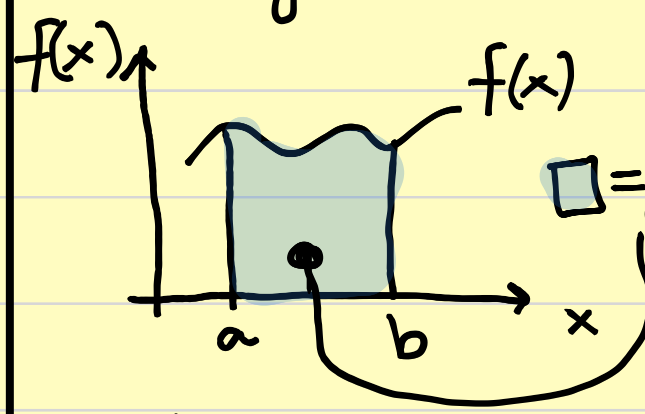
Wahrscheinlichkeit (W)

W. Funktionen

Grundlage



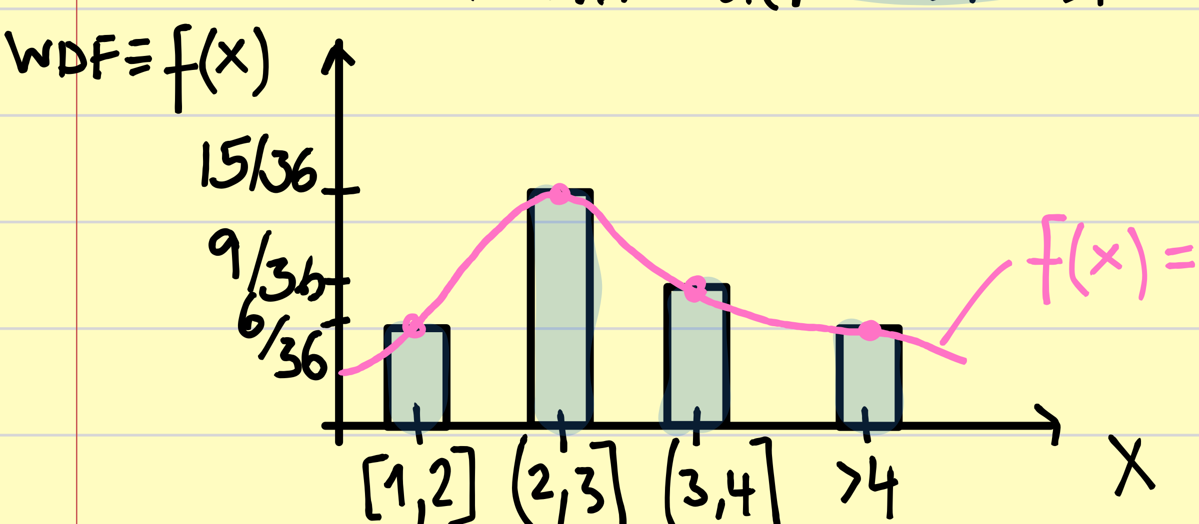
$\square = \int_a^b f(x)\,dx$

W. Dichtefunktion (WDF)

Beispiel.	X (Noten)	Häufigkeit vom Interval (*)
>4.0	6	6/36
(3,4]	9	9/36
(2,3]	15	15/36
[1,2]	6	6/36
	$\sum = 36$	$\sum = 1$

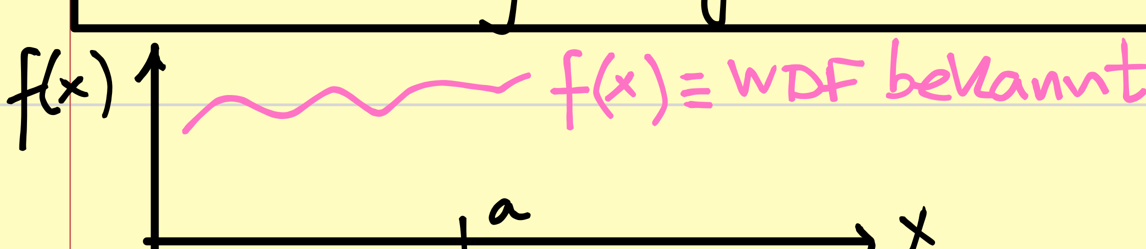
Interval (*)

- Die Häufigkeit vom Interval ergibt die WDF $\equiv f(x)$
- Die Fläche unter der WDF ist immer 1.



$\int_{-\infty}^{\infty} f(x)\,dx = \sum \text{Häufigkeiten} = 1$

W-Rechnung, angenommen die WDF ist bekannt.

