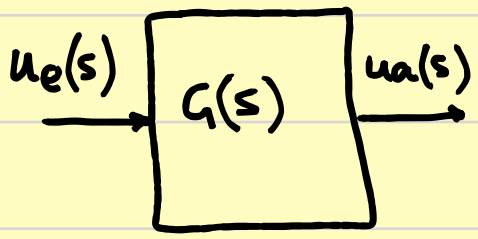
Beispiele / Klassifizierung von Regelkreisen nach Übertragungsfunktion

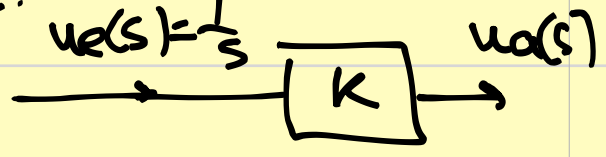


$$G(s) = \frac{u_a(s)}{u_e(s)} \qquad \frac{10}{5} = 2$$

1. PROPORTIONALER REGELKREIS (P)

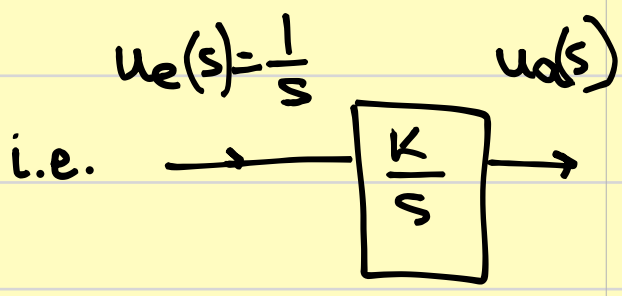
$$G(s) = K$$

i.e.



2. INTEGRALER REGELKREIS (I)

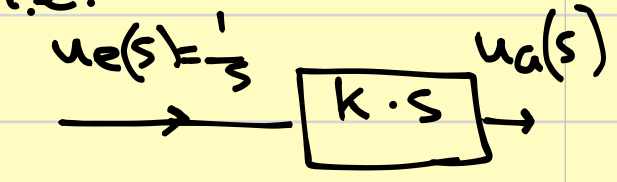
$$G(s) = \frac{K}{s}$$



3. DIFFERENZIALER REGELKREIS (D)

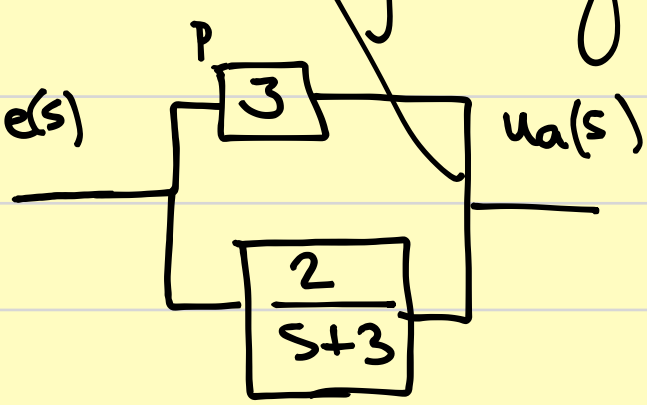
$$G(s) = K \cdot s$$

i.e.



Beispiel. Parallelschaltung von P & I Regelkreisgliedern. (PI)

