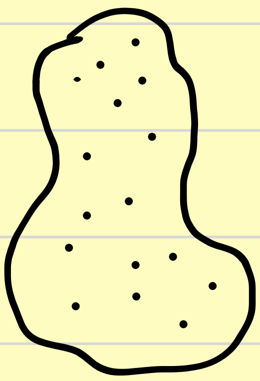
Dynamische Systeme 1D.



N

- $I(t)$ beschreibt die Anzahl mit Chlamydien infizierten Studierenden.
- Die Funktion $I(t)$ wird normiert auf $i(t) = \frac{I(t)}{N}$ damit $i(t) \in [0,1]$ bleibt.

- Es gibt zwei dynamische Effekte:
  - ANSTECKUNG.
  - GENESSUNG.

$$\boxed{\frac{di(t)}{dt} = \text{ANSTECKUNGSRATE} - \text{GENESSUNGSRATE}}$$

- ANSTECKUNG: β. WIEVIELE GESUNDE SICH PRO ZEITEINHEIT ANSTECKEN.
  $i \cdot (1-i)$. WAHRSCHEINLICHKEIT DAFÜR, DASS GESUND TRIFFT INFIZIERTER

ANSTECKUNGSRATE: $\beta i (1-i)$

- GENESSUNG: γ. WIE SCHNELL WERDE ICH GESUND

GENESSUNGSRATE: $\gamma \cdot i$

$$\frac{di(t)}{dt} = \beta i (1-i) - \gamma i = (\beta - \gamma) i - \beta i^2$$

$$\frac{di(t)}{(\beta-\gamma)i-\beta i^2} = dt \longrightarrow \frac{di(t)}{i(\beta-\gamma-\beta i)} = dt$$

$$\longrightarrow \int \frac{di(t)}{\beta i\left(\frac{\beta-\gamma}{\beta}-i\right)} = \int dt$$

Trennung der Veränderlichen: $\int \frac{dx}{x-1} = \ln|x-1| + c$

$$\boxed{\frac{1}{\beta i\left(\frac{\beta-\gamma}{\beta}-i\right)}} = \boxed{\frac{A}{i} + \frac{B}{\left(\frac{\beta-\gamma}{\beta}-i\right)}} = \frac{A\left(\frac{\beta-\gamma}{\beta}-i\right)+Bi}{i\left(\frac{\beta-\gamma}{\beta}-i\right)}$$

$$1 = A\left(\frac{\beta-\gamma}{\beta}-i\right) + Bi$$

$$i=\frac{\beta-\gamma}{\beta} \longrightarrow 1 = A\cancel{\left(\frac{\beta-\gamma}{\beta}-\frac{\beta-\gamma}{\beta}\right)} + B\cdot\frac{\beta-\gamma}{\beta} \longrightarrow B=\frac{\beta}{\beta-\gamma}$$

$$i=0 \longrightarrow 1 = A\left(\frac{\beta-\gamma}{\beta}-0\right) + \cancel{B\cdot 0} \longrightarrow A=\frac{\beta}{\beta-\gamma}$$

$$\boxed{\frac{1}{\beta i\left(\frac{\beta-\gamma}{\beta}-i\right)}} = \frac{\beta}{\beta-\gamma}\cdot\frac{1}{i} + \frac{\beta}{\beta-\gamma}\cdot\frac{1}{\frac{\beta-\gamma}{\beta}-i}$$

$$\int \frac{\beta}{\beta-\gamma}\left[\frac{1}{i}+\frac{1}{\frac{\beta-\gamma}{\beta}-i}\right]di(t) = \int dt \longrightarrow$$

$$\rightarrow \frac{\beta}{\beta-\gamma}\left[\ln|i| - \ln\left|\frac{\beta-\gamma}{\beta} - i\right|\right] = t + C$$

$$\rightarrow \ln|i| - \ln\left|\frac{\beta-\gamma}{\beta} - i\right| = \frac{\beta-\gamma}{\beta}t + c$$

exponentieeren:

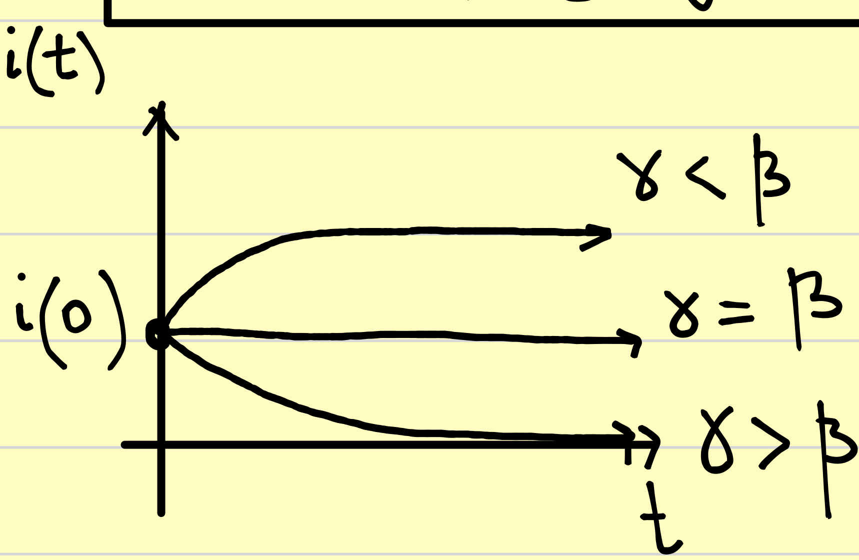
$$\frac{i}{\frac{\beta-\gamma}{\beta} - i} = e^{\frac{\beta-\gamma}{\beta}\cdot t} + C$$

