20240606_Regelungstechnik_BEL4

BODEDIAGRAMM

$$\boxed{\log(a\cdot b)=\log a+\log b}$$

1. $x_e \rightarrow \boxed{G_1(s)=0{,}5} \rightarrow \boxed{G_2(s)=\dfrac{1}{1+0{,}5s}} \rightarrow x_a \;\hat{=}\; \rightarrow \boxed{\dfrac{0{,}5}{1+0{,}5s}} \rightarrow$

$$G_1(j\omega)=0{,}5 \rightarrow |G_1(j\omega)|=0{,}5\;;\; \angle G_1(j\omega)=\arctan\frac{Im}{Re}=\arctan\frac{0}{0{,}5}=0$$

$$|G_1(j\omega)|_{dB}=20\log|G_1(j\omega)|=-6{,}02\,dB$$

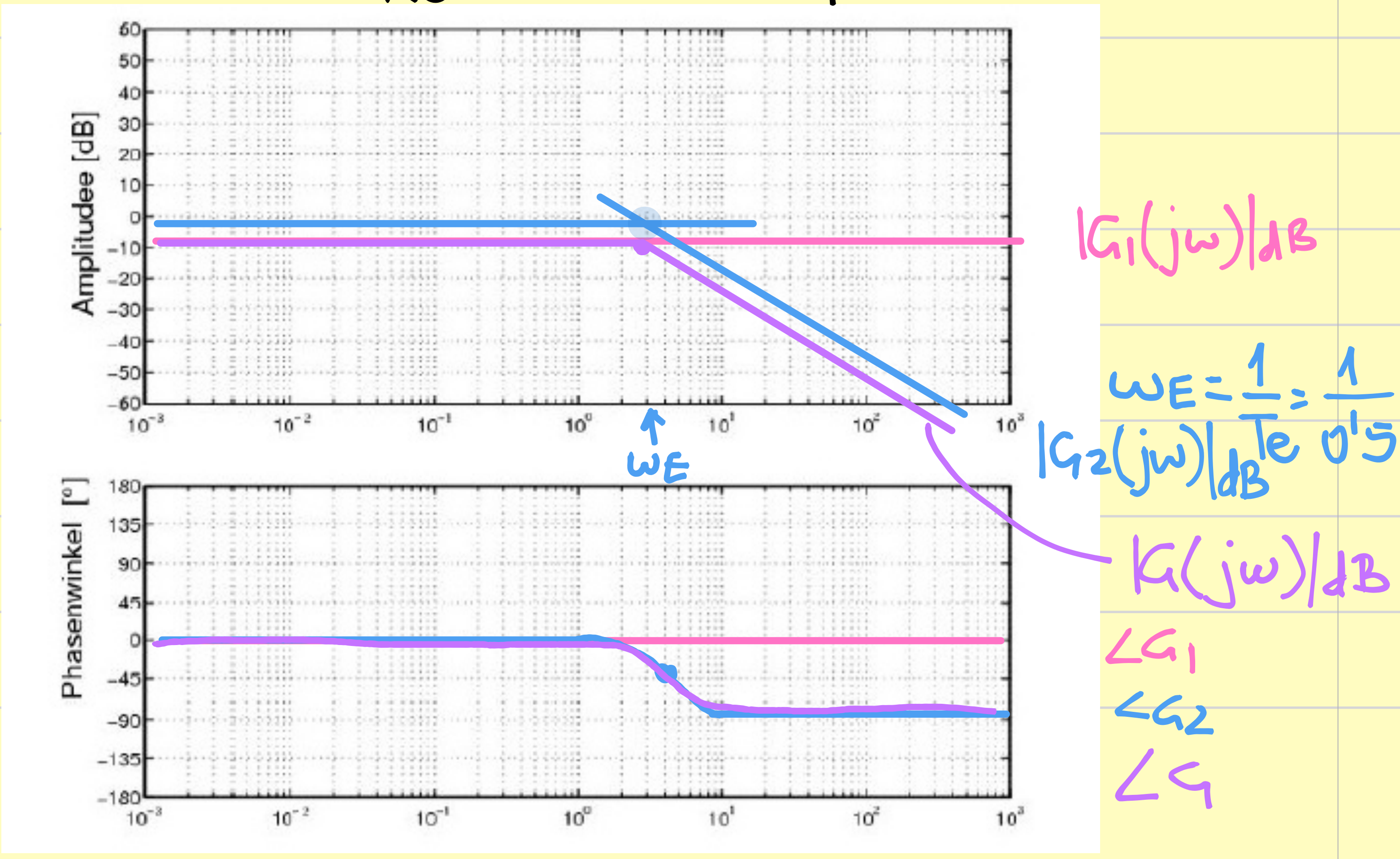
$$G_2(j\omega)=\frac{1}{1+0{,}5j\omega}\cdot\frac{1-0{,}5j\omega}{1-0{,}5j\omega}=\frac{1-0{,}5j\omega}{1^2+0{,}5^2\omega^2}$$