$$G_P(s) = 3 \qquad G_I(s) = \frac{2}{s+3}$$



$$G(s) = 3 + \frac{2}{s+3} = \frac{3s+11}{s+3}$$

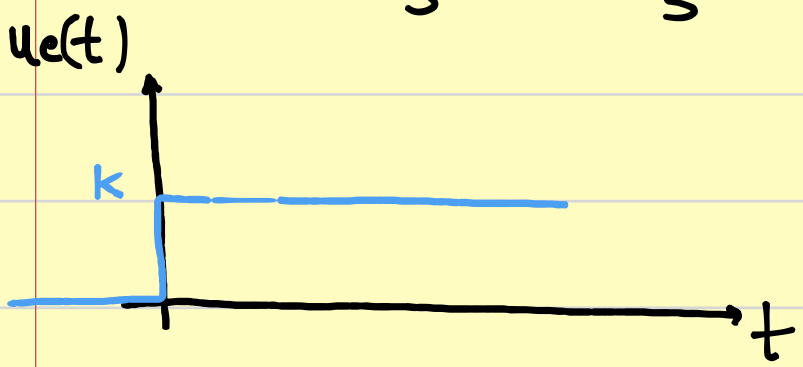
$$u_e(t) = k \longrightarrow u_e(s) = \frac{k}{s}$$

$$u_a(s) = u_e(s) \cdot G(s) = \frac{k}{s} \cdot \frac{3s+11}{s+3} = \frac{A}{s} + \frac{B}{s+3}$$

$$k(3s+11) = A(s+3) + Bs$$

$$\left.\begin{array}{l} \overset{*}{s=0} \rightarrow \quad 11k = 3A \longrightarrow A = \frac{11}{3}k \\ \overset{*}{s=-3} \rightarrow \quad 2k = -3B \longrightarrow B = \frac{-2}{3}k \end{array}\right\} \quad u_a(s) = \frac{11}{3}k \cdot \frac{1}{s} - \frac{2k}{3} \cdot \frac{1}{s+3}$$

$$\mathcal{L}^{-1}: \quad u_a(t) = \frac{11}{3}k - \frac{2k}{3} e^{-3t}$$



$$u_a(t=0) = 3K \qquad u_a(t \to \infty) = \frac{11}{3}k$$

4. PROPORTIONAL · INTEGRALER REGELKREIS (PI)

$$\boxed{G(s) = k_p + \frac{k_I}{s}}$$ (siehe Beispiel oben)

5. PROPORTIONAL · DIFFERENTIALER REGELKREIS (PD)

$$\boxed{G(s) = k_p + k_D \cdot s}$$

Beispiel. $u_e(t) = k$ bitte bei einer PD. Schaltung von $G_P(s) = 5$ und $G_D(s) = 3s$, $u_a(t)$ ermitteln.

$$G(s) = G_P(s) + G_D(s) = 5 + 3s$$

$$u_a(s) = u_e(s) \cdot G(s) = \frac{k}{s} \cdot (5 + 3s) = 5k \cdot \frac{1}{s} + 3k$$

$$u_a(t) = 5k + 3k\,\delta(t) \qquad \delta(t) = \text{DIRACFUNKTION}$$

$$u_e(t) = k \rightarrow \mathcal{L}(u_e(t)) = \frac{k}{s} = \int_0^\infty e^{-st}\, u_e(t)\, dt$$

6. PROPORTIONAL. INTEGRAL. DIFFERENTIALER REGELKREIS (PID)

Parallelschaltung von P + I + D

$$G(s) = k_P + \frac{k_I}{s} + k_D \cdot s$$

Beispiel: $u_e(t) = k$; $G_P = 3$, $G_I = \frac{2}{s}$, $G_D = 5s$
bitte $u_a(t)$ ermitteln im PID Schaltbetrieb.

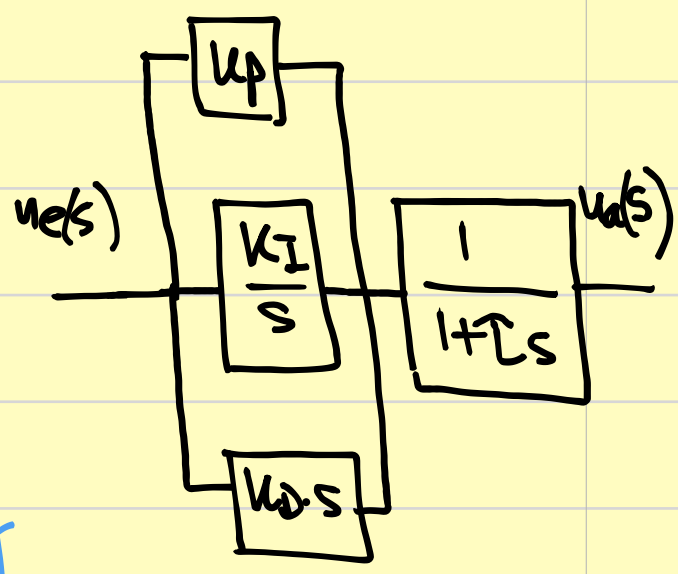
$$u_a(s) = \left[3 + \frac{2}{s} + 5s\right] \cdot \frac{k}{s} = \frac{3k}{s} + \frac{2k}{s^2} + 5k$$

