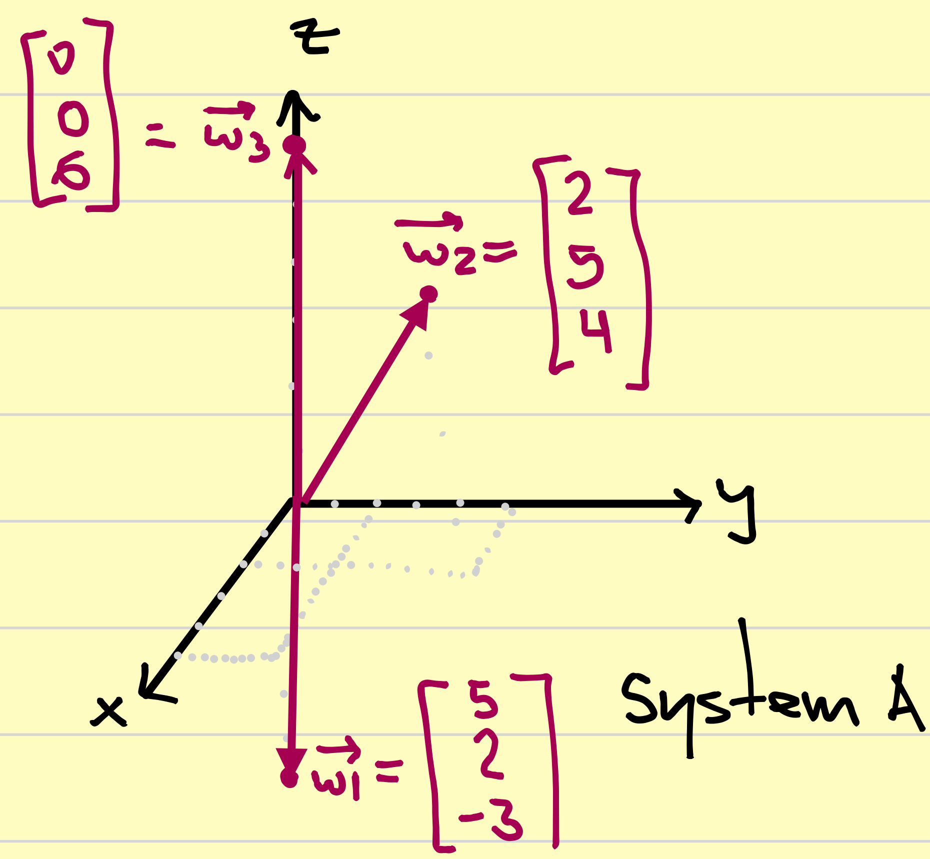
1 EIGENVEKTOREN & EIGENWERTE



$$\underset{\text{Matrix = System}}{A}\ \underset{\text{Eigenvektoren}}{\vec{v}} = \overset{\text{Eigenwerte}}{\lambda}\ I \rightarrow \det[A-\lambda I] = 0$$

Beispiel:
$$A = \begin{array}{cc} & \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ & \begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{bmatrix} \begin{array}{c} x \\ y \\ z \end{array} \end{array}$$

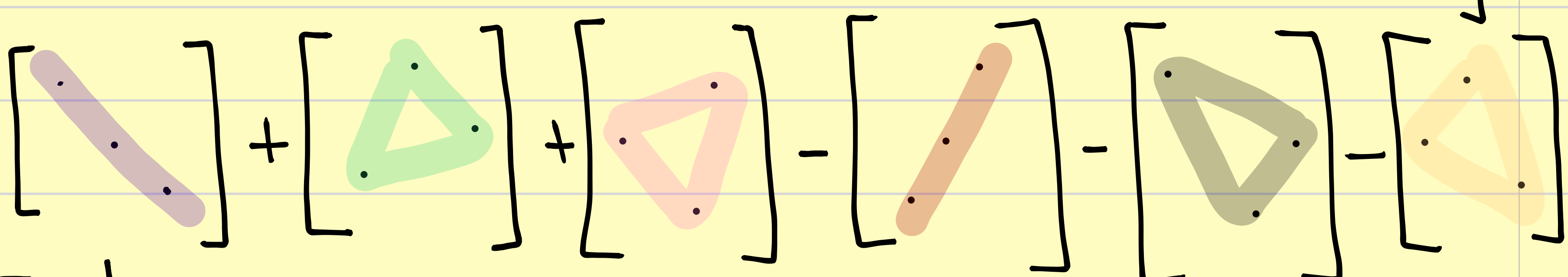
A) EIGENWERTE

$$\det[A-\lambda I] = 0 \rightarrow \det\left[\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right] = 0$$

$$\rightarrow \det\begin{bmatrix} 5-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ -3 & 4 & 6-\lambda \end{bmatrix} = 0 \rightarrow$$

$$\Big[(5-\lambda)(5-\lambda)(6-\lambda) + 2\cdot\cancel{0}\cdot(-3) + 2\cdot\cancel{4}\cdot 0 - (-3)\cancel{(5-\lambda)}\cdot 0 - 0\cdot\cancel{4}(5-\lambda) - 2\cdot 2\cdot(6-\lambda)\Big] = 0 \quad (1)$$

det =



$$\det\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} = a\cdot e\cdot i + d\cdot h\cdot c + b\cdot f\cdot g - g\cdot e\cdot c - b\cdot d\cdot i - a\cdot h\cdot f$$

