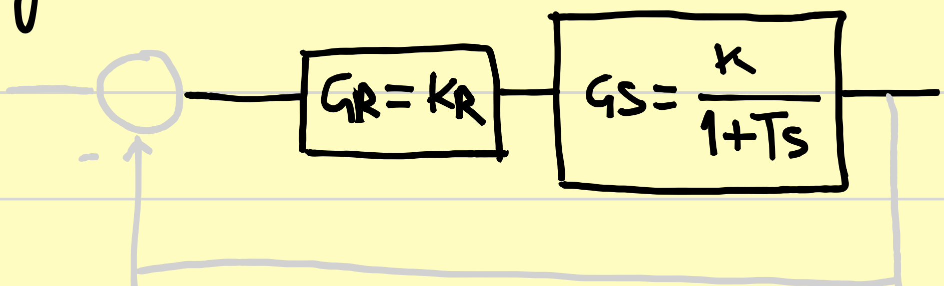
20240516_Regelungstechnik_BEL4

Übung 1



a) $$G_g(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{\frac{K \cdot K_R}{1+Ts}}{1+\frac{K \cdot K_R}{1+Ts}} = \frac{K \cdot K_R}{1+K \cdot K_R + Ts} = \frac{K \cdot K_R}{T\left[s+\frac{1+K \cdot K_R}{T}\right]}$$

$$= \frac{K \cdot K_R}{T} \cdot \frac{1}{s+\frac{1+KK_R}{T}}$$

STABILITÄTSKRITERIEN

Angenommen $K, K_R, T \in \mathbb{R}^+ \rightarrow \frac{1+KK_R}{T} > 0 \rightarrow KK_R > -1$

IMMER ERFÜLLT

b)

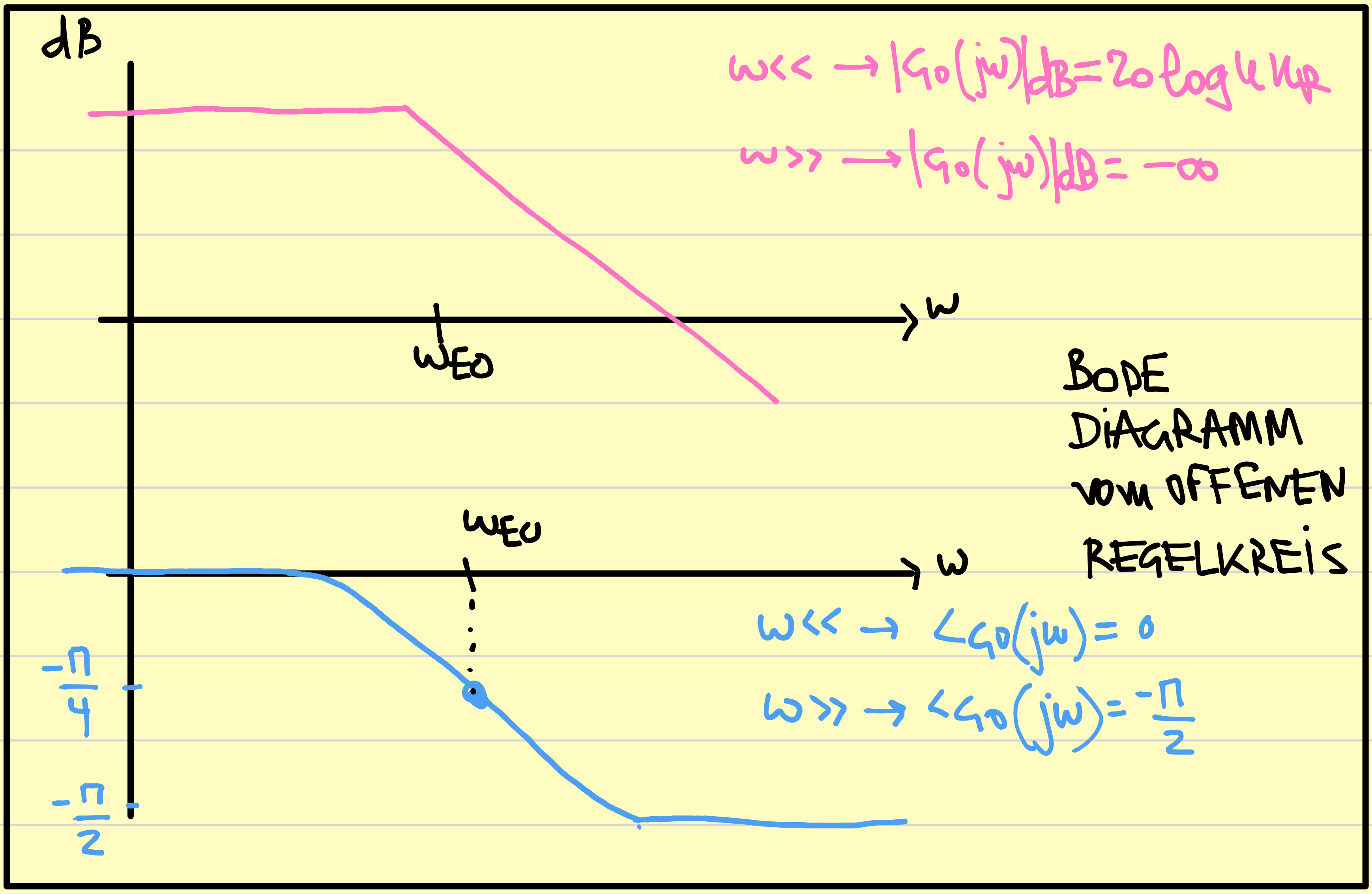
b.1) FREQUENZGANG vom OFFENEN RK:

