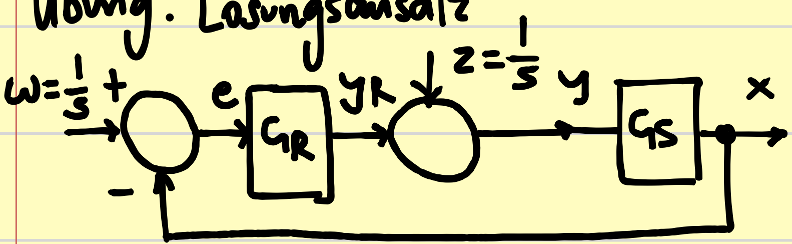
Übung. Lösungsansatz



$$G_R = 0{,}3 + \frac{0{,}2}{s+0{,}3}$$

$$G_S = \frac{0{,}9}{s+1}$$

a) Führungsverhalten

$$G_W = \frac{G_R \cdot G_S}{1+G_R G_S} = \frac{\left(0{,}3+\frac{0{,}2}{s+0{,}3}\right)\cdot\frac{0{,}9}{s+1}}{1+\left(0{,}3+\frac{0{,}2}{s+0{,}3}\right)\cdot\frac{0{,}9}{s+1}} =$$

$$= \frac{\left(0{,}3(s+0{,}3)+0{,}2\right)\cdot 0{,}9}{(s+0{,}3)(s+1)+\left(0{,}3(s+0{,}3)+0{,}2\right)\cdot 0{,}9} =$$

$$= \frac{0{,}27s+0{,}261}{s^2+1{,}57s+0{,}561} = \frac{A}{(s+0{,}55)}+\frac{B}{(s+1{,}02)} \rightarrow (*)$$

$$s^* = \frac{-1{,}57 \pm \sqrt{(1{,}57)^2-4\cdot 0{,}561}}{2} = \frac{-1{,}57\pm 0{,}47}{2} = \begin{cases} -0{,}55 \\ -1{,}02 \end{cases}$$

STABIL!

$$(*)\quad 0{,}27s+0{,}261 = A(s+1{,}02)+B(s+0{,}55)$$

$$s^* = -1{,}02 \rightarrow -0{,}0144 = B\cdot(-0{,}47) \rightarrow B = 0{,}03$$

$$s^* = -0{,}55 \rightarrow 0{,}1125 = A\cdot(0{,}47) \rightarrow A = 0{,}24$$

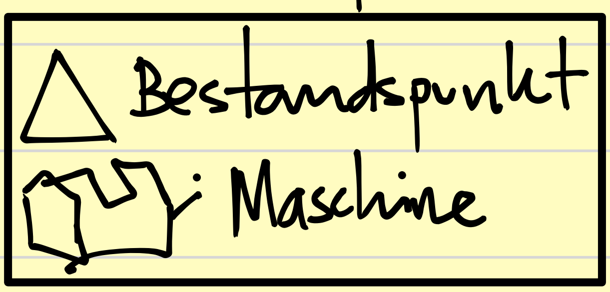
$$G_W = \frac{0{,}24}{s+0{,}55}+\frac{0{,}03}{s+1{,}02} = \frac{x}{w} \rightarrow x = \frac{1}{s}\cdot\left[\frac{0{,}24}{s+0{,}55}+\frac{0{,}03}{s+1{,}02}\right]$$

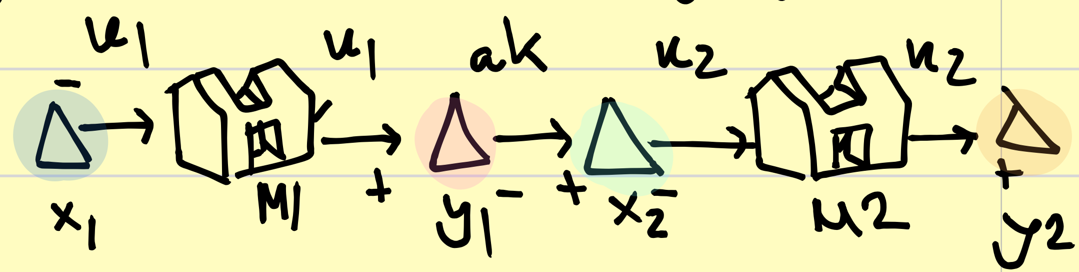
b) Störverhalten

$$G_Z = \frac{G_S}{1+G_R G_S} = \frac{\frac{0{,}9}{1+s}}{1+\left(0{,}3+\frac{0{,}2}{s+0{,}3}\right)\cdot\frac{0{,}9}{1+s}} = \frac{0{,}9(s+0{,}3)}{(s+0{,}3)(s+1)+0{,}9\left[0{,}3(s+0{,}3)+0{,}2\right]}$$