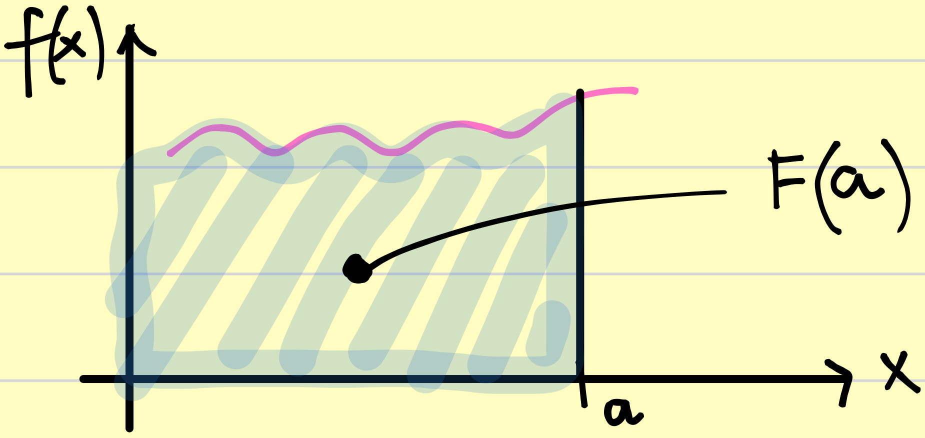


a) Was ist die W dafür, dass X kleiner/gleich „a“ ist?

Antwort: $P(X \leq a) = F(a) = \int_{-\infty}^{a} f(x)\,dx \equiv$ W. dafür, dass $X \leq a$ ist

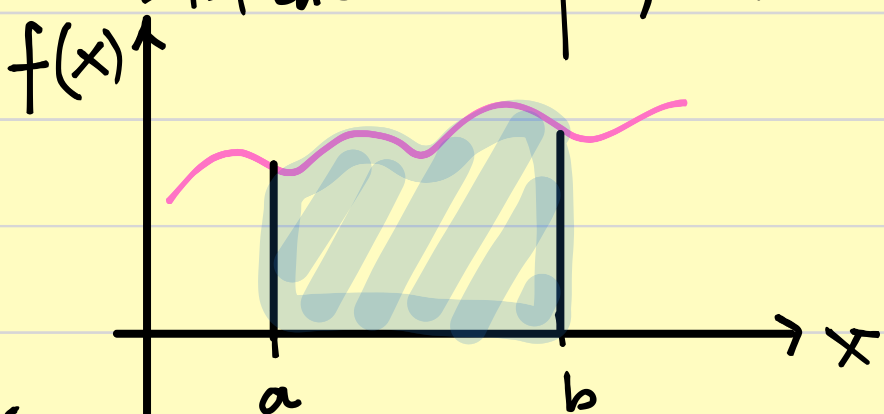
P kommt von „Probability“

Kummulative Funktion



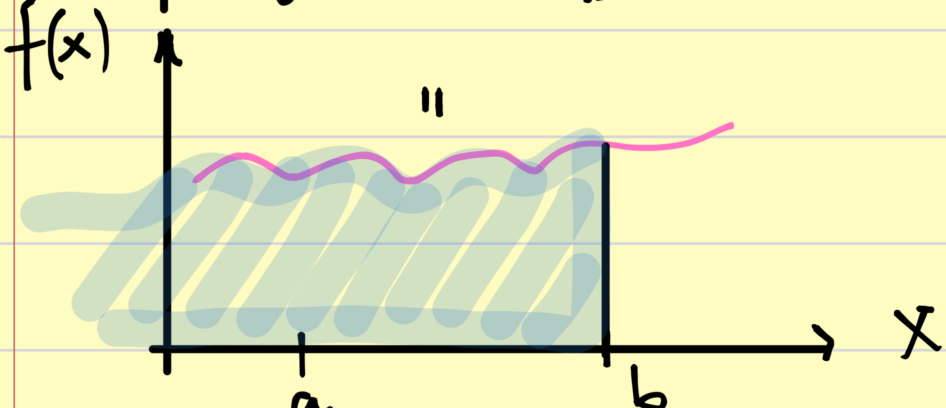
$$F(a) = P(X \leq a)$$

b) Was ist die W. dafür, dass X kleiner/gleich „b“ UND größer/gleich „a“ ist?



