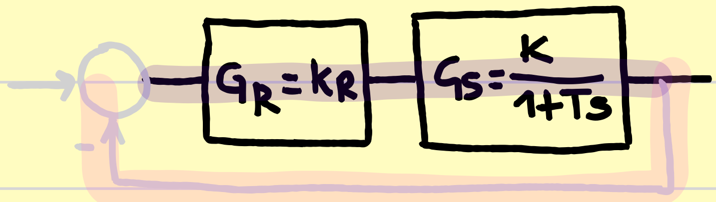
20231212_Regelungstechnik_BEL4

Übung 1. V2023/204



DAS OFFENE / GESCHLOSSENE RK·SYSTEM

Übertragungsfunktion vom offenen System

a) $G_g(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{\frac{K\cdot K_R}{1+Ts}}{1+\frac{K\cdot K_R}{1+Ts}} = \frac{K\cdot K_R}{1+K\cdot K_R+Ts} =$

Übertragungsfunktion vom geschlossenen System.

$= \frac{K\cdot K_R}{T\left(s+\frac{1+K\cdot K_R}{T}\right)} = \frac{K\cdot K_R}{T}\cdot\frac{1}{s+\frac{1+K K_R}{T}}$

STABILITÄTSKRITERIEN:

Angenommen $K, K_R, T \in \mathbb{R}^+$ $\rightarrow \frac{1+K K_R}{T} > 0 \rightarrow K K_R > -1$

IMMER ERFÜLLT

b)

b.1) FREQUENZGANG vom OFFENEN RK:

$G_0(s) = \frac{K K_R}{1+Ts} \rightarrow G_0(j\omega) = \frac{K K_R}{1+j\omega T}\cdot\frac{1-j\omega T}{1-j\omega T} = \frac{K K_R}{1+\omega^2 T^2}(1-j\omega T)$

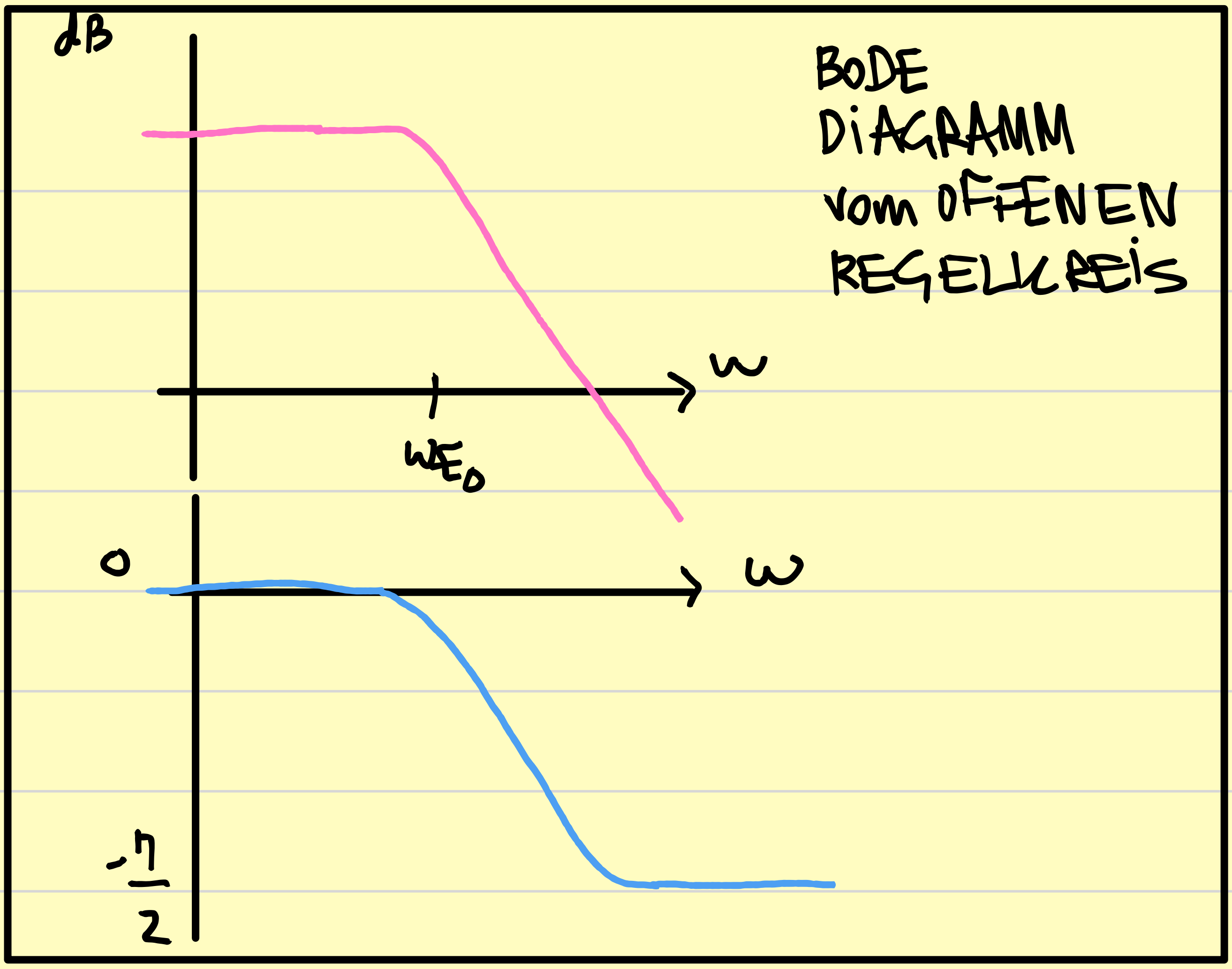
(•) $(1+j\omega T)(1-j\omega T) = 1\cdot 1 - j\omega T + j\omega T - j^2\omega^2 T^2 = 1+\omega^2 T^2$

