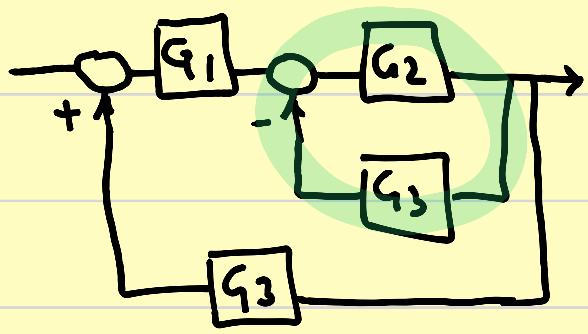
20240411_Regelungstechnik_BEL4

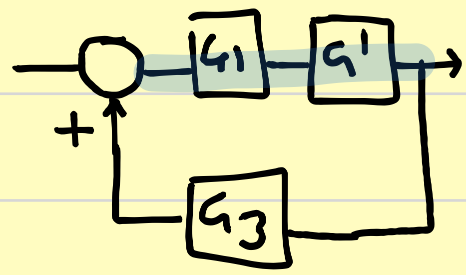
Übung. BD



$$G_1 = \frac{1}{1+2s} \quad G_2 = \frac{1}{1+s} \quad G_3 = \frac{1}{2+s}$$

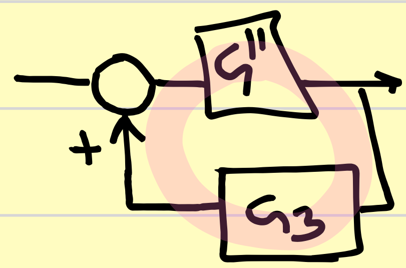
$$\equiv$$

Schritt 1.



$$G' = \frac{G_2}{1+G_2G_3}$$

Schritt 2.



$$G'' = G_1 G' = \frac{G_1 G_2}{1+G_2G_3}$$

Schritt 3.

$$G''' = \frac{G''}{1-G''G_3}$$

$$G''' = \frac{G_1G_2 \cdot \frac{1}{1+G_2G_3}}{1-\frac{G_1G_2G_3}{1+G_2G_3}} = \frac{G_1G_2}{1+G_2G_3-G_1G_2G_3}$$

$$= \frac{\frac{1}{1+2s}\cdot\frac{1}{1+s}}{1+\frac{1}{1+s}\cdot\frac{1}{2+s}-\frac{1}{1+2s}\cdot\frac{1}{1+s}\cdot\frac{1}{2+s}} =$$

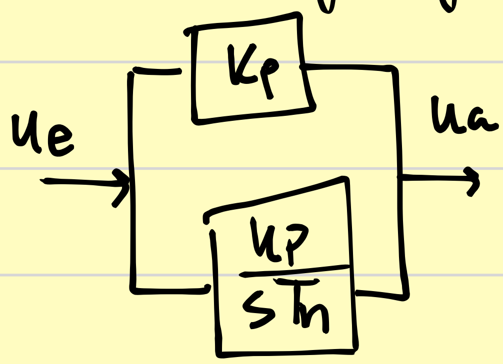
$$= \frac{2+s}{(1+2s)(1+s)(2+s)+(1+2s)-1}$$

$$G(j\omega) = \dots$$

5. BD . PI Regler

PI. Glied = Parallelschaltung P & I Glieder

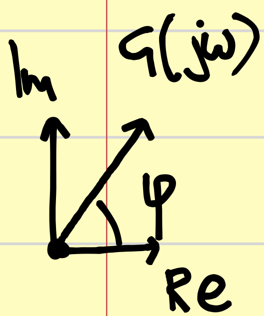
Übertragungsfunktion: $G(s) = K_p + \frac{K_p}{sT_n} = K_p\left(1 + \frac{1}{sT_n}\right)$