$$= \frac{0'9\,(s+0'3)}{(s+0'55)(s+1'02)} = \frac{A'}{s+0'55} + \frac{B'}{s+1'02} \to \dots$$

Regelung von einfachen logistischen Wertschöpfungskette





M1

$$\dot{x}_1 = -k_1 x_1$$

$$\dot{y}_1 = k_1 x_1 - ak\, y_1$$

Die Änderung des Bestands in y_1, ist proportional zum Bestand in y_1 & x_1

M2

$$\dot{x}_2 = -k_2 x_2 + ak\, y_1$$

$$\dot{y}_2 = +k_2 x_2$$

Laplace:

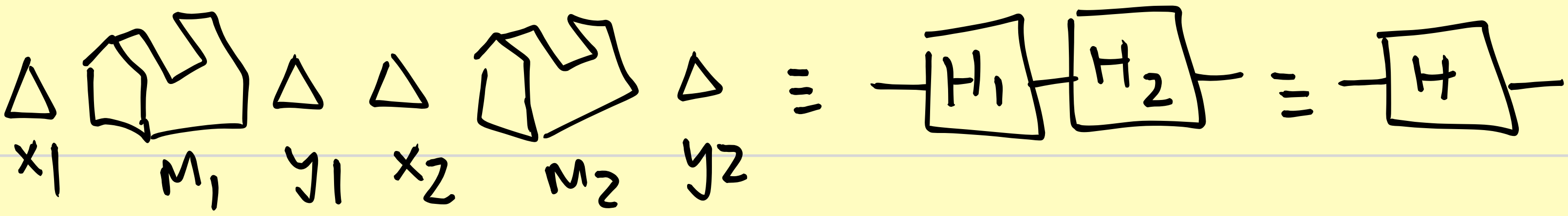
M1

$$\left.\begin{aligned} sX_1(s) &= -k_1 x_1(s) + x_1(0) \\ sY_1(s) &= k_1 x_1(s) - ak\,Y_1(s) + y_1(0) \end{aligned}\right\} (\bullet)$$

M2

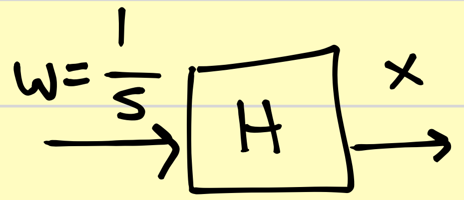
$$\left.\begin{aligned} sX_2(s) &= -k_2 x_2(s) + ak\,Y_1(s) + x_2(0) \\ sY_2(s) &= k_2 x_2(s) + y_2(0) \end{aligned}\right\} (\bullet\bullet)$$

$$\left.\begin{aligned} (\bullet)\quad H_1(s) &= \frac{Y_1(s)}{X_1(s)} = \frac{k_1}{s+ak} \\ (\bullet\bullet)\quad H_2(s) &= \frac{Y_2(s)}{X_2(s)} = \frac{k_2}{s} \end{aligned}\right\} H(s) = H_1(s) \cdot H_2(s) = \frac{k_1 k_2}{s(s+ak)}$$

