20210602_Wirtschaftsinformatik_MV2

June 2, 2021

1 Bayes'sche Theorem

[1]:
```
"""Das Bayes'sche Theorem ist ein fundamentales Theorem in der Bayes'schen␣
↪Statistik, da es von den Bayes'schen Methoden verwendet wird, um␣
↪Wahrscheinlichkeiten, die Glaubensgrade sind, nach Erhalt neuer Daten zu␣
↪aktualisieren. Bei zwei Ereignissen A und B wird die bedingte␣
↪Wahrscheinlichkeit von A unter der Voraussetzung, dass B wahr ist, wie folgt␣
↪ausgedrückt:"""
```

[1]:
```
"Das Bayes'sche Theorem ist ein fundamentales Theorem in der Bayes'schen
Statistik, da es von den Bayes'schen Methoden verwendet wird, um
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aktualisieren. Bei zwei Ereignissen A und B wird die bedingte Wahrscheinlichkeit
von A unter der Voraussetzung, dass B wahr ist, wie folgt ausgedrückt:"
```

[2]:
```
from IPython.display import Image
# Load image from local storage
Image(filename = '/Users/h4/Desktop/Screenshot 2021-06-02 at 09.13.22.png')
```

[2]:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

[3]:
```
"""Die Wahrscheinlichkeit des Beweises P(B) kann mit dem Gesetz der␣
↪Gesamtwahrscheinlichkeit berechnet werden. Wenn"""
```

[3]:
```
'Die Wahrscheinlichkeit des Beweises P(B) kann mit dem Gesetz der
Gesamtwahrscheinlichkeit berechnet werden. Wenn'
```

[6]:
```
from IPython.display import Image
# Load image from local storage
Image(filename = '/Users/h4/Desktop/Screenshot 2021-06-02 at 09.13.32.png')
```

[6]:

$$\{A_1, A_2, \ldots, A_n\}$$

[7]:
```
"""eine Partition des Stichprobenraums ist, der die Menge aller Ergebnisse␣
↪eines Experiments ist, dann:"""
```

[7]:
```
'eine Partition des Stichprobenraums ist, der die Menge aller Ergebnisse eines
Experiments ist, dann:'
```

[8]:
```
from IPython.display import Image
# Load image from local storage
Image(filename = '/Users/h4/Desktop/Screenshot 2021-06-02 at 09.13.38.png')
```

[8]:

$$P(B) = P(B \mid A_1)P(A_1) + P(B \mid A_2)P(A_2) + \cdots + P(B \mid A_n)P(A_n) = \sum_i P(B \mid A_i)P(A_i)$$

2 Bayesian Netzwerk

[9]:
```
"""Ein Bayes'sches Netzwerk ist ein probabilistisches grafisches Modell (eine␣
↪Art statistisches Modell), das einen Satz von Variablen und deren bedingte␣
↪Abhängigkeiten über einen gerichteten azyklischen Graphen (DAG) darstellt.␣
↪Bayes'sche Netze sind ideal, um ein aufgetretenes Ereignis zu nehmen und die␣
↪Wahrscheinlichkeit vorherzusagen, dass eine von mehreren möglichen bekannten␣
↪Ursachen der beitragende Faktor war."""
```

[9]:
```
"Ein Bayes'sches Netzwerk ist ein probabilistisches grafisches Modell (eine Art
statistisches Modell), das einen Satz von Variablen und deren bedingte
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Bayes'sche Netze sind ideal, um ein aufgetretenes Ereignis zu nehmen und die
Wahrscheinlichkeit vorherzusagen, dass eine von mehreren möglichen bekannten
Ursachen der beitragende Faktor war."
```

2.1 Beispiel 1

[10]:
```
import pandas as pd
data = pd.DataFrame(data={'fruit': ["banana", "apple", "banana", "apple",␣
 ↪"banana","apple", "banana",
                                    "apple", "apple", "apple", "banana",␣
 ↪"banana", "apple", "banana",],
                           'tasty': ["yes", "no", "yes", "yes", "yes", "yes",␣
 ↪"yes",
                                    "yes", "yes", "yes", "yes", "no", "no",␣
 ↪"no"],
                           'size': ["large", "large", "large", "small", "large",␣
 ↪"large", "large",
                                    "small", "large", "large", "large",␣
 ↪"large", "small", "small"]})
print(data)
```

```
     fruit tasty   size
0   banana   yes  large
1    apple    no  large
2   banana   yes  large
3    apple   yes  small
4   banana   yes  large
5    apple   yes  large
6   banana   yes  large
7    apple   yes  small
8    apple   yes  large
9    apple   yes  large
10  banana   yes  large
11  banana    no  large
12   apple    no  small
13  banana    no  small
```