$$i = \left(\frac{\beta-\gamma}{\beta} - i\right)\cdot\left(e^{\frac{\beta-\gamma}{\beta}t} + C\right)$$

$$= \frac{\beta-\gamma}{\beta}\cdot e^{\frac{\beta-\gamma}{\beta}t} - i\cdot e^{\frac{\beta-\gamma}{\beta}\cdot t} + \frac{\beta-\gamma}{\beta}\cdot c - i\cdot C \rightarrow$$

$$\rightarrow i\left[1 + e^{\frac{\beta-\gamma}{\beta}\cdot t} + c\right] = \frac{\beta-\gamma}{\beta}\cdot e^{\frac{\beta-\gamma}{\beta}t} + \frac{\beta-\gamma}{\beta}\cdot c$$

$$\boxed{i(t) = \frac{\frac{\beta-\gamma}{\beta}\cdot e^{\frac{\beta-\gamma}{\beta}t} + \frac{\beta-\gamma}{\beta}\cdot c}{1 + C + e^{\frac{\beta-\gamma}{\beta}t}}}$$



Beispiel: $\gamma = 2 \quad \beta = 3$

$$\frac{di(t)}{dt} = 3i(1-i) - 2i = 3i - 3i^2 - 2i = i - 3i^2 \rightarrow$$

$$\rightarrow \frac{di(t)}{i-3i^2} = dt \rightarrow \frac{di(t)}{i(1-3i)} = dt$$

$$\frac{1}{i(1-3i)} = \frac{1}{i \cdot 3\left(\frac{1}{3}-i\right)} = \frac{A}{i} + \frac{B}{\frac{1}{3}-i}$$

$$\frac{1}{3} = A\left(\frac{1}{3}-i\right) + B \cdot i$$

$$i = \frac{1}{3} \rightarrow \frac{1}{3} = A \cdot \left(\frac{1}{3} - \frac{1}{3}\right) + B \cdot \frac{1}{3} \rightarrow B = 1$$

$$i = 0 \rightarrow \frac{1}{3} = A \cdot \left(\frac{1}{3} - 0\right) + B \cdot 0 \rightarrow A = 1$$

$$\frac{1}{i(1-3i)} = \frac{1}{i} + \frac{1}{\frac{1}{3}-i}$$

$$\int \frac{di}{i(1-3i)} = \int dt \rightarrow \int \frac{di}{i} + \int \frac{di}{\frac{1}{3}-i} = \int dt \rightarrow$$

$$\rightarrow \ln|i| - \ln\left|\frac{1}{3}-i\right| = t + c \rightarrow$$

$$\rightarrow \frac{i}{\frac{1}{3}-i} = e^t + c \rightarrow i = \left(\frac{1}{3}-i\right)\left(e^t + c\right)$$

$$\rightarrow \quad i = \frac{1}{3}e^{t} + \frac{1}{3}\cdot c - i\,e^{t} - i\cdot c$$

$$\rightarrow \quad i\left[1 + e^{t} + c\right] = \frac{1}{3}e^{t} + \frac{1}{3}c$$

$$\rightarrow \quad i(t) = \frac{\frac{1}{3}e^{t} + \frac{1}{3}c}{1 + c + e^{t}}$$

$$\lim(t \to \infty) = \frac{1}{3}$$

$$t = 0 \rightarrow i(t=0) = \frac{\frac{1}{3} + \frac{1}{3}c}{1 + c + 1}$$

Beispiel: $\beta = 2 \quad \gamma = 3$

$$\frac{di(t)}{dt} = 2i(1-i) - 3i = 2i - 2i^2 - 3i = -i - 2i^2$$

$$\frac{-di(t)}{i(1+2i)} = dt$$

$$\frac{-1}{i(1+2i)} = \frac{-1}{2i\left(\frac{1}{2}+i\right)} = \frac{A}{i} + \frac{B}{\frac{1}{2}+i}$$

$$-\frac{1}{2} = A\left(\frac{1}{2}+i\right) + B\,i$$

$$i = -\frac{1}{2} \rightarrow -\frac{1}{2} = \cancel{A\left(\frac{1}{2} - \frac{1}{2}\right)} + B\left(-\frac{1}{2}\right) \rightarrow B = 1$$

$$i = 0 \rightarrow -\frac{1}{2} = A\cdot\frac{1}{2} + B\cdot 0 \longrightarrow A = -1$$

$$\int \frac{-di(t)}{i(1+2i)} = \int -\frac{1}{i}\,di + \int \frac{1}{\frac{1}{2}+i}\,di = \int dt \longrightarrow$$

$$\longrightarrow -\ln|i| + \ln\left|\frac{1}{2}+i\right| = t + c \longrightarrow$$

$$\longrightarrow \frac{i}{\frac{1}{2}+i} = e^{t} + C \longrightarrow i = \frac{1}{2}e^{t} + \frac{1}{2}C + i\left[e^{t}+C\right]$$

$$\longrightarrow \quad i(t) = \frac{\frac{1}{2}e^{t} + \frac{1}{2}C}{1 - e^{t} - c}$$