1. Lineare regression. Bitte die lineare regression ermitteln und vorhersagen wie das Gewicht sein wird bei 13 liter Bierkonsum.

(12 Punkte)

	Gewicht
10	70
15	80
17	90
18	92
x	y

$$y = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} =$$

$$\bar{x} = \frac{10+15+17+18}{4} = 15$$

$$\bar{y} = \frac{70+80+90+92}{4} = 83$$

$$= \frac{(10-15)(70-83)+(15-15)(80-83)+ + (17-15)(90-83)+(18-15)(92-83)}{(10-15)^2+(15-15)^2+(17-15)^2+(18-15)^2} =$$

$$= \frac{65+0+14+27}{25+0+4+9} = \frac{106}{38} = 2'789$$

$$\bar{y} = b_0 + b_1 \bar{x} \longrightarrow b_0 = 83 - 2'789 \cdot 15 = 41'157$$

$$\boxed{y = 41'157 + 2'789 \cdot x}$$ ✓

$$x = 13 \longrightarrow y = 41'157 + 2'789 \cdot 13 = 77'414$$ ✓

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2. Berechnen Sie die Kovarianz der oberen Daten. (8 PUNKTE)

$$cov(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{(10-15)(70-83)+(15-15)(80-83)+ +(17-15)(90-83)+(18-15)(92-83)}{4-1} =$$

$$= \frac{106}{3} = 35'333$$

Berechnen Sie die Kovarianzmatrix. (4 PUNKTE)

$$\text{KOV. MATRIX} = A = \begin{bmatrix} VAR_X & COV(X,Y) \\ COV(Y,X) & VAR_Y \end{bmatrix} =$$

$$VAR_X = \frac{\sum(x_i - \bar{x})^2}{n} = \frac{(10-15)^2+(15-15)^2+(17-15)^2+(18-15)^2}{4} =$$

$$= \frac{25+0+4+9}{4} = 9'5$$

$$VAR_Y = \frac{\sum(y_i - \bar{y})^2}{n} = \frac{(70-83)^2+(80-83)^2+(90-83)^2+(92-83)^2}{4} =$$

$$= \frac{169+9+49+81}{4} = 77$$

$$\boxed{A = \begin{bmatrix} 9'5 & 35'33 \\ 35'33 & 77 \end{bmatrix}}$$

✓

3 VARIABELN

$$\text{KOV MATRIX} = \begin{bmatrix} VAR_X & COV(X,Y) & COV(X,Z) \\ COV(Y,X) & VAR_Y & COV(Y,Z) \\ COV(Z,X) & COV(Z,Y) & VAR_Z \end{bmatrix}$$

3. Eigenwerte von $A = \begin{bmatrix} 0 & -6 \\ 1 & 5 \end{bmatrix}$ (4 PUNKTE)

$$\det[A-\lambda I]=0 \rightarrow \det\left(\begin{bmatrix} 0 & -6 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)=0 \rightarrow$$

$$\rightarrow \det\left(\begin{bmatrix} -\lambda & -6 \\ 1 & 5-\lambda \end{bmatrix}\right)=0 \rightarrow$$

$$-\lambda(5-\lambda) - 1.(-6) = 0 \rightarrow -5\lambda + \lambda^2 + 6 = 0$$

$$\lambda = \frac{5 \pm \sqrt{5^2 - 4.6}}{2} = \frac{5 \pm 1}{2} = \begin{cases} 3 = \lambda_1 \\ 2 = \lambda_2 \end{cases}$$

Eigenvektoren dazu. [8 PUNKTE]

$$A.\vec{v_1} = \lambda_1 \vec{v_1} \rightarrow \begin{bmatrix} 0 & -6 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = 3 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$$

$$0.v_{11} - 6.v_{12} = 3.v_{11} \rightarrow v_{11} = -2v_{12}$$

$$1.v_{11} + 5.v_{12} = 3v_{12} \rightarrow v_{11} = -2v_{12}$$

$$v_{11} = 1 \;;\; v_{12} = -0'5 \rightarrow \vec{v_1} = \begin{bmatrix} 1 \\ -0'5 \end{bmatrix}$$

$$A.\vec{v_2} = \lambda_2 \vec{v_2} \rightarrow \begin{bmatrix} 0 & -6 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = 2 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$$

$$\left.\begin{aligned} -6v_{22} &= 2v_{21} \rightarrow v_{21} = -3v_{22} \\ v_{21} + 5v_{22} &= 2v_{22} \rightarrow v_{21} = -3v_{22} \end{aligned}\right\}$$

