4. PD P-GLIED MIT VERZÖGERUNG

Übertragungsfunktion $G(s) = \frac{k_P}{1+T_1 s}$

Frequenzgang lautet $G(j\omega) = \frac{k_P}{1+T_1 j\omega}$

Daraus folgt $G(j\omega) = \frac{k_P}{|1+j\omega T_1|} \cdot e^{-j \cdot \arctan\left(\frac{\omega T_1}{1}\right)}$

$$|G(j\omega)| = \frac{k_P}{\sqrt{1+(\omega T_1)^2}} \rightarrow \log|G(j\omega)| = \log k_P - \frac{1}{2}\log\left(1+(\omega T_1)^2\right)$$

$$\tan\varphi = -\omega T_1 \rightarrow \varphi = -\arctan(\omega T_1)$$

ABD: $\omega << \rightarrow \omega T_1 << 1 \rightarrow \log|G(j\omega)| \simeq \log k_P$

$\omega >> \rightarrow \omega T_1 >> 1 \rightarrow \log|G(j\omega)| \simeq \log k_P - \log(\omega T_1)$ (**)

(**)das ist eine Linie mit negativen Steigung.

Beide Asymptoten schneiden sich für ω_E (ECKFREQUENZ)

$$\log|G(j\omega_E)| = \log k_P = \log k_P - \log(\omega_E T_1)$$

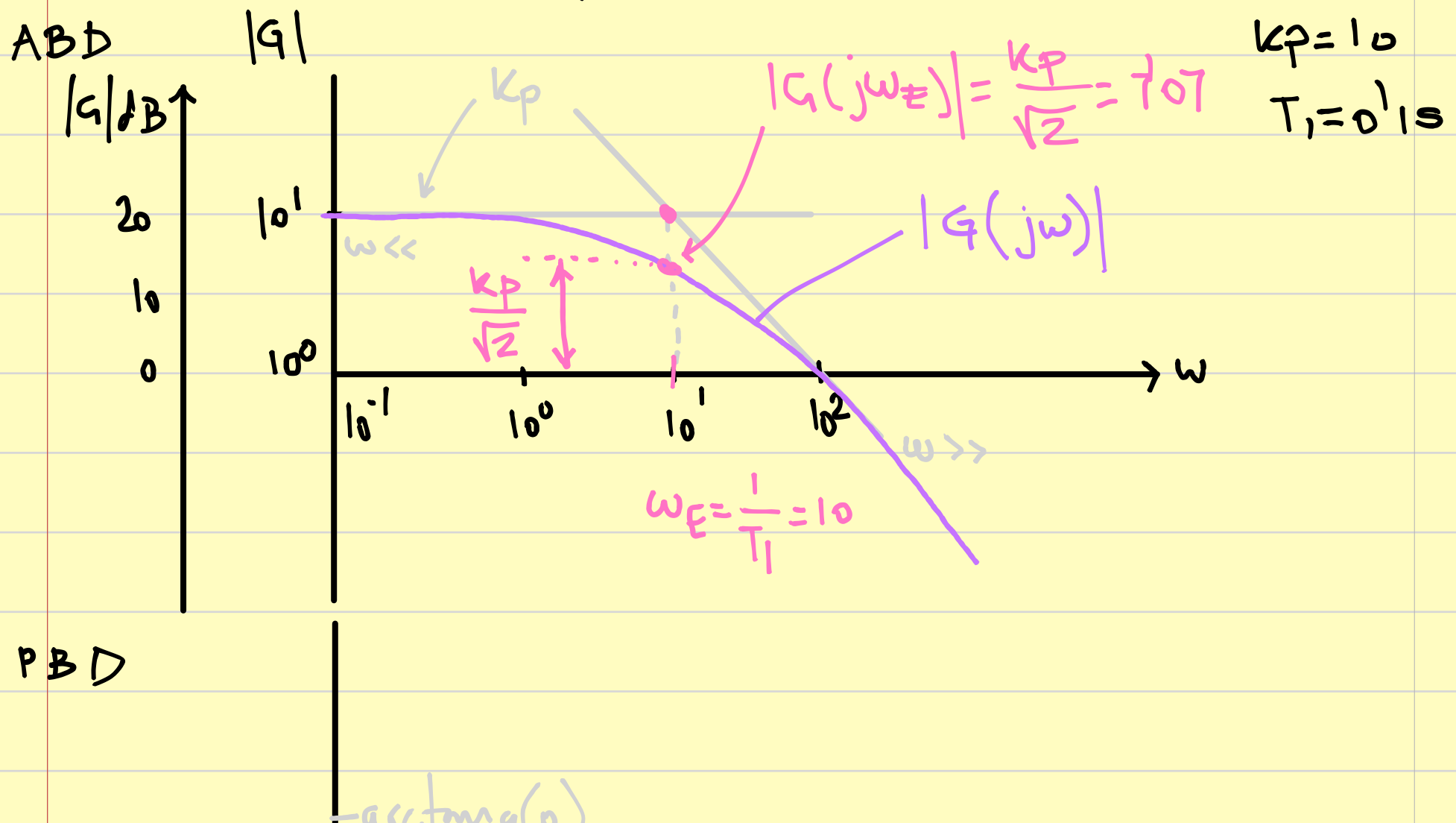
$$\rightarrow \omega_E = \frac{1}{T_1}$$