[12]:
```
# A1 -- fruit
# A2 -- size
# B  -- tasty
# A1-->B<--A2

# P(B|A1)
```

[13]:
```
!pip install pgmpy
```

```
Requirement already satisfied: pgmpy in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (0.1.14)
Requirement already satisfied: joblib in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (0.17.0)
Requirement already satisfied: statsmodels in
```

```
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (0.12.0)
Requirement already satisfied: pandas in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (1.1.3)
Requirement already satisfied: scipy in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (1.5.2)
Requirement already satisfied: pyparsing in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (2.4.7)
Requirement already satisfied: torch in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (1.8.1)
Requirement already satisfied: scikit-learn in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (0.23.2)
Requirement already satisfied: networkx in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (2.5)
Requirement already satisfied: tqdm in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (4.50.2)
Requirement already satisfied: numpy in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pgmpy) (1.20.3)
Requirement already satisfied: patsy>=0.5 in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from statsmodels->pgmpy)
(0.5.1)
Requirement already satisfied: pytz>=2017.2 in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pandas->pgmpy)
(2019.3)
Requirement already satisfied: python-dateutil>=2.7.3 in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from pandas->pgmpy) (2.8.1)
Requirement already satisfied: typing-extensions in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from torch->pgmpy)
(3.7.4.3)
Requirement already satisfied: threadpoolctl>=2.0.0 in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from scikit-learn->pgmpy)
(2.1.0)
Requirement already satisfied: decorator>=4.3.0 in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from networkx->pgmpy)
(4.4.2)
Requirement already satisfied: six in
/Users/h4/opt/anaconda3/lib/python3.8/site-packages (from
patsy>=0.5->statsmodels->pgmpy) (1.15.0)
```

[14]:
```
# Bayes Netzwerk Model aufstellen

from pgmpy.models import BayesianModel

model = BayesianModel([('fruit', 'tasty'), ('size', 'tasty')]) # fruit -> tasty␣
 ↪<- size
```

[16]:
```
# Parameter müssen ermittelt werden. Counts = Häufigkeit
```

```
from pgmpy.estimators import ParameterEstimator

pe = ParameterEstimator(model, data)

print('\n', pe.state_counts(('fruit'))) #unconditional
print('\n', pe.state_counts(('tasty'))) #conditional
```

```
         fruit
apple        7
banana       7

 fruit apple       banana
size  large small  large small
tasty
no      1.0   1.0    1.0   1.0
yes     3.0   2.0    5.0   0.0
```

[20]:
```
# Conditional Probability Distribution. CPD = Bedingte Wahrscheinlichkeiten

from pgmpy.estimators import MaximumLikelihoodEstimator

mle = MaximumLikelihoodEstimator(model, data)

print(mle.estimate_cpd('fruit'))  # unconditional
print(mle.estimate_cpd('tasty'))  # conditional
```

```
+---------------+-----+
| fruit(apple)  | 0.5 |
+---------------+-----+
| fruit(banana) | 0.5 |
+---------------+-----+
+------------+--------------+--------------------+---------------------+--------
-------+
| fruit      | fruit(apple) | fruit(apple)       | fruit(banana)       |
fruit(banana) |
+------------+--------------+--------------------+---------------------+--------
-------+
| size       | size(large)  | size(small)        | size(large)         |
size(small)   |
+------------+--------------+--------------------+---------------------+--------
-------+
| tasty(no)  | 0.25         | 0.3333333333333333 | 0.16666666666666666 | 1.0
|
+------------+--------------+--------------------+---------------------+--------
-------+
| tasty(yes) | 0.75         | 0.6666666666666666 | 0.8333333333333334  | 0.0
|
```

```
+-----------+-------------+------------------+--------------------+--------
-------+
```

