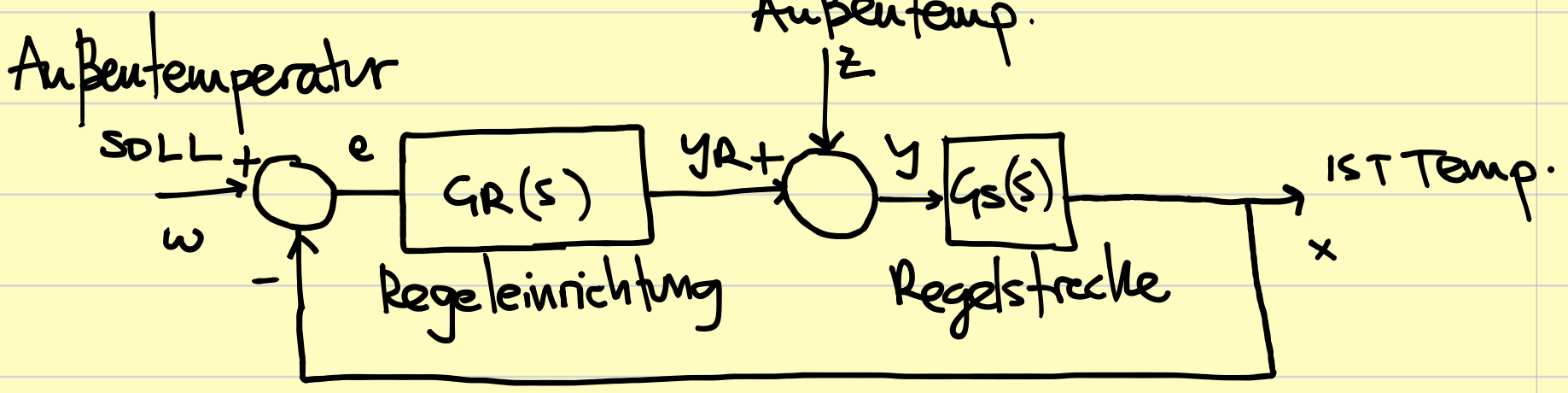
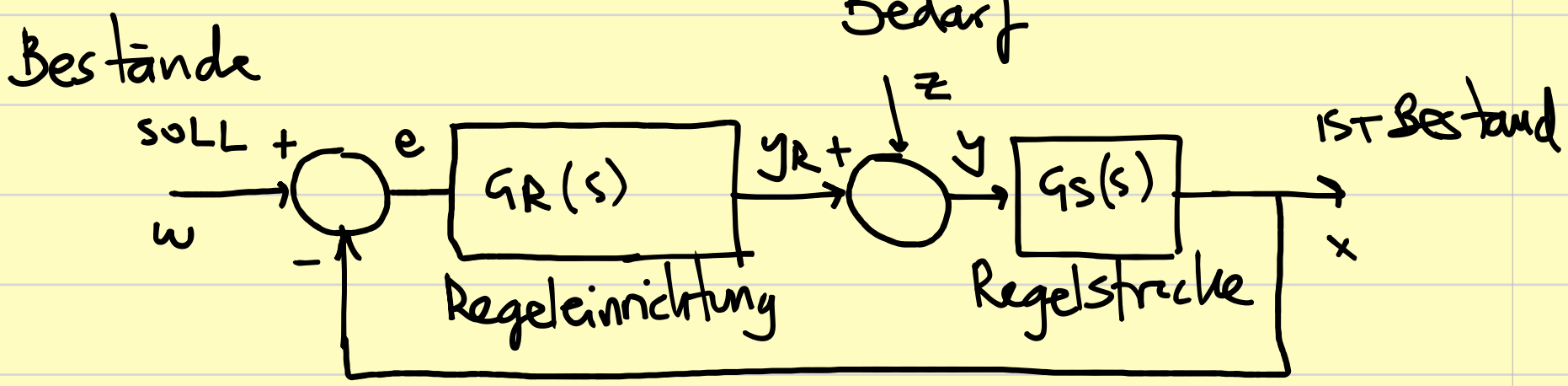
Logistik — Regelungstechnik