$|G_0(j\omega)| = \frac{K K_R}{1+\omega^2 T^2}\sqrt{1+\omega^2 T^2} = \frac{K K_R}{\sqrt{1+\omega^2 T^2}} = K K_R\left(1+\omega^2 T^2\right)^{-1/2}$

$|G_0(j\omega)|_{dB} = 20\cdot\log K K_R - 20\cdot\frac{1}{2}\log\left(1+\omega^2 T^2\right)$

$\angle G_0(j\omega) = \arctan\left[\frac{-\omega T}{1}\right] = \arctan\left[-\omega T\right]$

$\rightarrow$ BODE ...



b.2) FREQUENZGANG vom GESCHLOSSENEM REGELKREIS

$$G_g(s) = \frac{K K_R}{T} \cdot \frac{1}{s + \frac{1+KK_R}{T}} \rightarrow G_g(jw) = \frac{KK_R}{T} \cdot \frac{1}{\frac{1+KK_R}{T} + jw} \cdot \frac{\frac{1+KK_R}{T} - jw}{\frac{1+KK_R}{T} - jw}$$

$$G_g(jw) = \frac{KK_R}{T} \cdot \frac{1}{\left(\frac{1+KK_R}{T}\right)^2 + w^2} \cdot \left(\frac{1+KK_R}{T} - jw\right)$$

