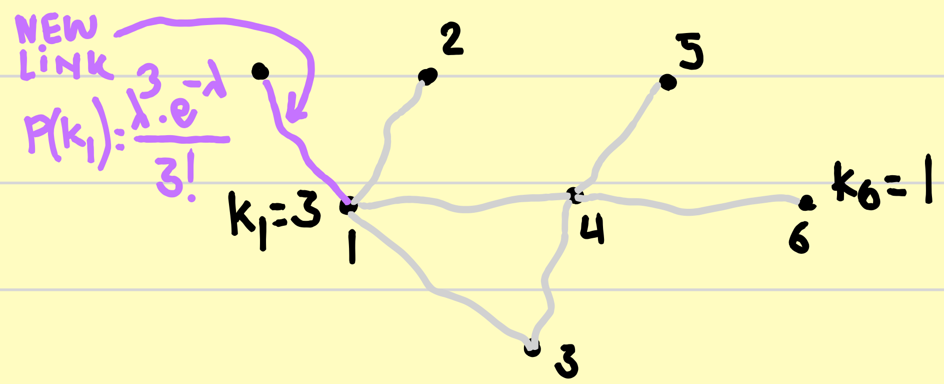
RANDOM NETWORKS

Remainder from statistics



- Given a random variable (X), it is described by a Poisson distribution.
- The probability that a node has "K" neighbours in a rando network is given by a Poisson distribution with parameter "λ":

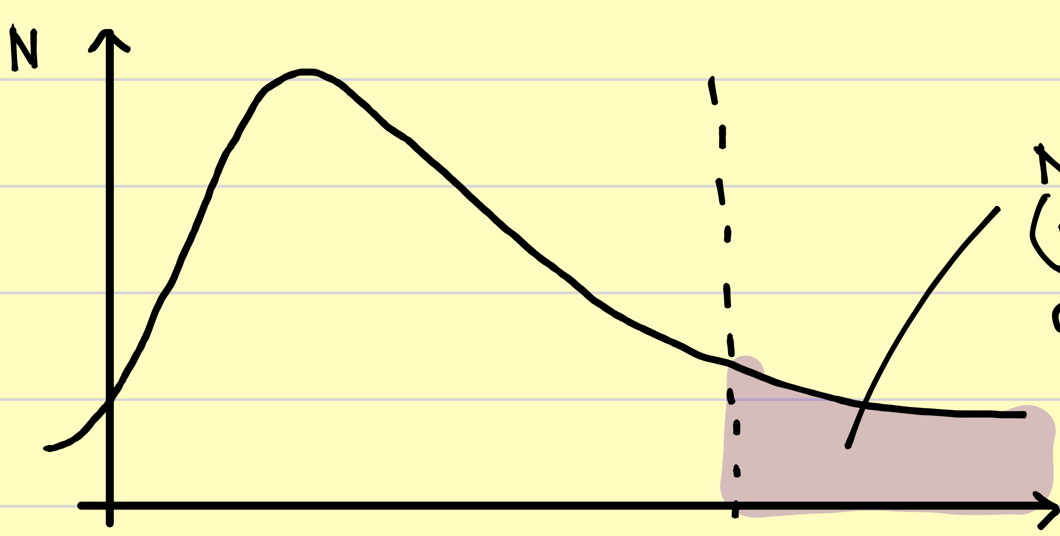
$$f(k,\lambda) = P(X=k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

