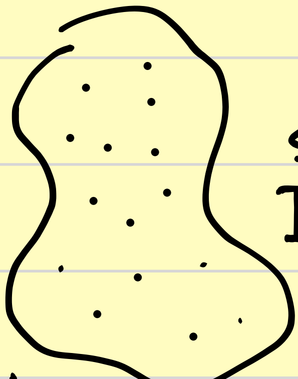
20250515_WIN6_Business_Intelligence

Dynamic Systems

1D & 2D Systems.

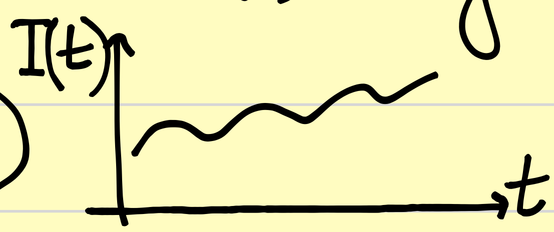
$\boxed{\text{SEX \& MATH}}$

· 1D. Transmission of sexual disseases (Chlamydia)



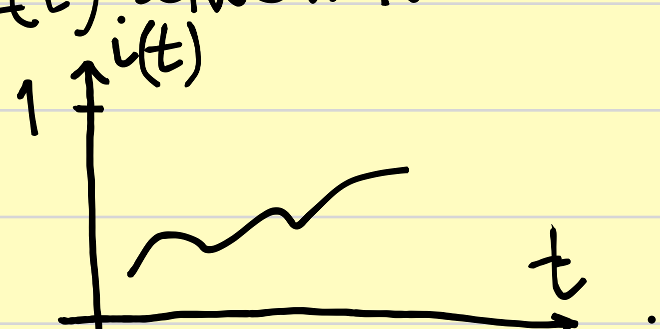
N

At the time point "t" the number of infected students is given by the function $I(t)$.



Because the population is constant (N individuals), we can normalize $I(t)$ between 0 and 1. We do so by dividing $I(t)$ between N.

$$i(t) = \frac{I(t)}{N} \in [0,1]$$



We would like to understand the dynamics of $i(t)$.

For this, we define two processes:

1) Infection. Every infected person can infect other individuals with Rate "β". The frequency, that an infected individual meets a healthy one is $i \cdot (1-i)$.

$$\boxed{\text{INFECTION RATE} = \beta \cdot i \cdot (1-i)}$$

2) Curation. Every infected person can get cured with rate γ. As a result:

$$\boxed{\text{CURATION RATE} = \gamma \cdot i}$$

The dynamic system describes $i(t)$ when time changes:

$$\frac{di(t)}{dt} = \text{Infection rate} - \text{Curation rate}.$$

$$\frac{di(t)}{dt} = \beta \cdot i \cdot (1-i) - \gamma \cdot i = \beta \cdot i - \beta \cdot i^2 - \gamma \cdot i$$

$$\frac{di(t)}{dt} = (\beta - \gamma) i - \beta \cdot i^2$$

We define:

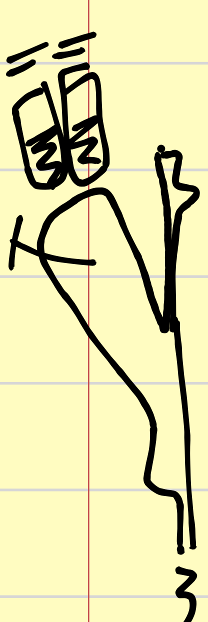
$$r = \beta - \gamma$$

$$K = \frac{r}{\beta} = \frac{\beta - \gamma}{\beta} = 1 - \frac{\gamma}{\beta} \equiv \beta = \frac{r}{K}$$

$$\frac{di(t)}{dt} = r \cdot i - \frac{r}{K} i^2 = r \cdot i \left[1 - \frac{i}{K}\right]$$

$$\boxed{\frac{di(t)}{dt} = r \cdot i \left[1 - \frac{i}{K}\right]}$$

Logistic Equation



1. $$\frac{d(\ln x)}{dx} = \frac{1}{x} \rightarrow \int \frac{1}{x} dx = \ln|x| + C$$

