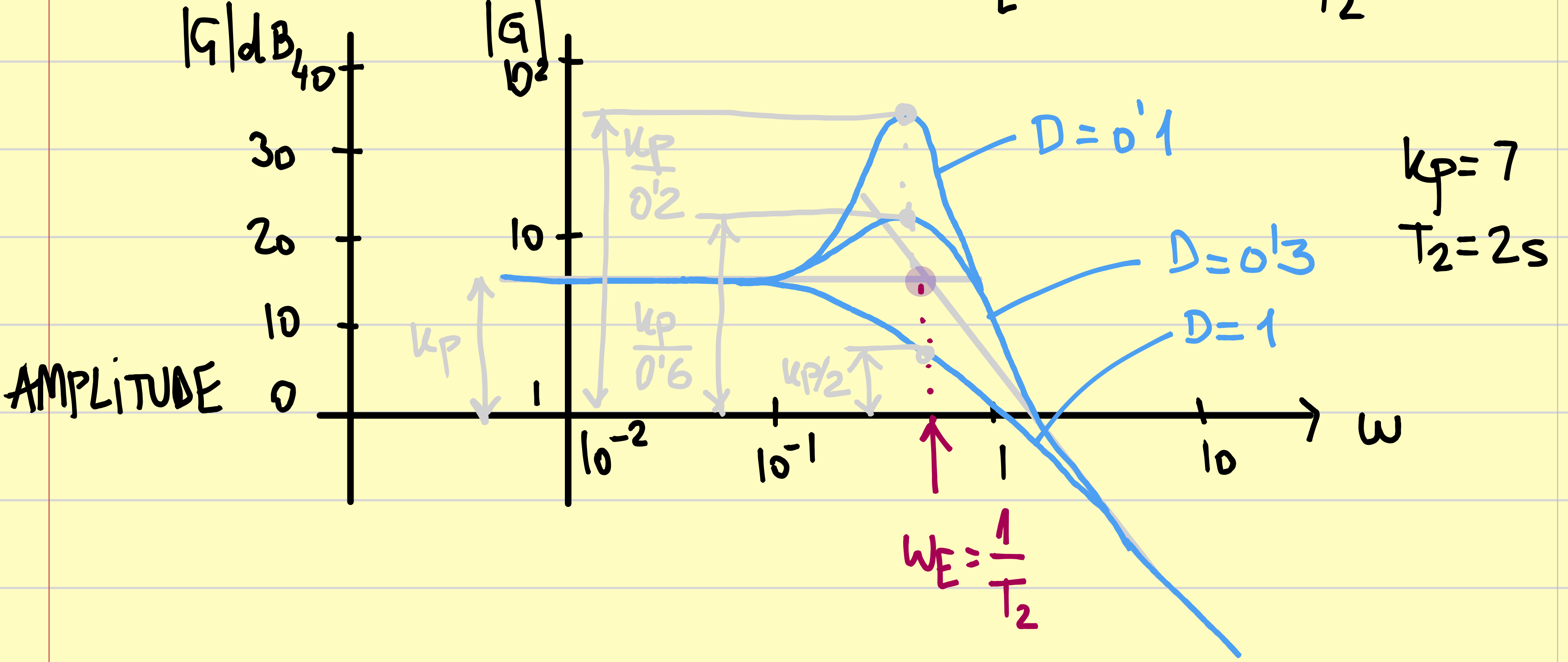
$\omega \to 0 \to \log|G(j\omega)| \cong \log K_p$ GERADE

$\omega \to \infty \to (\omega T_2)^2 \gg 1$ und $(\omega T_2)^2 \gg \omega T_1$

$$\log|G(j\omega)| \cong \log K_p - \frac{1}{2}\log\left[\left((\omega T_2)^2\right)^2\right] =$$

$$= \log k_p - 2\log(\omega T_2)$$ GERADE MIT NEGATIVEN STEIGUNG

$$\omega_E \to \log k_p = \log k_p - 2\log(\omega_E T_2) \to \omega_E = \frac{1}{T_2}$$



$$|G(j\omega)|_{\omega=\omega_E} = \frac{k_p}{\sqrt{\left(1-(\omega_E T_2)^2\right)^2 + (\omega_E T_1)^2}} = \frac{k_p \cdot T_2}{T_1} = \frac{k_p}{2D}$$