[21]:
```
# Bayes'sche Netzwerk
# Calibrate all CPDs of `model` using MLE:

model.fit(data, estimator=MaximumLikelihoodEstimator)
```

[22]:
```
from pgmpy.estimators import BayesianEstimator

est = BayesianEstimator(model, data)

print(est.estimate_cpd('tasty', equivalent_sample_size=10, prior_type='BDeu'))
```

```
+-----------+--------------------+--------------------+--------------------+--
------------------+
| fruit     | fruit(apple)       | fruit(apple)       | fruit(banana)      |
fruit(banana)       |
+-----------+--------------------+--------------------+--------------------+--
------------------+
| size      | size(large)        | size(small)        | size(large)        |
size(small)         |
+-----------+--------------------+--------------------+--------------------+--
------------------+
| tasty(no) | 0.34615384615384615 | 0.4090909090909091 | 0.2647058823529412 |
0.6428571428571429  |
+-----------+--------------------+--------------------+--------------------+--
------------------+
| tasty(yes) | 0.6538461538461539  | 0.5909090909090909 | 0.7352941176470589 |
0.35714285714285715 |
+-----------+--------------------+--------------------+--------------------+--
------------------+
```

2.2 Beispiel 2

[23]:
```
# Import notwendige Packete
import numpy as np
import pandas as pd
from pgmpy.models import BayesianModel
from pgmpy.estimators import BayesianEstimator

# generate data
data = pd.DataFrame(np.random.randint(low=0, high=2, size=(5000, 4)),␣
 ↪columns=['A', 'B', 'C', 'D'])
model = BayesianModel([('A', 'B'), ('A', 'C'), ('D', 'C'), ('B', 'D')]) #␣
 ↪D-->C<--A-->B-->D
```

```
model.fit(data, estimator=BayesianEstimator, prior_type="BDeu") # default␣
 ↪equivalent_sample_size=5
for cpd in model.get_cpds():
    print(cpd)
```

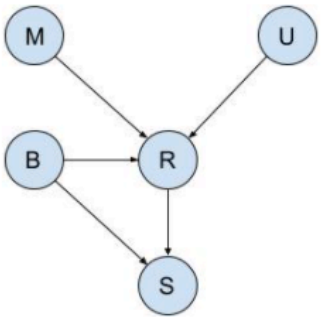
```
+------+----------+
| A(0) | 0.484615 |
+------+----------+
| A(1) | 0.515385 |
+------+----------+
+------+--------------------+--------------------+
| A    | A(0)               | A(1)               |
+------+--------------------+--------------------+
| B(0) | 0.49732014017728304 | 0.4978678038379531 |
+------+--------------------+--------------------+
| B(1) | 0.502679859822717  | 0.502132196162047  |
+------+--------------------+--------------------+
+------+--------------------+--------------------+--------------------+-------
-------------+
| A    | A(0)               | A(0)               | A(1)               | A(1)
|
+------+--------------------+--------------------+--------------------+-------
-------------+
| D    | D(0)               | D(1)               | D(0)               | D(1)
|
+------+--------------------+--------------------+--------------------+-------
-------------+
| C(0) | 0.5116351547891128 | 0.528226631212927  | 0.48825979928043933 |
0.4888822711931705 |
+------+--------------------+--------------------+--------------------+-------
-------------+
| C(1) | 0.4883648452108872 | 0.47177336878707304 | 0.5117402007195607  |
0.5111177288068295 |
+------+--------------------+--------------------+--------------------+-------
-------------+
+------+--------------------+--------------------+
| B    | B(0)               | B(1)               |
+------+--------------------+--------------------+
| D(0) | 0.5028106805862277 | 0.5055677072976735 |
+------+--------------------+--------------------+
| D(1) | 0.4971893194137723 | 0.4944322927023265 |
+------+--------------------+--------------------+
```

