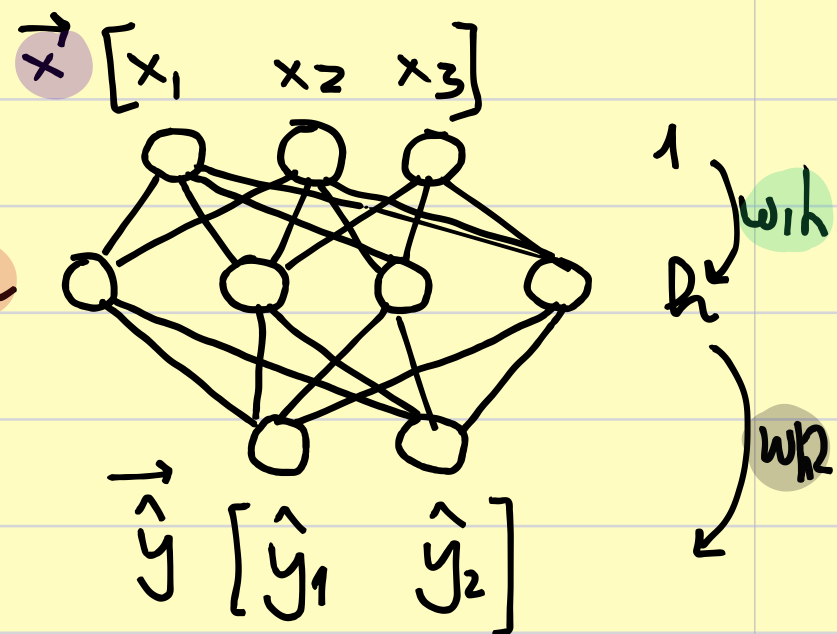
Beispiel 3. 3 Layer Perceptron · Forward pass [3,4,2]
Hidden Layer mit 4 Neuronen.

$\vec{x}$ [1x3]: $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ ← x_1, x_2, x_3

[3x4] w_{1h} [1x4] b_1

$$\begin{array}{|c|c|c|c|}\hline 1 & -1 & 1 & -5 \\ \hline 1 & 1 & 0 & 0 \\ \hline 0 & 1 & 1 & 1 \\ \hline 1 & 0 & 1 & -2 \\ \hline \end{array} \quad \begin{bmatrix} -1 \\ 3 \\ 5 \\ 3 \end{bmatrix} \xrightarrow{\text{ReLU}} \begin{bmatrix} 0 \\ 3 \\ 5 \\ 3 \end{bmatrix} \quad \hat{h}\,[1\times 4]$$



[4x2] w_{h2} [1x2] b_2 $\vec{\hat{y}}$

$$\begin{array}{|c|c|c|c|c|}\hline 1 & 1 & -1 & 0 & 0 \\ \hline 0 & 0 & 1 & -1 & 1 \\ \hline \end{array} \quad \begin{bmatrix} -2 \\ 3 \end{bmatrix} \xrightarrow{\text{ReLU}} \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$2 \cdot 1 + 1 \cdot (-1) + 3 \cdot 1 + (-5) = -1$
$2 \cdot 1 + 1 \cdot 1 + 3 \cdot 0 + 0 = 3$
$2 \cdot 0 + 1 \cdot 1 + 3 \cdot 1 + 1 = 5$
$2 \cdot 1 + 1 \cdot 0 + 3 \cdot 1 + (-2) = 3$