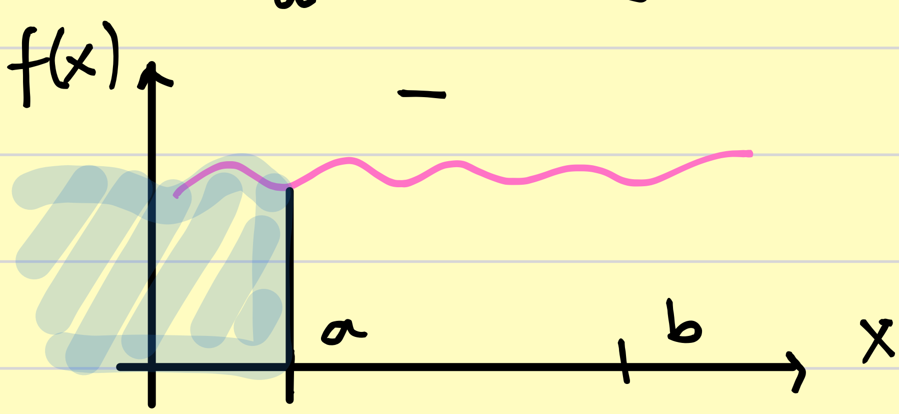
$$P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx$$

=



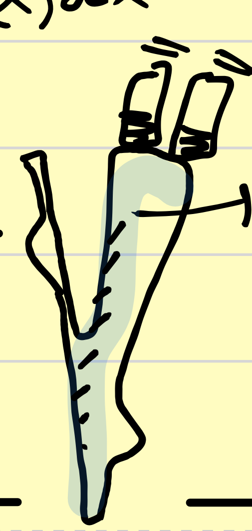
$$P(X \leq b) = \int_{-\infty}^{b} f(x)\,dx$$

−

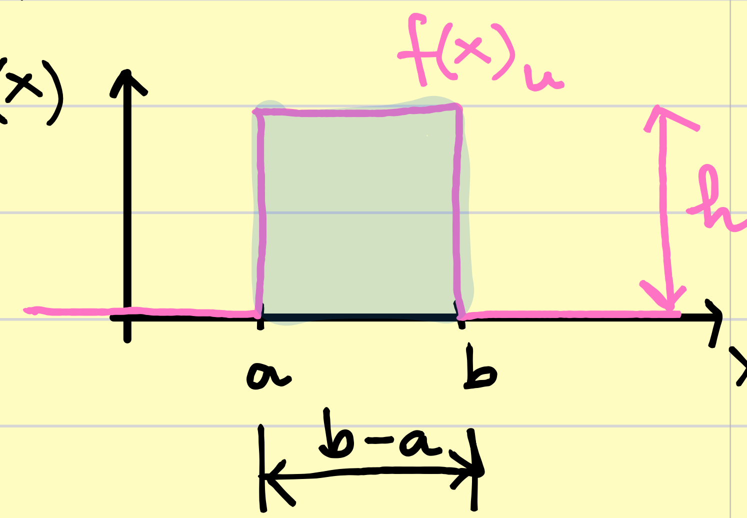


$$P(X \leq a) = \int_{-\infty}^{a} f(x)\,dx$$

$$\boxed{P(a \leq X \leq b) = P(X \leq b) - P(X \leq a)}$$



1. UNIFORM VERTEILUNG / WDF



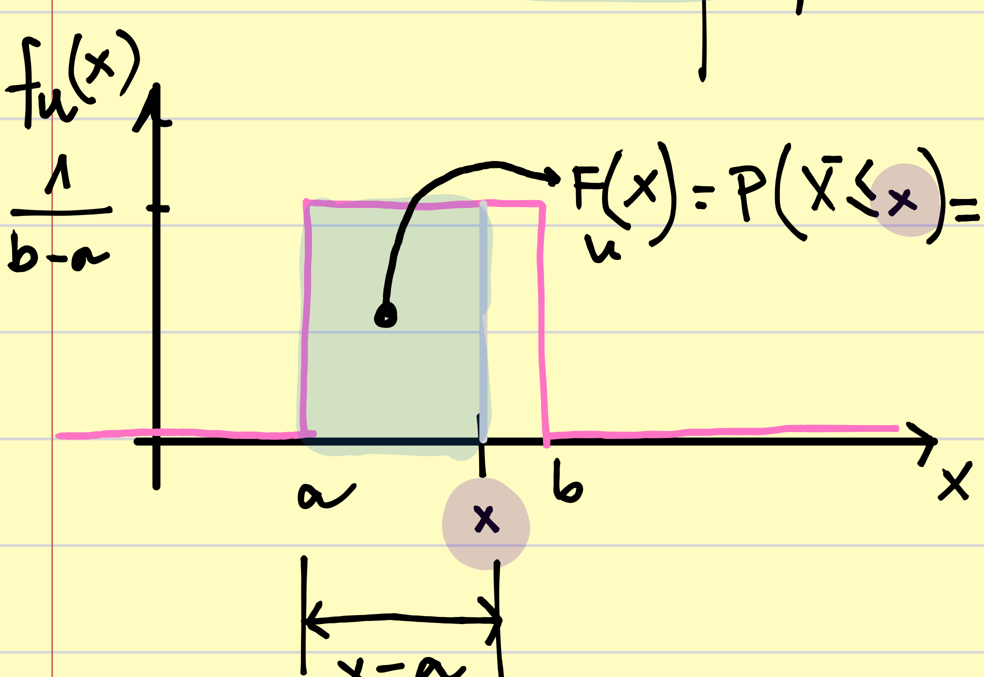
Fläche □ $= 1 = h \cdot (b-a) \rightarrow$