Bei der Beurteilung eines Regelkreises interessieren u.a.:

a) das dynamische Verhalten der Regelgröße x auf eine Sollwertänderung: **FÜHRUNGSVERHALTEN**.

$$G(s) = \frac{x(s)}{w(s)} \equiv \frac{\text{IST BESTAND}}{\text{SOLL BESTAND}}$$

b) die dynamische Reaktion der Regelgröße $\ddot{x}$ auf eine Störung $\ddot{z}$: **STÖRVERHALTEN**.

$$H(s) = \frac{x(s)}{z(s)} \equiv \frac{\text{IST BESTAND}}{\text{BEDARF}}$$

FÜHRUNGSÜBERTRAGUNGSFUNKTION:

$$y_R(s) = e \cdot G_R(s) = \big(w(s) - x(s)\big) \cdot G_R(s)$$
$$x(s) = y \cdot G_S(s) = \big(y_R(s) + z(s)\big) \cdot G_S(s)$$

$$x(s) = \Big[\big([w(s) - x(s)] \cdot G_R(s)\big) + z(s)\Big] \cdot G_S(s)$$

$$x(s) + x(s)\, G_R(s) G_S(s) = w(s) G_R(s) G_S(s) + z(s) G_S(s)$$

(1) $$\boxed{x(s)\big[1 + G_R(s) G_S(s)\big] = w(s) G_R(s) G_S(s) + z(s) G_S(s)}$$

SONDERFALL: $z(s) \equiv$ Bedarf $\equiv$ KONSTANT

$x = x_1,\ w = w_1 \rightarrow x_1(s)\big[1 + G_R(s) G_S(s)\big] = w_1(s) G_R(s) G_S(s) + z(s) G_S(s)$

$x = x_2,\ w = w_2 \rightarrow x_2(s)\big[1 + G_R(s) G_S(s)\big] = w_2(s) G_R(s) G_S(s) + z(s) G_S(s)$

$$\underbrace{\big[x_1(s) - x_2(s)\big]}_{\Delta x(s)}\big[1 + G_R(s) G_S(s)\big] = \underbrace{\big(w_1(s) - w_2(s)\big)}_{\Delta w(s)} G_R(s) G_S(s)$$

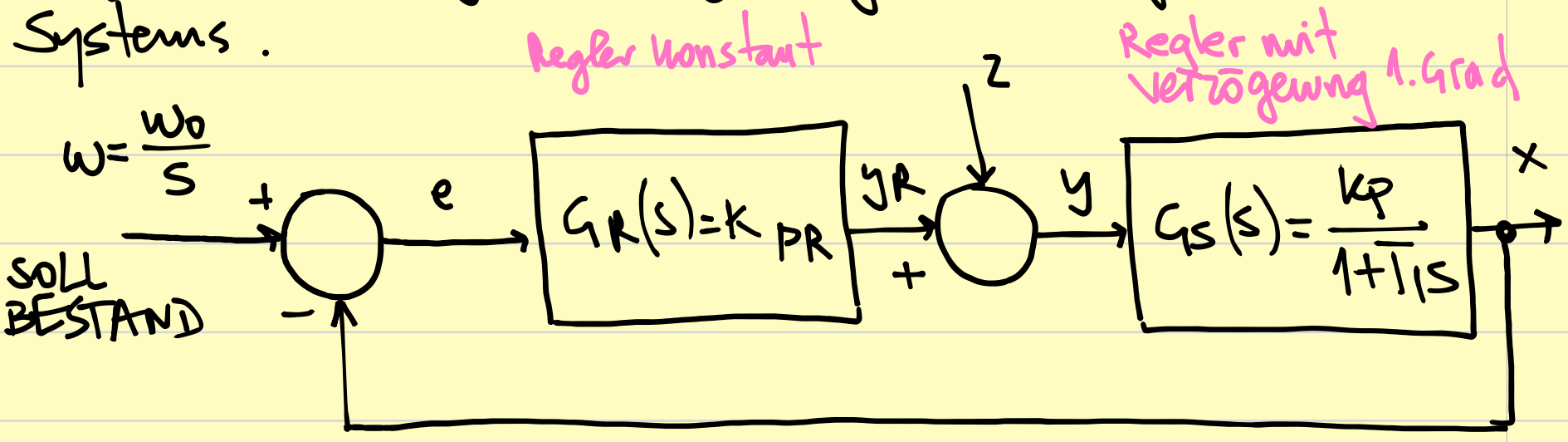
$$\boxed{G_w(s) = \frac{\Delta x(s)}{\Delta w(s)} = \frac{G_R(s) G_S(s)}{1 + G_R(s) G_S(s)}}$$ ✓