Frequenzgang: $G(j\omega) = K_p\left(1 + \frac{1}{j\omega T_n}\right) =$

$= K_p\left(1 - j\frac{1}{\omega T_n}\right)$

$Re(G(j\omega)) = K_p$; $Im(G(j\omega)) = \frac{-K_p}{\omega T_n}$



$|G(j\omega)| = \sqrt{Re^2 + Im^2} = \sqrt{K_p^2 + K_p^2\left(\frac{1}{\omega T_n}\right)^2} = K_p\sqrt{1 + \left(\frac{1}{\omega T_n}\right)^2} =$

$= K_p\left(1 + \left(\frac{1}{\omega T_n}\right)^2\right)^{1/2}$

$\boxed{\begin{array}{l} \log(a \cdot b) = \log a + \log b \\ \log a^b = b \log a \end{array}}$

$\log|G(j\omega)| = \log K_p + \frac{1}{2}\log\left(1 + \left(\frac{1}{\omega T_n}\right)^2\right)$

$\varphi = \arctan\left(\frac{Im}{Re}\right) = \arctan\left(\frac{-\frac{K_p}{\omega T_n}}{K_p}\right) = \arctan\left(\frac{-1}{\omega T_n}\right)$

$\omega \ll \quad \rightarrow \frac{1}{\omega T_n} \gg 1 \rightarrow \log|G(j\omega)| \approx \log K_p + \frac{1}{2}\log\left(\left(\frac{1}{\omega T_n}\right)^2\right)$

$= \log K_p + \frac{1}{2} \cdot 2\log\left((\omega T_n)^{-1}\right) =$

$= \log K_p - \log(\omega T_n)$ GERADE MIT NEG. STEIGUNG

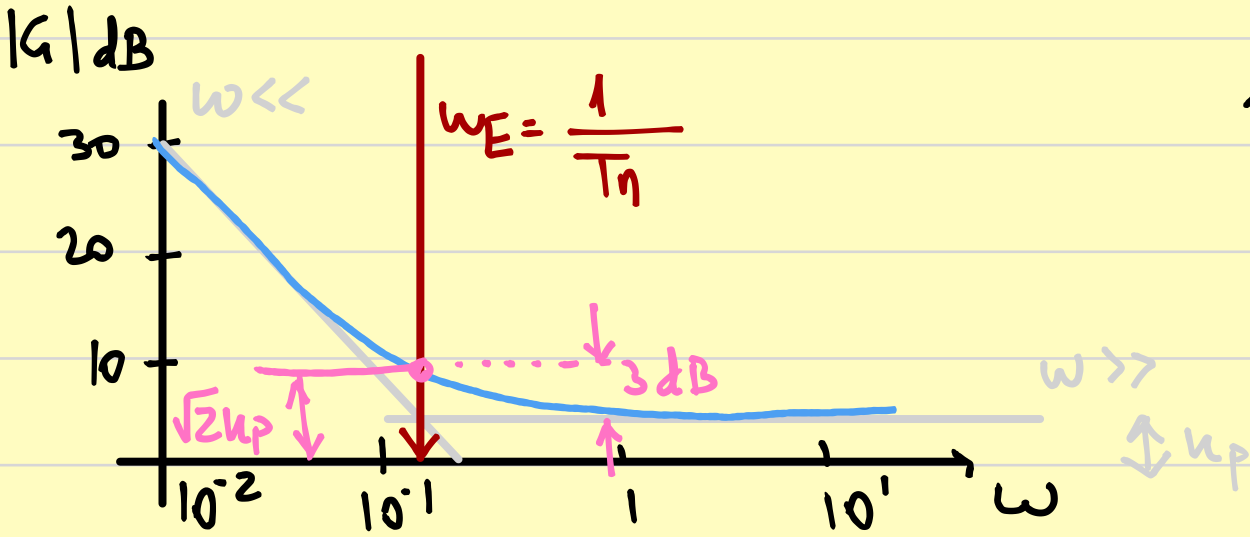
$\omega \gg \quad \rightarrow \quad \frac{1}{\omega T_n} \ll 1 \rightarrow \log|G(j\omega)| \simeq \log k_p$