$$\rightarrow |G_2(j\omega)|=\sqrt{Re^2+Im^2}=\frac{1}{1^2+0{,}5^2\omega^2}\cdot\sqrt{1^2+0{,}5^2\omega^2}=\left(1+0{,}25\omega^2\right)^{-1/2}$$

$$|G_2(j\omega)|_{dB}=20\log|G_2(j\omega)|=20\cdot-\frac{1}{2}\log\left(1+0{,}25\omega^2\right)$$

$$=-10\log\left(1+0{,}25\omega^2\right)$$

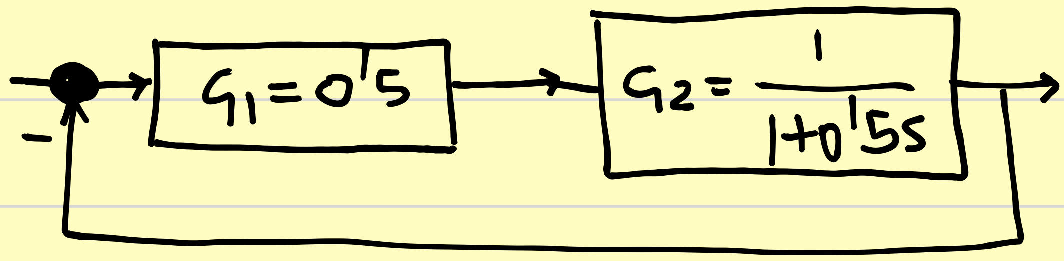
$$\angle G_2(j\omega)=\arctan\frac{Im}{Re}=\arctan\frac{-0{,}5\omega}{1}$$



$$|G(j\omega)|_{dB} = |G_1(j\omega)|_{dB} + |G_2(j\omega)|_{dB}$$

$$\angle G = \angle G_1 + \angle G_2$$

2.



Bodediagramm

$$G(s) = \frac{G_1 G_2}{1+G_1 G_2} = \frac{\frac{0'5}{1+0'5s}}{1+\frac{0'5}{1+0'5s}} = \frac{0'5}{1'5+0'5s} = \frac{0'5}{1'5}\cdot\frac{1}{1+\frac{0'5s}{1'5}}$$