REAL NETWORKS

Probability that a new node is connected to another with "k" neighbours

$$p_K = K^{-\gamma} \qquad \gamma \equiv \text{Exponent Degree}$$

POWER LAW

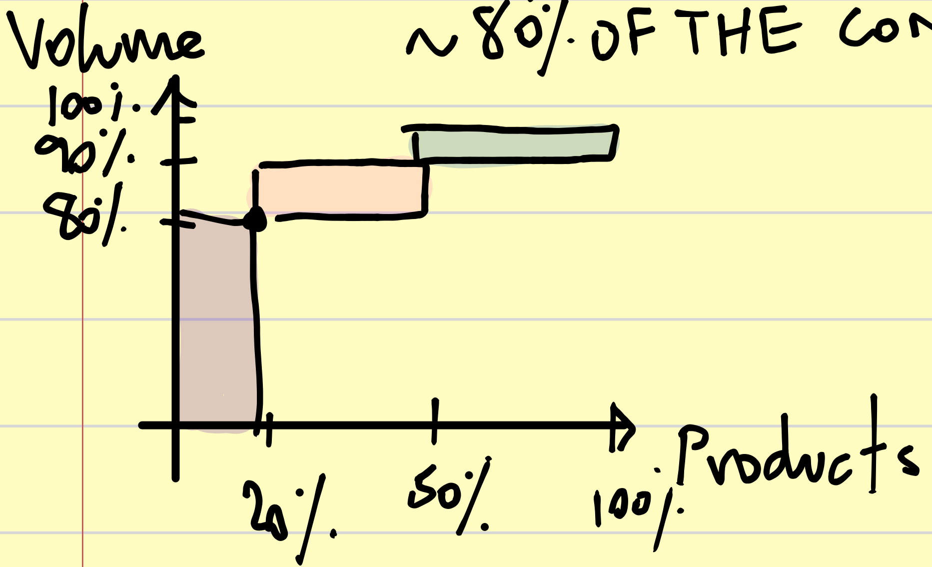


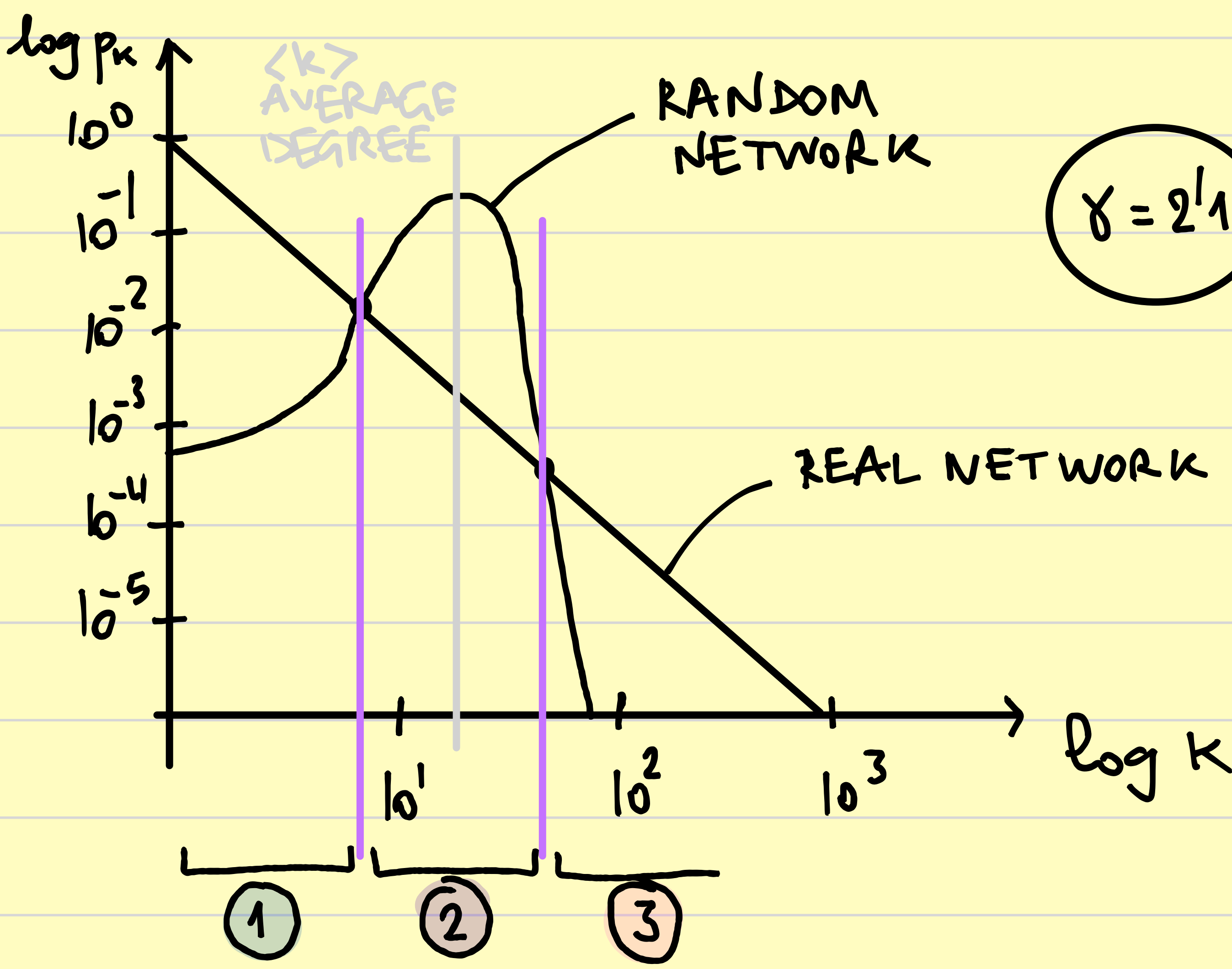
Nodes with high degree (with many neighbours) are called "HUBS".

K ≡ NODE DEGREE (# Neighbours)

Example: "Rich get richer" / Pareto

PARETO PRINCIPLE : 20% OF OCCURRENCES are RESPONSIBLE FOR ~80% OF THE CONSEQUENCES.





$\gamma = 2{,}1$

1 RANDOM << REAL

For small "k" the power law (Real network) is above the Poisson (Random Network), indicating that Real world network has many nodes with few neighbours.

2 RANDOM > REAL

For degrees "K" close to the average degree $\langle k \rangle$, the random network (Poisson) is above the power law, indicating that in a random network, there is an excess of nodes with average degree $\langle k \rangle$.