2.3 Beispiel 3

"""M : Eine Vorhersage von einem ML-Modell, das den Inhalt lesen kann und eine Punktzahl (Wahrscheinlichkeit) angibt, dass dieser Inhalt gekennzeichnet werden sollte. U : Ein anderer Benutzer kennzeichnet den Inhalt. B : Das Konto wurde zuvor wegen eines schlechten Inhalts gesperrt. R : Punktzahl (Wahrscheinlichkeit), dass der Inhalt von der Plattform entfernt werden sollte. S : Score (Wahrscheinlichkeit), dass das Konto gesperrt werden sollte. Gehen wir davon aus, dass die Wahrscheinlichkeiten für das Netzwerk wie folgt gegeben sind:"""

[27]:
```
from IPython.display import Image
# Load image from local storage
Image(filename = '/Users/h4/Desktop/Screenshot 2021-06-02 at 09.20.49.png')
```

[27]:



U=	T	F
P(U)	0.15	0.85

M=	T	F
P(M)	0.05	0.95

B=	T	F
P(B)	0.10	0.90

S=	T	F
P(S|RB)	0.40	0.60
P(S|R!B)	0.05	0.95
P(S|!RB)	0.12	0.88
P(S|!R!B)	0.02	0.98

R=	T	F
P(R|MBU)	0.95	0.05
P(R|MB!U)	0.90	0.10
P(R|M!BU)	0.85	0.15
P(R|M!B!U)	0.76	0.24
P(R|!MBU)	0.18	0.82
P(R|!MB!U)	0.06	0.94
P(R|!M!BU)	0.14	0.86
P(R|!M!B!U)	0.04	0.96

[29]:
```
from pgmpy.models import BayesianModel
from pgmpy.factors.discrete import TabularCPD
from pgmpy.inference import VariableElimination
import numpy as np

bayesNet = BayesianModel()

# Gestalten wird das Netzwerk

# Knoten + Verbindungen

bayesNet.add_node('M')
bayesNet.add_node('U')
bayesNet.add_node('R')
bayesNet.add_node('B')
bayesNet.add_node('S')

bayesNet.add_edge('M', 'R') #Verbindung geht von M an R
bayesNet.add_edge('U', 'R')
bayesNet.add_edge('B', 'R')
bayesNet.add_edge('B', 'S')
bayesNet.add_edge('R', 'S')

# Conditional Probabilities

# Die Knoten ohne "Eltern" haben 2^1 Wahrscheinlichkeiten

cpd_M = TabularCPD('M', 2, values = [[0.95], [0.05]]) # Knote = M kann 2␣
 ↪Variablen nehmen, Wahrhscheinlichkeit F, T.

cpd_U = TabularCPD('U', 2, values = [[0.85], [0.15]])

cpd_B = TabularCPD('B', 2, values=[[.90], [.10]])

# Knoten mit 1 Eltern haben 2^2 Wahrscheinlichkeiten

cpd_S = TabularCPD('S', #Knote
                   2,   #Anzahln Variablen von S
                   values = [[0.98, 0.88, 0.95, 0.6], [0.02, 0.12, 0.05, 0.4]],␣
 ↪#Bedingten Wahrscheinlichkeiten F, T
                   evidence=['R', 'B'], # die Knoten die Eltern von R
                   evidence_card=[2,2]) # die jeweiligen States von den␣
 ↪Evindence Knoten

# Knoten mit 2 Eltern haben 2^3 Wahrscheinlichkeiten
```

```
cpd_R = TabularCPD('R', 2,
                   values=[[0.96, .86, .94, .82, .24, .15, .10, .05], [.04, .
 ↪14, .06, .18, .76, .85, .90, .95]],
                   evidence=['M', 'B', 'U'],
                   evidence_card=[2, 2,2])

bayesNet.add_cpds(cpd_M, cpd_U, cpd_B, cpd_S, cpd_R)
```

[30]:
```
bayesNet.check_model()
print("Model is correct.")
```

```
Model is correct.
```

[31]:
```
solver = VariableElimination(bayesNet)
```

[32]:
```
result = solver.query(variables=['R'])
print(result)
```

```
Finding Elimination Order: :   0%|          | 0/4 [00:00<?, ?it/s]
  0%|          | 0/4 [00:00<?, ?it/s]
Eliminating: S:   0%|          | 0/4 [00:00<?, ?it/s]
Eliminating: M:   0%|          | 0/4 [00:00<?, ?it/s]
Eliminating: U:   0%|          | 0/4 [00:00<?, ?it/s]
Eliminating: B: 100%|     | 4/4 [00:00<00:00, 205.03it/s]
+------+----------+
| R    |   phi(R) |
+======+==========+
| R(0) |   0.9062 |
+------+----------+
| R(1) |   0.0938 |
+------+----------+
```