$$\omega_E = \frac{1}{\frac{0'5}{1'5}} = \frac{1'5}{0'5} = 3$$

$$G(s) = \frac{1}{3}\,\frac{1}{1+\frac{1}{3}s}$$

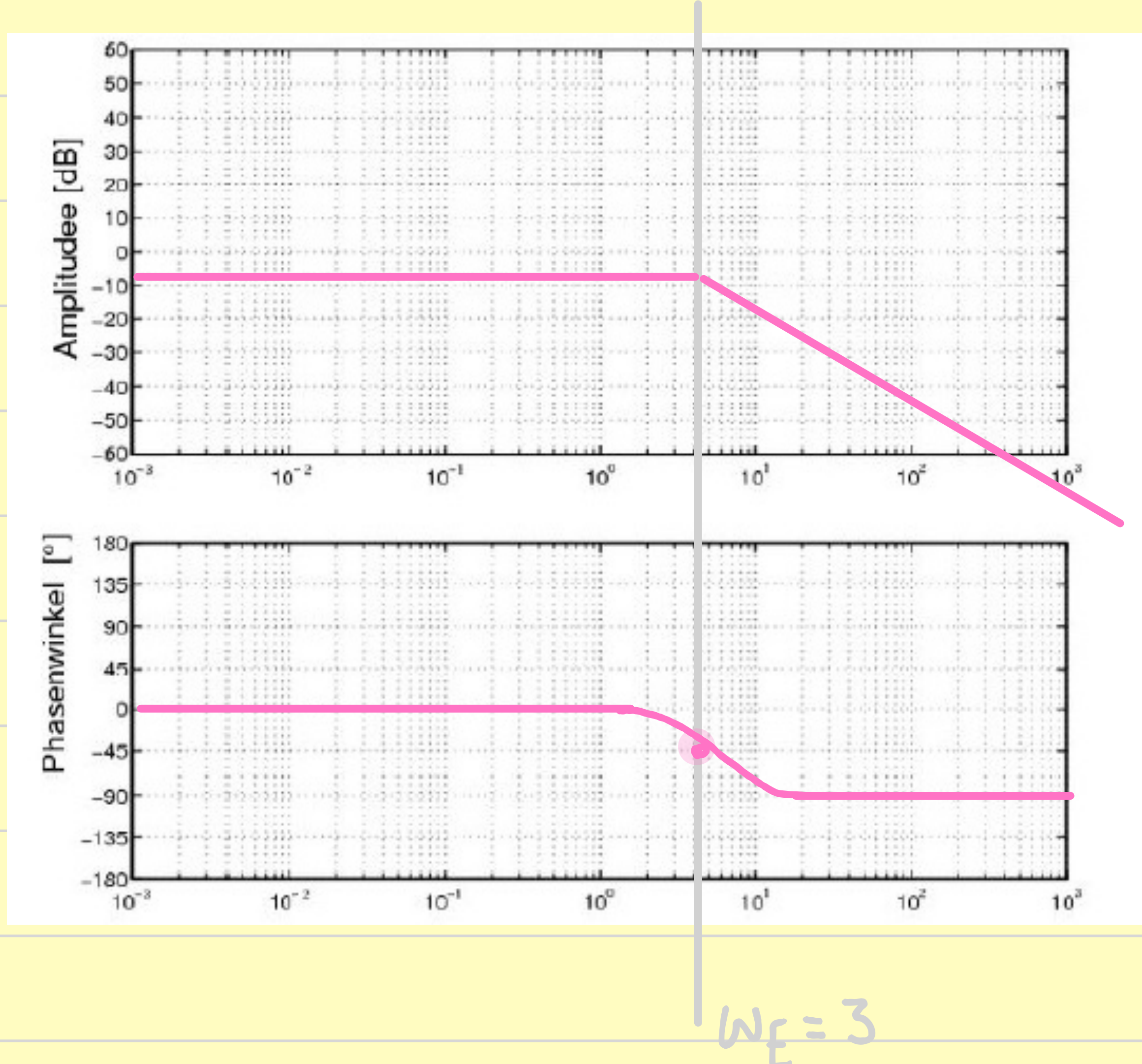
$$G(j\omega) = \frac{1}{3}\cdot\frac{1}{1+\frac{j\omega}{3}}\cdot\frac{1-\frac{j\omega}{3}}{1-\frac{j\omega}{3}} = \frac{1}{3}\cdot\frac{\left(1-\frac{j\omega}{3}\right)}{1^2+\frac{\omega^2}{3^2}}$$

$$|G(j\omega)| = \frac{1}{3}\left(1^2+\frac{\omega^2}{3^2}\right)^{-1/2}$$

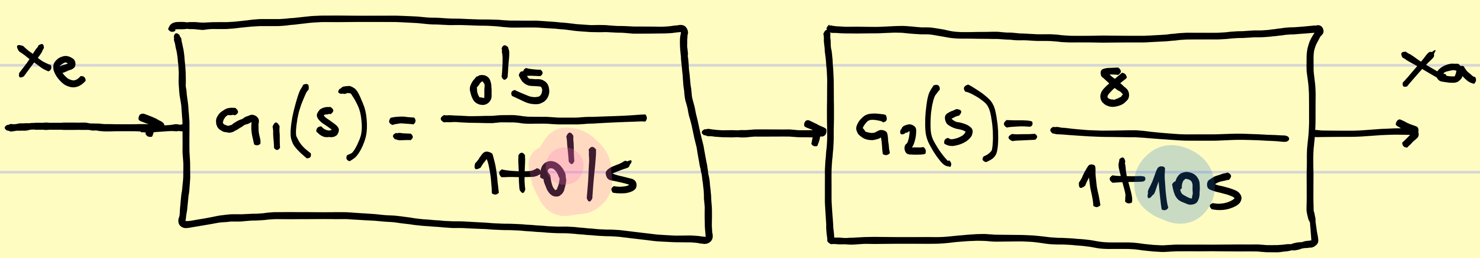
$$|G(j\omega)|_{dB} = 20\log|G(j\omega)| = 20\cdot\left[\log\frac{1}{3} + \frac{-1}{2}\log\left(1^2+\frac{\omega^2}{3^2}\right)\right] =$$

$$= -9'54 - 10\log\left(1^2+\frac{\omega^2}{3^2}\right)$$

$$\angle G(j\omega) = \arctan\left[\frac{-\omega}{3}\right]$$



3.