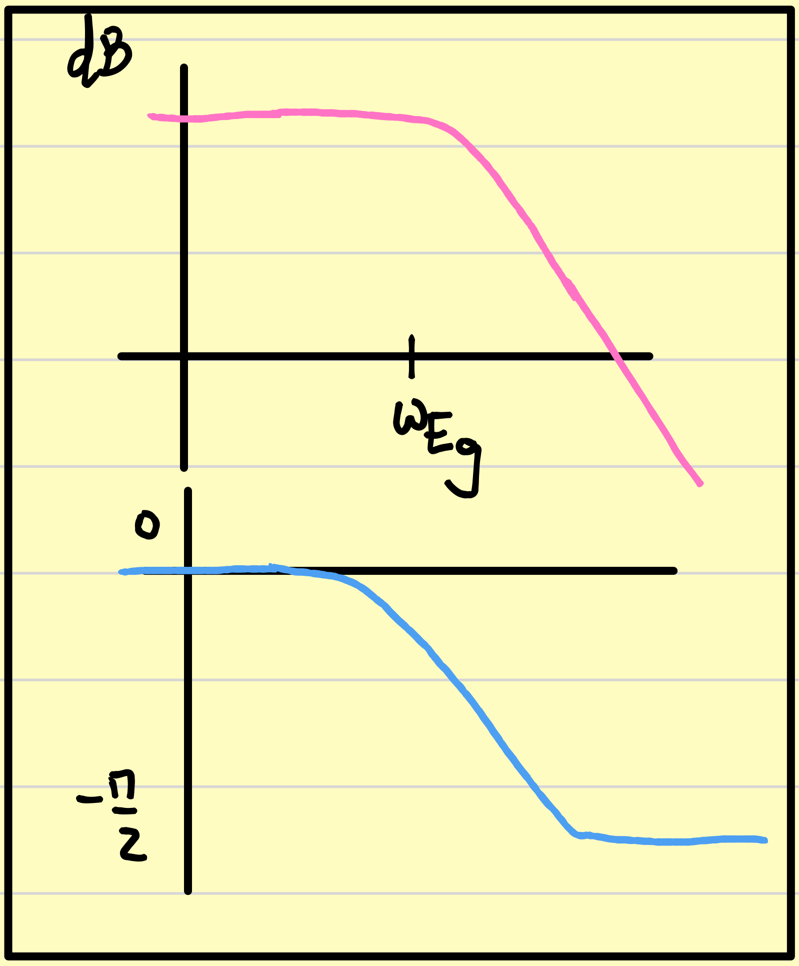
$$|G_g(jw)| = \frac{KK_R}{T} \cdot \frac{1}{\left(\frac{1+KK_R}{T}\right)^2 + w^2} \cdot \sqrt{\left(\frac{1+KK_R}{T}\right)^2 + w^2} =$$

$$= \frac{KK_R}{T} \cdot \left(\left(\frac{1+KK_R}{T}\right)^2 + w^2\right)^{-1/2}$$

$$\boxed{|G_g(jw)|_{dB} = 20 \log \frac{K_R K}{T} - 20 \cdot \frac{1}{2} \log\left(\left(\frac{1+KK_R}{T}\right)^2 + w^2\right)}$$

→ BODE...

$$\boxed{\angle G_g(j\omega) = \arctan\left(\frac{-\omega \cdot T}{1+kK_R}\right)}$$



BODE DIAGRAMM vom GESCHLOSSENEM REGELKREIS

Unterschiede: Qualitativ KEINE.

Eckfrequenz $\omega_{E0} \neq \omega_{Eg}$.

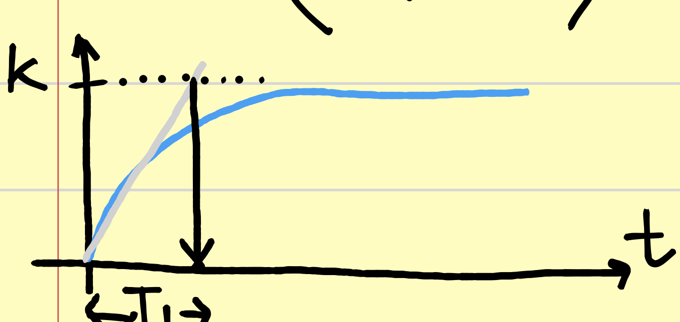
c) Übertragungsfunktion vom geschl. RK $G_g(s)$ ✓