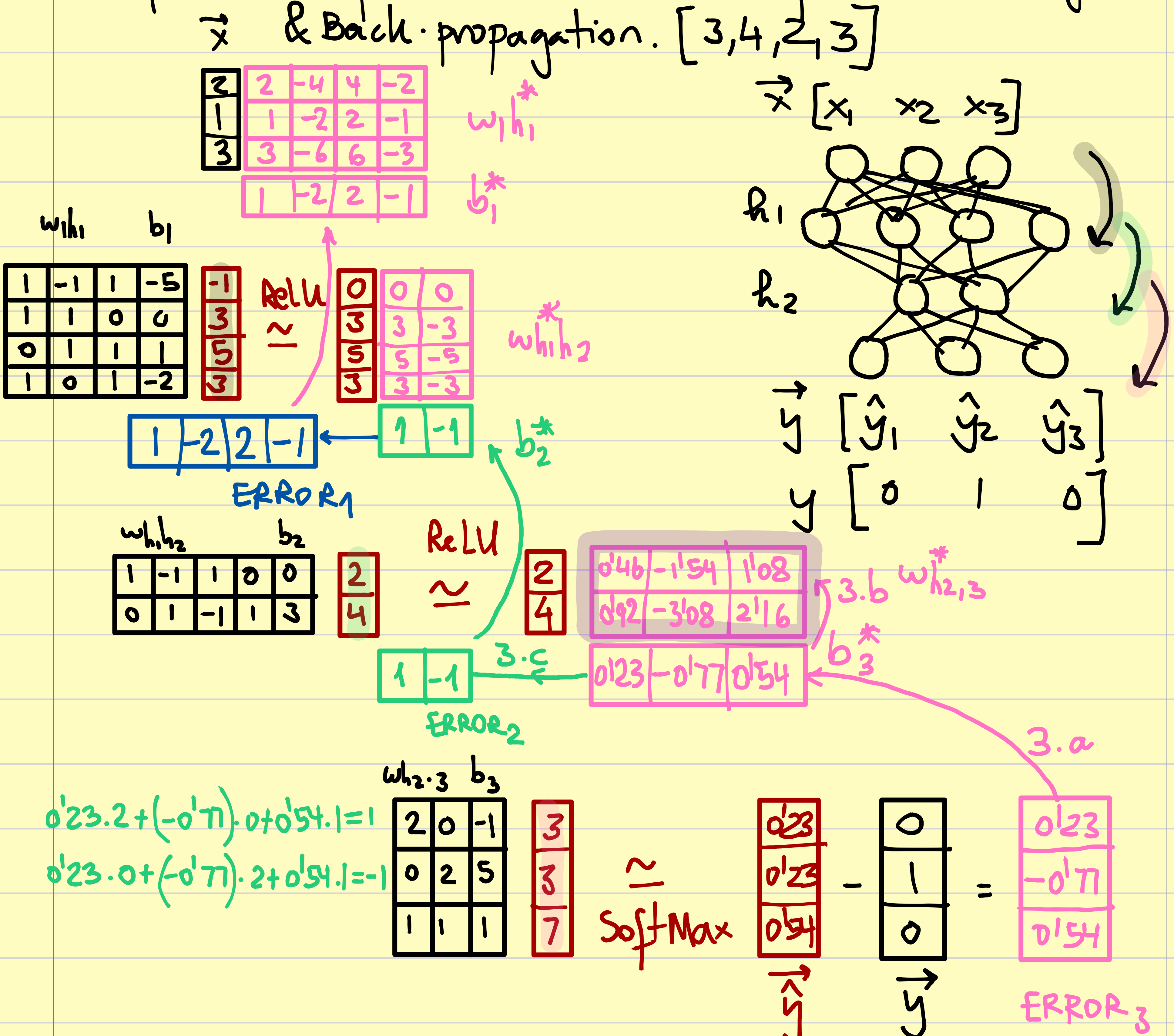
$0 \cdot 1 + 3 \cdot 1 + 5 \cdot (-1) + 3 \cdot 0 + 0 = -2$
$0 \cdot 0 + 3 \cdot 0 + 5 \cdot 1 + 3 \cdot (-1) + 1 = 3$

Entscheidungsfunktion: [letzten Layer]

$$\text{SOFTMAX}(\vec{x}): \quad \vec{x} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \to \text{SOFTMAX}(\vec{x}) = \begin{bmatrix} \frac{2}{2+1+3} \\ \frac{1}{2+1+3} \\ \frac{3}{2+1+3} \end{bmatrix} = \begin{bmatrix} 0'33 \\ 0'16 \\ 0'5 \end{bmatrix}$$

Nach der Umsetzung von Softmax, die Summe der Vektorelementen ergibt 1: wir kriegen eine Wahrscheinlichkeits distribution: erster Element x_1 hat eine Wahrscheinlichkeit von 0'33 einzutreffen, x_2 von 0'16, und x_3 von 0'5.

Beispiel 4. Perceptron mit 2 hidden Layers with Feedforward & Back·propagation. [3,4,2,3]



1. Feed forward Pass.

xh_1: $2 \cdot 1 + 1(-1) + 3 \cdot 1 + (-5) = \boxed{-1}$

$2.1 + 1.1 + 3.0 + 0 = 3$

$2.0 + 1.1 + 3.1 + 1 = 5$

$2.1 + 1.0 + 3.1 + (-2) = 3$

$h_1 h_2$: $0.1 + 3.(-1) + 5.1 + 3.0 + 0 = \boxed{2}$

$0.0 + 3.1 + 5.(-1) + 3.1 + 3 = 4$

$h_2 y$: $2.2 + 4.0 + (-1) = \boxed{3}$

$2.0 + 4.2 + (-5) = 3$

$2.1 + 4.1 + 1 = 7$

2. SoftMax Decission Layer.

$$\text{SoftMax}\left(\begin{bmatrix} 3 \\ 3 \\ 7 \end{bmatrix}\right) = \begin{bmatrix} \frac{3}{3+3+7} \\ \frac{3}{3+3+7} \\ \frac{7}{3+3+7} \end{bmatrix} = \begin{bmatrix} \frac{3}{13} \\ \frac{3}{13} \\ \frac{7}{13} \end{bmatrix} = \begin{bmatrix} 0'23 \\ 0'23 \\ 0'54 \end{bmatrix}$$