In der Eckfrequenz treffen sich beide Asymptoten:

$$\log|G(j\omega_E)| = \log k_p = \log(k_p) - \log(\omega_E T_n) \rightarrow \omega_E = \frac{1}{T_n}$$

$$|G(j\omega_E)| = k_p\sqrt{1+\left(\frac{1}{\omega_E T_n}\right)^2} = \sqrt{2}\,k_p \rightarrow |G(j\omega)|_{dB} = 20\log\sqrt{2} + 20\log k_p = \simeq 3dB + 20\log k_p$$

ABP



$T_n = 5s$

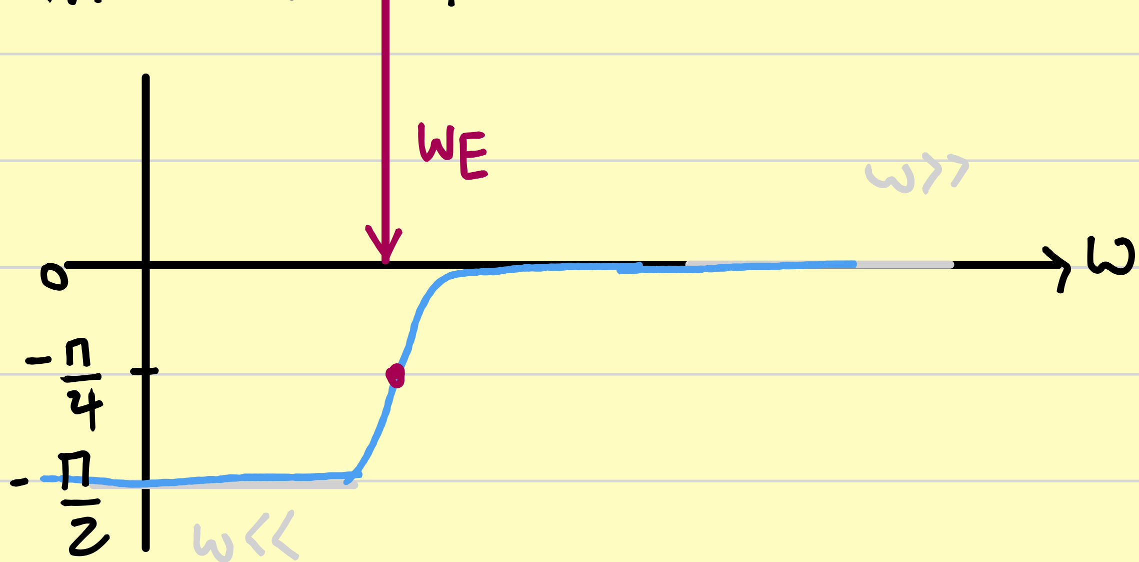
$k_p = 2$

$$\varphi = \arctan g\left(\frac{-1}{\omega T_n}\right)$$

$\omega \ll \quad \rightarrow \quad \varphi = \frac{-\pi}{2} = -90^\circ$

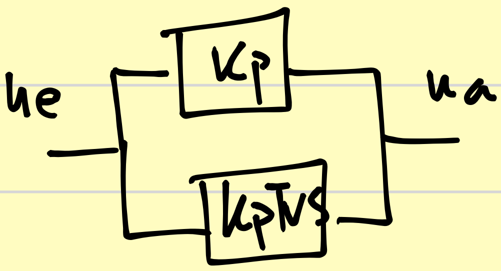
$\omega \gg \quad \rightarrow \quad \varphi = 0^\circ$

$\omega_E = \frac{1}{T_n} \rightarrow \varphi = -\frac{\pi}{4} = -45^\circ$



6. BD PD-GLIED

Parallelschaltung von P & D Glied.



$$G(s) = K_p + K_p T_v s = K_p(1 + T_v s)$$

Frequenzgang $G(j\omega) = K_p(1 + jT_v\omega)$

$Re(G(j\omega)) = K_p \quad Im(G(j\omega)) = K_p T_v \omega$

$$|G(j\omega)| = \sqrt{Re^2 + Im^2} = \sqrt{K_p^2 + (K_p T_v \omega)^2} = K_p\left(1 + (T_v\omega)^2\right)^{1/2}$$

$$\log|G(j\omega)| = \log K_p + \frac{1}{2}\log\left(1 + (T_v\omega)^2\right)$$

$$\varphi = \arctan g\left(\frac{Im}{Re}\right) = \frac{K_p T_v \omega}{K_p} = T_v\omega$$