$$G_0(s) = \frac{KK_R}{1+Ts} \rightarrow G_0(j\omega) = \frac{KK_R}{1+j\omega T} \cdot \frac{1-j\omega T}{1-j\omega T} = \frac{KK_R}{1+\omega^2T^2}(1-j\omega T)$$

$$\boxed{(1+j\omega T)(1-j\omega T) = 1 \cdot 1 - \cancel{j\omega T} + \cancel{j\omega T} - j^2\omega^2T^2 = 1+\omega^2T^2}$$

$$|G_0(j\omega)| = \frac{KK_R}{1+\omega^2T^2}\sqrt{1+\omega^2T^2} = KK_R(1+\omega^2T^2)^{-1/2}$$

$$\left.\begin{aligned} |G_0(j\omega)|_{dB} &= 20 \log KK_R - 20 \cdot \tfrac{1}{2} \log(1+\omega^2T^2) \\ \angle G_0(j\omega) &= \arctan\left[\frac{Im}{Re}\right] = \arctan\left[\frac{-\omega T}{1}\right] \end{aligned}\right\} \rightarrow \text{BODE}$$



b.2) FREQUENZGANG VOM GESCHLOSSENEM REGELKREIS

$$G_g(s) = \frac{KK_R}{T} \cdot \frac{1}{s + \frac{1+KK_R}{T}} \rightarrow G_g(j\omega) = \frac{KK_R}{T} \cdot \frac{1}{j\omega + \frac{1+KK_R}{T}}$$