$$\omega_E = \frac{1}{T_2}$$

$D = 1 \to |G(j\omega_E)| = \frac{k_p}{2}$

$D = 0'3 \to |G(j\omega_E)| = \frac{k_p}{0'6}$

$D = 0'1 \to |G(j\omega_E)| = \frac{k_p}{0'2}$

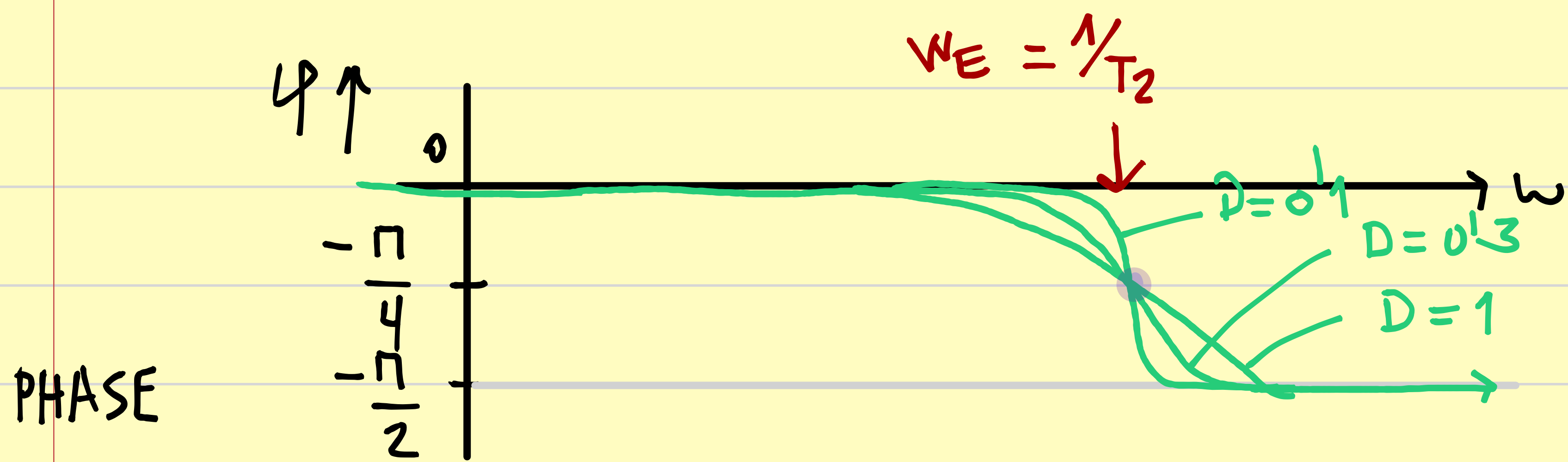
Wenn $D > 1$ ist, so lässt sich das PT_2-Glied in zwei PT_1-Glieder in Reihenschaltung zerlegen.

$$\tan\varphi = \frac{\operatorname{Im} G(j\omega)}{\operatorname{Re} G(j\omega)} = \frac{-\omega T_1}{1-(\omega T_2)^2} \rightarrow \varphi = -\operatorname{atan} \frac{\omega T_1}{1-(\omega T_2)^2}$$

$\omega \to 0 \;:\; \varphi = 0$

$\omega \to \infty \;:\; \varphi = \frac{-\pi}{2}$

$\omega = \omega_E = \frac{1}{T_2} \;:\; \varphi = \frac{-\pi}{4}$



DARSTELLUNG IN REIHE GESCHALTETER GLIEDER IN BODE-DIAGRAMM

Sind „n" Glieder mit Frequenzgängen $G_1(j\omega), G_2(j\omega), \dots, G_n(j\omega)$ in Reihe geschaltet, so ist der Gesamtfrequezgang gleich dem Produkt der einzelnen Frequenzgänge.