2. $$\frac{K}{x(1-x)} = \frac{A}{x} + \frac{B}{(1-x)} = \frac{A(1-x) + Bx}{x(1-x)}$$

$$x = 0 \rightarrow K = A(1-0) + B \cdot 0 \rightarrow K = A$$

$$x = 1 \rightarrow K = A(1-1) + B \cdot 1 \rightarrow K = B$$

$$\frac{di(t)}{i\left[1-\frac{i}{K}\right]} = r\,dt \rightarrow \int \frac{di(t)}{i\left(1-\frac{i}{k}\right)} = \int r\,dt$$

$$\rightarrow \int \frac{K\,di(t)}{i(K-i)} = \int r\,dt \longrightarrow (\bullet)$$

$$\boxed{\begin{array}{l} \dfrac{K}{i(K-i)} = \dfrac{A}{i} + \dfrac{B}{K-i} = \dfrac{A(K-i)+B\cdot i}{i(K-i)} \longrightarrow \\ \longrightarrow \quad i=K \rightarrow K = A(K-K)+B.K \longrightarrow K = B\cdot K \longrightarrow B=1 \\ \qquad\; i=0 \rightarrow K = A(K-0)+B.0 \longrightarrow K = A.K \longrightarrow A=1 \\ \dfrac{K}{i(K-i)} = \dfrac{1}{i} + \dfrac{1}{K-i} \end{array}}$$

$$\rightarrow (\bullet) \rightarrow \int \frac{1}{i}\,di(t) + \int \frac{1}{K-i}\,di(t) = \int r\,dt \rightarrow$$

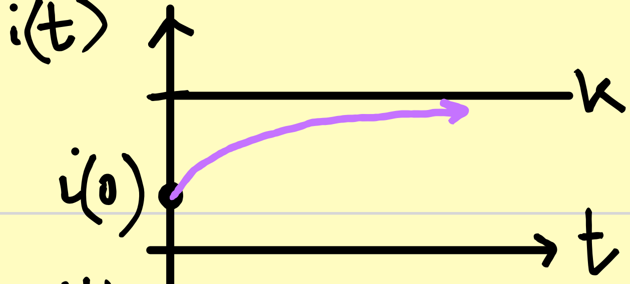
$$\rightarrow \ln|i| - \ln|K-i| = rt + C$$

Exponentiate:

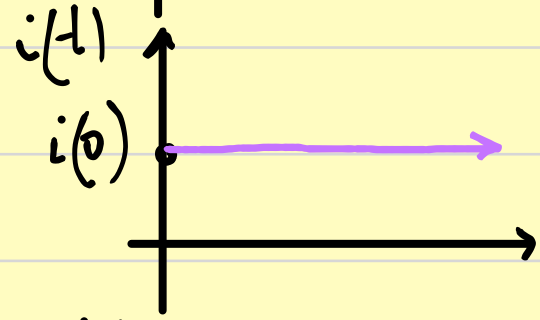
$$\left.\begin{array}{l} \dfrac{i}{K-i} = A e^{rt} \\ A = \dfrac{i(0)}{K-i(0)} \end{array}\right\} \quad \boxed{i(t) = \frac{K}{1+\left[\frac{K-i(0)}{i(0)}\right]e^{-rt}}}$$

✓

$\boxed{r>0}$ We infect faster than we cure.



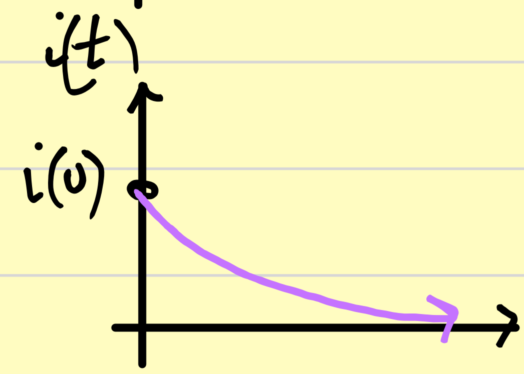
$\boxed{r=0}$ we infect at the same rate as we cure.



$$i(t) = \frac{K}{1+\left[\frac{K-i(0)}{i(0)}\right]} = \frac{\cancel{K}\cdot i(0)}{\cancel{i(0)}+\cancel{K}-\cancel{i(0)}} = i(0)$$