d) Anstiegszeit des geschlossenen RK. t_r

Anstiegszeit: 2023|011

$$x_a(s) = \frac{K}{s} - \frac{KT_1}{1+sT_1}$$

$$x_a(t) = K\left(1 - T_1 e^{-t/T_1}\right)$$



$$x_a(s) = \frac{1}{s} \cdot G_g(s) = \frac{K_R K}{T} \cdot \frac{1}{s} \cdot \frac{1}{s + \frac{KK_R+1}{T}}$$

$$= \frac{A}{s} + \frac{B}{s + \frac{1+KK_R}{T}}$$

$$\frac{KK_R}{T} = A\left(s + \frac{1+KK_R}{T}\right) + Bs$$

$$s=0 \rightarrow \quad A=\frac{kk_R}{1+kk_R}$$

$$s=-\frac{1+kk_R}{T} \rightarrow \quad \frac{kk_R}{T}=-B\,\frac{1+kk_R}{T} \rightarrow B=\frac{-kk_R}{1+kk_R}$$

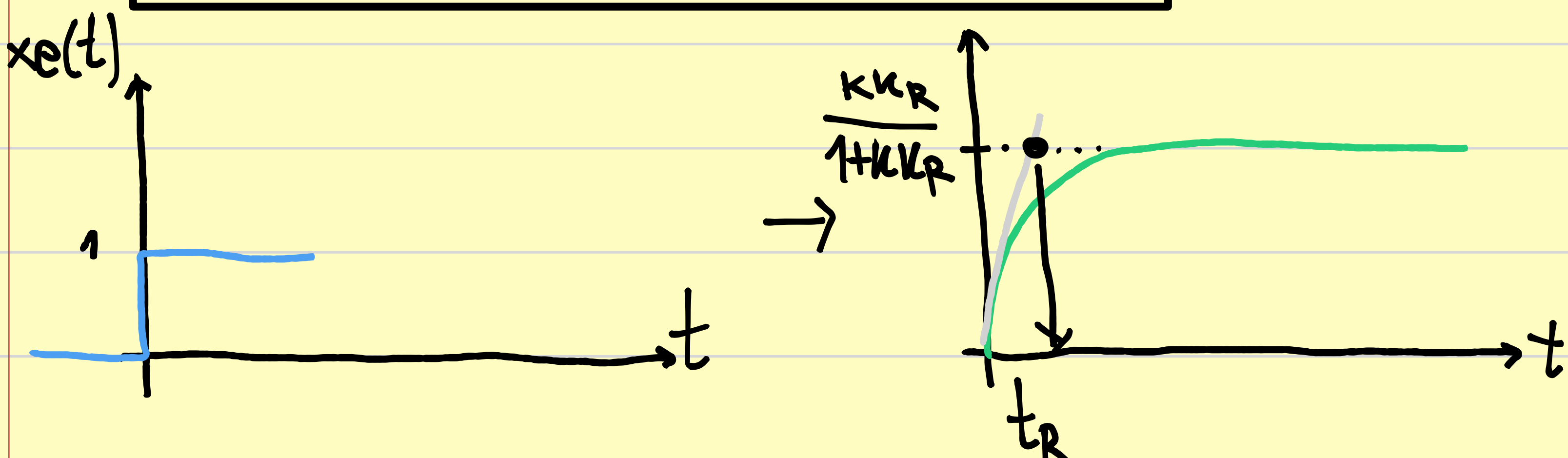
$$x_a(s)=\frac{kk_R}{1+kk_R}\left(\frac{1}{s}-\frac{1}{s+\frac{1+kk_R}{T}}\right)=$$

$$=\frac{kk_R}{1+kk_R}\left(\frac{1}{s}-\frac{1}{\frac{1+kk_R}{T}}\cdot\frac{1}{1+\frac{T}{1+kk_R}s}\right)$$

$$x_a(t)=\frac{kk_R}{1+kk_R}\left(1-\frac{T}{1+kk_R}\cdot\frac{1+k_Rk}{T}\,e^{-t/(T/1+kk_R)}\right)$$

$$x_a(t)=\frac{kk_R}{1+kk_R}\left(1-e^{\frac{-t\cdot(1+kk_R)}{T}}\right)$$

$$\boxed{t_R=\frac{T}{1+kk_R}\ ;\quad \text{ANSTIEGSZEIT}}$$



e) Überschwingung gibt es nicht.

