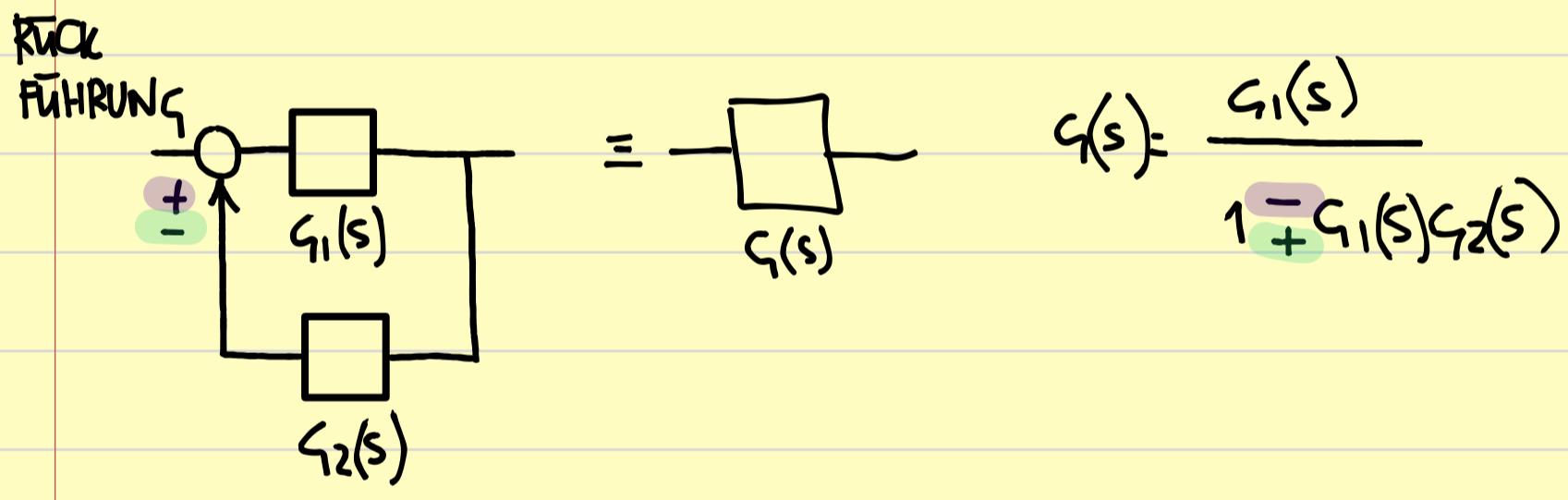
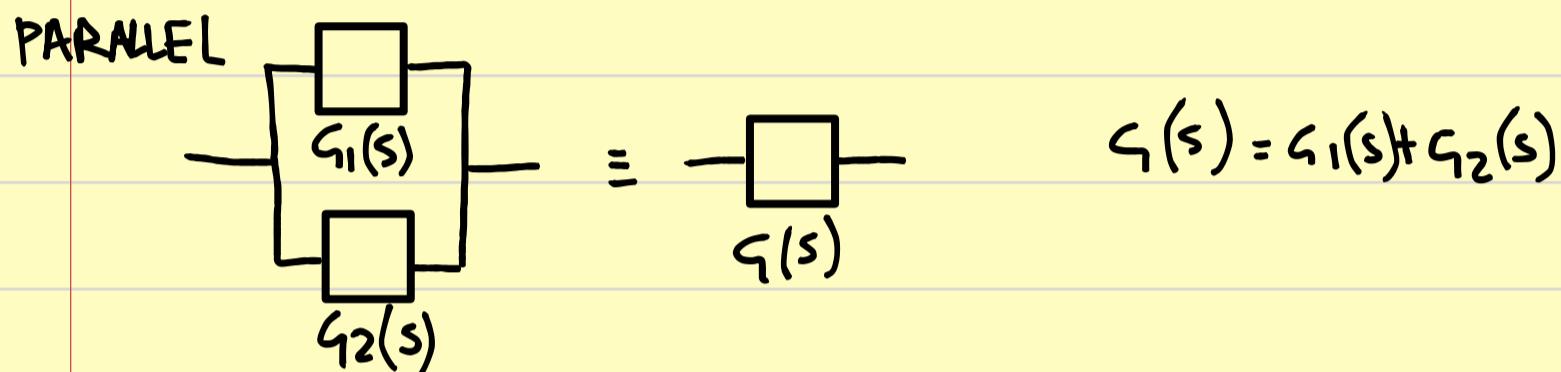
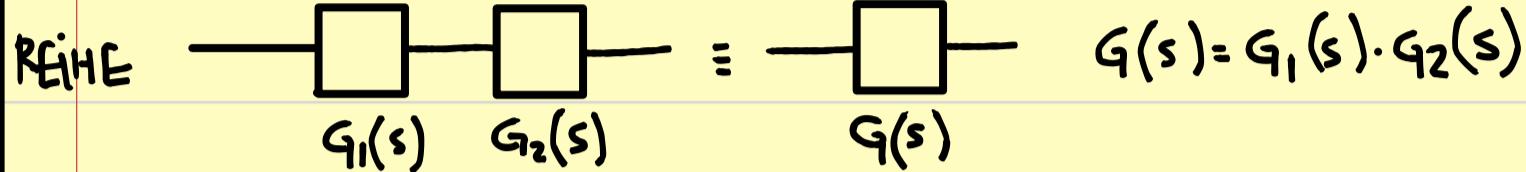


