20220113_Finanzmathematik_FAT1

1 EIGENVEKTOREN EIGENWERTE

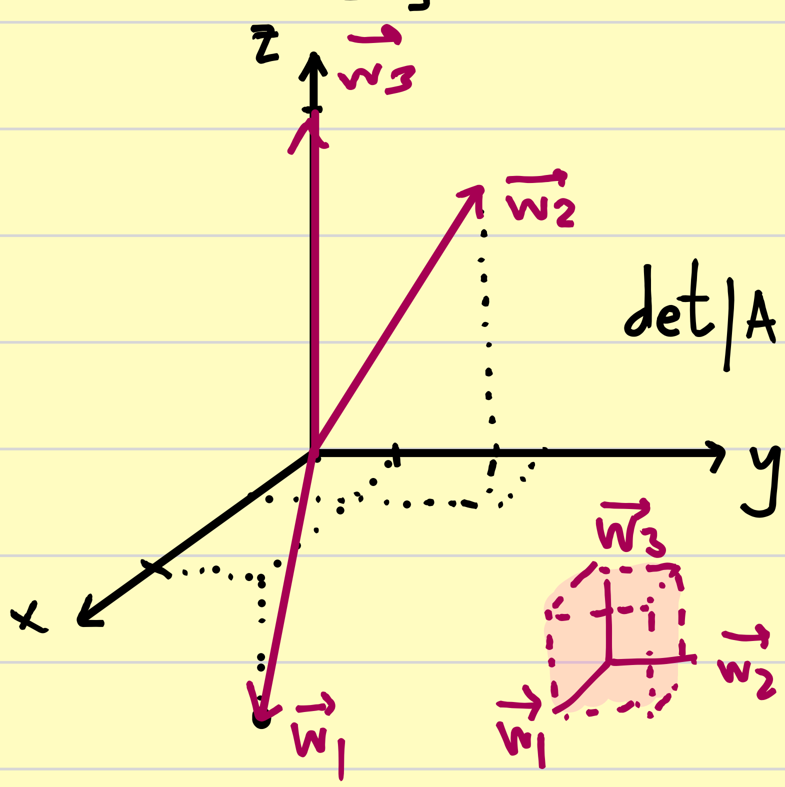
$$\boxed{A\vec{v} = \lambda I} \equiv \det[A-\lambda I]=0$$

System   Eigenvectoren   Eigenwerte   Id.Matix

Beispiel:

$$A=\begin{bmatrix}5&2&0\\2&5&0\\-3&4&6\end{bmatrix} \vdots \quad \vec{w}_1=\begin{bmatrix}5\\2\\-3\end{bmatrix};\ \vec{w}_2=\begin{bmatrix}2\\5\\4\end{bmatrix};\ \vec{w}_3=\begin{bmatrix}0\\0\\6\end{bmatrix}$$

1. Eigenwerte



$\det|A| = $ volumen

$$\det[A-\lambda I]=0$$

$$\det\left[\begin{bmatrix}5&2&0\\2&5&0\\-3&4&6\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right]=0$$

$$\det\begin{bmatrix}5-\lambda&2&0\\2&5-\lambda&0\\-3&4&6-\lambda\end{bmatrix}=0$$

$$\Big[(5-\lambda)(5-\lambda)(6-\lambda)+\cancel{2\cdot 0\cdot 3}+\cancel{2\cdot 4\cdot 0}+$$

$$-\cancel{(-3)\cdot(5-\lambda)\cdot 0}\quad-\cancel{0\cdot 4}\cdot(5-\lambda)-2\cdot 2\cdot(6-\lambda)\Big]=0 \qquad (1)$$

$(\cdot)$

$$(\cdot)\quad \boxed{\det\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} = a\cdot e\cdot i + d\cdot h\cdot c + b\cdot f\cdot g - \\ - c\cdot e\cdot g - b\cdot d\cdot i - f\cdot a\cdot h}$$

$$(1)\quad (6-\lambda)\left[(5-\lambda)^2 - 4\right] = 0$$

$$\lambda = 6$$

$$\left[25 + \lambda^2 - 10\lambda - 4\right] = 0 \rightarrow \left[\lambda^2 - 10\lambda + 21\right] = 0$$

$$\lambda = \frac{-(-10) \pm \sqrt{100 - 4\cdot 21}}{2} = \frac{10 \pm 4}{2} = \begin{cases} 7 \\ 3 \end{cases}$$

$$\boxed{\lambda_1 = 3 \qquad \lambda_2 = 6 \qquad \lambda_3 = 7}$$

2. EIGENVEKTOREN

$$(A - \lambda I)\vec{v} = 0 \longrightarrow \begin{bmatrix} 5-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ -3 & 4 & 6-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{0}$$

$$\left.\begin{aligned} (5-\lambda)x + 2y + 0z &= 0 \\ 2x + (5-\lambda)y + 0z &= 0 \\ -3x + 4y + (6-\lambda)z &= 0 \end{aligned}\right| \quad (2)$$

$$\boxed{\lambda_1 = 3} \quad \left.\begin{aligned} 2x + 2y &= 0 \\ 2x + 2y &= 0 \end{aligned}\right| \rightarrow x = -y \;\Big|\; 3x + 4x - 3z = 0$$

$$-3x + 4y + 3z = 0$$

$$7x - 3z = 0 \rightarrow x = \frac{3z}{7}$$

Setzen wir $x = 1 \to y = -1 \to z = \frac{7}{3}$

$$\boxed{\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 7/3 \end{bmatrix}}$$

$\boxed{\lambda_2 = 6}$

(2) $(5-6)x + 2y = 0$ | $x = 2y$ | $x = y = 0$

$2x + (5-6)y = 0$ | $2x = y$ | $z = 1$

$-3x + 4y + (6-5)z = 0$

$$\boxed{\vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

$\boxed{\lambda_3 = 7}$

(2) $(5-7)x + 2y = 0$ | $x = y$ | $x = y = z$

$2x + (5-7)y = 0$

$3x - 4y - (6-7)z = 0$ | $-x + z = 0$

$$\boxed{\vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

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2 INVERSE MATRIX

$$M \cdot M^{-1} = I \longrightarrow \boxed{M^{-1} = \frac{1}{\det M} \cdot \mathrm{Adj}(M)}$$

Inverse Matrix

Beispiel:

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 7 & 2 \\ 5 & 6 & 3 \end{bmatrix} \longrightarrow M^{-1} = \frac{1}{\det M} \cdot \mathrm{Adj}(M)$$

$$\det M = \left[(1 \cdot 7 \cdot 3) + (2 \cdot 2 \cdot 5) + (4 \cdot 6 \cdot 3) - (3 \cdot 7 \cdot 5) - (4 \cdot 2 \cdot 3) - (6 \cdot 2 \cdot 1) \right] = \dots$$

$$\mathrm{Adj}\, M = \begin{bmatrix} (7 \cdot 3 - 2 \cdot 6) & -(2 \cdot 3 - 3 \cdot 6) & +(2 \cdot 2 - 3 \cdot 7) \\ -(4 \cdot 3 - 2 \cdot 5) & (1 \cdot 3 - 3 \cdot 5) & -(1 \cdot 2 - 3 \cdot 4) \\ +(4 \cdot 6 - 7 \cdot 5) & -(1 \cdot 6 - 2 \cdot 5) & (1 \cdot 7 - 4 \cdot 2) \end{bmatrix} = \dots$$