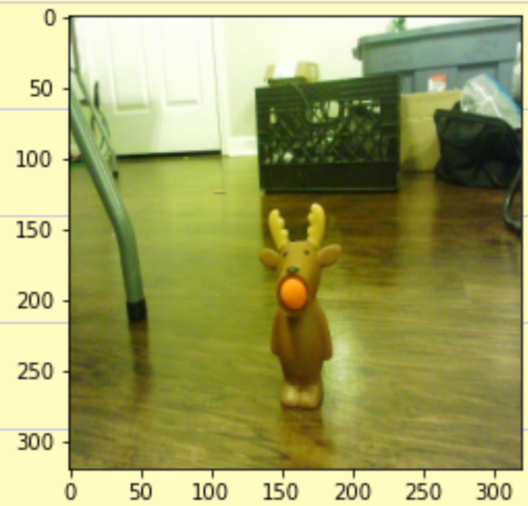
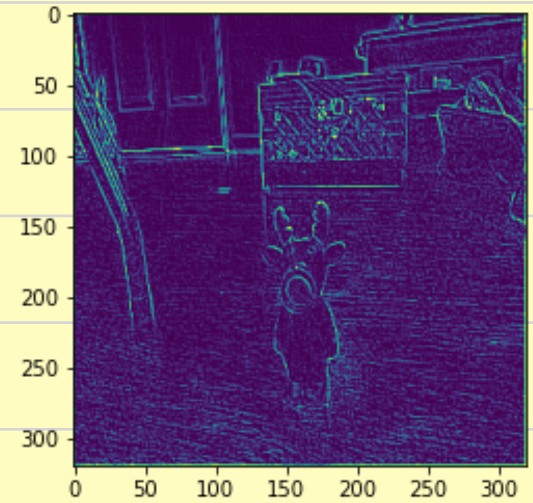
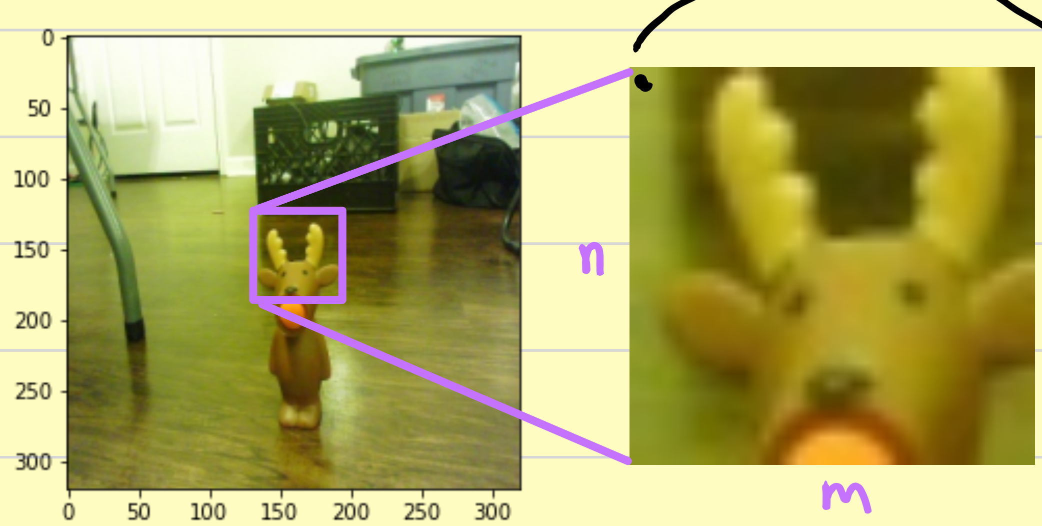
Deep learning by hand (Part 2) CONVOLUTION