## Grundkonzepte.

### 1 Verbindungen von Regelkreisglieder



### 2 LAPLACE TRANSFORM

$$\mathcal{L}[u(t)] = \int_0^{\infty} e^{-st} u(t) dt$$

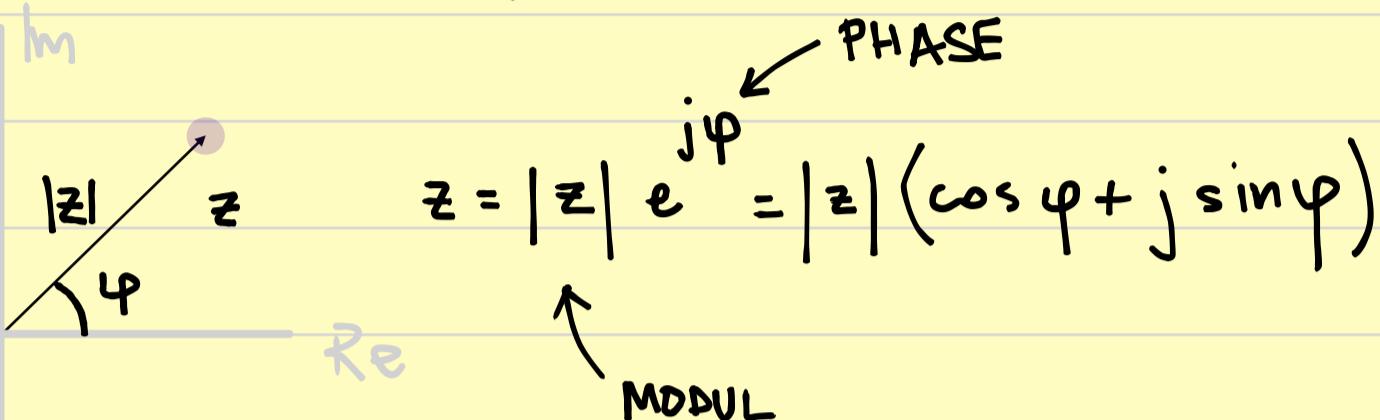
$$u(t) = \mathcal{L}^{-1}\left[G(s) = A \cdot \frac{1}{s+B}\right] = A \cdot e^{-Bt}$$

$$\rightarrow u(t) = k \rightarrow \mathcal{L}(u(t)) = \frac{k}{s} \quad (\text{Eingangsfunktion!})$$

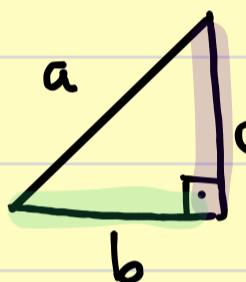
$$n(t) = \alpha^{-1} [G(s) = A] = A \delta(t) \quad \delta(t) \text{ = DIRAC FUNC.}$$

3

### KOMPLEXE ZAHLEN



PYTHAGORAS

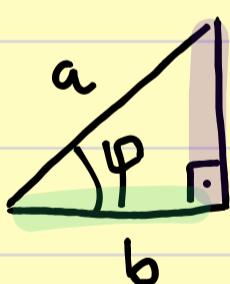


$$a^2 = b^2 + c^2$$

$$|z| = \sqrt{\operatorname{Re}|z|^2 + \operatorname{Im}|z|^2} \equiv |z| = \sqrt{\operatorname{Re}|z|^2 + \operatorname{Im}|z|^2}$$

$$\rightarrow |z| = \sqrt{\operatorname{Re}|z|^2 + \operatorname{Im}|z|^2}$$

TANGENTE



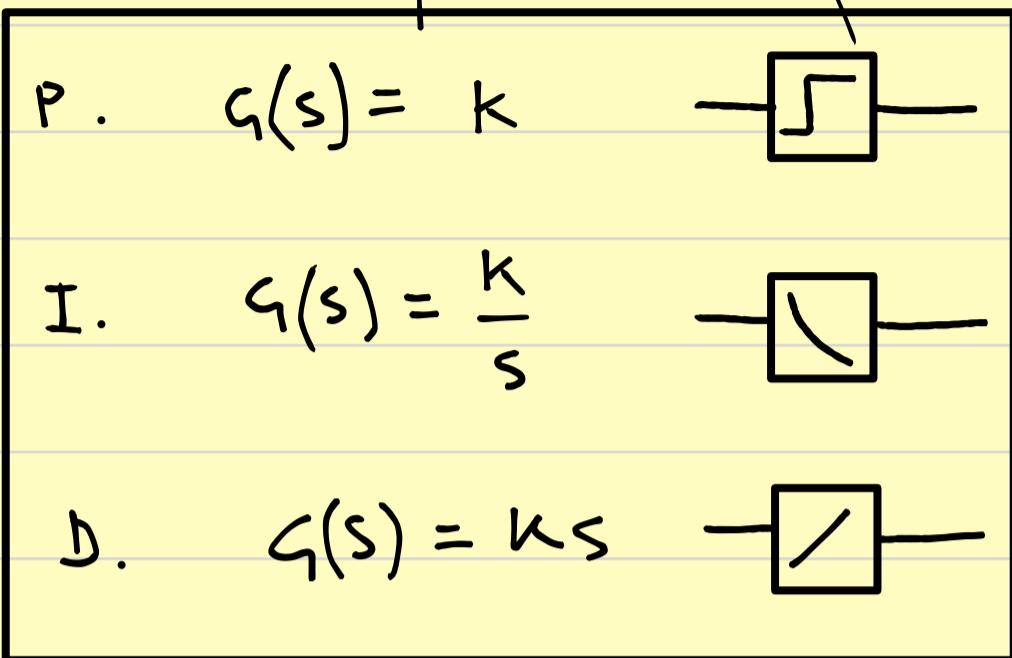
$$\operatorname{tg} \varphi = \frac{c}{b}$$

$$\operatorname{tg} \varphi = \frac{\operatorname{Im}|z|}{\operatorname{Re}|z|}$$