$v_{21}=1$; $v_{22}=-0'33$ $\rightarrow$ $\vec{v}_2=\begin{bmatrix}1\\-0'33\end{bmatrix}$ ✓

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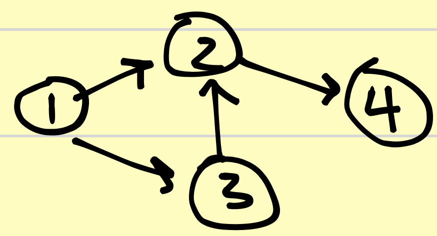
4. Laplacian, APL & CC vom Netzwerk G ermitteln [12 PUNKTE]

G :



$$\mathcal{L}=D-A=\begin{bmatrix}2&&&\\&3&&0\\&&2&\\0&&&1\end{bmatrix}-\begin{bmatrix}0&1&1&0\\1&0&1&1\\1&1&0&0\\0&1&0&0\end{bmatrix}=\begin{bmatrix}2&-1&-1&0\\-1&3&-1&-1\\-1&-1&2&0\\0&-1&0&1\end{bmatrix}$$

$$APL=\frac{1}{4\cdot(4-1)}\cdot\Big[(1+1+2)+(1+1+1)+(1+1+2)+(2+1+2)\Big]=$$

$$=\frac{1}{12}\Big[4+3+4+5\Big]=\frac{16}{12}=\frac{4}{3}=1'33$$

$$CC=\frac{1}{4}\sum\frac{2L_i}{k_i(k_i-1)}=\frac{1}{4}\left[\frac{2\cdot1}{2(2-1)}+\frac{2\cdot1}{3(3-1)}+\frac{2\cdot1}{2(2-1)}+0\right]=$$

$$=\frac{1}{4}\left[1+\frac{1}{3}+1\right]=\frac{3'33}{4}=0'8325$$

$$CC=\frac{1}{N}\sum\frac{2L_i}{k_i(k_i-1)}$$

$N \equiv$ NUMBER of NODES

$L_i \equiv$ # BEZIEHUNGEN ZW. DEN NACHBARN VOM NODE i

$k_i \equiv$ # NACHBARN NODE i

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5. K Means Cluster. Bitte Position der 2 Lager für die Werke mittels clustering coefficient ermitteln.

[12 PUNKTE]

	X	Y
w_1	0	3
w_2	1	4
w_3	4	1
w_4	5	0
w_5	7	3

$A\{w_1, w_2\}$ $\quad\quad$ $B\{w_3, w_4, w_5\}$

$$z_A = \left[\frac{0+1}{2}, \frac{3+4}{2}\right] = \left[\frac{1}{2}, \frac{7}{2}\right] \quad\quad z_B = \left[\frac{4+5+7}{3}, \frac{1+0+3}{3}\right] = \left[\frac{16}{3}, \frac{4}{3}\right]$$

Abstände

$$d_{w_1,A} = \sqrt{\left(0-\frac{1}{2}\right)^2 + \left(3-\frac{7}{2}\right)^2} = 0'707 \quad d_{w_1,B} = \sqrt{\left(0-\frac{16}{3}\right)^2 + \left(3-\frac{4}{3}\right)^2} = 5'587$$

$$d_{w_2,A} = \sqrt{\left(1-\frac{1}{2}\right)^2 + \left(4-\frac{7}{2}\right)^2} = 0'707 < d_{w_2,B} = \sqrt{\left(1-\frac{16}{3}\right)^2 + \left(4-\frac{4}{3}\right)^2}$$

$$d_{w_3,A} = \sqrt{\left(4-\frac{1}{2}\right)^2 + \left(1-\frac{7}{2}\right)^2} > d_{w_3,B} = \sqrt{\left(4-\frac{16}{3}\right)^2 + \left(1-\frac{4}{3}\right)^2}$$

$$d_{w_4,A} = \sqrt{\left(5-\frac{1}{2}\right)^2 + \left(0-\frac{7}{2}\right)^2} > d_{w_4,B} = \sqrt{\left(5-\frac{16}{3}\right)^2 + \left(0-\frac{4}{3}\right)^2}$$

$$d_{w_5,A} = \sqrt{\left(7-\frac{1}{2}\right)^2 + \left(3-\frac{7}{2}\right)^2} > d_{w_5,B} = \sqrt{\left(7-\frac{16}{3}\right)^2 + \left(3-\frac{4}{3}\right)^2}$$

$A\{w_1, w_2\}$ $\quad\quad$ $B\{w_3, w_4, w_5\}$

$z_A = [0'5, 3'5]$ $\quad\quad$ $z_B = [5'33, 1'33]$