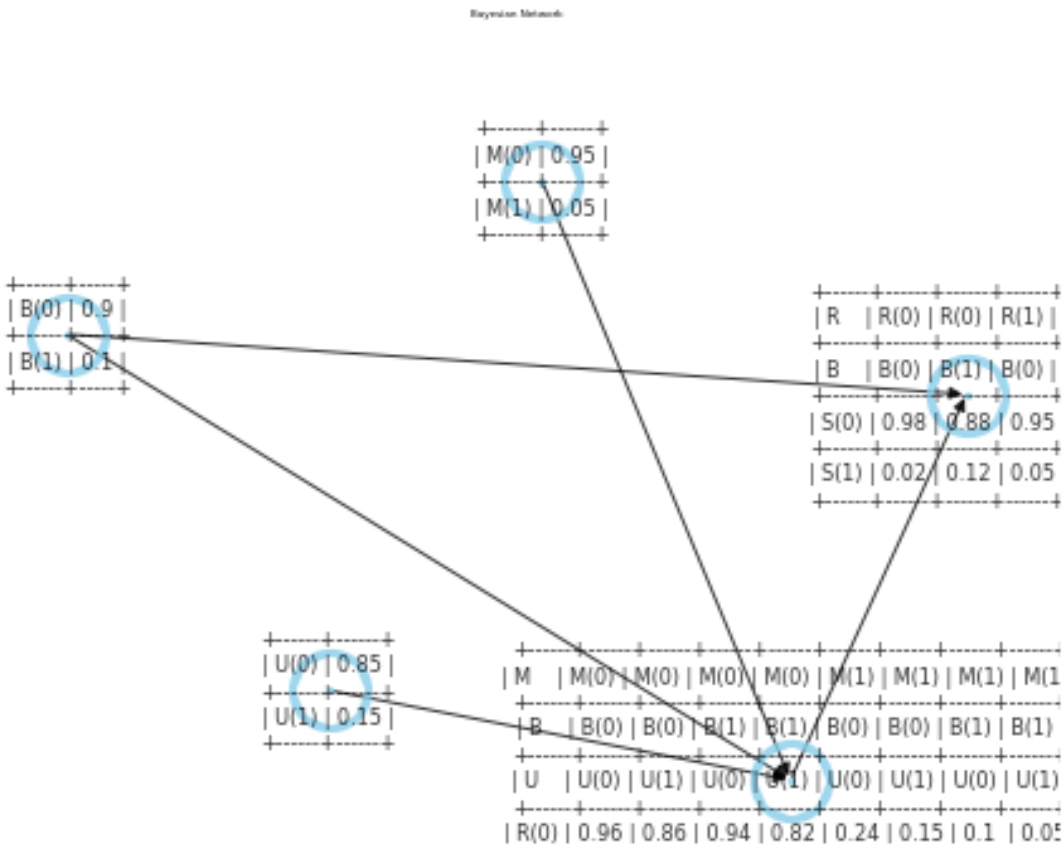
[33]:
```
"""P(R)=
         P(R|MBU)P(M)P(B)P(U)+
         P(R|MBU)P(M)P(B)P(!U)+
         P(R|MBU)P(M)P(!B)P(U)+
         P(R|MBU)P(M)P(!B)P(!U)+
         P(R|MBU)P(!M)P(B)P(U)+
         P(R|MBU)P(!M)P(B)P(!U)+
         P(R|MBU)P(!M)P(!B)P(U)+
         P(R|MBU)P(!M)P(!B)P(!U) ---
         [Using total probability theorem as R depends on M, B, U] =
         0.950.050.10.15+
         0.90.050.10.85+
         0.850.050.90.15+
```

```
        0.760.050.90.85+
        0.180.950.10.15+
        0.060.950.10.85+
        0.140.950.90.15+
        0.040.950.90.85=
        0.09378
"""
```