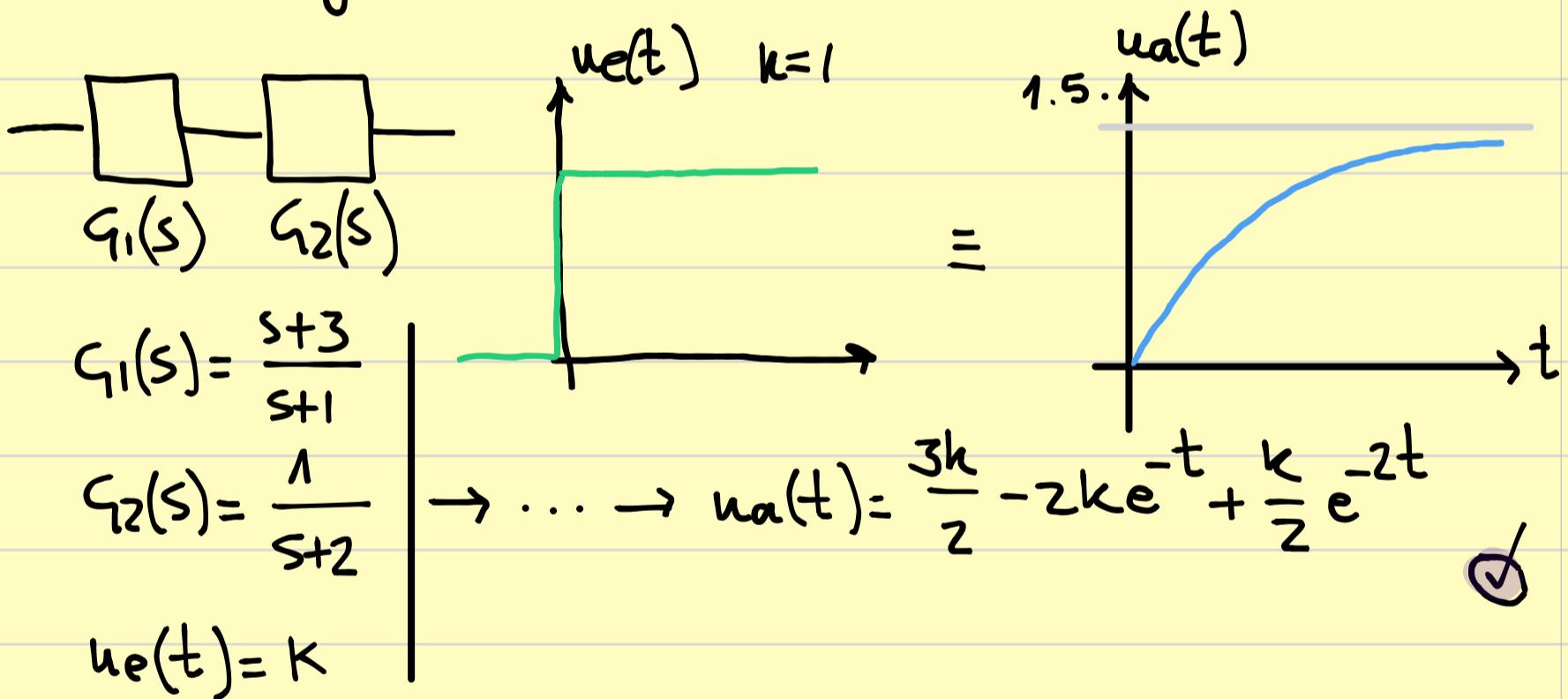
$$\rightarrow \varphi = \operatorname{atan} \left( \frac{\operatorname{Im}|z|}{\operatorname{Re}|z|} \right)$$

$$z = \frac{1}{b+jc} \rightarrow z = \frac{1}{b+jc} \cdot \frac{b-jc}{b-jc} = \frac{b-jc}{b^2+c^2} = \frac{1}{b^2+c^2} (b-jc)$$

## 4 Klassifizierung von Regelkreisgliedern



siehe Vorlesung 20231017



$$G(s) = G_1(s) \cdot G_2(s) = \frac{s+3}{(s+1)(s+2)}$$

BODE DIAGRAMM ...

Schritt 1.  $G(s)$  in dem Frequenzgang

$$\begin{aligned}
 G(j\omega) &= \frac{j\omega + 3}{(j\omega + 1)(j\omega + 2)} = \frac{3 + j\omega}{-\omega^2 + 3j\omega + 2} = \\
 &= \frac{3 + j\omega}{(2 - \omega^2) + 3j\omega} \cdot \frac{(2 - \omega^2) - 3j\omega}{(2 - \omega^2) - 3j\omega} = \\
 &= \frac{6 - 3\omega^2 - 9j\omega + 2j\omega - j\omega^3 + 3\omega^2}{(2 - \omega^2)^2 + 9\omega^2} = \\
 &= \frac{6 - 7j\omega - j\omega^3}{4 - 4\omega^2 + \omega^4 + 9\omega^2} =
 \end{aligned}$$

$$G(j\omega) = \frac{1}{\omega^4 + 5\omega^2 + 4} \cdot [6 - j\omega(7 - \omega^2)]$$

$$|z| = \sqrt{\operatorname{Re}|z|^2 + \operatorname{Im}|z|^2}$$

$$\operatorname{Re}(G(j\omega)) = \frac{6}{\omega^4 + 5\omega^2 + 4}; \quad \operatorname{Im}(G(j\omega)) = \frac{-\omega(7 - \omega^2)}{\omega^4 + 5\omega^2 + 4}$$