(1) $(5-\lambda)^2 \cdot (6-\lambda) - 4(6-\lambda) = 0 \rightarrow$

$$\underbrace{\left[(5-\lambda)^2 - 4\right]} \cdot (6-\lambda) = 0 \rightarrow \lambda = 6$$

$$25 + \lambda^2 - 10\lambda - 4 = 0 \rightarrow \lambda^2 - 10\lambda + 21 = 0$$

$$\lambda = \frac{-(-10) \pm \sqrt{100 - 4.21}}{2} = \frac{10 \pm \sqrt{16}}{2} = \begin{cases} 7 \\ 3 \end{cases}$$

$$\boxed{\lambda_1 = 3 \quad \lambda_2 = 6 \quad \lambda_3 = 7}$$

B) EIGENVEKTOREN

$$(A - \lambda I)\vec{v} = 0 \rightarrow \begin{bmatrix} 5-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ -3 & 4 & 6-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\left.\begin{aligned} (5-\lambda)x + 2y + 0.z &= 0 \\ 2x + (5-\lambda)y + 0.z &= 0 \\ -3x + 4y + (6-\lambda).z &= 0 \end{aligned}\right| \quad (2)$$

$\boxed{\lambda_1 = 3}$

(2)

$$\left.\begin{aligned} (5-3)x + 2y &= 0 \\ 2x + (5-3)y &= 0 \\ -3x + 4y + 3z &= 0 \end{aligned}\right| \left.\begin{aligned} 2x + 2y &= 0 \\ 2x + 2y &= 0 \end{aligned}\right| \quad x = -y$$

$$3x+4x-3z=0 \rightarrow x=\frac{3z}{7}$$

$$x=1 \rightarrow y=-1 \rightarrow z=\frac{7}{3} \rightarrow \vec{v_1}=\begin{bmatrix} 1 \\ -1 \\ 7/3 \end{bmatrix}$$

$$x=2 \rightarrow y=-2 \rightarrow z=\frac{14}{3} \rightarrow \ldots$$

$$\boxed{\lambda_2=6} \quad (2)$$

$$\left.\begin{aligned} (5-6)x+2y&=0 \\ 2x+(5-6)y&=0 \\ -3x+4y+(6-5)z&=0 \end{aligned}\right| \left.\begin{aligned} x&=2y \\ 2x&=y \end{aligned}\right| \; x=y=0$$

$$z=1 \longrightarrow \vec{v_2}=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\lambda_3=7}$$

$$\left.\begin{aligned} (5-7)x+2y&=0 \\ 2x+(5-7)y&=0 \end{aligned}\right| \; x=y$$

$$3x-4y-(6-7)z=0 \quad \Big| \; x=z$$

$$x=1 \rightarrow y=1 \rightarrow z=1 \rightarrow \vec{v_3}=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{\lambda_1=3,\ \lambda_2=6,\ \lambda_3=7 \ ; \ \vec{v_1}=\begin{bmatrix} 1 \\ -1 \\ 7/3 \end{bmatrix} ; \vec{v_2}=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ; \vec{v_3}=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

② KOVARIANZMATRIX

X	Y	Z
6	1	7
3	-1	2
2	0	4

$$K = \begin{bmatrix} \sigma_{XX}^2 & \sigma_{XY}^2 & \sigma_{XZ}^2 \\ \sigma_{YX}^2 & \sigma_{YY}^2 & \sigma_{YZ}^2 \\ \sigma_{ZX}^2 & \sigma_{ZY}^2 & \sigma_{ZZ}^2 \end{bmatrix}$$

$$\sigma_{XX}^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} \quad \text{VARIANZ}$$

$$\sigma_{XY}^2 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1} \quad \text{CO VARIANZ}$$

$$\sigma_{XZ}^2 = \frac{\sum (x_i - \bar{x})(z_i - \bar{z})}{n-1} \quad \text{CO VARIANZ}$$

$$\sigma_{YZ}^2 = \frac{\sum (y_i - \bar{y})(z_i - \bar{z})}{n-1} \quad \text{CO VARIANZ}$$

$$\bar{x} = \frac{6+3+2}{3} = \frac{11}{3} \; ; \; \bar{Y} = \frac{1-1+0}{3} = 0 \; ; \; \bar{Z} = \frac{7+2+4}{3} = \frac{13}{3}$$

$$\sigma_{xy}^2 = \frac{\left(6 - \frac{11}{3}\right)^2 + \left(3 - \frac{11}{3}\right)^2 + \left(2 - \frac{11}{3}\right)^2}{2}$$

$$\sigma_{XY}^2 = \frac{\left(6 - \frac{11}{3}\right)(1-0) + \left(3 - \frac{11}{3}\right)(-1-0) + \left(2 - \frac{11}{3}\right)(0-0)}{2} = \sigma_{YX}^2$$

$$\sigma_{XZ}^2 = \sigma_{ZX}^2$$

$$\sigma_{YZ}^2 = \sigma_{ZY}^2$$

$$K = \begin{bmatrix} \sigma_{XX}^2 & \sigma_{XY}^2 & \sigma_{XZ}^2 \\ \sigma_{YX}^2 & \sigma_{YY}^2 & \sigma_{YZ}^2 \\ \sigma_{ZX}^2 & \sigma_{ZY}^2 & \sigma_{ZZ}^2 \end{bmatrix} \quad \text{(Übung)}$$

WARUM IST ES FÜR WI WICHTIG?

Die Eigenwerte der Kovarianzmatrix sind die Hauptkomponenten des Systems. Die Hauptkomponenten (Principal Components - PCA) beschreiben die Variabilität vom System mit weniger Daten als das gesamte.

i.e.	KPI_1	KPI_2	$\dots$	KPI_n
t_1				
t_2				
t_3				
$\vdots$				
t_n				

→ PCA Analyse → wichtigsten Merkmalen vom System ohne die ..n Komponenten zuliefern.