[33]:
```
'P(R) =P(R|MBU)P(M)P(B)P(U)+P(R|MBU)P(M)P(B)P(!U)+P(R|MBU)P(M)P(!B)P(U)
+P(R|MBU)P(M)P(!B)P(!U)+P(R|MBU)P(!M)P(B)P(U)+P(R|MBU)P(!M)P(B)P(!U)
+P(R|MBU)P(!M)P(!B)P(U)+P(R|MBU)P(!M)P(!B)P(!U) --- [Using total probability
theorem as R depends on M, B, U] =0.950.050.10.15+0.90.050.10.85+0.850.050.90.15
+0.760.050.90.85+0.180.950.10.15+0.060.950.10.85
+0.140.950.90.15+0.040.950.90.85 =0.09378\n'
```

[34]:
```
import networkx as nx
import pylab as plt

labeldict = {}
labeldict["M"] = cpd_M
labeldict["U"] = cpd_U
labeldict["B"] = cpd_B
labeldict["R"] = cpd_R
labeldict["S"] = cpd_S

nx.draw(bayesNet,
        labels=labeldict,
        with_labels=True,
        font_size=8,
        node_size=10,
        node_color="skyblue",
        node_shape="o",
        alpha=0.8,
        linewidths=30,
        pos=nx.spring_layout(bayesNet))

plt.title("Bayesian Network", fontsize=4, y=1.12)

plt.show()
```

```
Finding Elimination Order: : 100%|      | 4/4 [02:23<00:00, 35.82s/it]
```



[35]:
```
"""Lassen Sie uns die Wahrscheinlichkeit für "Inhalt sollte von der Plattform␣
↪entfernt werden, wenn unser ML-Modell ihn markiert" ermitteln"""
```

[35]:
```
'Lassen Sie uns die Wahrscheinlichkeit für "Inhalt sollte von der Plattform
entfernt werden, wenn unser ML-Modell ihn markiert" ermitteln'
```

[37]:
```
"""P(R|M)=0.95*0.1*0.15 + 0.9*0.1*0.85 +0.85*0.9*0.15 + 0.76*0.9*0.85 = 0.
↪7869"""
```

[37]:
```
'P(R|M)=0.95*0.1*0.15 + 0.9*0.1*0.85 +0.85*0.9*0.15 + 0.76*0.9*0.85 = 0.7869'
```

[38]:
```
result = solver.query(variables=['R'], evidence={'M': 1})
print("R| M", result)
```

```
Finding Elimination Order: :   0%|          | 0/3 [00:00<?, ?it/s]
  0%|          | 0/3 [00:00<?, ?it/s]
Eliminating: S:   0%|          | 0/3 [00:00<?, ?it/s]
Eliminating: U:   0%|          | 0/3 [00:00<?, ?it/s]
Finding Elimination Order: : 100%|      | 3/3 [00:00<00:00, 159.15it/s]
Eliminating: B: 100%|      | 3/3 [00:00<00:00, 205.52it/s]
```

```
R| M +------+----------+
| R    |   phi(R) |
+======+==========+
| R(0) |   0.2131 |
+------+----------+
| R(1) |   0.7869 |
+------+----------+
```

[39]:
```
"""Zum Beispiel finden wir "Konto sollte gesperrt werden, da es zuvor gesperrt␣
↪wurde"""
```

[39]:
```
'Zum Beispiel finden wir "Konto sollte gesperrt werden, da es zuvor gesperrt
wurde'
```

[40]:
```
"""P(S|B)"""

result = solver.query(variables=['S'], evidence={'M':1})
print("S|B", result)
```

```
Finding Elimination Order: :   0%|          | 0/3 [00:00<?, ?it/s]
  0%|          | 0/3 [00:00<?, ?it/s]
Eliminating: U:   0%|          | 0/3 [00:00<?, ?it/s]
Eliminating: R:   0%|          | 0/3 [00:00<?, ?it/s]
Eliminating: B: 100%|     | 3/3 [00:00<00:00, 268.33it/s]
```

```
S|B +------+----------+
| S    |   phi(S) |
+======+==========+
| S(0) |   0.9237 |
+------+----------+
| S(1) |   0.0763 |
+------+----------+
```

[ ]:
```

```