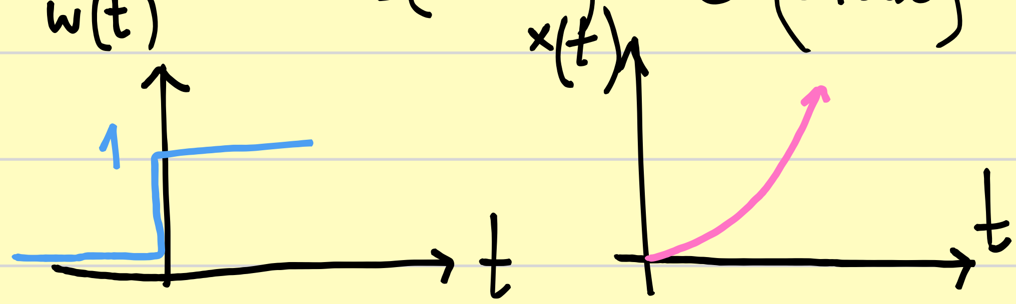


Regelung vom System:

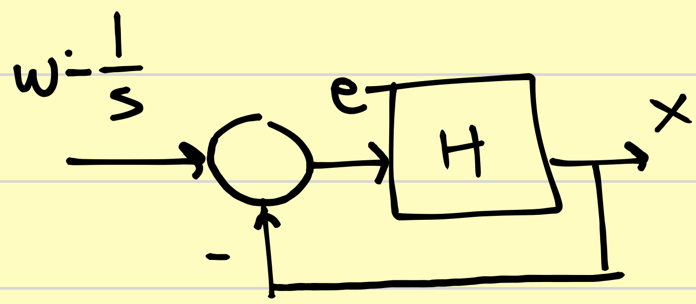
a) OHNE REGLER:



$$x(s) = \underbrace{\frac{1}{s}}_{w(t)} \cdot \frac{k_1 k_2}{s(s+ak)} = \frac{k_1 k_2}{s^2(s+ak)} \rightarrow x(t) = \frac{k_1 k_2}{(ak)^2}\left(e^{-akt} - 1 + t\right)$$



b) REGELUNG:



$$x(s) = \frac{1}{s} \cdot \frac{H}{1+H} = \frac{1}{s} \cdot \frac{k_1 k_2}{s(s+ak)+k_1 k_2} =$$

$$= \frac{k_1 k_2}{s\left(s^2 + aks + k_1 k_2\right)}$$

$$s^* = \frac{-ak \pm \sqrt{(ak)^2 - 4k_1 k_2}}{2}$$

KONDITION FÜR STABILITÄT IST STETS GEGEBEN! $(\mathrm{Re}\, s^* < 0)$

i.e.

$k = k_1 = k_2 = 1$ (ideale Bestandspunkte)

$a = \frac{1}{3}$ (Verlust im Bestand)

$$s^* = \frac{-\frac{1}{3} \pm \sqrt{\frac{1}{9} - 4}}{2} = -\frac{1}{6} \pm j\,1{,}97$$

$$x(s) = \frac{1}{s\left(s+\frac{1}{6}-j1'97\right)\left(s+\frac{1}{6}+j1'97\right)} = \frac{A}{s} + \frac{B}{\left(s+\frac{1}{6}-j1'97\right)} + \frac{C}{\left(s+\frac{1}{6}+j1'97\right)}$$

$$1 = A\left(s^2+\frac{1}{36}+1'97^2\right) + Bs\left(s+\frac{1}{6}+j1'97\right) + Cs\left(s+\frac{1}{6}-j1'97\right)$$

$$s^* = 0 \rightarrow \quad A = \frac{1}{\frac{1}{36}+1'97^2} = \frac{1}{\frac{1}{36}+3'8809} = 0'256$$

$$s^* = \frac{-1}{6}-j1'97 \rightarrow 1 = C\left(\frac{-1}{6}-j1'97\right)\left(-j1'97\right)2 \rightarrow$$

$$\rightarrow 1 = 2C\left(-3'8809+j0'328\right) \rightarrow$$

$$\rightarrow C = \frac{1}{-7'7618+j0'656} \cdot \frac{-7'7618-j0'656}{-7'7618-j0'656}$$