$-\boxed{G_1}-\boxed{G_2}- \dots -\boxed{G_n}-$

$$G(s) = \prod_{i=1}^{n} G_i(s)$$

$$G(j\omega) = \prod_{i=1}^{n} G_i(j\omega) \quad (*)$$

Zur Darstellung im Bode-Diagramm wird $G(j\omega)$ zerlegt:

$$G(j\omega)=|G(j\omega)|\cdot e^{j\varphi(\omega)}$$

Angewand (*) wird:

$$G(j\omega) = |G_1(j\omega)|\, e^{j\varphi_1(\omega)}\, |G_2(j\omega)|\, e^{j\varphi_2(\omega)} \cdots G_n(j\omega)\, e^{j\varphi_n(\omega)} =$$

$$= |G_1(j\omega)|\,|G_2(j\omega)|\cdots|G_n(j\omega)|\, e^{j(\varphi_1+\varphi_2+\dots+\varphi_n)}$$

Es folgt dann:

$$|G(j\omega)| = |G_1(j\omega)|\,|G_2(j\omega)|\cdots|G_n(j\omega)|$$

$$\varphi(\omega) = \varphi_1(\omega)+\varphi_2(\omega)+\dots+\varphi_n(\omega) \quad (***)$$

Infolge der logaritmischen Darstellung des Amplitudganges:

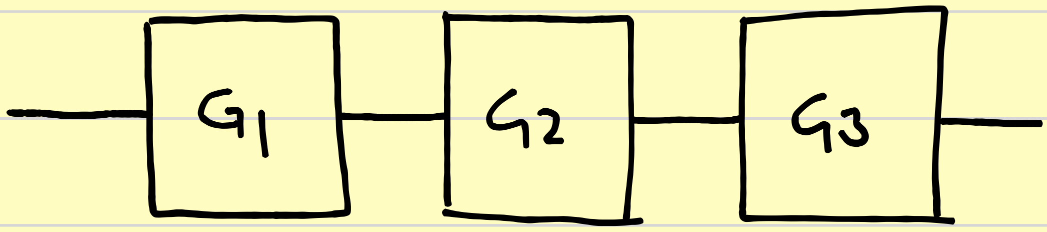
$$\log|G(j\omega)| = \log|G_1(j\omega)|+\log|G_2(j\omega)|+\dots+\log|G_n(j\omega)|$$

$$|G(j\omega)|_{dB} = \sum_{i=1}^{n} |G_i(j\omega)|_{dB} \quad (**)$$

Der Amplitudengang des Gesamtfrequenzganges $|G(j\omega)|_{dB}$ ergibt sich durch einfache Addition der einzelnen Ordinaten der Amplitudengänge $|G_i(j\omega)|_{dB}$ (**).

Das gleiche gilt auch für die Asymptoten (***)

Beispiel. Bitte Bode-Diagramm von folgenden Gliedern darstellen



$$G_1(s) = \frac{k_{p1}}{1+sT_1} \qquad k_{p1} = 2 \qquad T_1 = 5\,s$$

$$G_2(s) = \frac{k_{p2}}{1+sT_2} \qquad k_{p2} = 4 \qquad T_2 = 1\,s$$

$$G_3(s) = k_{p3}(1+sT_v) \qquad k_{p3} = 8 \qquad T_v = 0'25\,s$$

$$G(s) = G_1 \cdot G_2 \cdot G_3 = \frac{64(1+0'25s)}{(1+5s)(1+s)} = \frac{A}{(1+5s)} + \frac{B}{(1+s)}$$