$\omega \ll \; \rightarrow \; T_v\omega \ll 1 \rightarrow \log|G(j\omega)| = \log K_p + \frac{1}{2}\log(1) = \log K_p$

$\omega \gg \; \rightarrow \; T_v\omega \gg 1 \rightarrow \log|G(j\omega)| = \log K_p + \frac{1}{2}\log(T_v\omega)^2 =$

$= \log K_p + \log(T_v\omega)$

GERADE mit POS. NEIGUNG.

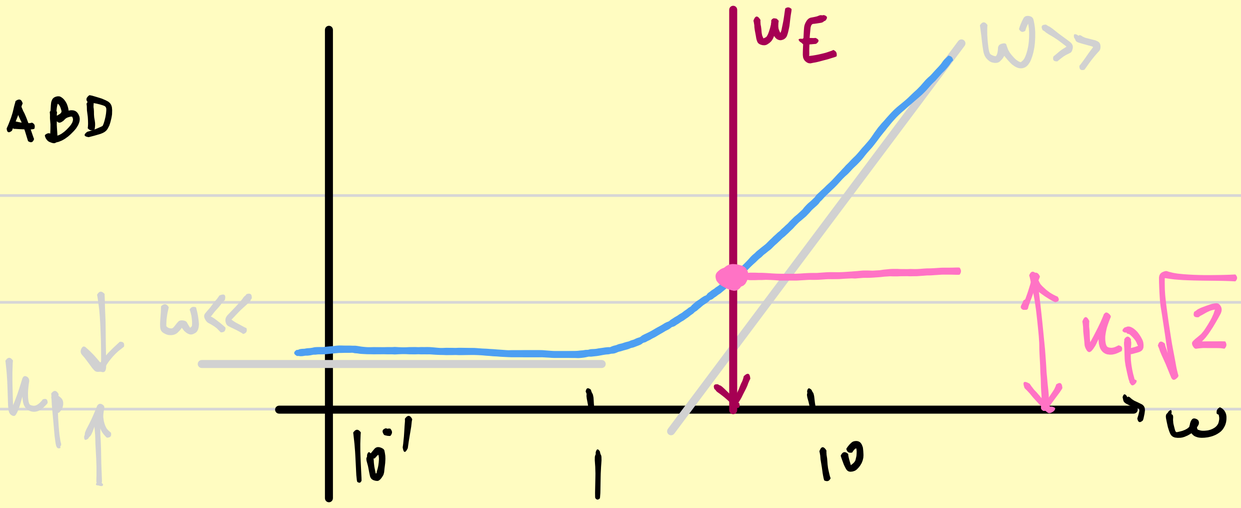
In der Eckfrequenz beide Asymptoten treffen sich: $\log K_p = \log K_p + \log(T_v\omega_E)$

$\rightarrow \omega_E = \frac{1}{T_v}$

$\rightarrow |G(j\omega_E)| = K_p \cdot \sqrt{1 + (\omega_E T_v)^2} = K_p \cdot \sqrt{2}$