$\rightarrow h = \frac{1}{b-a}$

$$\boxed{f_u(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{sonst} \end{cases}}$$

$$\boxed{M_1 = \frac{a+b}{2} \qquad \sqrt{M_2} = \frac{b-a}{\sqrt{12}}}$$

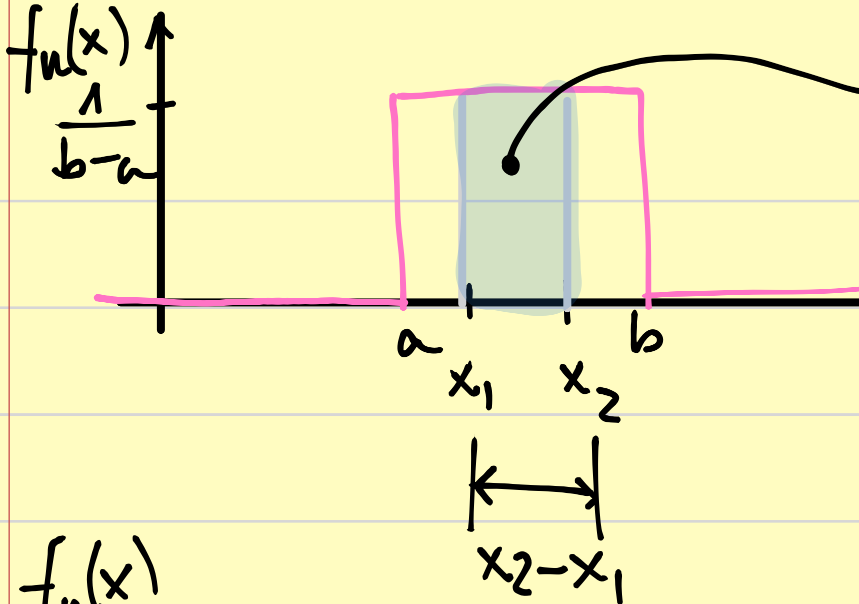
Was ist die W. dafür, dass die Variable $X \le x$ ist?



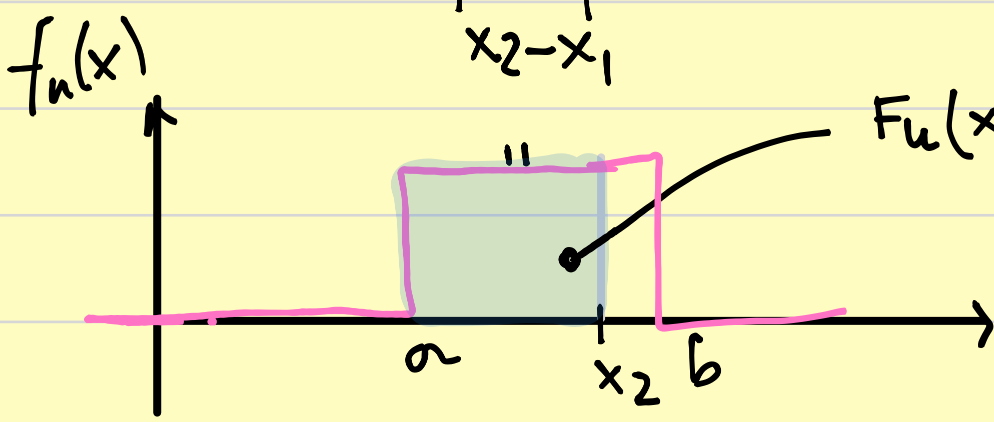
$$F_u(x) = P(X \le x) = \int_{-\infty}^{x} f_u(x)\,dx = \int_{a}^{x} f_u(x)\,dx = (x-a)\cdot\frac{1}{b-a}$$