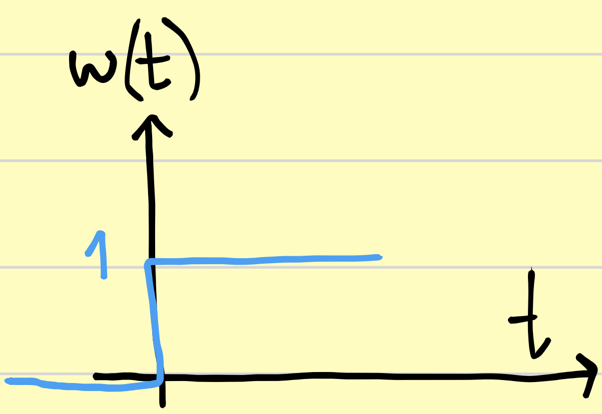
$$= \frac{1}{60'24+0'43}\left(-7'7618-j0'656\right)$$

$$s^* = \frac{-1}{6}+j1'97 \rightarrow 1 = B\left(\frac{-1}{6}-j1'97\right)\left(j1'97\right)2 \rightarrow$$

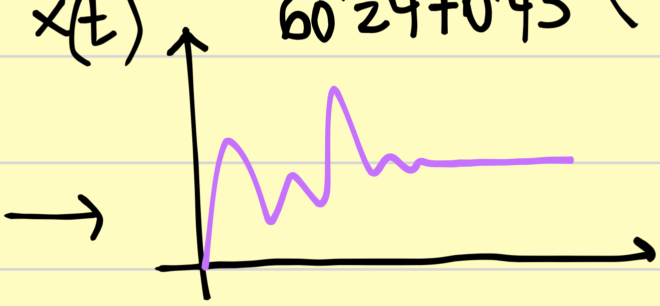
$$\rightarrow 1 = 2B\left(3'8809-j0'328\right) \rightarrow$$

$$\rightarrow B = \frac{1}{7'768-j0'656} \cdot \frac{7'7618+j0'656}{7'7618+j0'656}$$

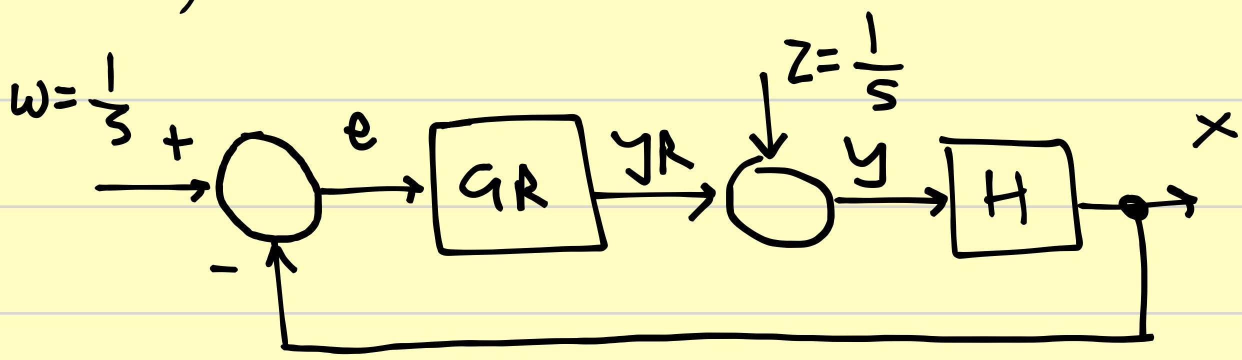
$$= \frac{1}{60'24+0'43}\left(7'7618+j0'656\right)$$



$\longrightarrow$



c) REGELEINRICHTUNG



$$x(s) = \frac{1}{s}\cdot\frac{G_R \cdot H}{1+G_R H} = \frac{1}{s}\cdot\frac{\frac{k_R}{1+Ts}\cdot\frac{k_1 k_2}{s(s+ak)}}{1+\frac{k_R}{1+Ts}\cdot\frac{k_1 k_2}{s(s+ak)}} =$$

$$G_R = \frac{k_R}{1+Ts}$$

$$= \frac{k_R k_1 k_2}{s\left[(1+Ts)s(s+ak)+k_R k_1 k_2\right]} =$$

$$= \frac{k_R k_1 k_2}{s\left[Ts^3+s^2(ak+T)+s\,akT+k_R k_1 k_2\right]}$$

i.e

$k_R = k_1 = k_2 = k = 1$

$a = \frac{1}{3}$

$T = 0'9$

$$= \frac{1}{s\left[0'9s^3+s^2\cdot 1'2+s\cdot 0'3+1\right]} = \ldots$$

STABIL!