$$\omega_{E1} = \frac{1}{0'1} = 10 \; ; \; \omega_{E2} = \frac{1}{10} = 0'1$$

$$G_1(j\omega) = \frac{0'5}{1+0'1\,j\omega} \cdot \frac{1-0'1\,j\omega}{1-0'1\,j\omega} = 0'5 \cdot \frac{1-0'1\,j\omega}{1^2+0'1^2\omega^2}$$

$$|G_1(j\omega)| = \ldots = 0'5\left(1^2+0'1^2\,\omega^2\right)^{-1/2}$$

$$\rightarrow |G_1(j\omega)|_{dB} = 20\log|G_1(j\omega)| = 20\log 0'5 - 20\,\frac{1}{2}\cdot\log\left(1^2+0'1^2\omega^2\right)$$

$$= -6'02 - 10\log\left(1+0'01\,\omega^2\right)$$

$$\angle G_1(j\omega) = \arctan\left[\frac{-0'1\,\omega}{1}\right]$$

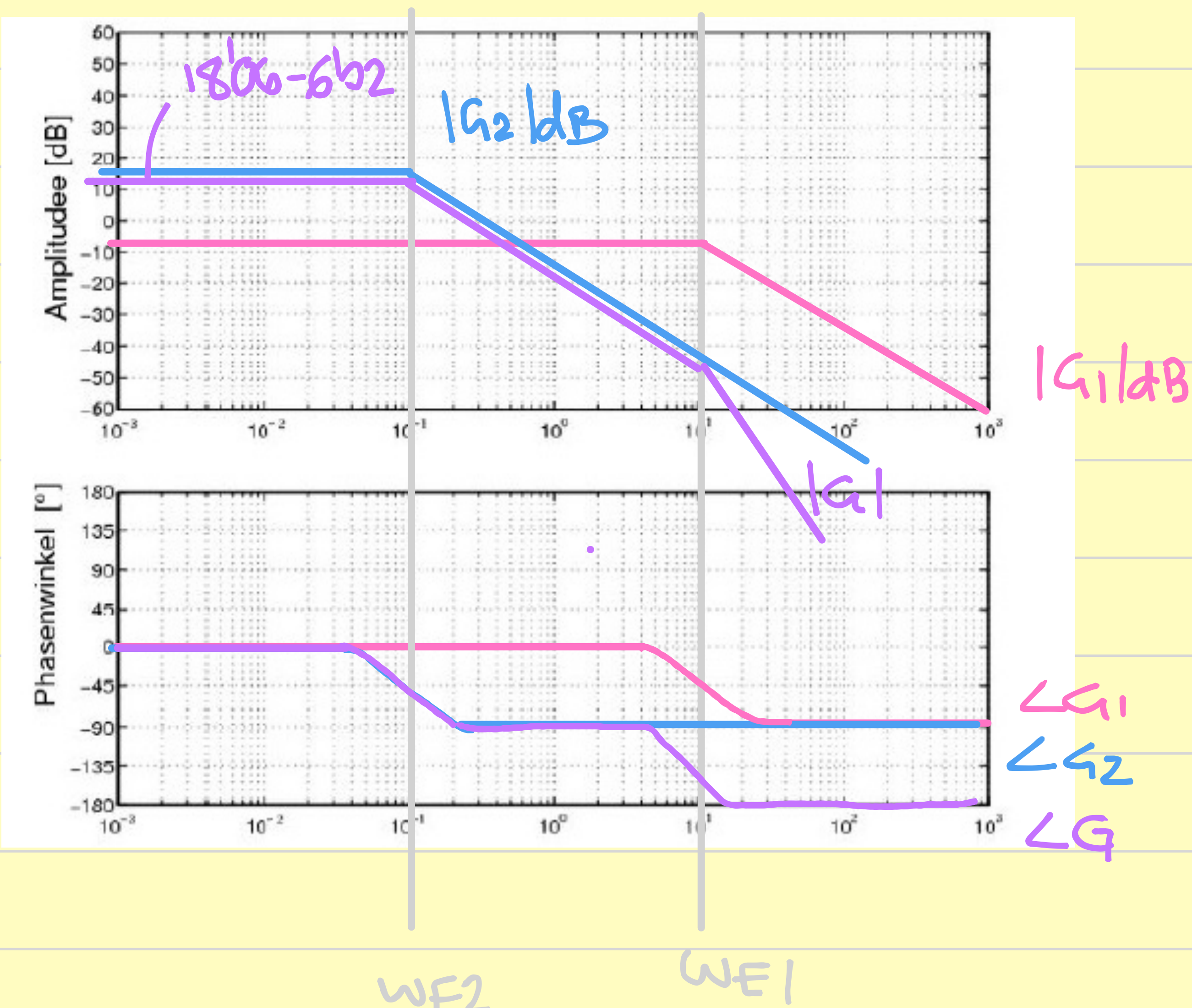
$$G_2(jw) = \frac{8}{1+10jw} \cdot \frac{1-10jw}{1-10jw} = \frac{8(1-10jw)}{1^2+10^2w^2}$$