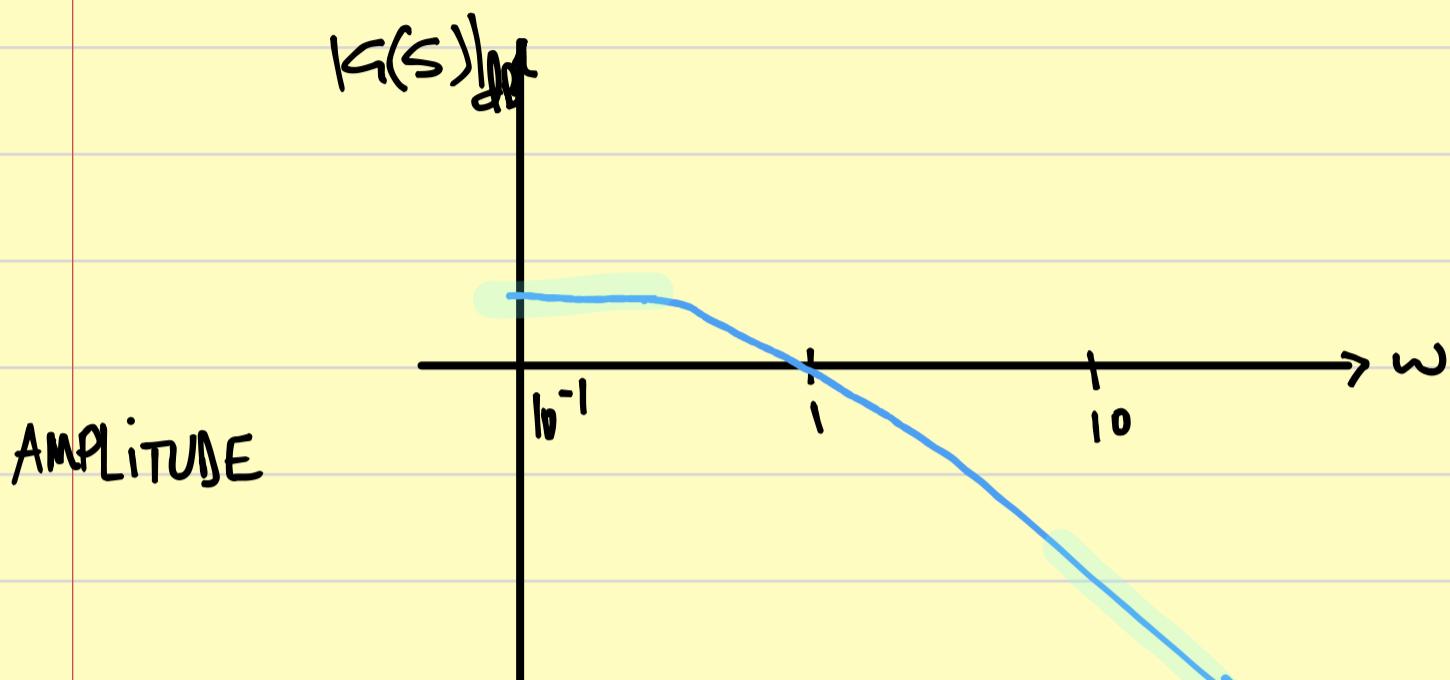
$$|G(j\omega)| = \sqrt{\frac{6^2}{(\omega^4 + 5\omega^2 + 4)^2} - \frac{(\omega(7 - \omega^2))^2}{(\omega^4 + 5\omega^2 + 4)^2}} =$$

$$= \frac{1}{\omega^4 + 5\omega^2 + 4} \sqrt{6^2 - \omega^2(49 - 14\omega^2 + \omega^4)}$$

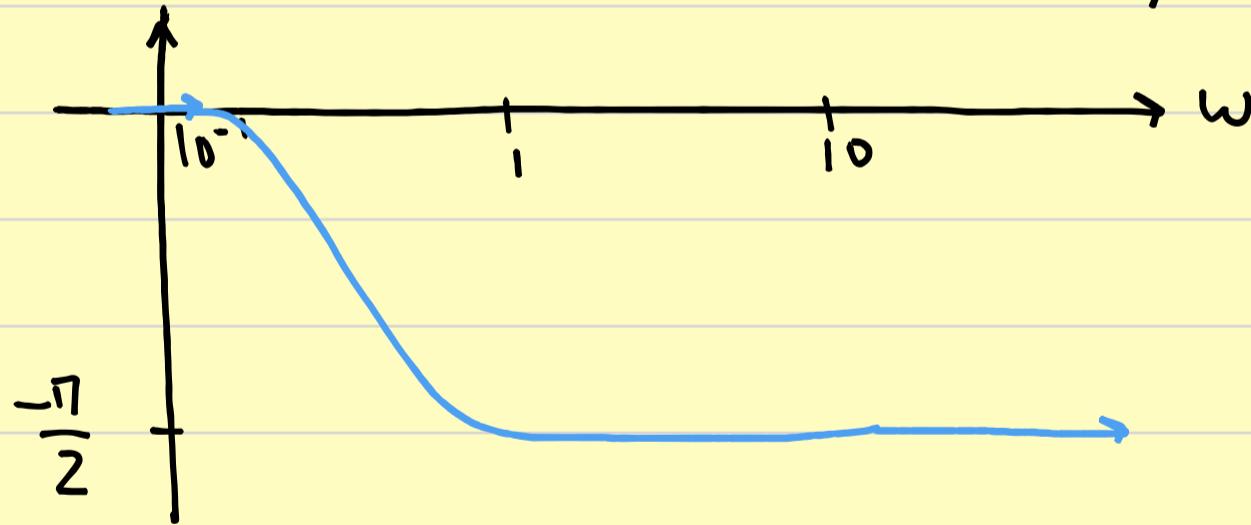
$$= \frac{1}{\omega^4 + 5\omega^2 + 4} \sqrt{\omega^6 + 14\omega^4 - 49\omega^2 + 36}$$