$$\boxed{F_u(x) = \frac{x-a}{b-a}}$$

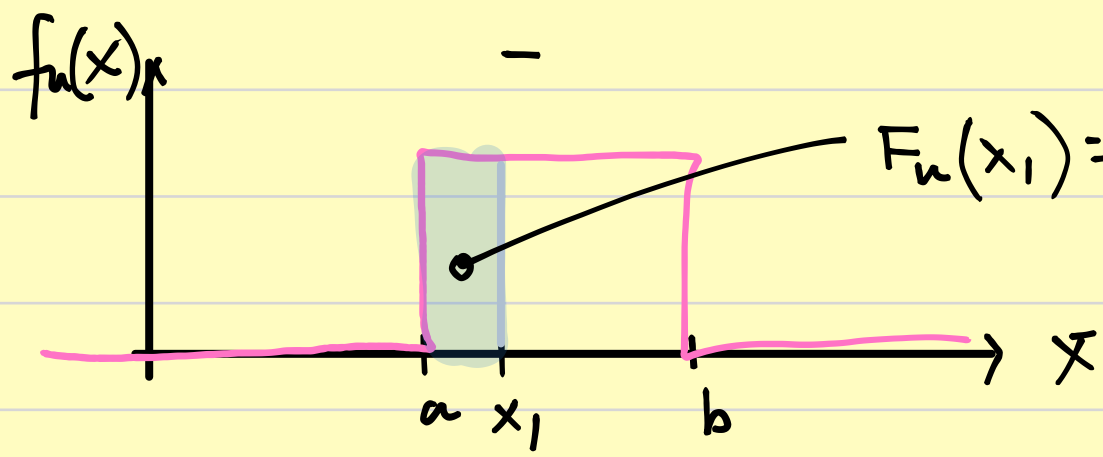
Was ist die W. dafür, dass die Variable $X \le x_2$ UND $X \ge x_1$ ist?



$$\boxed{P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b-a}}$$



$$F_u(x_2) = P(X \leq x_2) = \frac{x_2 - a}{b-a}$$



$$F_u(x_1) = P(X \leq x_1) = \frac{x_1 - a}{b-a}$$

$$P(x_1 \leq X \leq x_2) = P(X \leq x_2) - P(X \leq x_1)$$

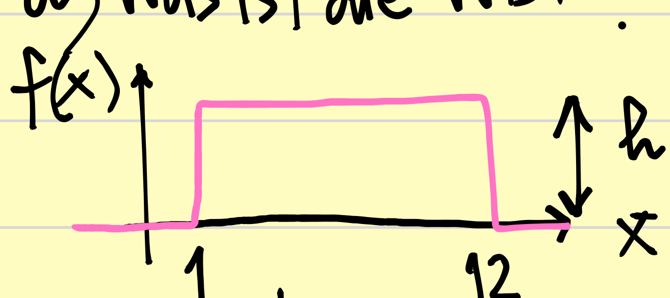
Übung. Uniformverteilung.

Die Wartezeit an einer Bushaltestelle ist UNIFORM verteilt:

$X \sim U[1, 12]$ Minuten.

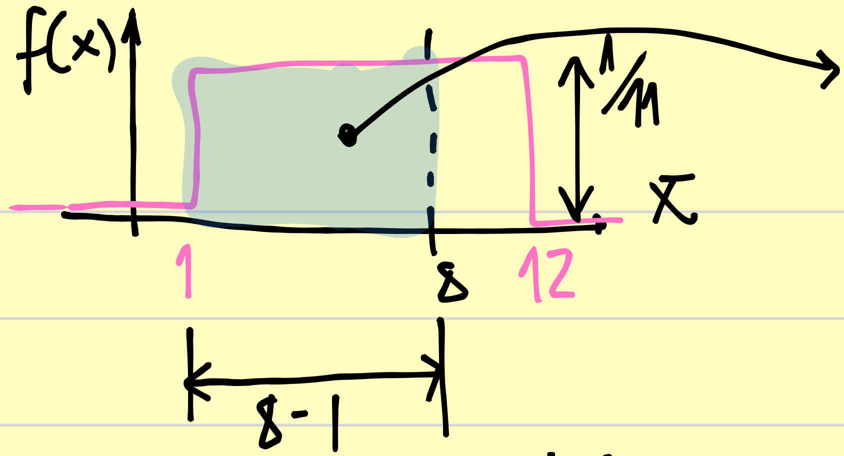
Wartezeit

a) Was ist die WDF?