$|G(j\omega_E)|_{dB} = 20 \log K_p + 3dB$

ABD



$K_p = 2$
$T_V = 0{,}2\,s$

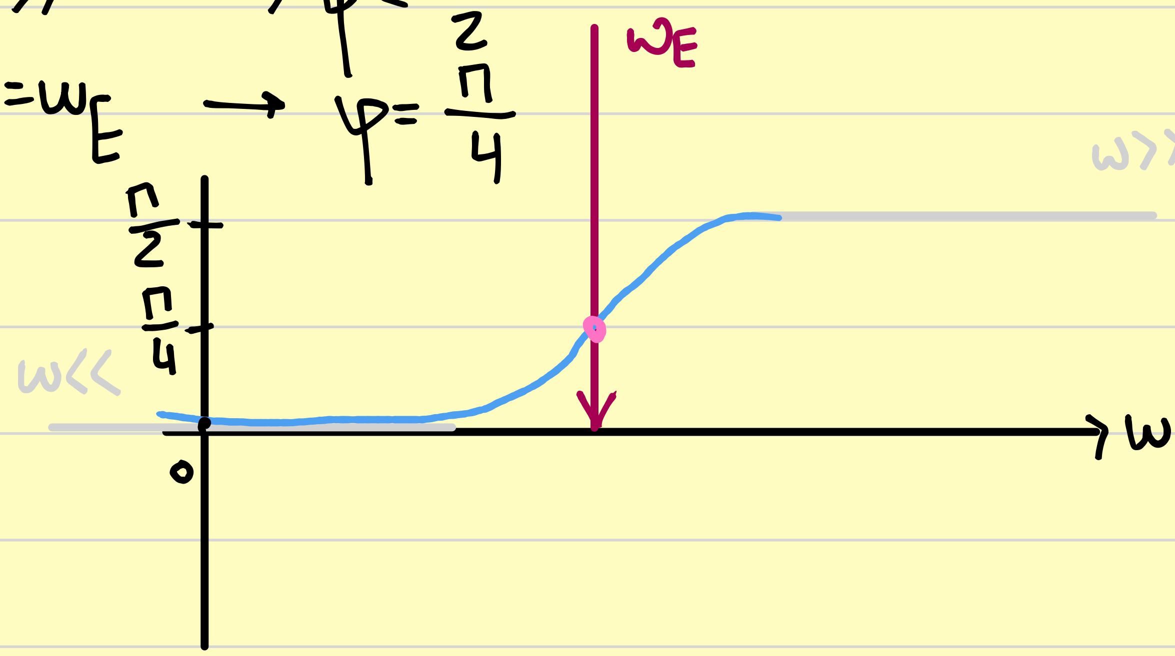
$$\varphi = \arctan(\omega T_V)$$

$\omega \ll \quad \rightarrow \quad \varphi = 0^\circ$

$\omega \gg \quad \rightarrow \quad \varphi = \frac{\pi}{2}$

$\omega = \omega_E \quad \rightarrow \quad \varphi = \frac{\pi}{4}$

PBD



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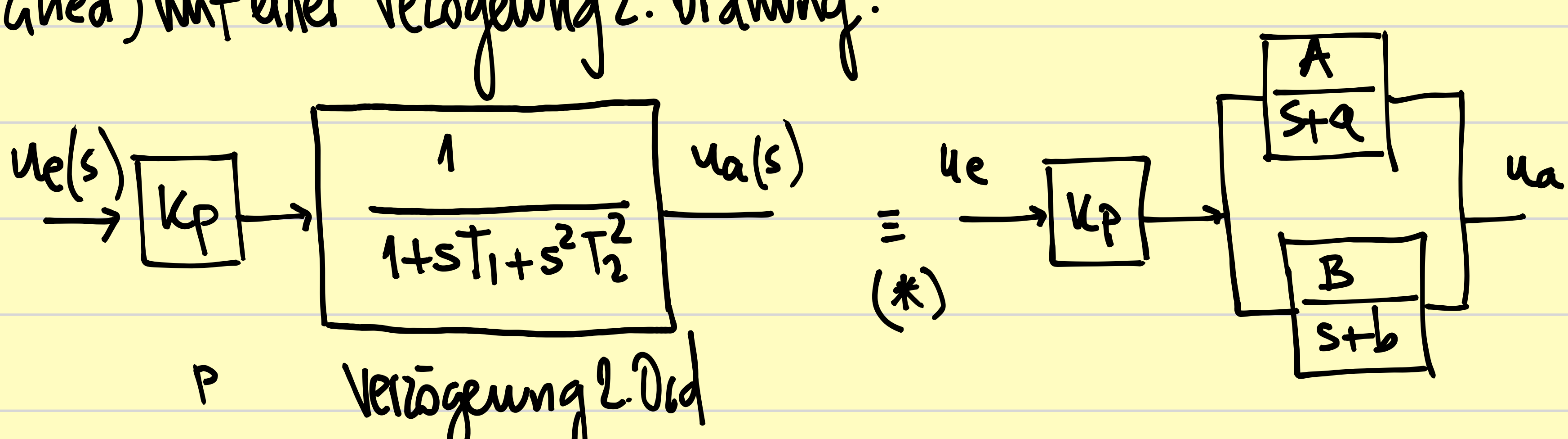
7. BD PT_2 - GLIED