$$|G(w=0)| = 1^{\circ} 5 \rightarrow |G(w=0)|_{dB} = 20 \log 1^{\circ} 5$$

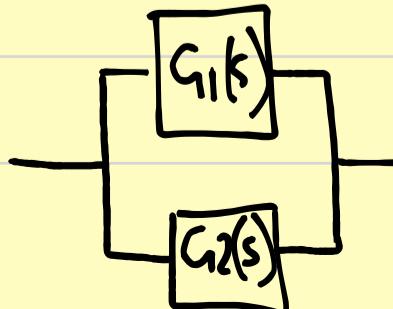
$$|G(w=\infty)| \approx -\frac{1}{\omega} \rightarrow -\infty$$



$$\varphi = \text{atan} \left( \frac{\text{Im}(G(j\omega))}{\text{Re}(G(j\omega))} \right) = \text{atan} \left( \frac{-\omega(7-\omega^2)}{6} \right)$$



Übung . bitte die obere Aufgabe mit Parallelschaltung der Glieder lösen ...



$$G_1(s) = \frac{s+3}{s+1} ; G_2(s) = \frac{1}{s+2} ; u_e(t) = k$$

$$G(s) = G_1(s) + G_2(s) = \frac{s+3}{s+1} + \frac{1}{s+2} = \frac{(s+3)(s+2) + s+1}{(s+1)(s+2)}$$

$$u_a(s) = u_e(s) \cdot G(s) = \frac{k}{s} \cdot \frac{(s+3)(s+2) + s+1}{(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

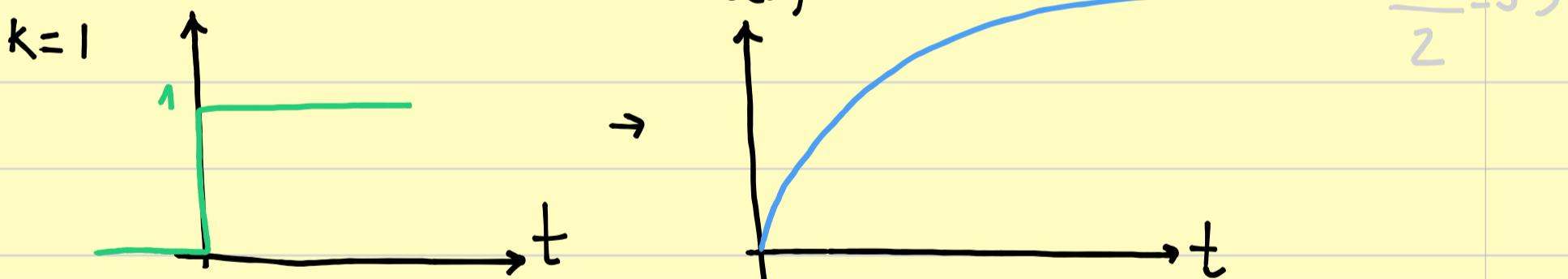
$$k \cdot [(s+3)(s+2) + s+1] = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

$$s=0 \rightarrow k \cdot 7 = A \cdot 2 \rightarrow A = \frac{7k}{2}$$

$$s=-1 \rightarrow k \cdot 2 = B \cdot (-1) \rightarrow B = -2k$$

$$s=-2 \rightarrow k \cdot (-1) = C \cdot 2 \rightarrow C = \frac{-k}{2}$$

$$u_a(t) = \mathcal{L}^{-1}(u_a(s)) = \frac{7k}{2} e^{-t} - 2k e^{-t} - \frac{k}{2} e^{-2t}$$



### BODE DIAGRAMM

$$G(s) = \frac{(s+3)(s+2) + s+1}{(s+1)(s+2)} = \frac{s^2 + 6s + 7}{s^2 + 3s + 2}$$

$$G(j\omega) = \frac{(-\omega^2 + 7) + 6j\omega}{(-\omega^2 + 2) + 3j\omega} \cdot \frac{(-\omega^2 + 2) - 3j\omega}{(-\omega^2 + 2) - 3j\omega} =$$

$$= \frac{(-\omega^2 + 7)(-\omega^2 + 2) + 18\omega^2 + j[-\omega^2 + 2] - 3(-\omega^2 + 7)}{(-\omega^2 + 2)^2 + 9\omega^2}$$