3. Back Propagation.

a) Transpose the error vector as new bias.

b) We DOT. PRODUCT of the transposed error and the output of the previous layer.

$w^*_{h_2,3}$: $2.0'23 = 0'46$

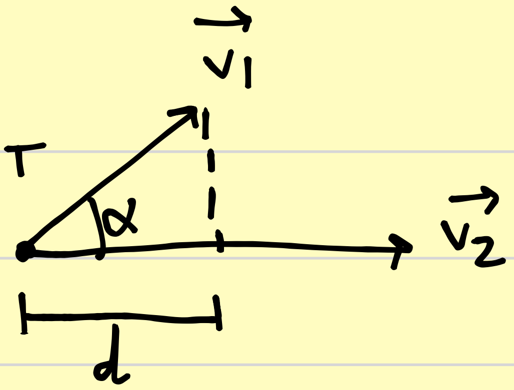
DOT PRODUCT

$2.(-0'77) = -1'54$

$2.(0'54) = 1'08$

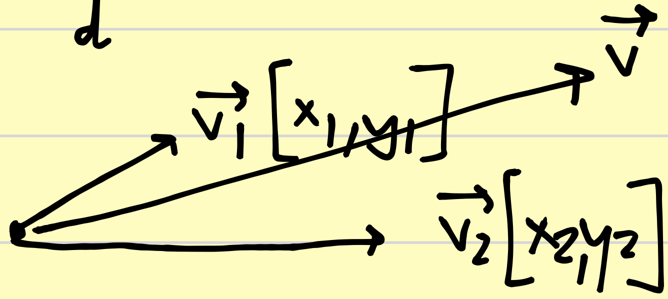
c) We multiply the transposed error with the weight matrix of the current Layer.

SCALAR PRODUCT



$$\vec{v_1} \cdot \vec{v_2} = |\vec{v_1}||\vec{v_2}| \cdot \cos\alpha = d$$

DOT PRODUCT



$$\vec{v_1} \odot \vec{v_2} = [x_1 \cdot x_2 \, , y_1 y_2] = \vec{v}$$

$$\vec{x} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\overset{*}{w_1 h_1}$ $\quad$ $\overset{*}{b_1}$

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ -4 & -2 & -6 & -2 \\ 4 & 2 & 6 & 2 \\ 2 & -1 & 3 & -1 \end{bmatrix} \begin{bmatrix} 15 \\ -30 \\ 30 \\ 11 \end{bmatrix} \underset{\sim}{\text{ReLu}} \begin{bmatrix} 15 \\ 0 \\ 30 \\ 11 \end{bmatrix}$$

$\overset{*}{w_{h_1 h_2}}$ $\quad$ $\overset{*}{b_2}$

$$\begin{bmatrix} 0 & 3 & 5 & 3 & 1 \\ 0 & -3 & -5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 184 \\ -184 \end{bmatrix} \underset{\sim}{\text{ReLu}} \begin{bmatrix} 184 \\ 0 \end{bmatrix}$$

$\overset{*}{w_{h23}}$

$$\begin{bmatrix} 0'46 & 0'92 & 0'23 \\ -1'54 & -3'08 & -0'77 \\ 1'08 & 2'16 & 0'54 \end{bmatrix} \begin{bmatrix} 84'87 \\ -284'13 \\ 199'26 \end{bmatrix} \underset{\text{Soft Max}}{\sim} \underbrace{\begin{bmatrix} 0'15 \\ 0'5 \\ 0'35 \end{bmatrix}}_{\hat{y}^*} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 0'15 \\ -0'5 \\ 0'35 \end{bmatrix}}_{\text{ERROR}_3^*}$$

$$\text{SoftMax} \begin{bmatrix} 84'87 \\ -284'13 \\ 199'26 \end{bmatrix} = \frac{1}{586'26} \begin{bmatrix} 84'87 \\ 284'13 \\ 199'26 \end{bmatrix} = \begin{bmatrix} 0'15 \\ 0'5 \\ 0'35 \end{bmatrix}$$

$84'87 + 284'13 + 199'26 = 586'26$