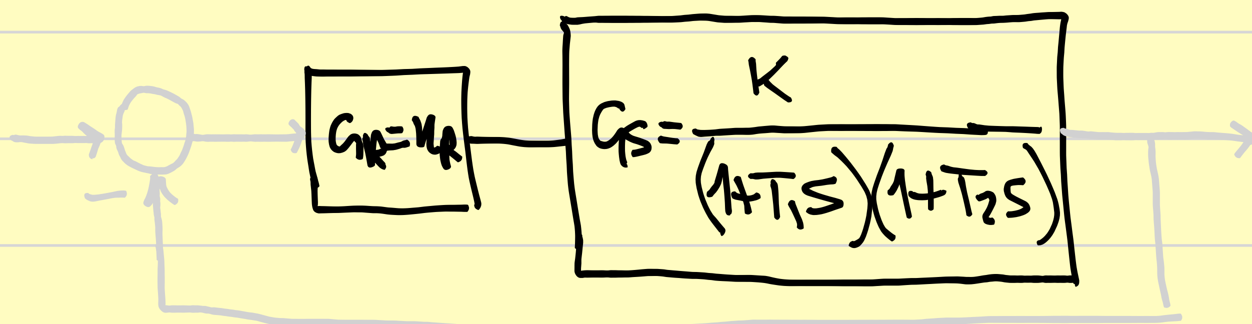
$$t_R = \frac{T}{1+kk_R} < 2 \quad \text{KONDITION}$$

$$1+kk_R > 0 \quad \text{(Stabilität)}$$

$$\Rightarrow \quad T < 2(1+kk_R)$$

Die Verzögerung vom System darf nicht die Obergrenze von $2(1+kk_R)$ überschreiten.

Übung 2. V20231204



a) $$G_g(s) = \frac{\frac{kk_R}{(1+T_1s)(1+T_2s)}}{1+\frac{kk_R}{(1+T_1s)(1+T_2s)}} = \frac{kk_R}{(1+T_1s)(1+T_2s)+kk_R} =$$

$$= \frac{kk_R}{(1+kk_R)+(T_1+T_2)s+T_1T_2s^2}$$

$$s = \frac{-(T_1+T_2) \pm \sqrt{(T_1+T_2)^2 - 4T_1T_2(1+kk_R)}}{2T_1T_2}$$

$$-(T_1+T_2) \pm \sqrt{(T_1+T_2)^2 - 4T_1T_2(1+kk_R)} < 0$$

(bei negativ, immer negativ)

$$-(T_1+T_2)+\sqrt{(T_1+T_2)^2-4T_1T_2(1+KK_R)} < 0$$

$$\cancel{(T_1+T_2)^2}-4T_1T_2(1+KK_R) < \cancel{(T_1+T_2)^2}$$

$$4T_1T_2(1+KK_R) > 0 \rightarrow \text{WENN } T_1,T_2,K,K_R \in \mathbb{R}^+ \rightarrow \text{IMMER STABIL}$$

b) $\boxed{\text{Analog V20231107}}$

b.1) Bode Diagramm des offenen RKs.

$$G_0(s)=\frac{KK_R}{(1+T_1s)(1+T_2s)}=\frac{KK_R}{1+(T_1+T_2)s+T_1T_2s^2}$$

$$G_0(j\omega)=\frac{KK_R}{1+(T_1+T_2)j\omega+T_1T_2(j\omega)^2}=\frac{KK_R}{(1-T_1T_2\omega^2)+(T_1+T_2)\omega j}$$

$$=\frac{KK_R}{(1-T_1T_2\omega^2)+(T_1+T_2)\omega j}\cdot\frac{(1-T_1T_2\omega^2)-(T_1+T_2)\omega j}{(1-T_1T_2\omega^2)-(T_1+T_2)\omega j}=$$