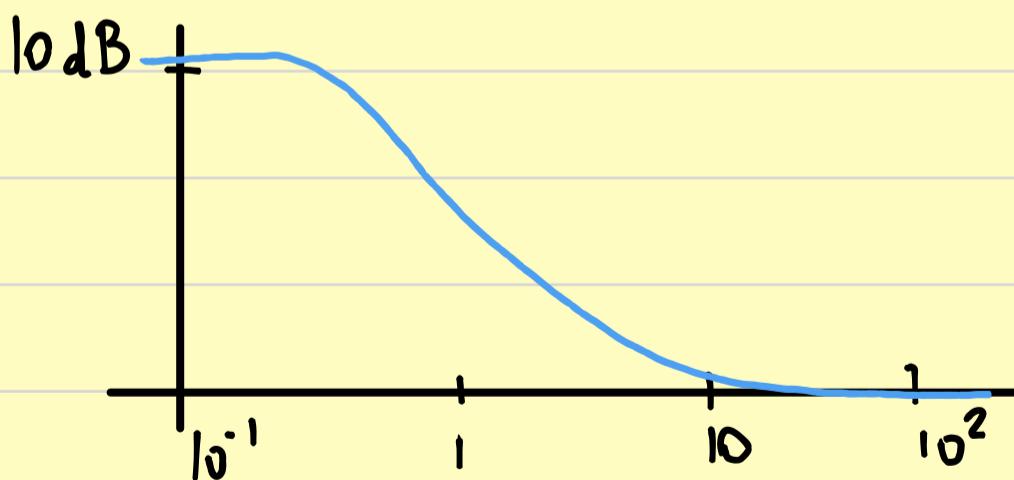
$$= \frac{\omega^4 - 9\omega^2 + 14 + 18\omega^2 + j[6\omega^2 + 12 + 3\omega^2 - 21]}{\omega^4 - 4\omega^2 + 4 + 9\omega^2} =$$

$$G(j\omega) = \frac{\omega^4 + 9\omega^2 + 14}{\omega^4 + 5\omega^2 + 4} + j \frac{-3\omega^2 - 9}{\omega^4 + 5\omega^2 + 4}$$

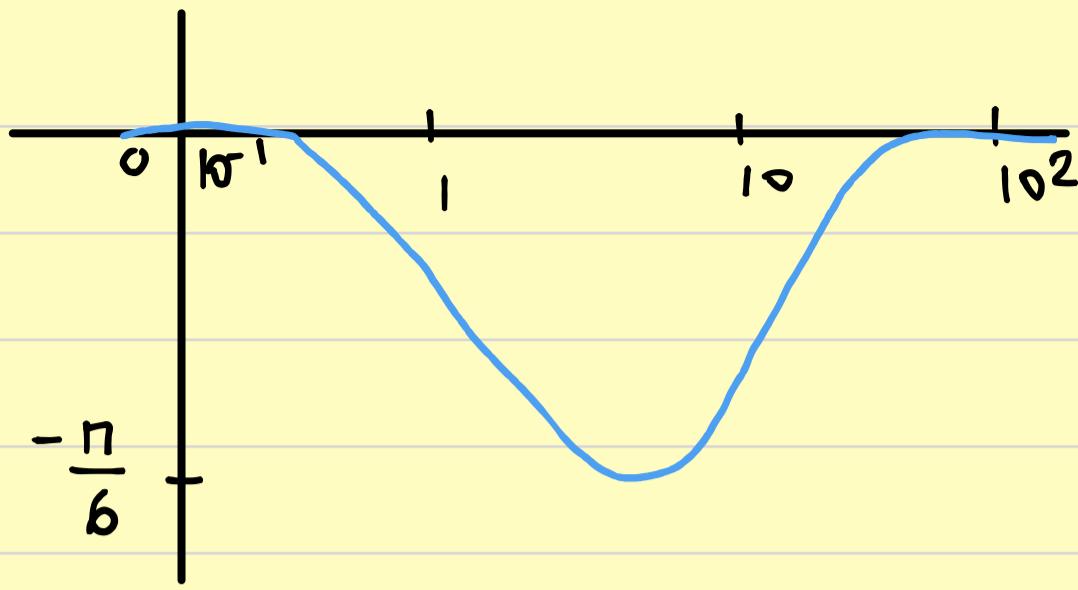
Re(G(j\omega))      Im(G(j\omega))

$$|G(j\omega)| = \sqrt{\operatorname{Re}(G(j\omega))^2 + \operatorname{Im}(G(j\omega))^2} =$$

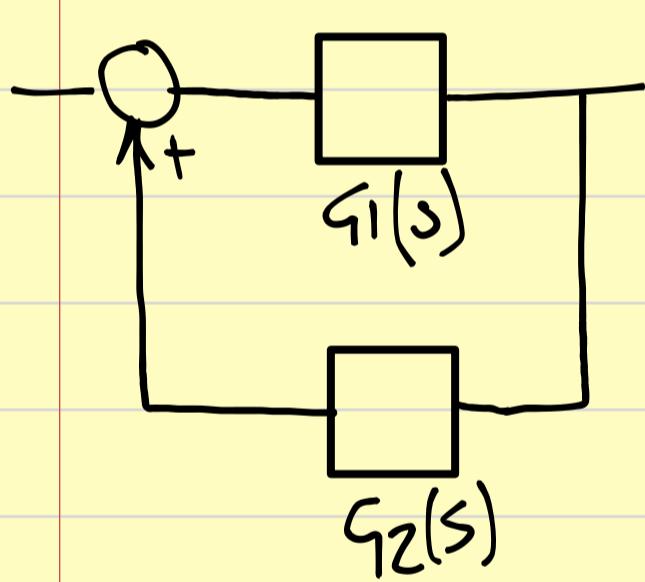
$$= \frac{1}{\omega^4 + 5\omega^2 + 4} \sqrt{(\omega^4 + 9\omega^2 + 14)^2 - (-3\omega^2 - 9)^2} =$$



$$\varphi = \operatorname{atan} \left( \frac{\operatorname{Im} G(j\omega)}{\operatorname{Re} G(j\omega)} \right) = \operatorname{atan} \left( \frac{-3\omega^2 - 9}{\omega^4 + 9\omega^2 + 14} \right)$$