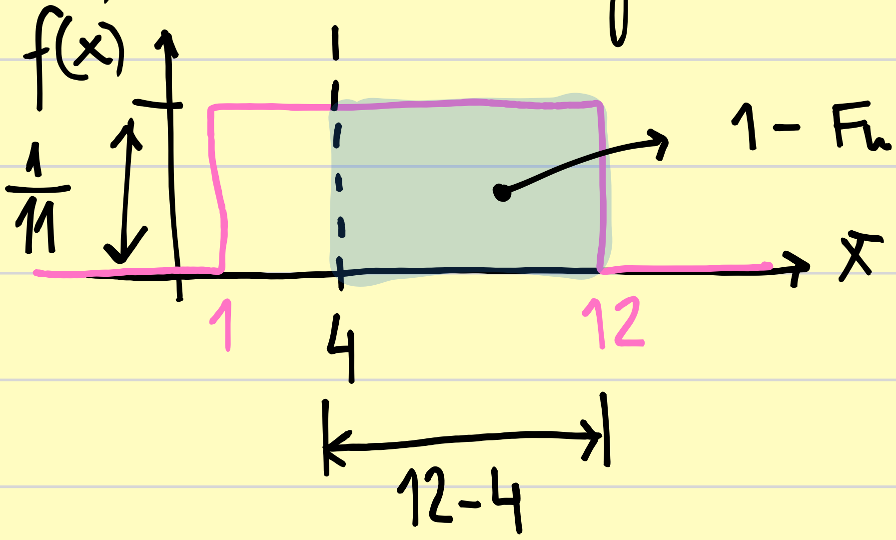
$$h = \frac{1}{12-1} = \frac{1}{11} \longrightarrow f(x) = \begin{cases} \frac{1}{11} & 1 \leq x \leq 12 \\ 0 & \text{sonst} \end{cases}$$

b) Was ist die W. dafür, dass die Wartezeit $X \leq 8$ Minuten ist?



$$F_u(8) = P(X \le 8) = (8-1)\cdot\frac{1}{11} = \frac{7}{11}$$

c) Was ist die W. dafür, dass die Wartezeit $X > 4$ Minuten ist?



$$1 - F_u(4) = P(X \ge 4) = (12-4)\cdot\frac{1}{11} = \frac{8}{11}$$

d) Was ist M_1 & $\sqrt{M_2}$?

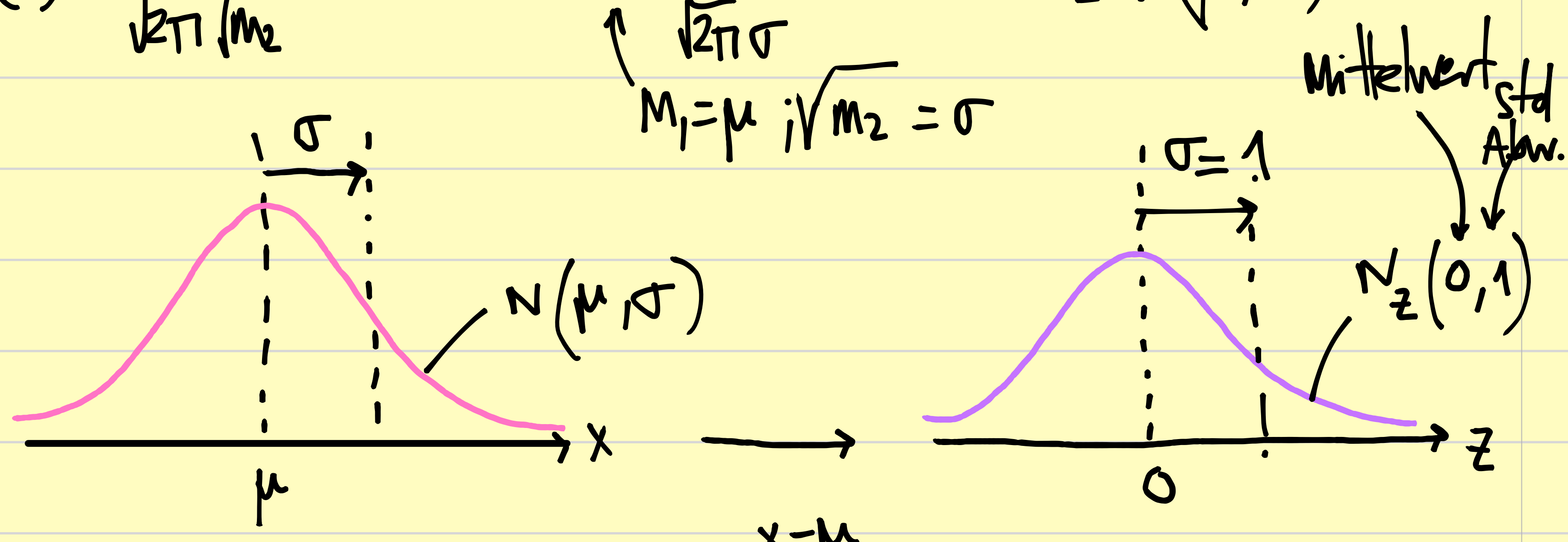
$$M_1 = \frac{a+b}{2} = \frac{12+1}{2} = \frac{13}{2} \qquad \sqrt{M_2} = \frac{b-a}{\sqrt{12}} = \frac{12-1}{\sqrt{12}} = \frac{11}{\sqrt{12}}$$