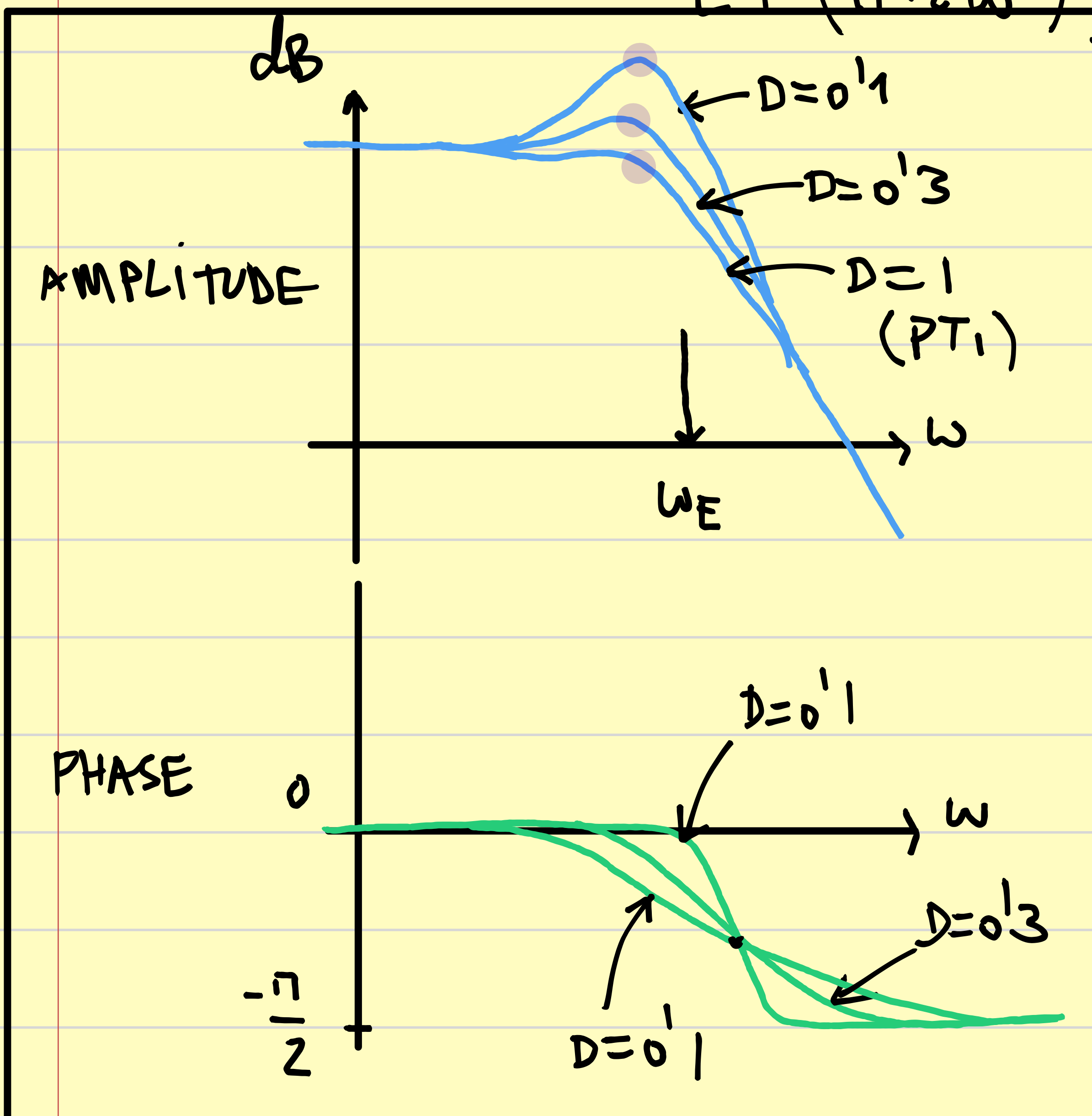
$$=\frac{KK_R}{(1-T_1T_2\omega^2)^2+(T_1+T_2)^2\omega^2}\left[(1-T_1T_2\omega^2)-(T_1+T_2)\omega j\right]$$