Als PT_2-Glied bezeichnet man ein proportionales Übertragungsverhalten (P-Glied) mit einer Verzögerung 2. Ordnung.



(*)

$$\boxed{\frac{1}{1+sT_1+s^2T_2^2} = \frac{A}{s+a} + \frac{B}{s+b}}$$

$$\boxed{\text{Dämpfung} \doteq D = \frac{T_1}{2T_2}}$$

$$G(s) = \frac{K_p}{1 + sT_1 + s^2T_2^2}$$

Frequenzgang: $G(j\omega) = \frac{K_p}{1 + j\omega T_1 - (\omega T_2)^2} = \frac{K_p}{1-(\omega T_2)^2} + \frac{K_p}{\omega T_1} j$

$$Re\left(G(j\omega)\right) = \frac{K_p}{1-(\omega T_2)^2} \;;\; Im\left(G(j\omega)\right) = \frac{K_p}{\omega T_1}$$

$$|G(j\omega)| = \sqrt{Re^2 + Im^2} = \sqrt{\frac{K_p^2}{\left(1-(\omega T_2)^2\right)^2} + \frac{K_p^2}{(\omega T_1)^2}} = K_p\left[\left(1-(\omega T_2)^2\right)^2 + (\omega T_1)^2\right]^{-\frac{1}{2}}$$

$$\log|G(j\omega)| = \log K_p - \frac{1}{2}\log\left[\left(1-(\omega T_2)^2\right)^2 + (\omega T_1)^2\right]$$