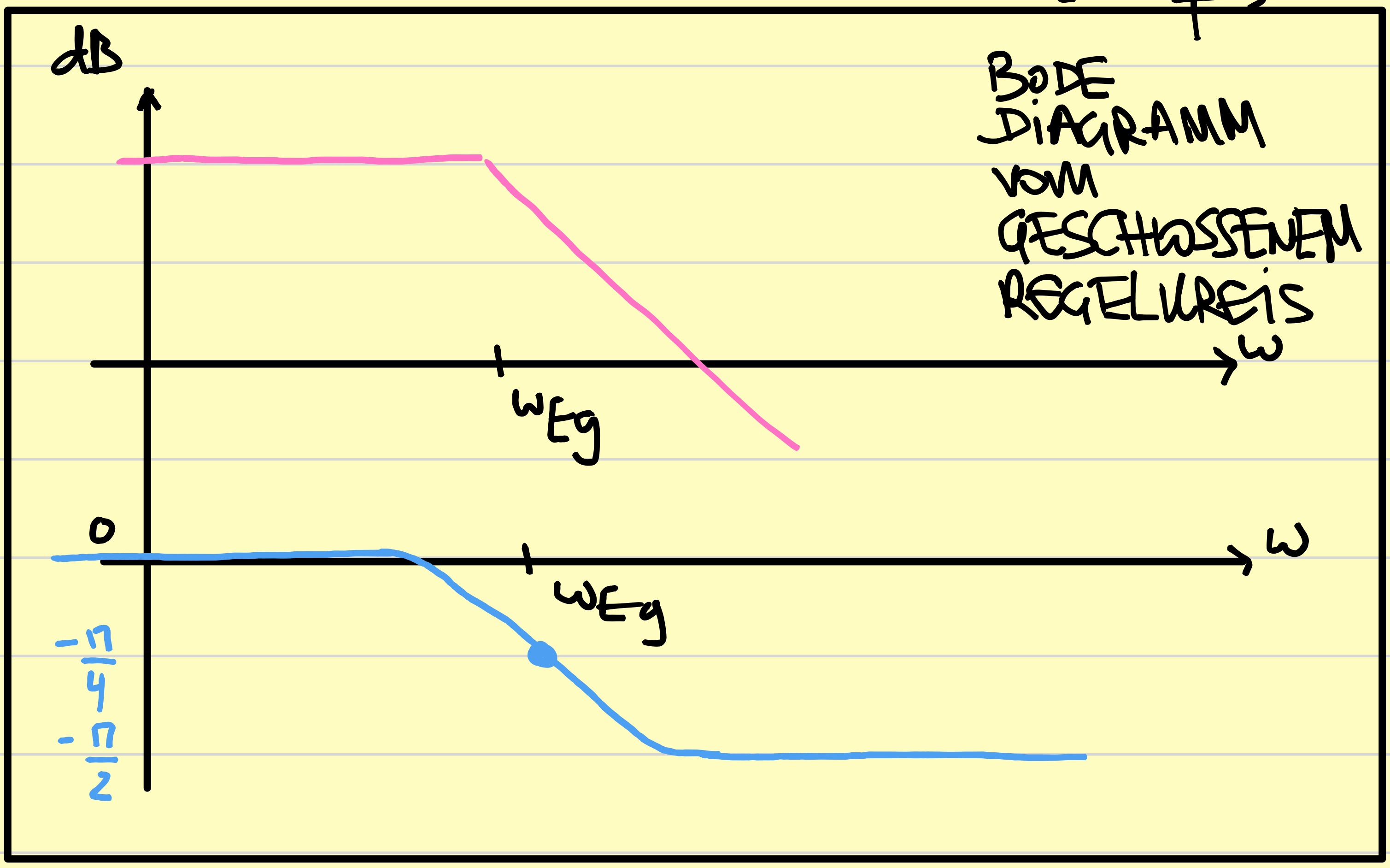
$$G_g(j\omega) = \frac{KK_R}{T} \cdot \frac{1}{\frac{1+KK_R}{T} + j\omega} \cdot \frac{1 + \frac{KK_R}{T} - j\omega}{1 + \frac{KK_R}{T} - j\omega} =$$

$$= \frac{KK_R}{T} \cdot \frac{1}{\left(\frac{1+KK_R}{T}\right)^2 + \omega^2} \cdot \left(1 + \frac{KK_R}{T} - j\omega\right)$$

$$|G_g(j\omega)| = \frac{KK_R}{T} \left[\left(\frac{1+KK_R}{T}\right)^2 + \omega^2\right]^{-1/2}$$

$$\left.\begin{array}{l} |G_g(j\omega)|_{dB} = 20 \log \frac{KK_R}{T} - 20 \, \frac{1}{2} \log\left[\left(\frac{1+KK_R}{T}\right)^2 + \omega^2\right] \end{array}\right\} \rightarrow \text{BODE}$$

$$\angle G_g(j\omega) = \arctan\left[\frac{Im}{Re}\right] = \arctan\left[\frac{-\omega}{1+\frac{kK_R}{T}}\right]$$