---

2. NORMALVERTEILUNG / WDF

$$f(x) = \frac{1}{\sqrt{2\pi}\sqrt{M_2}}\, e^{-\frac{(x-M_1)^2}{2M_2}} = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} = N(\mu,\sigma)$$

$M_1 = \mu \,;\, \sqrt{M_2} = \sigma$



$$z = \frac{x-\mu}{\sigma}$$

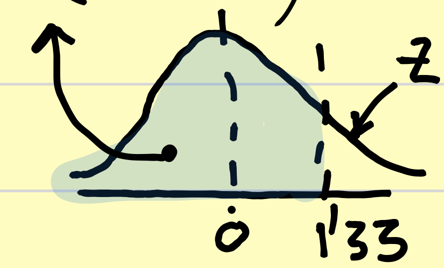
Verteilungstabellen

Standardnormalverteilung

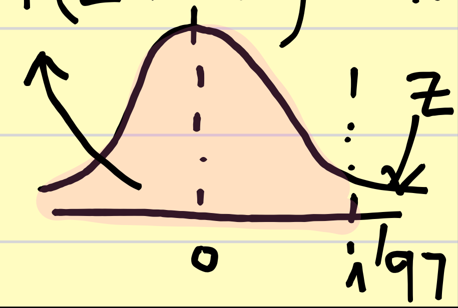
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

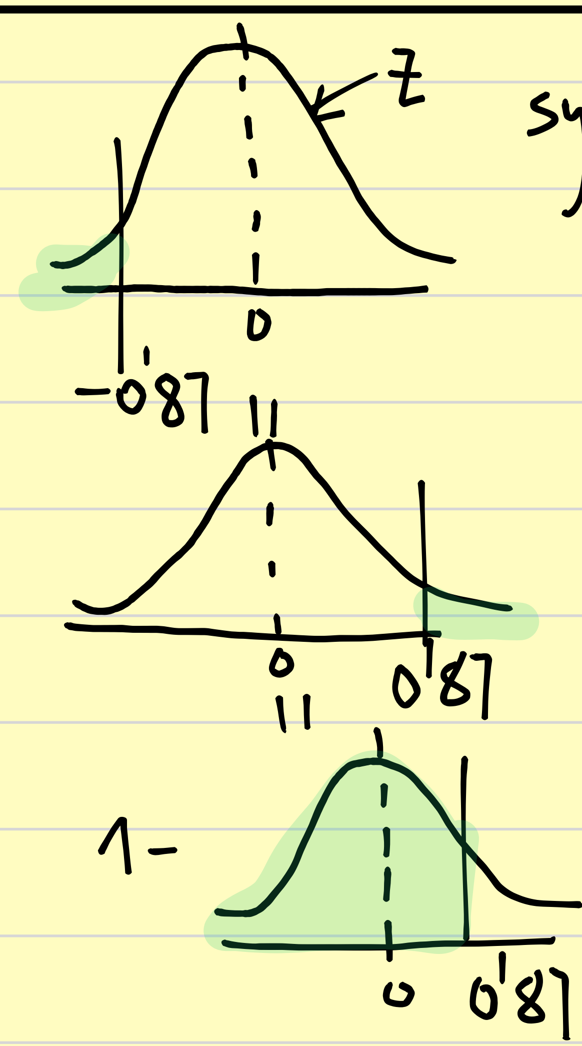
wird in die Prüfung ggf. zur Verfügung gestellt.