$$64 + 16s = A(1+s) + B(1+5s)$$

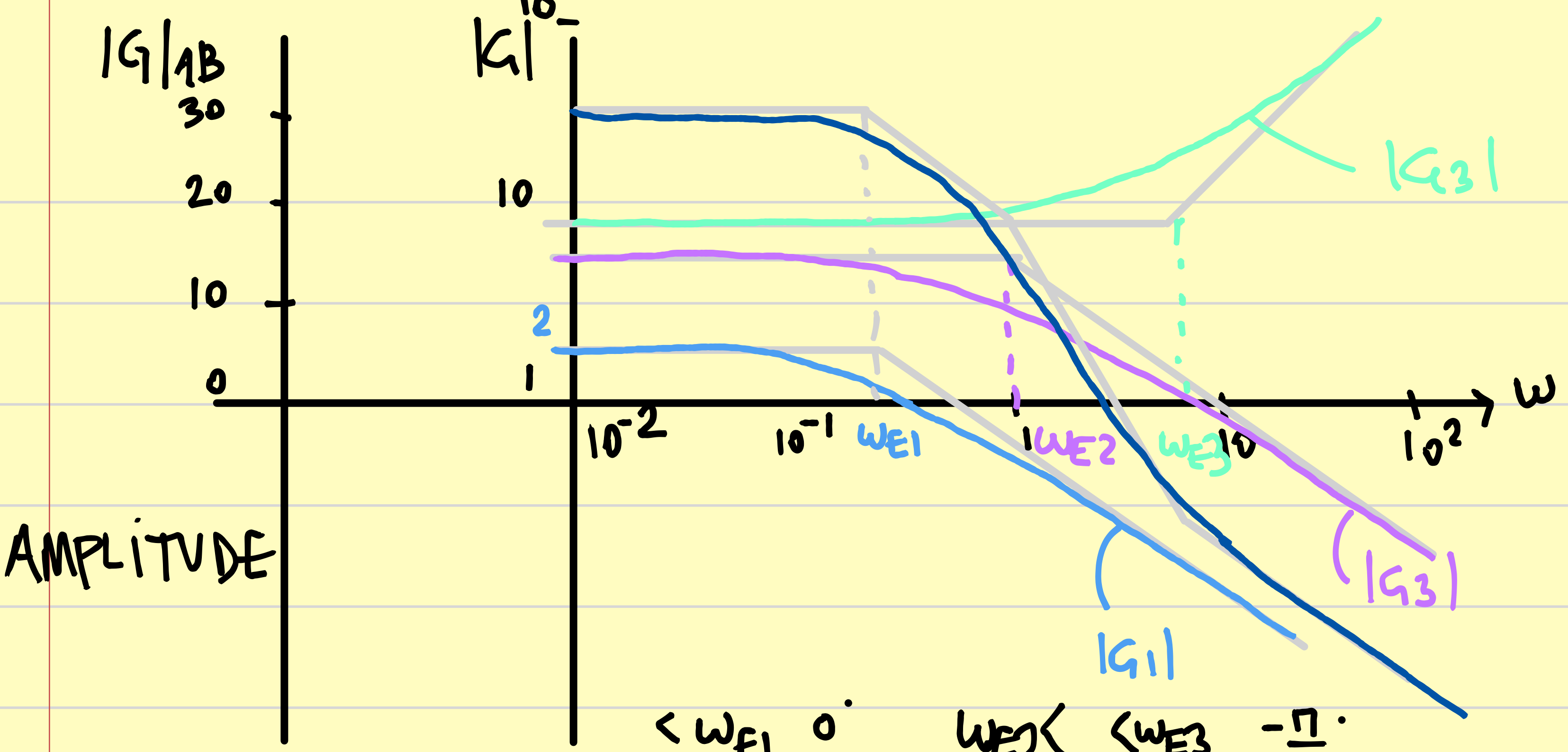
$$s = -1 \rightarrow 48 = 4B \rightarrow B = 12 \; ; \; s = -0'2 \rightarrow 60'8 = 0'8A \rightarrow A = 76$$

$$G(s) = \frac{76}{1+5s} + \frac{12}{1+s} \equiv \dots$$

bereits bekannt

Zunächst werden die Asymptoten der einzelnen Amplitudengänge gezeichnet (siehe oben):

$$\omega_{E1} = \frac{1}{T_1} = 0'2\,s^{-1} \; ; \; \omega_{E2} = \frac{1}{T_2} = 1\,s^{-1} \; ; \; \omega_{E3} = \frac{1}{T_V} = 4\,s^{-1}$$



$< \omega_{E1}$ $0°$ $\qquad \omega_{E2} < \quad < \omega_{E3}$ $-\frac{\pi}{2}$°

$\omega_{E1} < \quad < \omega_2$ $-\frac{\pi}{4}$° $\qquad > \omega_{E3}$ $\qquad -\frac{\pi}{4}$°

$$G_1(s) = \frac{2}{1+5s}$$

$$G(j\omega) = \frac{2}{1+5\omega j} \cdot \frac{1-5\omega j}{1-5\omega j} = \frac{2-10\omega j}{1+(5\omega)^2} \rightarrow |G(j\omega)| = \frac{1}{1+(5\omega)^2}\sqrt{4-(10\omega)^2}$$

$\omega \to 0 : |G(j\omega)| \simeq 2$

$\omega \to \infty : |G(j\omega)| = -\infty$