Unterschiede: Qualitativ KEINE

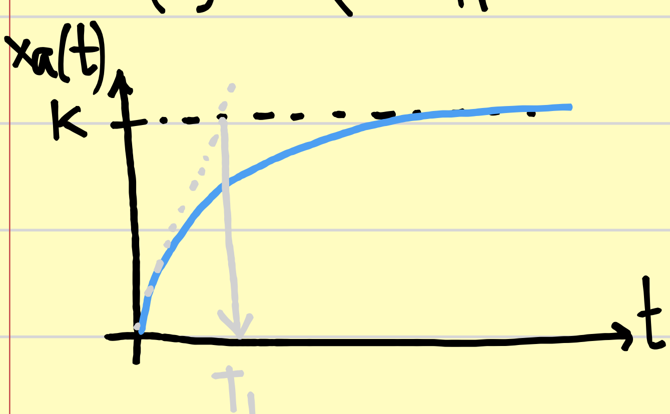
Eckfrequenz $\omega_{E_0} \neq \omega_{Eg}$

c) Übertragungsfunktion geschl. RK ✓

d) Anstiegszeit des geschlossenen RK. t_R

$$x_a(s) = \frac{K}{s} - \frac{KT_1}{1+sT_1}$$

$$x_a(t) = K\left(1 - T_1 e^{-t/T_1}\right)$$



$$x_a(s) = \frac{1}{s} G_g(s) = \frac{kK_R}{T}\cdot\frac{1}{s}\cdot\frac{1}{s+\frac{1+kK_R}{T}}$$

$$= \frac{A}{s} + \frac{B}{s+\frac{1+kK_R}{T}}$$

$$\frac{kK_R}{T} = A\left(s+\frac{1+kK_R}{T}\right) + Bs$$

$$s=0 \rightarrow A = \frac{KK_R}{1+KK_R}$$

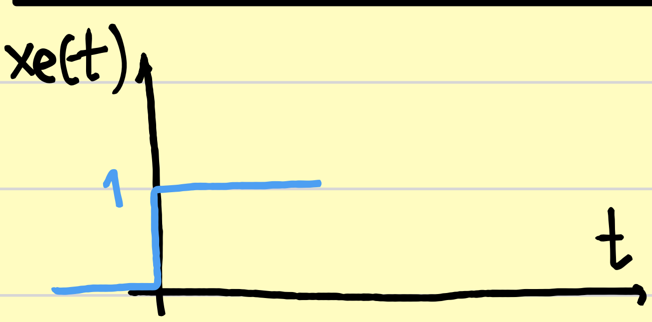
$$s = -\frac{1+KK_R}{T} \longrightarrow \frac{KK_R}{T} = -B\,\frac{1+KK_R}{T} \rightarrow B = \frac{-KK_R}{1+KK_R}$$

$$x_a(s) = \frac{KK_R}{1+KK_R}\left[\frac{1}{s} - \frac{1}{s+\frac{1+KK_R}{T}}\right] = \frac{KK_R}{1+KK_R}\left[\frac{1}{s} - \frac{1}{\frac{1+KK_R}{T}}\cdot\frac{1}{1+\frac{T}{1+KK_R}s}\right]$$

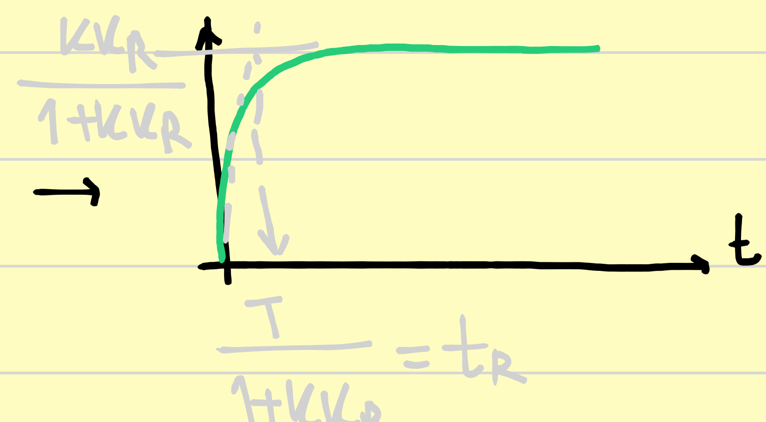
$$x_a(t) = \frac{KK_R}{1+KK_R}\left[1 - \frac{T}{1+KK_R}\cdot e^{-t/\left[\frac{T}{1+KK_R}\right]}\right] \longrightarrow$$