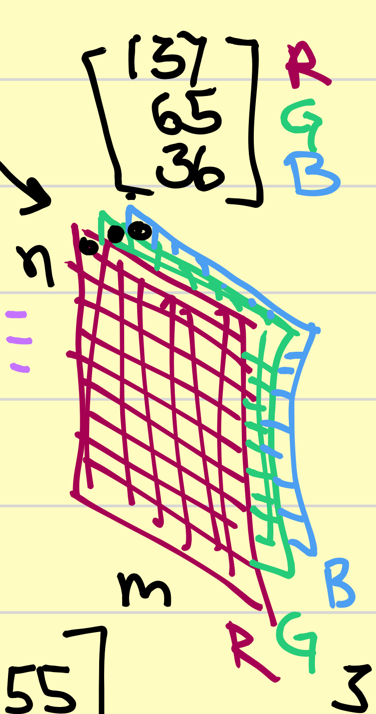




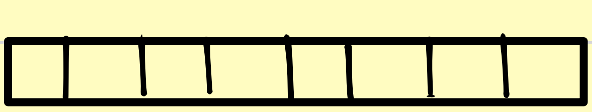
Background: convolution is a technique used in image processing. Is the process of summing each element of the input image with its local neighbours. The convolution process uses some filters to extract the information we are interested in. These filters are called "KERNELS".
A kernel is in this case a matrix that shapes the weights of convolution.



= Tensor of dim. = $n \times m \times 3$



The values of each channel are between [0,255]



1 byte = 8 bits $\{0,1\}$ $2^8 = 256$

Input filter or kernel.

- kernels can have many shapes & sizes.
- the kernels make up filters, which are a parameter in the convolutional layer: there are kernels for "attention", for "blurring", for "edge recognition", ...

CORNERS

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

This is an Edge recognition kernel because only the edges (corners) are non-zero.

ATTENTION

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

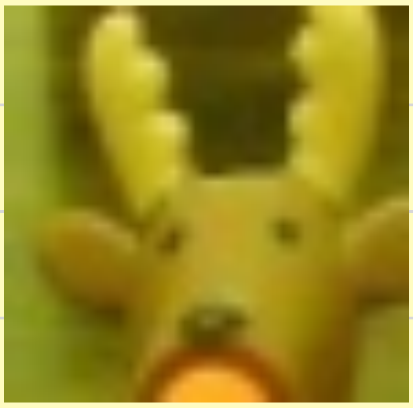
This is an Attention kernel because the middle is highlighted. (x4)

STRONGER ATTENTION

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

This is a stronger attention kernel. (x8)

Example:



$\rightarrow$

R	G	B
$\begin{bmatrix} 1 & 1 & 0 & 2 & 1 \\ 1 & 2 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} \; \end{bmatrix}$	$\begin{bmatrix} \; \end{bmatrix}$

Attention Kernel $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

$$(f \circ g)(x) = f(g(x))$$

CONVOLUTION STRIDE = 1

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \circ \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \rightarrow \begin{array}{l} 1.0 + 1.-1 + 0.0 + \\ 1.-1 + 2.4 + 1.-1 + \\ 0.0 + 1.-1 + 2.0 = 4 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \circ \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \rightarrow \begin{array}{l} 1.0 + 0.-1 + 2.0 + \\ 2.-1 + 1.4 + 1.-1 + \\ 1.0 + 2.-1 + 1.0 = -1 \end{array}$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \circ \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \rightarrow \begin{array}{l} 0.0 + 2.-1 + 1.0 + \\ 1.-1 + 4.1 + 0.-1 + \\ 2.0 + 1.-1 + 1.0 = 0 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix} \circ \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \rightarrow \begin{array}{l} 1.0 + 2.-1 + 1.0 + \\ 0.-1 + 4.1 + 2.-1 + \\ 1.0 + 2.-1 + 2.0 = -2 \end{array}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix} \circ \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \rightarrow \begin{array}{l} 2.0 + 1.-1 + 1.0 + \\ 1.-1 + 2.4 + 1.-1 + \\ 2.0 + 2.-1 + 1.0 = -1 \end{array}$$