$$\log|G(j\omega_E)| = \log k_P - \frac{1}{2}\log\left(1+\omega_E T_1\right) =$$

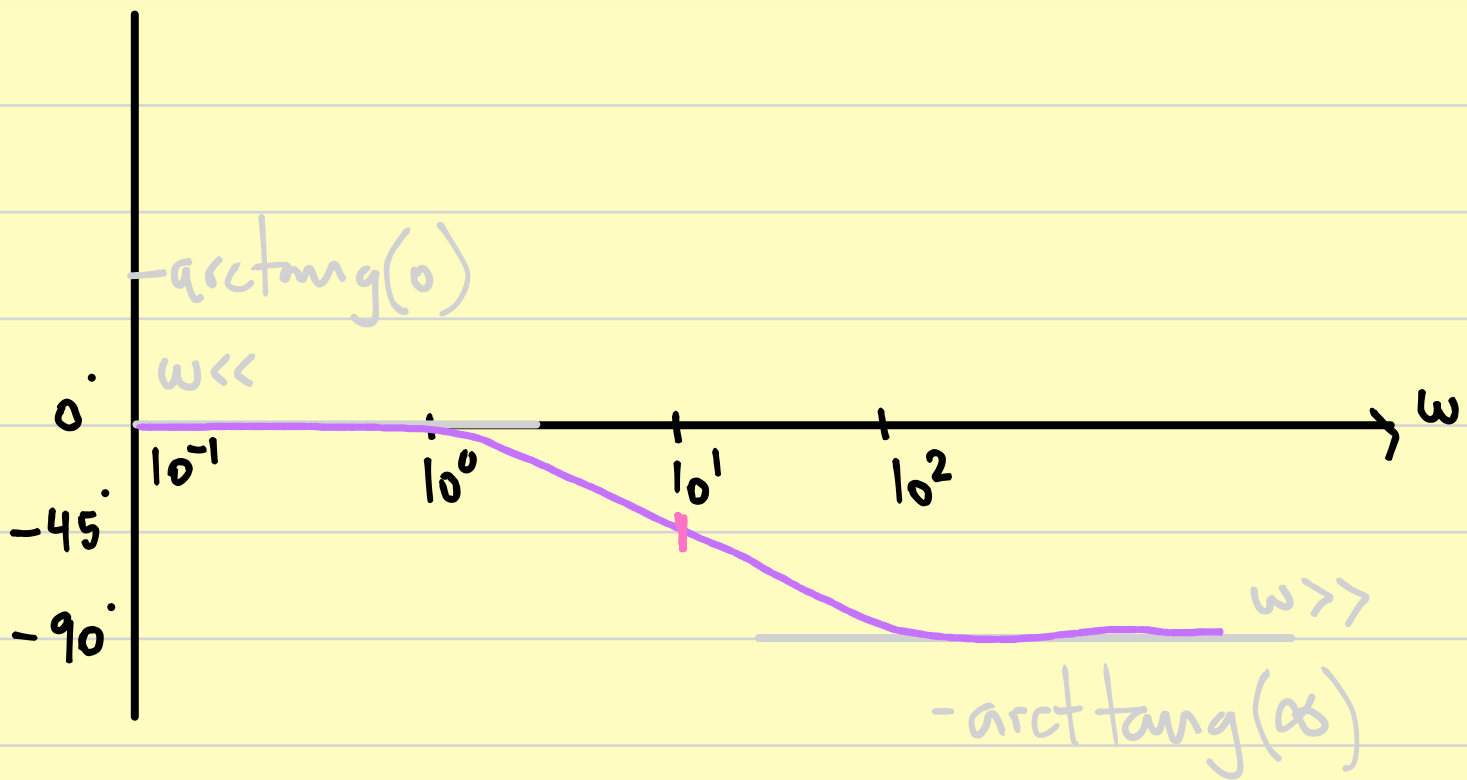
$$= \log k_P - \frac{1}{2}\log\left(1+\frac{T_1}{T_1}\right) =$$