STÖRÜBERTRAGUNGSFUNKTION

$$G_Z(s) = \frac{\Delta x(s)}{\Delta z(s)} = \frac{1}{\frac{1}{G_S(s)} + G_R(s)} = \frac{G_S(s)}{1 + G_R(s) G_S(s)}$$ ✓

Beispiel

Regeleinrichtung zur Regelung eines logistischen Systems.



$$x(s)\left[1 + K_{PR} \cdot \frac{K_P}{1 + T_1 s}\right] = \frac{w_0}{s} \cdot K_{PR} \cdot \frac{K_P}{1 + T_1 s} + z(s) \cdot \frac{K_P}{1 + T_1 s}$$

$$x(s)\left[\frac{1 + T_1 s + K_{PR} K_P}{1 + T_1 s}\right] = \frac{w_0 K_{PR} K_P}{s(1 + T_1 s)} + \frac{z_0}{s} \cdot \frac{K_P}{1 + T_1 s}$$

$z(s) = \frac{z_0}{s}$

$$x(s)\left[1 + T_1 s + K_{PR} K_P\right] = \frac{w_0 K_{PR} K_P}{s} + \frac{z_0 K_P}{s}$$

$$x(s) = \frac{w_0 K_{PR} K_P + z_0 K_P}{s(1 + K_{PR} K_P + T_1 s)} = K_P (w_0 K_{PR} + z_0) \cdot \frac{1}{s(1 + K_{PR} K_P + T_1 s)}$$

$s^* = 0$

$$1 + K_{PR} K_P + T_1 s^* = 0 \rightarrow s^* = \frac{-1 - K_{PR} K_P}{T_1}$$

$$x(s) = k_p(w_0 k_{PR} + z_0) \cdot \left[\frac{A}{s} + \frac{B}{s + \frac{1 + k_{PR} k_p}{T_1}} \right]$$

$$= k_p(w_0 k_{PR} + z_0) \cdot \frac{1}{s(1 + k_p k_{PR} + T_1 s)}$$

$k_p = k_{PR} = 1$

$w_0 = z_0 = 1$

$T_1 = 5$

$$x(s) = 2 \left[\frac{A}{s} + \frac{B}{s + \frac{2}{5}} \right] =$$

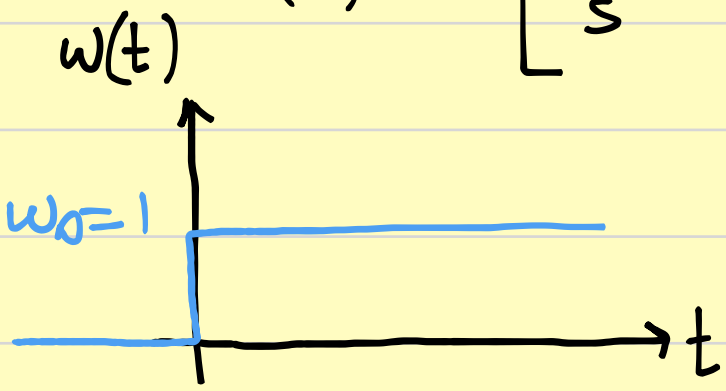
$$= 2 \cdot \frac{1}{s(2 + 5s)} = \frac{2}{5} \cdot \frac{1}{s\left(\frac{2}{5} + s\right)}$$