CONVOLUTED LAYER Red

$$\begin{bmatrix} 4 & -1 & 0 \\ -2 & -1 & -1 \\ 0 & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \circ \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \longrightarrow \begin{array}{l} 1 \cdot 0 + 1 \cdot -1 + 0 \cdot 0 \\ 2 \cdot -1 + 1 \cdot 4 + 1 \cdot -1 + \\ 2 \cdot 0 + -1 \cdot 1 + 0 \cdot 1 \quad = -1 \end{array}$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \longrightarrow \begin{array}{l} 0 \cdot 0 + 1 \cdot -1 + 2 \cdot 0 + \\ 1 \cdot -1 + 2 \cdot 4 + 2 \cdot -1 + \\ 1 \cdot 0 + 0 \cdot -1 + 0 \cdot 0 \quad = 0 \end{array}$$

PADDING

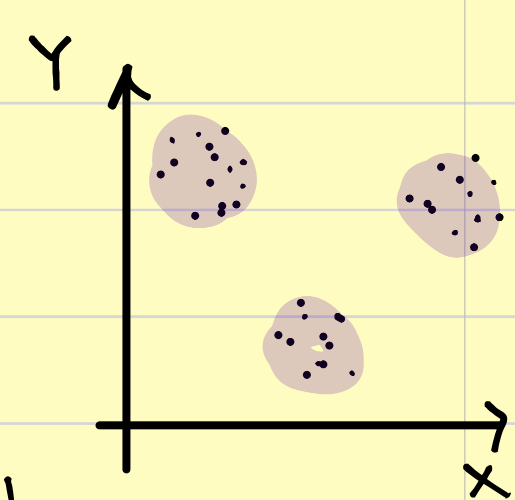
Red channel with padding = 1

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 2 & 3 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

PADDING = 1

p = 1

K-MEANS CLUSTERING



Clustering algorithm. Builds groups out of data.
Hypothesis: data are euclidean.
K is a parameter giving the number of groups.

DISADVANTAGE: we need to tell the algorythm, how many groups we want.

ADVANTAGE: fast & efficient.

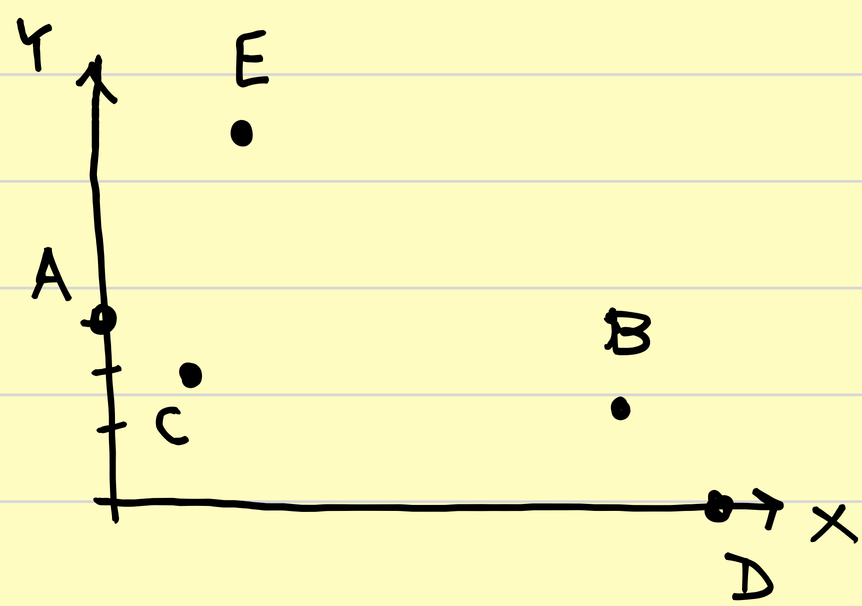
K-Means Algorythm:

- Step 0. Decide the number of clusters ..K..
- Step 1. Distribute the data points in ..K.. groups.
- Step 2. Find the centroid of each group.
- Step 3. Calculate the distance of the points to the centroids.
- Step 4. Cluster the points by their distance to the centroids.

Step 5. Repeat from Step 1 until the centroids are constant.