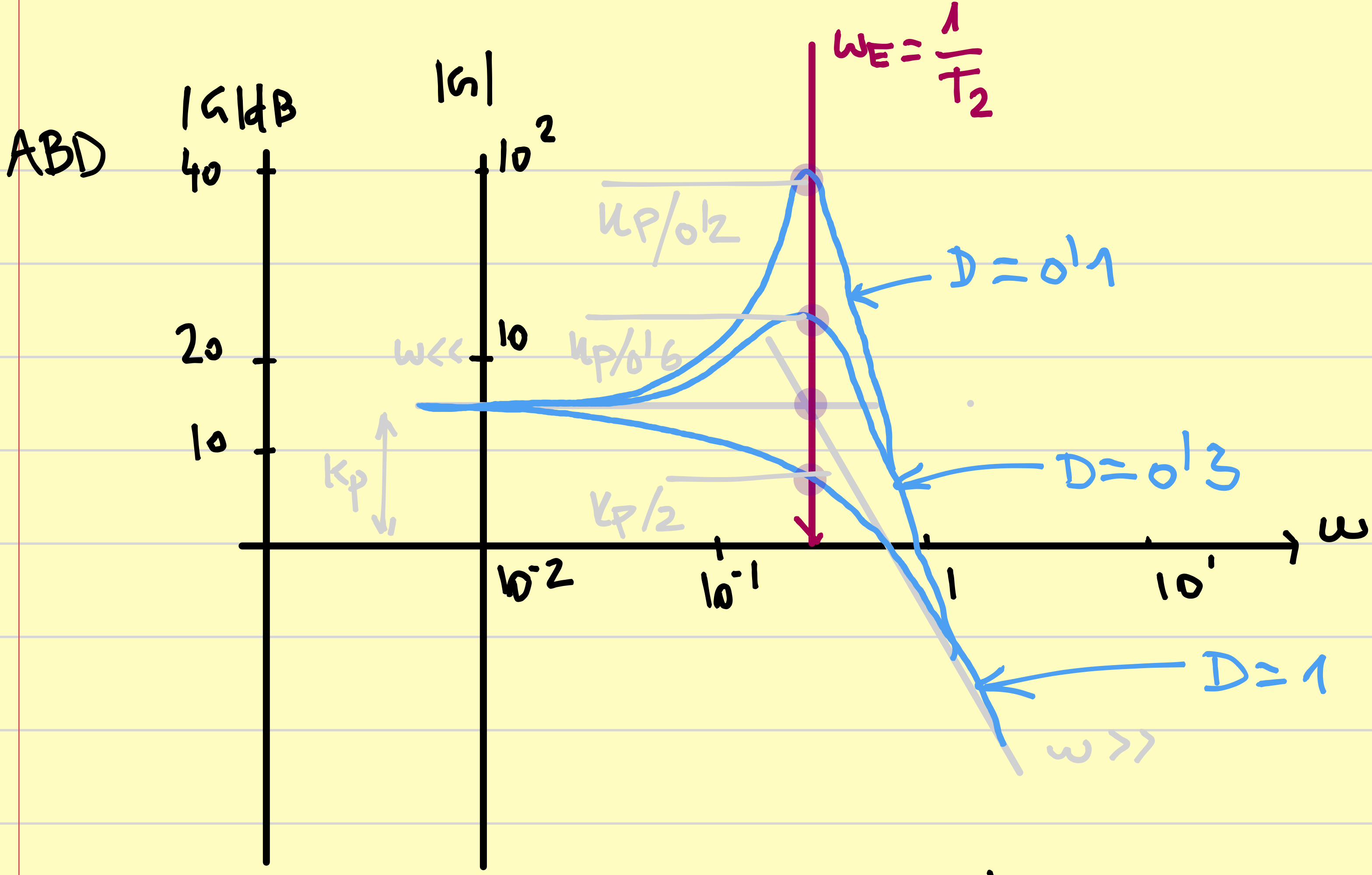
$\omega \ll \;\rightarrow\; \log|G(j\omega)| = \log K_p$

$\omega \gg \;\rightarrow\; (\omega T_2)^2 \gg 1 \;\&\; (\omega T_2)^2 \gg \omega T_1 \rightarrow$

$\rightarrow \log|G(j\omega)| = \log K_p - 2\log(\omega T_2)$ GERADE MIT NEG. NEIGUNG

In der Eckfrequenz treffen sich beide Asymptoten:

$$\omega_E \rightarrow |G(j\omega_E)| = \log K_p = \log K_p - 2\log(\omega_E T_2) \rightarrow \omega_E = \frac{1}{T_2}$$



$K_p = 7$

$T_2 = 2s$

$$|G(j\omega_E)| = K_p\left[\left(1-(\omega_E T_2)^2\right)^2 + (\omega_E T_1)^2\right]^{-\frac{1}{2}} \underset{\omega_E = \frac{1}{T_2}}{\overset{\uparrow}{=}} \frac{K_p \cdot T_2}{T_1} = \frac{K_p}{2D}$$

$D=1 \longrightarrow |G(j\omega_E)| = \frac{K_p}{2}$

$D=0{,}3 \rightarrow |G(j\omega_E)| = \frac{K_p}{0{,}6}$

$D=0{,}1 \rightarrow |G(j\omega_E)| = \frac{K_p}{0{,}2}$

Wenn $D>1$ ist, so lässt sich das PT_2-Glied in zwei PT_1-Glieder in Reihenschaltung zerlegen.

$$\tan\varphi = \frac{-\omega T_1}{1-(\omega T_2)^2} \rightarrow \varphi = -\operatorname{atan}\frac{\omega T_1}{1-(\omega T_2)^2}$$

$\omega \ll \longrightarrow \varphi = 0$

$\omega \gg \longrightarrow \varphi = -\frac{\pi}{2}$

$\omega = \omega_E \rightarrow \varphi = -\frac{\pi}{4}$