$P(Z \le 1'33) = 0'9082$



$P(Z \le 1'97) = 0'9756$





$P(Z \le -0'87) =$

$= P(Z \ge 0'87)$

symmetrisch

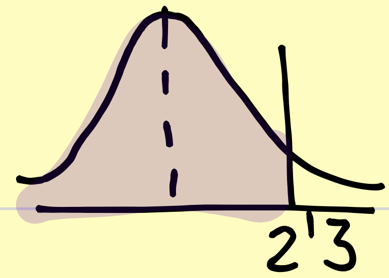
$= 1 - P(Z \le 0'87)$

$= 1 - 0'8078 =$

$= 0'1922$

Beispiele:

1) $P(Z \leq 2'3) = 0'9893$



2) $P(Z \leq -1) = P(Z \geq 1) = 1 - P(Z \leq 1) =$

$= 1 - 0'8413 = 0'1587$

3) $P(Z > -2'7) = P(Z \leq 2'7) = 0'9965$

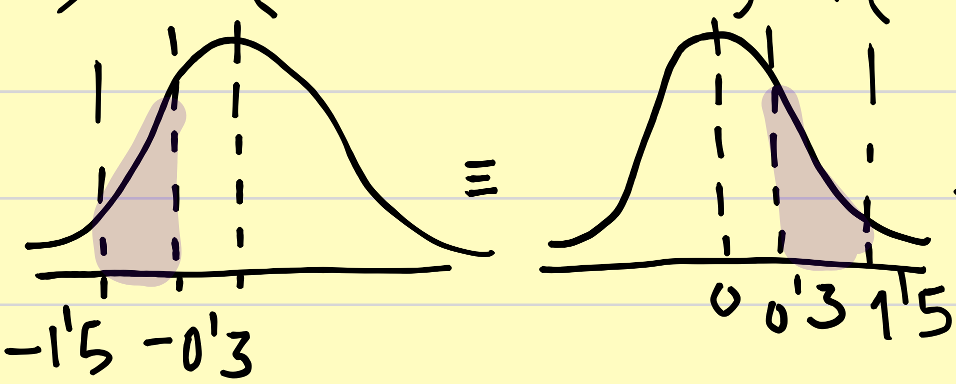
4) $P(1 \leq Z \leq 2) = P(Z \leq 2) - P(Z \leq 1) =$

$= 0'9772 - 0'8413 = 0'1359$

5) $P(-1'5 \leq Z \leq -0'3) = P(0'3 \leq Z \leq 1'5) =$

$= P(Z \leq 1'5) - P(Z < 0'3)$

$= 0'9332 - 0'6179$



Übung der Normierung:

Eine Variable X ist Normalverteilt $\sim N(\underset{M_1}{175}, \underset{\sqrt{m_2}}{8})$.

W. dafür, dass $X \geq 183$?

$$P(X \geq 183) = 1 - P(X \leq 183) = 1 - P\left(\frac{X-175}{8} \leq \frac{183-175}{8}\right) =$$

$$= 1 - P(Z \leq 1) = 1 - 0'8413 = 15'87\%$$

$$Z = \frac{X - M_1}{\sqrt{m_2}}$$

3. BINOMIALVERTEILUNG / WDF

3 Konditionen:
a) das Experiment unterliegt „N" UNABHÄNGIGE Versuche.
b) Jeder Versuch hat NUR ZWEI Ausgänge (Erfolg/Miserfolg)
c) Die W. vom Erfolg „p" ist konstant.

$$\boxed{P(X \leq x) = \binom{n}{x} p^{x} (1-p)^{n-x}}$$

n = Anzahl Versuche
x = Anzahl Erfolge
p = W. Erfolg

$$\boxed{\begin{aligned} M_1 &= n \cdot p \\ \sqrt{m_2} &= \sqrt{np(1-p)} \end{aligned}}$$

Notiz: $\binom{n}{x} = \frac{n!}{x!(n-x)!} = \frac{n(n-1)\cdot(n-2)\cdot\ldots\cdot 2\cdot 1}{[x(x-1)(x-2)\ldots 2\cdot 1][(n-x-1)(n-x-2)\ldots 2\cdot 1]}$

Beispiel: Die W. dafür, dass ein Techniker das Projekt erfolgreich beendet ist 80%. Wenn eine Variabel X die Anzahl Techniker beschreibt, welcher aus einer Gruppe von 10 Techniker das Projekt erfolgreich beendet, ermitteln Sie M_1 & $\sqrt{m_2}$ eines erfolgreichen Projektabschlusses.

Konditionen: a) $N=10$ ✓; b) Erfolg/Miserfolg ✓
c) $p=0'8$ ist konstant ✓

$$M_1 = n \cdot p = 10 \cdot 0'8 = 8$$

$$\sqrt{M_2} = \sqrt{n \cdot p \cdot (1-p)} = \sqrt{10 \cdot 0'8 (1-0'8)} = 1'26$$

NOTIZ: Wenn „n“ sehr groß ist, tendiert die Binomialverteilung zu einer Normalverteilung.