$\mathcal{L}^{-1}$: $u_a(t) = 3k + 2kt + 5k\,\delta(t)$

7. Erweitertes PID Regelkreis mit „Filter" / „Dämpfung"

$$G(s) = \left[K_p + \frac{K_I}{s} + K_D \cdot s \right] \cdot \frac{1}{1+Ts}$$

PID Regler | I Regler mit Dämpfung



Beispiel: $u_e(t) = k$ $\quad G_p = 4$ $\quad G_I = \frac{3}{s}$ $\quad G_D = 2s$ $\quad G_{FILTER} = \frac{1}{1+10s}$

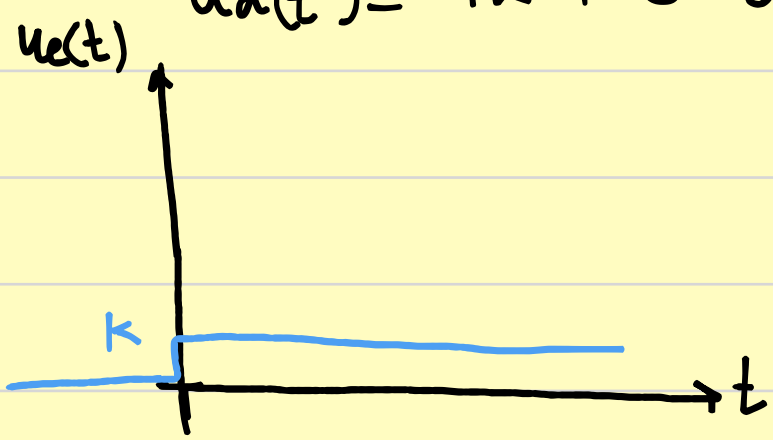
a)

ohne FILTER PID Regler:

$$\delta(t) = \begin{cases} 1 & t=0 \\ 0 & t \neq 0 \end{cases}$$

$$u_a(s) = \frac{k}{s}\left[4 + \frac{3}{s} + 2s\right] = \frac{4k}{s} + \frac{3k}{s^2} + 2k$$

$$u_a(t) = 4k + 3kt + 2k\,\delta(t)$$



$u_a(t=0) = 6k \qquad u_a(t \to \infty) = \infty$

b)

MIT FILTER & PID Regler:

$$u_a(s) = \frac{k}{s}\left[4 + \frac{3}{s} + 2s\right] \cdot \frac{1}{1+10s} =$$

$$= \frac{k}{s} \cdot \frac{2s^2+4s+3}{s} \cdot \frac{1}{1+10s} = \frac{A}{s^2} + \frac{B}{1+10s}$$

$$k \cdot \left(2s^2+4s+3\right) = A\left(1+10s\right) + Bs^2$$

$$s^* = 0 \rightarrow \quad 3k = A \quad \rightarrow \quad A = 3k$$

$$s^* = \frac{-1}{10} \rightarrow k \cdot \left[\frac{2}{100} - \frac{4}{10} + 3\right] = \frac{B}{100} \rightarrow B = 262\,k$$

$$u_a(s) = \frac{3k}{s^2} + \frac{262k}{1+10s} \rightarrow u_a(t) = 3kt + 262k\,e^{-10t}$$