Übung: die oberen Gliedern in Rückkopplungsschaltung setzen und  $u_a(t)$  bzw. Bode-Diagramm ermitteln.  
(Gegenkopplung)



$$G(s) = \frac{G_1(s)}{1 - G_1(s)G_2(s)} = \frac{\frac{s+3}{s+1}}{1 - \frac{s+3}{s+1} \cdot \frac{1}{s+2}}$$

$$u_a(s) = u_e(s) \cdot G(s) = \frac{\frac{k}{s} \cdot \frac{s+3}{s+1}}{\frac{(s+1)(s+2) - (s+3)}{(s+1)(s+2)}} = \frac{\frac{k}{s} \cdot (s+3)(s+2)}{(s+1)(s+2) - (s+3)} =$$

$$= \frac{k \cdot (s+3)(s+2)}{s[s^2 + 3s + 2 - s - 3]} = \frac{k(s+3)(s+2)}{(s^2 + 2s - 1)s}$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow s = \frac{-2 \pm \sqrt{4 + 4}}{2} = \begin{cases} -1 + 2\sqrt{2} = s_1 \\ -1 - 2\sqrt{2} = s_2 \end{cases}$$

$$s_1 = 1'8, s_2 = -3'8$$

$$u_a(s) = \frac{k(s+3)(s+2)}{(s^2 + 2s - 1)s} = \frac{k(s+3)(s+2)}{(s-s_1)(s-s_2)s} = \frac{k(s+3)(s+2)}{(s-1'8)(s+3'8)s}$$

$$= \frac{A}{s} + \frac{B}{(s-1'8)} + \frac{C}{(s+3'8)}$$

$$k(s+3)(s+2) = A(s-1'8)(s+3'8) + Bs(s+3'8) + Cs(s-1'8)$$

$$s=0 \rightarrow k \cdot 6 = A(-6'84) \rightarrow A = -0'87k$$

$$s=1'8 \rightarrow k \cdot 18'24 = B \cdot 10'08 \rightarrow B = 1'81k$$

$$s=-3'8 \rightarrow k \cdot 1'44 = C \cdot (-5'6) \rightarrow C = -0'26k$$

$$u_a(t) = -0'87k + 1'81k e^{1'8t} - 0'26k e^{-3'8t}$$

$$G(s) = \frac{\frac{s+3}{s+1}}{1 - \frac{s+3}{s+1} \cdot \frac{1}{s+2}} = \frac{\frac{s+3}{s+1}}{(s+1)(s+2) - (s+3)} = \frac{(s+3)}{(s+1) \cdot (s^2 + 2s - 1)} =$$

$$= \frac{s+3}{(s+1)(s-1'8)(s+3'8)}$$

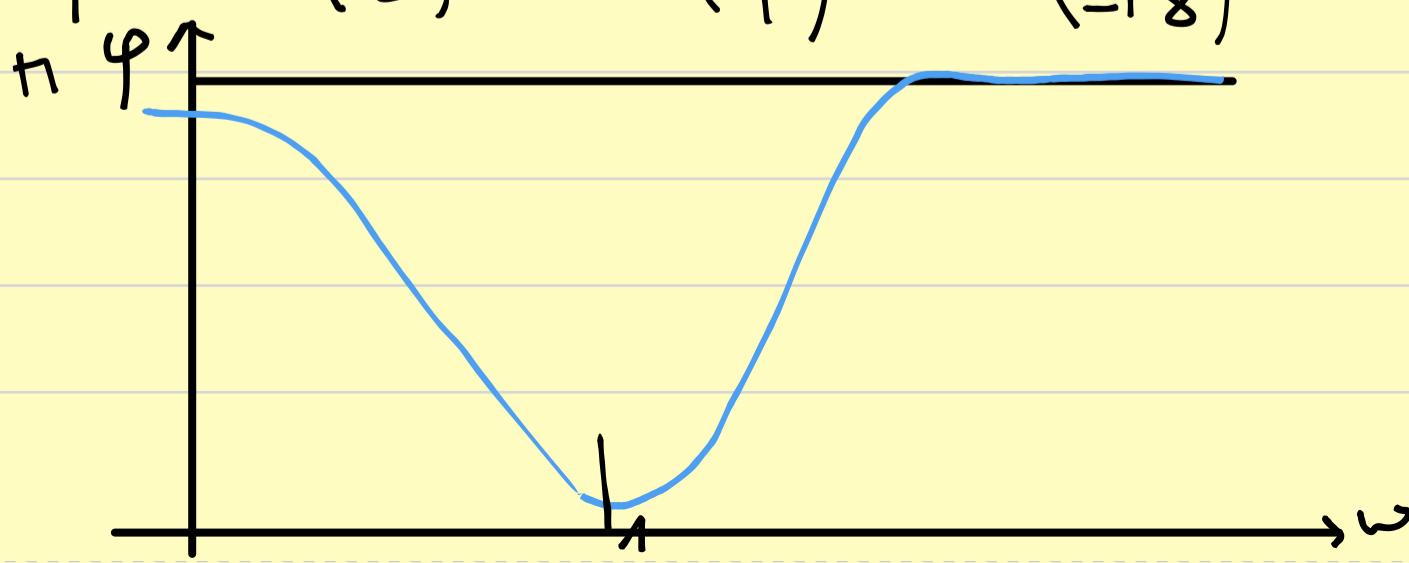
$$G(j\omega) = \frac{j\omega + 3}{(j\omega + 1)(j\omega - 1'8)(j\omega + 3'8)}$$

$$|G(j\omega)| = \frac{|j\omega + 3|}{|j\omega + 1||j\omega - 1'8||j\omega + 3'8|} = \\ = \frac{\sqrt{9 + \omega^2}}{\sqrt{1 + \omega^2} \sqrt{1'8^2 + \omega^2} \sqrt{3'8^2 + \omega^2}}$$