$r=0$

$\boxed{r<0}$ We infect slower than we cure



Example separation of variables for Integration:

$$\frac{7}{(x-2)(3x-1)} = \frac{7}{(x-2)\,3\cdot\left(x-\frac{1}{3}\right)} = \frac{7}{3}\cdot\frac{1}{(x-2)\left(x-\frac{1}{3}\right)} = \frac{A}{x-2}+\frac{B}{x-\frac{1}{3}}$$

$$= \frac{A\left(x-\frac{1}{3}\right)+B(x-2)}{(x-2)\left(x-\frac{1}{3}\right)}$$

$$\frac{7}{3} = A\left(x-\frac{1}{3}\right)+B(x-2) \rightarrow$$

$$\rightarrow \quad x=\frac{1}{3} \rightarrow \frac{7}{3} = B\left(\frac{1}{3}-2\right) \rightarrow \frac{7}{3} = B\cdot\frac{-5}{3} \rightarrow B = \frac{-7}{5}$$

$$x=2 \rightarrow \frac{7}{3} = A\left(2-\frac{1}{3}\right) \rightarrow \frac{7}{3} = A\cdot\frac{5}{3} \rightarrow A = \frac{7}{5}$$

$$\frac{7}{(x-2)(3x-1)} = \frac{7}{5}\left(\frac{1}{x-2}-\frac{1}{x-\frac{1}{3}}\right)$$

$$\int \frac{7}{(x-2)(3x-1)}\,dx = \frac{7}{5}\left[\int\frac{1}{x-2}\,dx - \int\frac{1}{x-\frac{1}{3}}\,dx\right] =$$

$$= \frac{7}{5}\left[\ln|x-2| - \ln\left|x-\frac{1}{3}\right|\right] + C$$

$$\int \frac{3}{(x-1)(2x-5)}\,dx$$

$$\frac{3}{(x-1)(2x-5)} = \frac{3}{(x-1)\,2\left(x-\frac{5}{2}\right)} = \frac{3}{2}\cdot\frac{1}{(x-1)\left(x-\frac{5}{2}\right)} = \frac{A}{x-1} + \frac{B}{x-\frac{5}{2}}$$

$$= \frac{A\left(x-\frac{5}{2}\right)+B(x-1)}{(x-1)\left(x-\frac{5}{2}\right)}$$

- $\frac{3}{2} = A\left(x-\frac{5}{2}\right) + B(x-1)$

$$x=1 \longrightarrow \frac{3}{2} = A\left(1-\frac{5}{2}\right) + \cancel{B(1-1)}^{\,0} \rightarrow \frac{3}{2} = A\left(\frac{-3}{2}\right) \rightarrow A=-1$$

$$x=\frac{5}{2} \longrightarrow \frac{3}{2} = \cancel{A\cdot\left(\frac{5}{2}-\frac{5}{2}\right)}_{\,0} + B\left(\frac{5}{2}-1\right) \rightarrow \frac{3}{2} = B\left(\frac{3}{2}\right) \rightarrow B=1$$

$$\frac{3}{(x-1)(2x-5)} = \frac{-1}{x-1} + \frac{1}{x-\frac{5}{2}}$$

$$\int \frac{3\,dx}{(x-1)(2x-5)} = \int \frac{-dx}{x-1} + \int \frac{dx}{x-\frac{5}{2}} = -\ln|x-1| + \ln\left|x-\frac{5}{2}\right| + C$$

· 2D System $\boxed{\text{LOVE \& MATH}}$

ROMEO - JULIA

$R(t)$ $\quad$ $J(t)$

- Romeo & Julia interact, so the love functions for each other $R(t)$ & $J(t)$ depend mutually on each other.

To describe this interaction we need four parameters:

a. describes the intrinsic love of Romeo to Julia.

$a>0$: Romeo increases love to Julia independent of Julia's behaviour.

$a<0$: Romeo decreases love to Julia independent of Julia's behaviour.

b. describes the influence of Julia on Romeo's love.

$b>0$: Romeo loves Julia more if she is charming.

$b<0$: Romeo loves Julia more if she rejects him.

(1) $$\frac{dR(t)}{dt} = a\,R(t) + b\,J(t)$$

For c and d is the same changing the names

(2) $$\frac{dJ(t)}{dt} = c\,R(t) + d\,J(t).$$

$$\frac{dR(t)}{dt} = a R(t) + b J(t)$$
$$\frac{dJ(t)}{dt} = c R(t) + d J(t)$$

$$X(t) = \begin{bmatrix} R(t) \\ J(t) \end{bmatrix} \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\boxed{\frac{dX(t)}{dt} = A \cdot X(t)}$$

General dynamic equation for 2 variables interacting.