③ RANDOM << REAL

For large "K" the power law is again above the Poisson curve. This difference is particularly visible in the curve, indicating that the probability of observing a high degree node (HUB) is several orders of magnitude higher in a real network than in a random one.

The role of the exponent γ

- The properties of the real network are a function of the degree exponent "γ".
- $\boxed{\text{Almost ALL real networks have } \gamma > 2.}$
- $\gamma \leq 2$. ANOMALOUS REGIME.

The number of links connected to the largest hub grows faster than the size of the network.

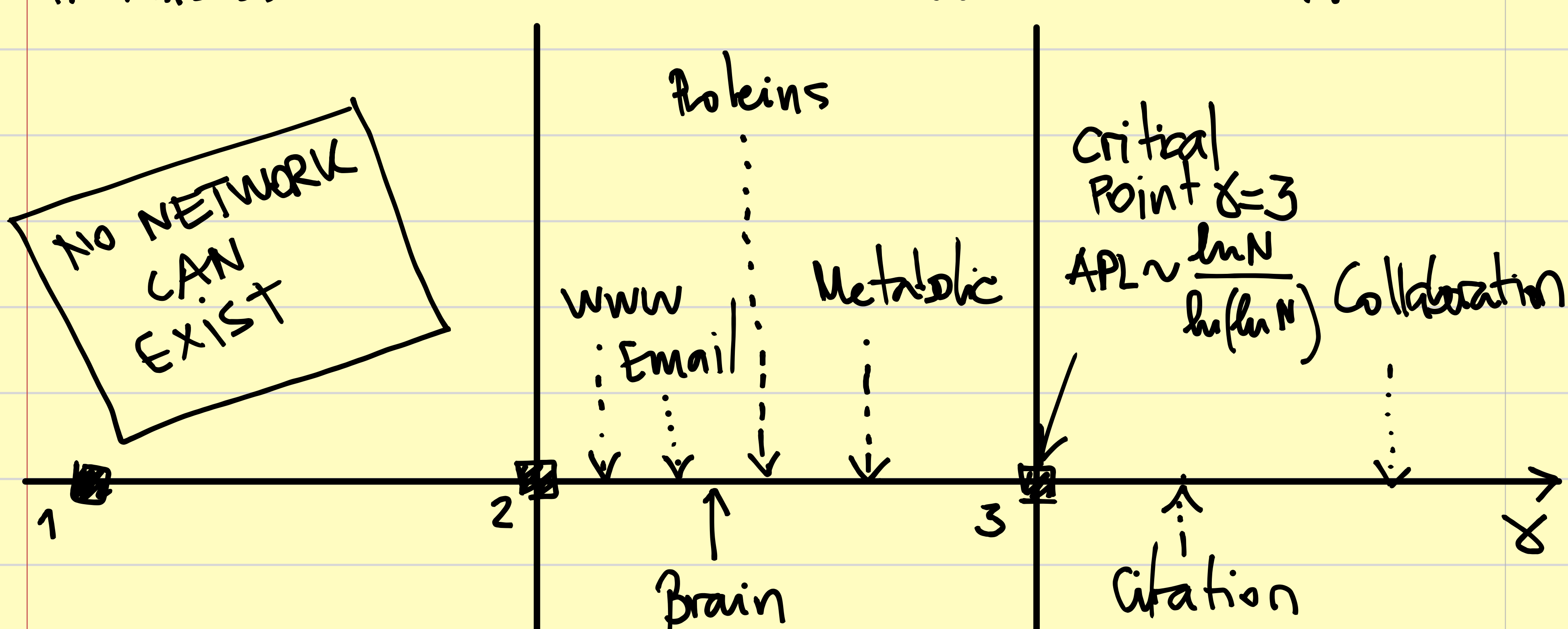
This means that for a sufficiently big network ($N \to \infty$) the degree of the largest hub <u>must</u> exceed the total number of nodes. There are not enough nodes to connect and the network disgregates in smaller ones.

- $2 < \gamma < 3$. SCALE FREE REGIME (SFN)

In this case, as we have seen, the spreading dynamics are independent of the network structure $\langle k^2 \rangle \to \infty$.

- $\gamma > 3$. RANDOM REGIME

$\langle k \rangle$ Finite
$\langle k^2 \rangle$ Finite $\to$ Organization dynamics are not as robust as with SFN.



ANOMALOUS REGIME	SCALE FREE REGIME	RANDOM REGIME
$\langle k \rangle \to \infty$	$\langle k \rangle$ Finite	$\langle k \rangle$ Finite
$\langle k^2 \rangle \to \infty$	$\langle k^2 \rangle \to \infty$	$\langle k^2 \rangle$ Finite
Maximum Degree grows faster than N!!	$APL \sim \ln(\ln N)$	$APL \sim \frac{\ln N}{\ln \langle k \rangle}$
	Scale Free Network (SFN)	Random Network

(!)

i.e.

APL of a NETWORK.

N is known
APL is calculated

$\rightarrow$ APL can be compared to these values, APL $\sim \ln(\ln N)$

More Network Science : (Barabási, 2016)