$$|G_2(jw)| = 8\left(1+10^2w^2\right)^{-1/2}$$

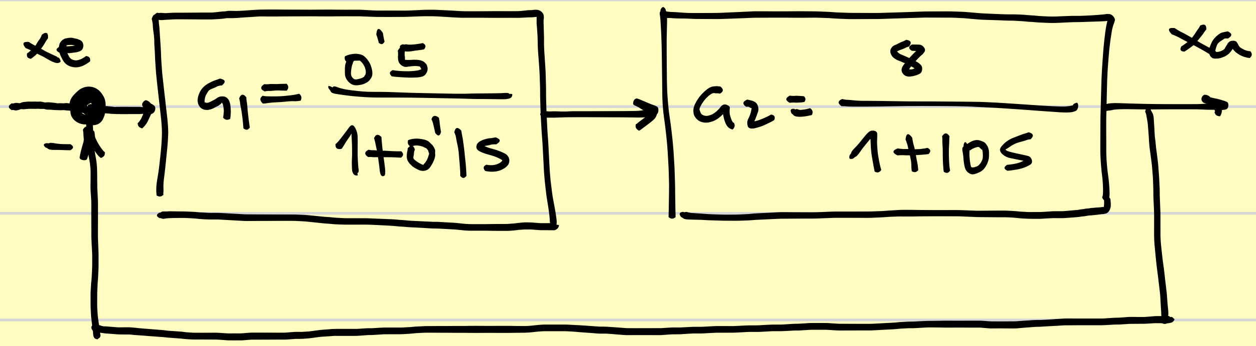
$$\rightarrow |G_2(jw)|_{dB} = 20\log 8 - 20\frac{1}{2}\log\left(1+10^2w^2\right) =$$

$$= 18'06 - 10\log\left(1+10^2w^2\right)$$

$$\angle G_2(jw) = \arctan\left[\frac{-10w}{1}\right]$$



4.



$$G(s) = \frac{\frac{0'5}{1+0'1s} \cdot \frac{8}{1+10s}}{1+\frac{0'5}{1+0'1s} \cdot \frac{8}{1+10s}} = \frac{4}{(1+0'1s)(1+10s)+4}$$

$$s = 1 + 10s + 0'1s + s^2 + 4 = s^2 + 10'1s + 5$$

$$s^* = \frac{-10'1 \pm \sqrt{10'1^2 - 4.5}}{2} = \frac{-10'1 \pm 9'05}{2} = \begin{cases} -0'52 \\ -9'57 \end{cases}$$

$$G(s) = \frac{4}{(s+0'52)(s+9'57)} = \longrightarrow \overset{\mathcal{L}^{-1}}{\cdots} \longrightarrow \frac{A}{(s+0'52)} + \frac{B}{(s+9'57)} \to x(t)$$

$$\longrightarrow \overset{s=j\omega}{\cdots} \longrightarrow \text{Bodediagramm}$$

$$= \xrightarrow{x_e} \underset{G_1}{\boxed{4}} \longrightarrow \underset{G_2}{\boxed{\frac{1}{s+0'52}}} \longrightarrow \underset{G_3}{\boxed{\frac{1}{s+9'57}}} \longrightarrow$$

(!) Fehler: $\omega_{E2} = \frac{1}{0'52}$

$$\frac{1}{s+0'52} = \frac{1}{0'52} \cdot \frac{1}{1 + \frac{1}{0'52}s} \longrightarrow \omega_{E2} = \frac{1}{\frac{1}{0'52}} = 0'52$$