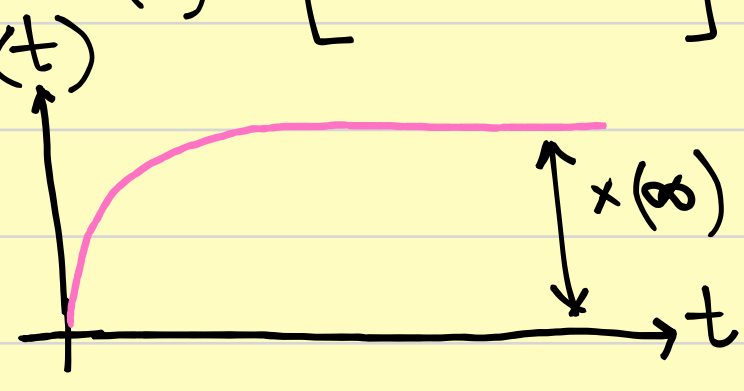
$$2 \left[\frac{A\left(s + \frac{2}{5}\right) + B(s)}{s\left(s + \frac{2}{5}\right)} \right] = 2 \cdot \frac{1}{5} \cdot \frac{1}{s\left(s + \frac{2}{5}\right)}$$

$$s^* = 0 \rightarrow \frac{2}{5} A = \frac{2}{5} \rightarrow A = 1$$

$$s^* = \frac{-2}{5} \rightarrow \frac{-2}{5} B = \frac{1}{5} \rightarrow B = -1$$

$$x(s) = 2 \left[\frac{1}{s} - \frac{1}{s + \frac{2}{5}} \right] \rightarrow x(t) = 2\left[1 - e^{-\frac{2}{5}t} \right]$$





$t = 0 \rightarrow x(0) = 2[1 - 1] = 0$

$t = \infty \rightarrow x(\infty) = 2[1 - 0] = 2$

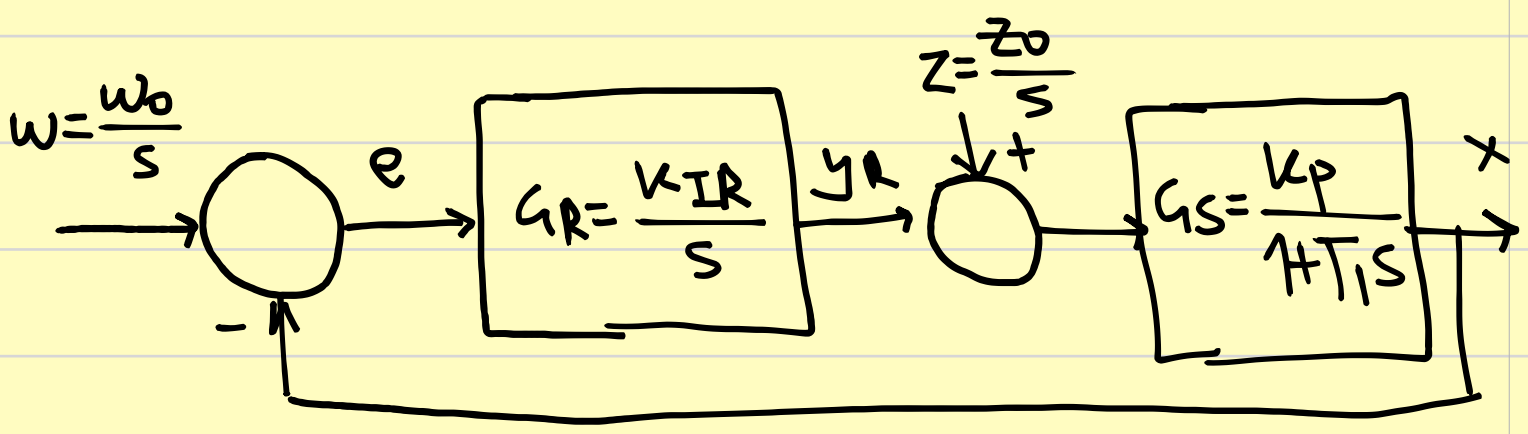
Störverhalten:

$$G_{Z(s)} = \frac{x(s)}{z(s)} = \frac{G(s)}{1 + G_R(s) G_S(s)} = \frac{K_P}{1 + K_{PR} K_P + T_1 s}$$

$$z(s) = \frac{z_0}{s}$$

$$x(s) = \frac{K_P \cdot z_0}{s(1 + K_{PR} K_P + T_1 s)} = \dots \text{(wie oben)}$$

Beispiel.



$$x(s)\left[1 + \frac{K_{IR}}{s} \cdot \frac{K_P}{1 + T_1 s}\right] = \frac{w_0}{s} \frac{K_{IR}}{s} \frac{K_P}{1 + T_1 s} + \frac{z_0}{s} \cdot \frac{K_P}{1 + T_1 s}$$

$$x(s)\left[s(1 + T_1 s) + K_{IR} \cdot K_P\right] = \frac{w_0 K_{IR} K_P}{s} + z_0 K_P$$

$$x(s) = \frac{z_0 K_P s + w_0 K_{IR} K_P}{K_{IR} K_P + s(1 + T_1 s)} = \frac{w_0 K_{IR} K_P + z_0 K_P s}{T_1 s^2 + s + K_{IR} K_P}$$

$$= \frac{w_0 K_{IR} K_P + z_0 K_P s}{T_1} \cdot \frac{1}{s^2 + \frac{1}{T_1} s + \frac{K_{IR} K_P}{T_1}}$$

$$s^* \rightarrow \frac{\frac{-1}{T_1} \pm \sqrt{\left(\frac{1}{T_1}\right)^2 - \frac{4 K_{IR} K_P}{T_1}}}{2} = \frac{-\frac{1}{1/4}}{2} = -2 \quad \text{(doppelt)}$$