In order to understand the qualitative behaviour of the system, we diagonalize our Equation by finding the eigenvalues and eigenvectors of the matrix.

$$\boxed{\text{The general solution of the equation is: } \vec{x}(t)=\begin{bmatrix} R(t) \\ J(t) \end{bmatrix} = c_1 \cdot \vec{v_1}(t) \cdot e^{\lambda_1 t} + c_2 \cdot \vec{v_2}(t) e^{\lambda_2 t}}$$

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \det[A-\lambda I]=0 \rightarrow \det\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}=0$$

$$\rightarrow (a-\lambda)(d-\lambda)-cb=0 \rightarrow \lambda^2-\lambda(a+d)+(ad-cb)=0$$

$$\lambda=\frac{a+d\pm\sqrt{(a+d)^2-4(ad-cb)}}{2}=\frac{a+d\pm\sqrt{a^2+d^2-2ad+4cb}}{2}=$$

$$\lambda=\frac{a+d\pm\sqrt{(a-d)^2+4cb}}{2} = \begin{cases} \lambda_1 \rightarrow \vec{v_1} \\ \lambda_2 \rightarrow \vec{v_2} \end{cases}$$

Examples:

1) The behaviour of Julia does not influence Romeo. $b=0$
The behaviour of Romeo does not influence Julia. $c=0$

$$\lambda=\frac{a+d\pm\sqrt{(a-d)^2+0}}{2}=\frac{a+d\pm(a-d)}{2}= \begin{cases} \lambda_1=a \\ \lambda_2=d \end{cases}$$

$$A \cdot \vec{v_1} = \lambda_1 \vec{v_1} \rightarrow \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = a \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} \rightarrow$$

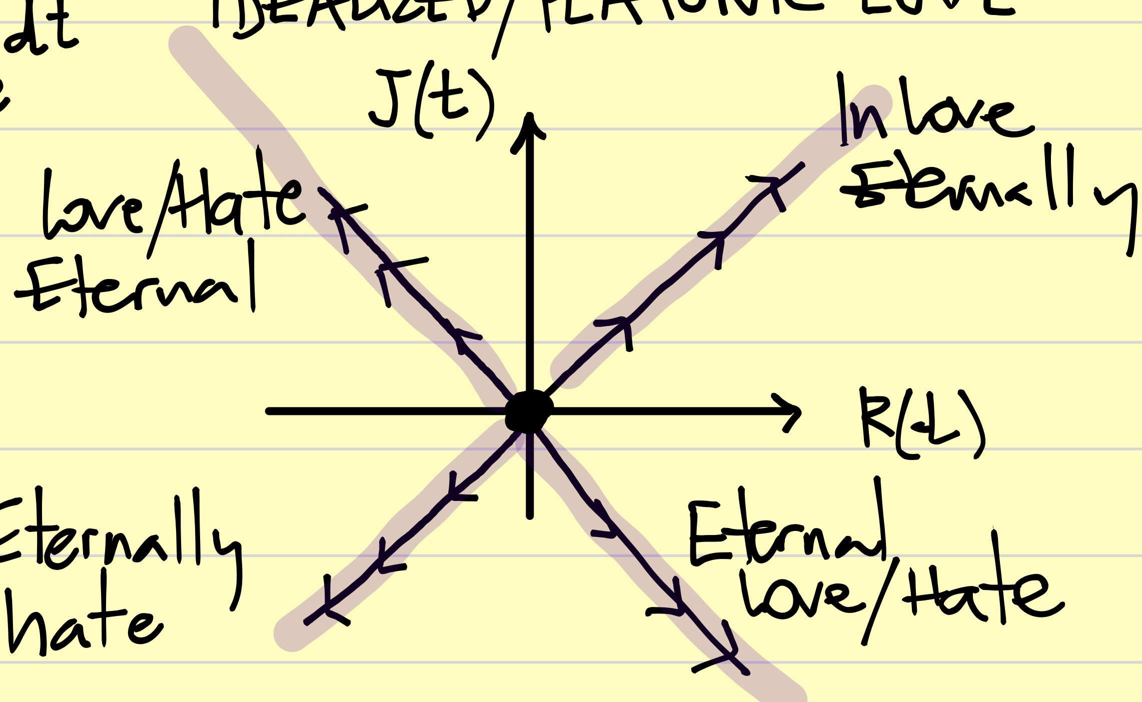
$$\rightarrow \left. \begin{array}{l} a v_{11} = a v_{11} \\ d v_{12} = a v_{12} \longrightarrow v_{12} = 0 \end{array} \right\} v_{11} = 1$$

The behaviour of Romeo & Julia are independent.

$$X(t) = c_1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{at} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{dt} = \begin{bmatrix} R(t) \\ J(t) \end{bmatrix}$$

$$R(t) = c_1 e^{at}$$
$$J(t) = c_2 e^{dt}$$

IDEALIZED/PLATONIC LOVE



2) Romeo likes charming Julia.
Julia likes asshole Romeo.