$$\boxed{\frac{1}{s+8} = \frac{1}{8}\cdot\frac{1}{1+\frac{s}{8}}}$$

$$t_R = \text{ANSTIEGSZEIT} = t_R = \frac{T}{1+KK_R}$$



→



e) Überschwingung gibt es nicht.

$$t_R = \frac{T}{1+KK_R} < 2$$

$$1+KK_R > 0 \quad \text{(Stabilität)} \checkmark$$

$$\Big\}\; T < 2(1+KK_R)$$

Die Verzögerung vom System darf nicht die Obergrenze von $2(1+KK_R)$ überschreiten.

Übung 2.

a) $G_g(s) = \dfrac{\dfrac{KK_R}{(1+T_1s)(1+T_2s)}}{1+\dfrac{KK_R}{(1+T_1s)(1+T_2s)}} = \dfrac{KK_R}{(1+KK_R)+(T_1+T_2)s+T_1T_2s^2}$

$$s = \frac{-(T_1+T_2) \pm \sqrt{(T_1+T_2)^2 - 4T_1T_2(1+KK_R)}}{2T_1T_2}$$

(bei negativ, immer negativ) $\rightarrow$ stabil.

$$-(T_1+T_2) + \sqrt{(T_1+T_2)^2 - 4T_1T_2(1+KK_R)} < 0$$

$$\cancel{(T_1+T_2)^2} - 4T_1T_2(1+KK_R) < \cancel{(T_1+T_2)^2}$$

$-4T_1T_2(1+KK_R) < 0 \longrightarrow$ immer so wenn $T_1, T_2, K, K_R \in \mathbb{R}^+$

b)

b.1) BD offener RK

$$G_0(s) = \frac{KK_R}{1+(T_1+T_2)s+T_1T_2s^2} \longrightarrow G_0(j\omega) = \frac{KK_R}{1+(T_1+T_2)j\omega+T_1T_2(j\omega)^2} =$$

$$= \frac{KK_R}{(1-T_1T_2\omega^2)+(T_1+T_2)j\omega} \cdot \frac{1-T_1T_2\omega^2-(T_1+T_2)j\omega}{1-T_1T_2\omega^2-(T_1+T_2)j\omega}$$