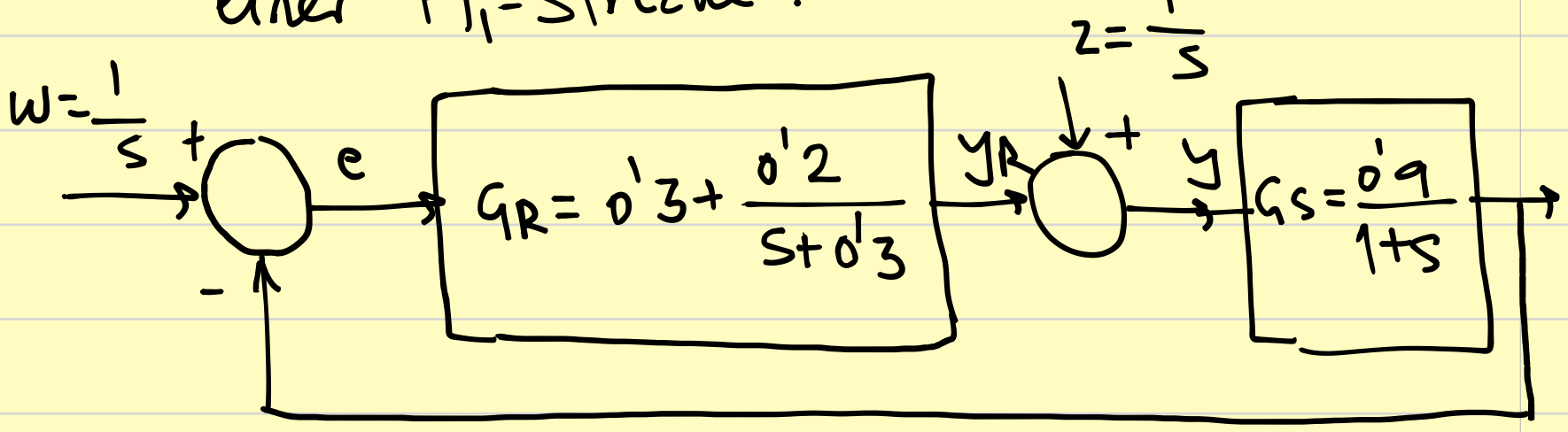
$K_{IR} = K_P = 1 = w_0 = z_0$

$$\left(\frac{1}{T_1}\right)^2 = \frac{4}{T_1} \rightarrow T_1 = \frac{1}{4}$$

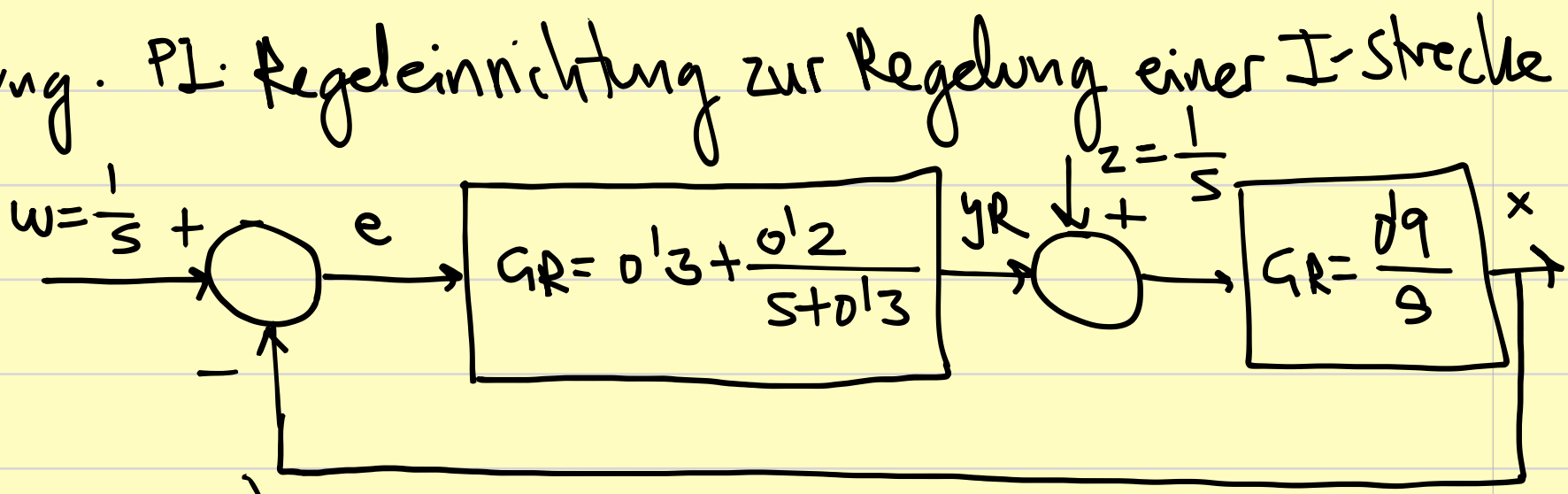
$$x(s) = \frac{1+2s}{1/4} \cdot \frac{1}{(s+2)^2}$$

$$z_0 = 0 \rightarrow x(s) = \frac{4}{(s+2)^2} \rightarrow x(t) = \dots$$

Übung. Bitte Führungs- und Störverhalten der folgenden PI-Einrichtung zur Regelung einer PT_1-Strecke:



Übung. PI: Regeleinrichtung zur Regelung einer I-Strecke



(gleichen Fragen)