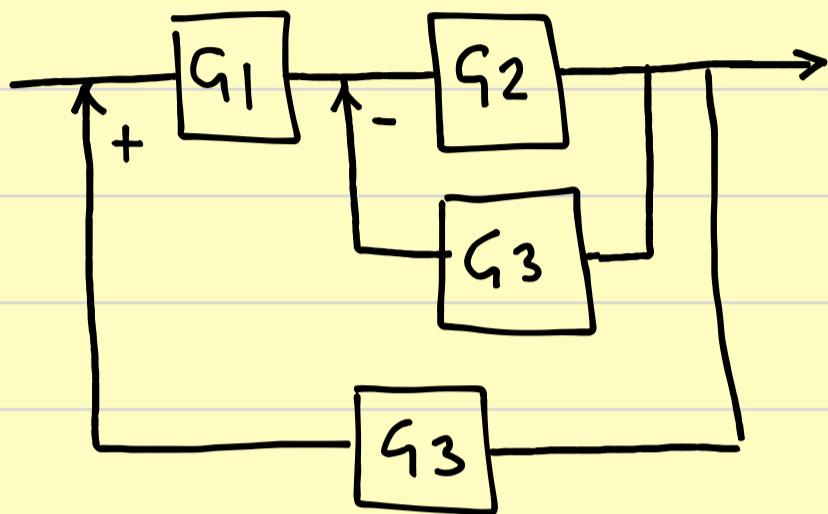
$$|G(j\omega)|_{dB} = 20 \cdot \frac{1}{2} \left[ \log(9 + \omega^2) - \log(1 + \omega^2) - \log(1'8^2 + \omega^2) - \log(3'8^2 + \omega^2) \right]$$



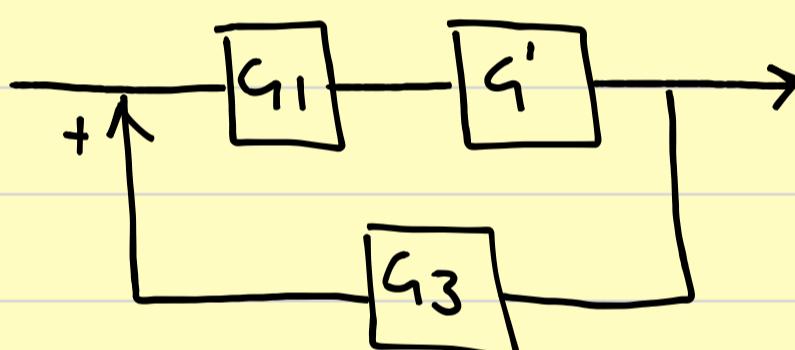
$$\varphi = \text{atan}\left(\frac{\omega}{3}\right) - \text{atan}\left(\frac{\omega}{1}\right) - \text{atan}\left(\frac{\omega}{-1'8}\right) - \text{atan}\left(\frac{\omega}{3'8}\right)$$



Übung. bitte ermitteln Sie die u(t) und das Bode-Diagramm  
 wenn  $G_1 = \frac{1}{1+2s}$ ,  $G_2 = \frac{1}{1+s}$ ,  $G_3 = \frac{1}{2+s}$  in folgender  
 Konfiguration:

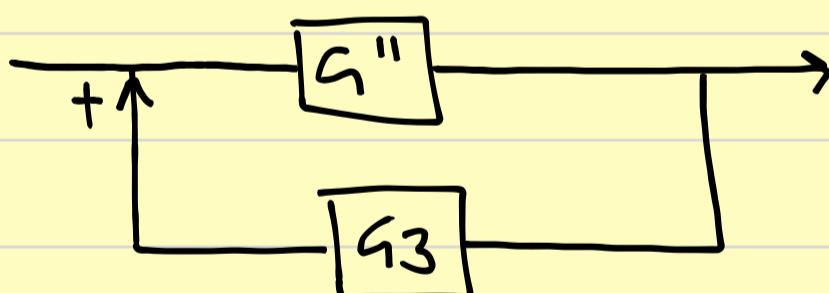


|||



$$G' = \frac{G_2}{1+G_2G_3}$$

|||



$$G'' = \frac{G_1 G_2}{1+G_2G_3}$$

$$= G_1 \cdot G'$$

|||



$$G''' = \frac{G_1 G_2}{1+G_2G_3} \cdot \frac{1 - \frac{G_1 G_2 G_3}{1+G_2G_3}}{1+G_2G_3}$$

$$G''' = \frac{\frac{G_1 G_2}{1+G_2 G_3}}{\frac{1+G_2 G_3 - G_1 G_2 G_3}{1+G_2 G_3}} = \frac{G_1 G_2}{1+G_2 G_3 - G_1 G_2 G_3}$$

$$\left. \begin{array}{l} G_1 = \frac{1}{1+2s} \\ G_2 = \frac{1}{1+s} \\ G_3 = \frac{1}{2+s} \end{array} \right| \quad G''' = \frac{\frac{1}{1+2s} \cdot \frac{1}{1+s}}{1 + \frac{1}{1+s} \cdot \frac{1}{2+s} - \frac{1}{1+2s} \cdot \frac{1}{1+s} \cdot \frac{1}{2+s}} =$$

$$= \frac{\frac{1}{(1+2s)(1+s)}}{(1+2s)(1+s)(2+s) + (1+2s) - 1} =$$

$$= \frac{(2+s)}{(1+2s)(1+s)(2+s) + 2s} = \dots$$