$$|G(j\omega_E)| = \frac{K_P}{\sqrt{2}} = 0{,}707\,K_P$$

ABD

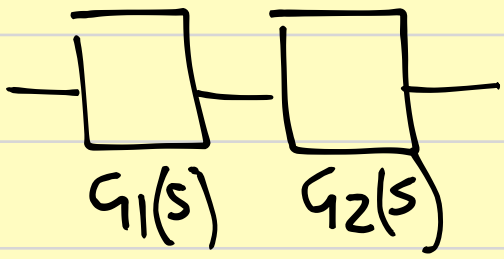


PBD



$\omega = \omega_E \rightarrow \varphi = -45°$

Übung. Zeichnen Sie das BD von folgenden Gliedern.



$$G_1(s) = \frac{s+3}{s+1} \qquad G_2(s) = \frac{1}{s+2}$$

Schritt 1. Übertragungsfunktion $G(s) = \frac{s+3}{(s+1)(s+2)}$

Schritt 2. Frequenzgang. $G(j\omega) = \frac{3+j\omega}{(1+j\omega)(2+j\omega)} =$

$$= \frac{3+j\omega}{2-\omega^2+3j\omega} \cdot \frac{2-\omega^2-3j\omega}{2-\omega^2-3j\omega} =$$

$$= \frac{6-3\omega^2-9j\omega+2\omega j-\omega^3 j+3\omega^2}{(2-\omega^2)^2+9\omega^2} =$$

$$= \frac{6-j(7\omega+\omega^3)}{4-4\omega^2+\omega^4+9\omega^2} = \frac{1}{\omega^4+5\omega^2+4}\left[6-j\omega(7-\omega^2)\right]$$

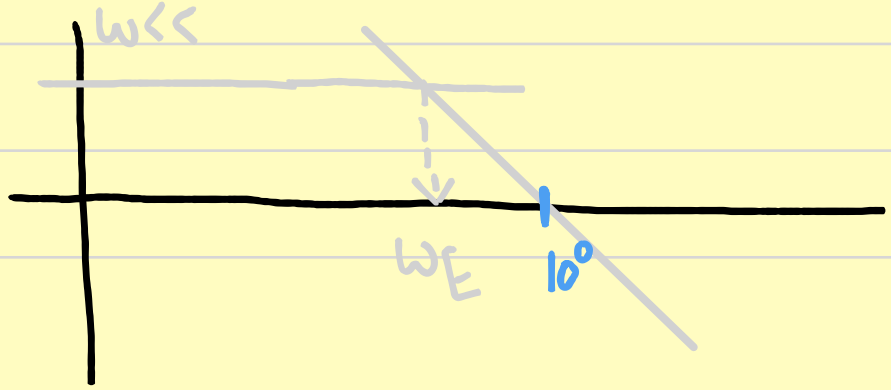
$$|G(j\omega)| = \sqrt{Re^2(G(j\omega))+Im^2(G(j\omega))} = \sqrt{\frac{6^2}{(\omega^4+5\omega^2+4)^2} - \frac{(\omega(7-\omega^2))^2}{(\omega^4+5\omega^2+4)^2}} =$$

$$= \frac{1}{\omega^4+5\omega^2+4}\sqrt{-\omega^6+14\omega^4-49\omega^2+36}$$

$\omega << \rightarrow |G(\omega<<)| = \frac{6}{4} = 1'5 \rightarrow |G(\omega<<)|_{dB} = 20\log 1'5$

$\omega >> \rightarrow |G(\omega>>)| = \frac{-1}{\omega} \rightarrow -0 \rightarrow |G(\omega>>)|_{dB} = -\infty$