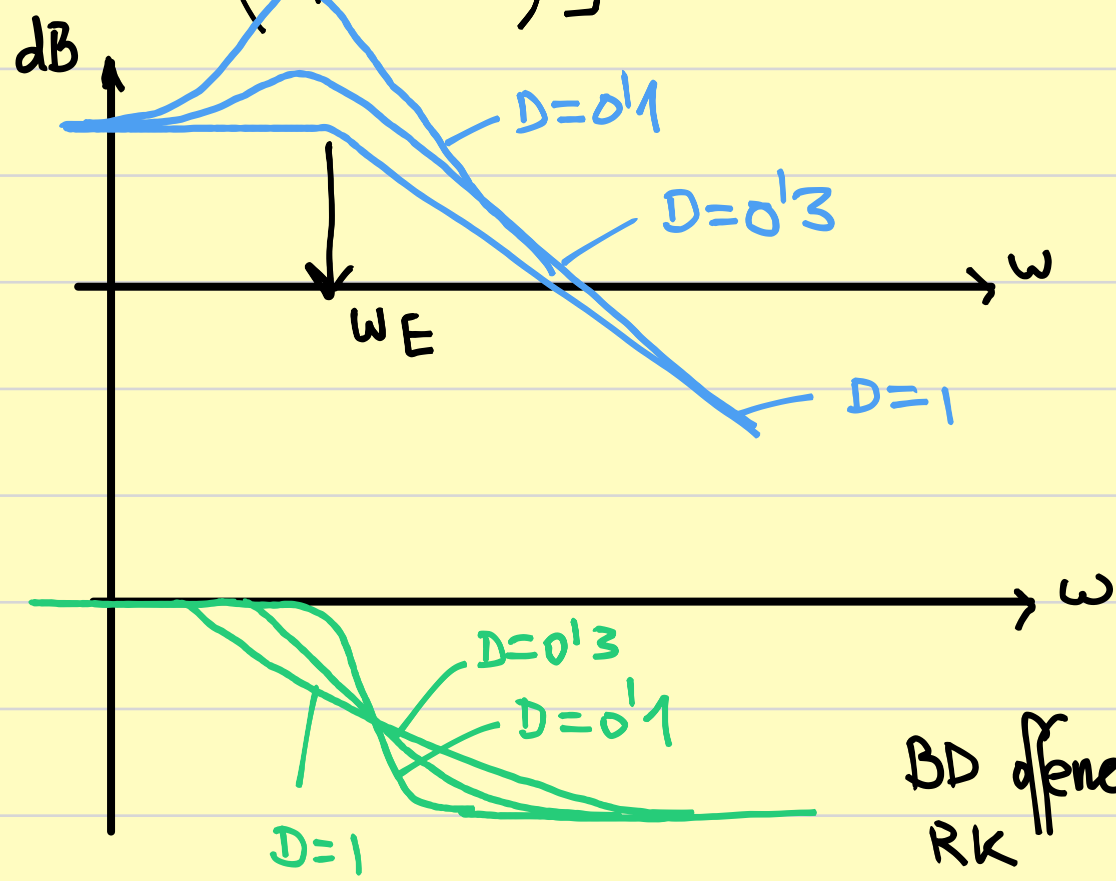
$$= \frac{KK_R}{(1-T_1T_2\omega^2)+(T_1+T_2)^2\omega^2}\left[(1-T_1T_2\omega^2)-(T_1+T_2)j\omega\right]$$

$$|G_0(j\omega)| = KK_R\left[(1-T_1T_2\omega^2)^2+(T_1+T_2)^2\omega^2\right]^{-1/2}$$

$$|G_0(j\omega)|_{dB} = 20\log kK_R - 20\cdot\frac{1}{2}\log\left[\left(1-T_1T_2\omega^2\right)^2+\left(T_1+T_2\right)^2\omega^2\right]$$

$$\angle G_0(j\omega) = \arctan\left[\frac{-\omega\left(T_1+T_2\right)}{1-\left(T_1T_2\omega^2\right)}\right]$$

$$\boxed{\begin{gathered} H(s)=\frac{K}{1+s\alpha_1+s^2\alpha_2^2} \\ \omega_E=\frac{1}{\alpha_2}\;;\; D=\frac{\alpha_1}{2\alpha_2}\end{gathered}}$$



BD offener RK

$$\omega_E = \frac{1}{\alpha_2} = \frac{1}{\sqrt{T_1T_2}}$$

b.2) BD geschlossener RK.

e) Überschwingung darf maximal 10% sein.
Anstiegszeit darf maximal 2sekunden werden.
Sprungfunktion $x_e(t)=1$

$$G_g(s) = \frac{kK_R}{\left(1+kK_R\right)+\left(T_1+T_2\right)s+\left(T_1T_2\right)s^2}$$

$$D = \frac{T_1 + T_2}{2\sqrt{T_1 T_2}}$$

$$|G(j\omega)|_{\omega = \omega_{Eg}} = \frac{K K_R}{2D} < 1{,}1$$

10% des Eingangs

Überschwingung

$$\frac{K K_R}{2\,\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}} < 1{,}1 \rightarrow \boxed{K K_R < \frac{1{,}1\,(T_1 + T_2)}{\sqrt{T_1 T_2}} \quad \text{KONDITION}}$$

$$\text{Anstiegszeit}: \; t_R = \frac{1}{\omega_{Eg}} = \boxed{\sqrt{T_1 T_2} < 2 \quad \text{KONDITION}}$$