- $a = -0'5$. Romeo does not find Julia attractive
- $b = 1$. If Julia is charming, Romeo falls in love.
- $c = -1$. Julia does not find Rome attractive.
- $d = -0'5$. Julia rejects Romeo when he shows love. (she likes bad guys).

$$\lambda = \frac{(a+d) \pm \sqrt{(a-d)^2+4cb}}{2} = \frac{-1 \pm \sqrt{0-4}}{2} = -0'5 \pm i$$

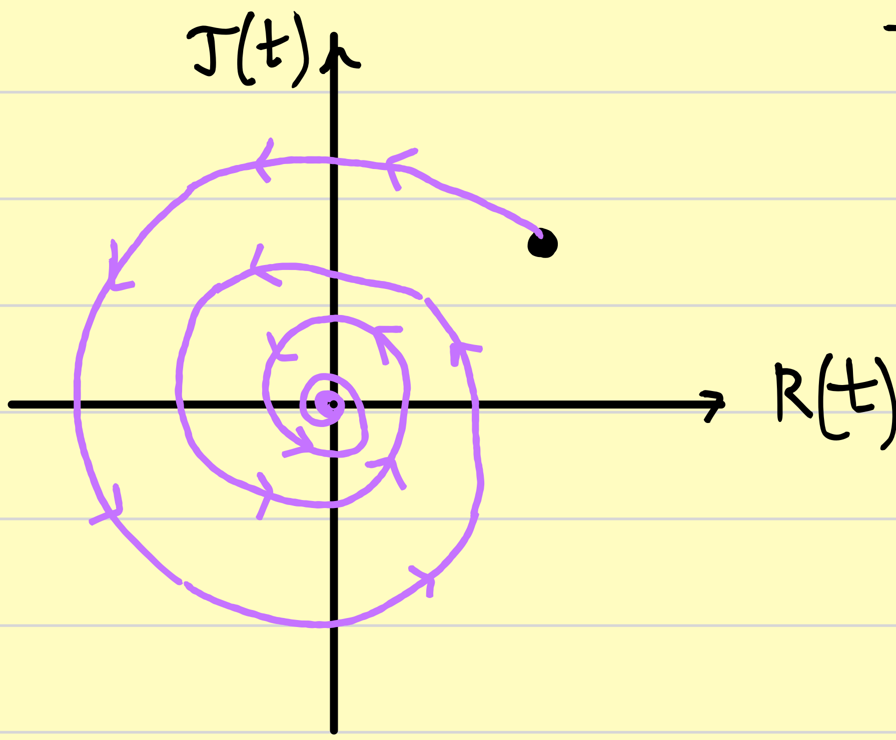
$$\lambda_1 = -0'5 + i \qquad \lambda_2 = -0'5 - i$$

$$A\vec{v_1} = \lambda_1 \vec{v_1} \rightarrow \begin{bmatrix} -0'5 & 1 \\ -1 & -0'5 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = (-0'5+i) \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} \rightarrow$$

$$\rightarrow \begin{aligned} -0'5 v_{11} + v_{12} &= (-0'5+i) v_{11} \\ -v_{11} - 0'5 v_{12} &= (-0'5+i) v_{12} \end{aligned} \rightarrow$$

$$\rightarrow \begin{aligned} v_{12} &= i\, v_{11} \\ v_{12} &= i\, v_{11} \end{aligned} \rightarrow \begin{aligned} v_{11} &= 1 \\ v_{12} &= i \end{aligned} \qquad \vec{v_1} = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\begin{bmatrix} R(t) \\ J(t) \end{bmatrix} = e^{-0'5t} \left[A \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix} + B \begin{bmatrix} -\sin(t) \\ \cos(t) \end{bmatrix} \right]$$



3) JULIA is an ICE WOMAN. $c = d = 0$

ROMEO is FIRE . $a, b > 0$

$$\lambda = \frac{a \pm \sqrt{(a)^2}}{2} = \begin{cases} \lambda_1 = a \\ \lambda_2 = 0 \end{cases}$$

$$\vec{x}(t) = \begin{bmatrix} R(t) \\ J(t) \end{bmatrix} = c_1 \cdot \vec{v_1}(t) \cdot e^{at} + c_2 \cdot \vec{v_2}(t)$$

$$\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = a\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} \longrightarrow \begin{array}{l} v_{11} = 1 \\ v_{12} = 0 \end{array} \rightarrow \vec{v_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = 0\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} \longrightarrow \begin{array}{l} v_{21} = \\ v_{22} = \end{array} \Big| \text{degenerate} \quad \vec{v_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\vec{x}(t) = c_1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{at} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$