ABD



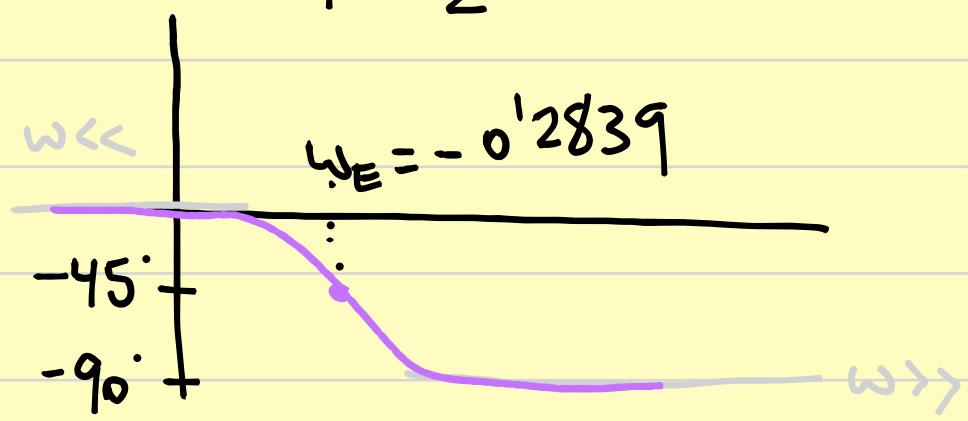
ω_E:

$$\frac{-1}{\omega_E} = 20\log 1'5 \rightarrow \omega_E = -0'2839\ s^{-1}$$

$$\varphi = \text{arctang}\left(\frac{Im}{Re}\right) = \text{atan}\left(\frac{-\omega(7-\omega^2)}{6}\right) = \text{atan}\left(\frac{\omega^3-7\omega}{6}\right)$$

$\omega << \rightarrow \varphi = 0$

$\omega >> \rightarrow \varphi = \frac{-\pi}{2}$



Übung. gleiche Aufgabe mit parallelen Schaltung.

$$G(s) = \frac{s+3}{s+1} + \frac{1}{s+2} = \frac{(s+3)(s+2)+(s+1)}{(s+1)(s+2)} = \frac{s^2+6s+7}{s^2+3s+2}$$

$$G(j\omega) = \frac{-\omega^2+7+j6\omega}{-\omega^2+2+j3\omega} \cdot \frac{-\omega^2+2-j3\omega}{-\omega^2+2-j3\omega} =$$

$$= \frac{(-\omega^2+7)(-\omega^2+2)+18\omega^2+j\left[6\omega(-\omega^2+2)-3\omega(-\omega^2+7)\right]}{(-\omega^2+2)^2+9\omega^2}$$

$$= \frac{\omega^4-9\omega^2+14+18\omega^2+j\left[-6\omega^2+12+3\omega^2-21\right]\omega}{\omega^4+5\omega^2+4}$$

$$G(jw) = \frac{w^4 + 9w^2 + 14}{w^4 + 5w^2 + 4} + j\frac{-3w^3 - 9w}{w^4 + 5w^2 + 4}$$

$j = \sqrt{-1}$

Re Im

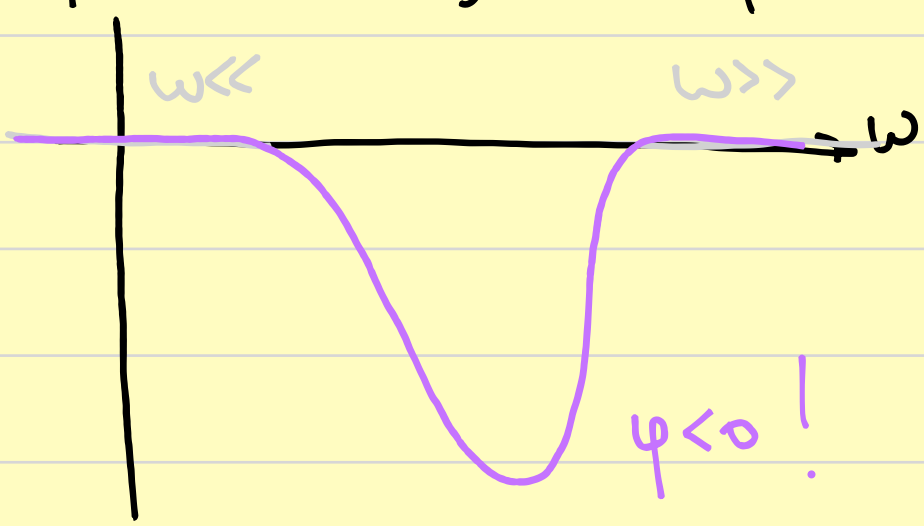
$$|G(jw)| = \sqrt{Re^2 + Im^2} = \frac{1}{w^4 + 5w^2 + 4}\sqrt{(w^4 + 9w^2 + 14)^2 - (3w^3 + 9w)^2}$$