Example.

	x	y
A	0	3
B	6	1
C	1	2
D	7	0
E	2	4



Step 0. $K=2$

Step 1. 1. $\{A, B, C\}$ 2. $\{D, E\}$

Step 2. Centroids

$$z_1 = \left\{ \frac{0+6+1}{3}, \frac{3+1+2}{3} \right\} = \{2'33, 2\}$$

$$z_2 = \left\{ \frac{7+2}{2}, \frac{0+4}{2} \right\} = \{4'5, 2\}$$

Step 3.

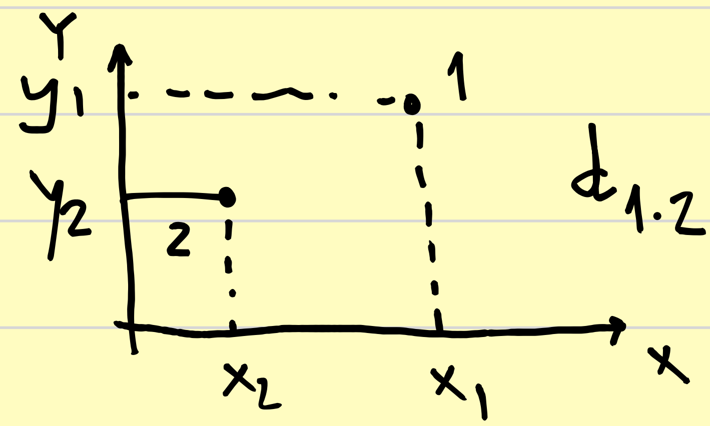
$$d(A,Z_1) = \sqrt{(0-2'33)^2+(3-2)^2} = 2'535 \;;\; d(A,Z_2) = \sqrt{(0-4'5)^2+(3-2)^2} = 4'61$$

$$d(B,Z_1) = 3'804 \;;\; d(B,Z_2) = 1'803$$

$$d(C,Z_1) = 1'33 \;;\; d(C,Z_2) = 4'5$$

$$d(D,Z_1) = 5,08 \;;\; d(D,Z_2) = 3'201$$

$$d(E,Z_1) = 2'027 \;;\; d(E,Z_2) = 3'201$$



$$d_{1.2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Step 4. New clusters. $1^*: \{A, C, E\}$ $2^*: \{B, D\}$

Step 2. centroids

$$z_1^* = \left[\frac{0+1+2}{3}, \frac{3+2+4}{3}\right] = [1,3]$$

$$z_2^* = \left[\frac{6+7}{2}, \frac{1+0}{2}\right] = [6'5, 0'5]$$

Step 3.

$$d(A,z_1^*) = 1 \;;\; d(A,z_2^*) = 6'96$$

$$d(B,z_1^*) = 5'38 \;;\; d(B,z_2^*) = 0'71$$

$$d(C,z_1^*) = 1 \;;\; d(C,z_2^*) = 5'7$$

$d(D, z_1^*) = 6'71$; $d(D, z_2^*) = 0'71$

$d(E, z_1^*) = 0'71$; $d(E, z_2^*) = 5'7$

Step 4. Clusters remain constant.

GROUP: $1^* \{A, C, E\}$ $2^* \{B, D\}$

CENTROIDS: $z_1^* [1, 3]$ $z_2^* [6'5, 0'5]$

Excercise. The position of 7 factories with different demands on raw-material is given by their coordinates (x, y). Every factory is delivered by one of TWO suppliers. To reduce the transport cost, the suppliers ask you to perform an analysis and recommend the optimal position of their supply points.

DATA.

	x	y	Demand
w_1	1	3	2
w_2	2	2	1
w_3	0	1	3
w_4	6	1	1
w_5	7	2	3
w_6	3	3	1

Hint: $1 \{w_1, w_2, w_4\}$ $z_1 = \left[\frac{1 \cdot 2 + 2 \cdot 1 + 6 \cdot 1}{3}, \frac{3 \cdot 2 + 2 \cdot 1 + 1 \cdot 1}{3} \right]$