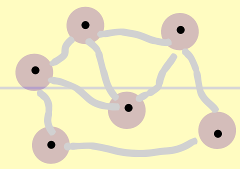
Mathematical formalization of Complex Network Dynamics



$G = \{N, E\}$

"Network Science" Barabási 2016

- Description of spreading Phenomena within complex networks: spreading of viruses, spreading of opinions, spreading of behavioural patterns $(CPD)_nA$ & $(DCP)_nA$ within organization.

2 Assumptions:
1. Individuals (nodes) can be in two different states:
   (S) "Susceptible" "Not yet infected"
   (I) "Infected".
2. Each individual can infect anyone else (HOMOGENEOUS MIXING HYPOTHESIS)

I SUCEPTIBLE-INFECTIOUS-SUSCEPTIBLE (SIS) Model

- We consider a behavioural pattern like a "disease" that spreads in a population (network) and allow for individuals to "forget" the behavioural pattern and abandon it. This means that people that have learnt $(CPD)_nA$ or $(DCP)_nA$ can forget it and stop using it.

Parameters:

I.1. $i(t)$: the fraction of individuals who have been "infected" (the fraction of people using $(CPD)_nA$ / $(DCP)_nA$ at a moment in time).

I.2. β: rate of infection (speed at which people learn)

I.3. $\langle k \rangle$: average network degree (average number of neighbours of a node)

I.4. $\mu . i(t)$: rate of forgetfulness (speed at which people unlearn)

$$(\mathrm{I.I}) \quad \underbrace{\frac{di(t)}{dt}}_{\text{SPEED OF INFECTION}} = \underbrace{\beta . \langle k \rangle . i(t) . (1 - i(t))}_{\text{SPEED OF LEARNING}} - \underbrace{\mu . i(t)}_{\text{SPEED OF FORGETTING}}$$

β: Infection Rate

$\langle k \rangle$: How well connected are the individuals

$i(t)$: fraction of infected individual

$1 - i(t)$: fraction of not infected individuals

μ: forget-fulness rate

...

(I.II) SOLUTION:

$$i(t) = \left[1 - \frac{\mu}{\beta\langle k \rangle}\right] . C . \frac{e^{(\beta\langle k \rangle - \mu)t}}{1 + Ce^{(\beta\langle k \rangle - \mu)t}}$$

$$C = \frac{i(t=0)}{1 - i(t=0) . \frac{\mu}{\beta\langle k \rangle}}$$

We are interested in what happens when $t \to \infty$.

$$i(t \to \infty) = \lim_{t \to \infty} i(t) = 1 - \frac{1}{R_0}$$

R_0 = Reproductive Number = Number of people being infected by an individual in a period of time $= \dfrac{\beta\langle k \rangle}{\mu}$

Explanation of R_0.

$$R_0 = \frac{\beta \langle k \rangle}{\mu}$$

← Infection Rate β
$\langle k \rangle$ connectivity

← Forgetfulness

$\boxed{R_0 > 1 \ : \ \beta \langle k \rangle > \mu}$: We get infected faster than we forget : The behavioural pattern spreads in the organization

$\boxed{R_0 < 1 \ : \ \beta \langle k \rangle < \mu}$: We get infected slower than we forget : The behavioural pattern dies out

Explanation:

$R_0 \uparrow\uparrow \equiv \beta \uparrow\uparrow \equiv$ Infection rate is high

↓

ACTION

↓

Bring $\langle k \rangle$ down : bring connectivity down : Keep distance!

$R_0 \uparrow\uparrow \equiv \langle k \rangle \uparrow\uparrow \equiv$ People are a lot in contact to each other

$R_0 \uparrow\uparrow \equiv \mu \downarrow\downarrow \equiv$ People recover slowly from the disease

↓

ACTION

↓

Develop vaccine

II

We allow individuals to transmit the behavioural pattern only to those they are in contact with. This is a more realistic approach that the SIS.

Parameters:

II.1. i_k : Fraction of the nodes with degree "k" that are infected among all other nodes with degree "k"

II.2. θ_k : Fraction of infected neighbours of a susceptible node with degree "k"

II.I $$\frac{di_k}{dt} = \beta \cdot i_k(1-i_k) \cdot \theta_k - \mu \cdot i_k$$

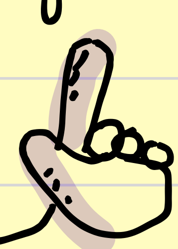
$\downarrow$

...

$\downarrow$

The condition for global spread of the behavioural pattern is given when the characteristic time to achieve $\frac{1}{e}$ fraction of the population infected given by τ is:

II.II $$\boxed{\tau = \frac{\langle k \rangle}{\beta \langle k^2 \rangle - \mu \langle k \rangle}}$$



$\langle k^2 \rangle$: heterogeneity : Standard Deviation of the degree distribution.

$I > 0 \rightarrow \beta\langle k^2\rangle - \mu\langle k\rangle > 0 \rightarrow$ The condition to predict if a behavioural pattern spreads on a certain organizational design:

$$\boxed{\lambda = \frac{\beta}{\mu} > \frac{\langle k\rangle}{\langle k^2\rangle} = \lambda_c}$$

(*) $\beta\langle k^2\rangle - \mu\langle k\rangle > 0 \rightarrow \beta\langle k^2\rangle > \mu\langle k\rangle \rightarrow \underset{\lambda}{\underset{\parallel}{\frac{\beta}{\mu}}} > \underset{\lambda_c}{\underset{\parallel}{\frac{\langle k\rangle}{\langle k^2\rangle}}}$

This means that:

- high λ means $\begin{cases} \beta\uparrow \text{ (high infection rate)} \\ \mu\downarrow \text{ (low forgetfulness rate)} \end{cases}$

the organizational culture transmits behavioural pattern rapidly and it does not forget it quickly.

Special case: Scale Free Network APL $\sim \ln(\ln N)$

In this case $\langle k^2\rangle \rightarrow \infty$

The behavioural pattern always propagates successfully in the culture.

$$\lambda = \frac{\beta}{\mu} > \frac{\langle k\rangle}{\langle k^2\rangle} = \frac{\langle k\rangle}{\infty} = 0$$

This is the real reason why managers should seek to achieve a scale free organizational design configuration.