$w << \rightarrow |G(w<<)| = \frac{14}{4} \rightarrow |G(w<<)|_{dB} \simeq 10\,dB$

$w >> \rightarrow |G(w>>)|_{dB} = 0$

$|G(w<<)|_{dB} = 20 \log_{10}\left(\frac{14}{4}\right)$

$\varphi = \arctan\left(\frac{Im}{Re}\right) = \operatorname{atan}\left(\frac{-3w^3 - 9w}{w^4 + 9w^2 + 14}\right)$



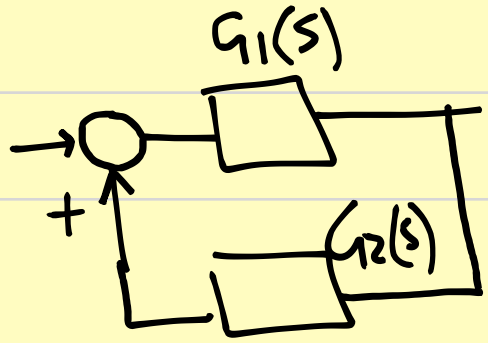
$w << \rightarrow \varphi = 0$

$w >> \rightarrow \varphi = 0$

Übung.

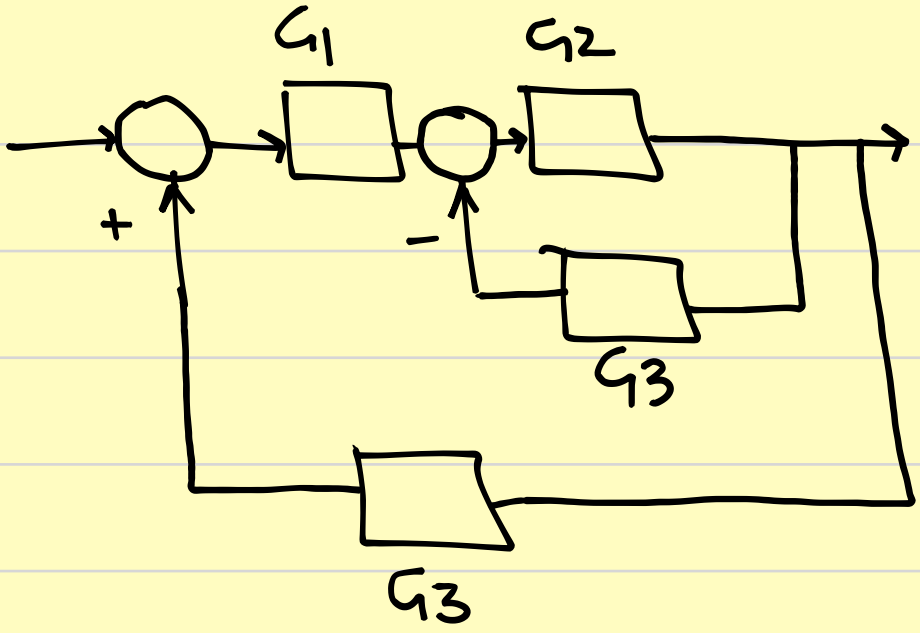
$G_1(s) = \frac{s+3}{s+2}$

$G_2(s) = \frac{1}{s+2}$



Bitte das BD der Rückkopplungsschaltung (Gegenkopplung) darstellen.

Übung.



Bitte das BD der Regelkreisglieder darstellen.

$G_1 = \frac{1}{1+2s}$

$G_2 = \frac{1}{1+s}$

$G_3 = \frac{1}{2+s}$