$$|G_0(j\omega)|=\frac{KK_R}{(1-T_1T_2\omega^2)^2+(T_1+T_2)^2\omega^2}\sqrt{(1-T_1T_2\omega^2)^2+(T_1+T_2)^2\omega^2}=$$

$$=KK_R\cdot\left((1-T_1T_2\omega^2)^2+(T_1+T_2)^2\omega^2\right)^{-1/2}$$

$$|G_0(j\omega)|_{dB} = 20\log KK_R - 20\,\frac{1}{2}\log\left[(1-T_1T_2\omega^2)^2+(T_1+T_2)^2\omega^2\right]$$

$$\angle G_0(j\omega) = \arctan\left[\frac{-\omega(T_1+T_2)}{1-(T_1T_2\omega^2)}\right]$$



V20231107

$$H(s) = \frac{K}{1+s\alpha_1+s^2\alpha_2^2}$$

$$\omega_E = \frac{1}{\alpha_2}\;;\; D = \frac{\alpha_1}{2\alpha_2}$$

BODE DIAGRAMM des OFFENEN REGELKREISES

b.2)

$$G_g(s) = \frac{KK_R}{(1+KK_R)+(T_1+T_2)s+(T_1T_2)s^2}$$

$$G_g(j\omega) = \frac{KK_R}{(1+KK_R)+(T_1+T_2)j\omega+(T_1T_2)(j\omega)^2} =$$

$$= \frac{KK_R}{(1+KK_R)-(T_1T_2)\omega^2+(T_1+T_2)\omega j} =$$

$$= \frac{KK_R}{(1+KK_R)-(T_1T_2)\omega^2+(T_1+T_2)\omega j} \cdot \frac{(1+KK_R)-(T_1T_2)\omega^2-(T_1+T_2)\omega j}{(1+KK_R)-(T_1T_2)\omega^2-(T_1+T_2)\omega j}$$

$$= \frac{KK_R}{\left[(1+KK_R)-T_1T_2\omega^2\right]^2+(T_1+T_2)^2\omega^2}\left[(1+KK_R)-(T_1T_2\omega^2)-(T_1+T_2)\omega j\right]$$

$$|G_g(j\omega)| = KK_R\left[\left[(1+KK_R)-T_1T_2\omega^2\right]^2+(T_1+T_2)^2\omega^2\right]^{-1/2}$$

$$|G_g(j\omega)|_{dB} = 20\log KK_R - 20\cdot\frac{1}{2}\cdot\left[\left[(1+KK_R)-T_1T_2\omega^2\right]^2+(T_1+T_2)^2\omega^2\right]$$

$$\angle G_g(j\omega) = \arctan\left[\frac{-(T_1+T_2)\omega}{(1+KK_R)-(T_1T_2\omega^2)}\right]$$

BODE DIAGRAM ...

e) Überschwingung maximal 10%.
Anstiegszeit soll maximal 2s
bei Sprungfunktion $x_e(t)=1$.

(••) $$G_g(s) = \frac{KK_R}{(1+KK_R)+(T_1+T_2)s+(T_1T_2)s^2}$$

$$D = \frac{T_1+T_2}{2\sqrt{T_1T_2}}$$

$$\boxed{\text{V2023/07} \quad |G(j\omega)|_{\omega=\omega_E} = \frac{KK_R}{2D}} < 1{,}1$$

10% des Eingang

Überschwingung

$$\frac{KK_R}{\cancel{2}\cdot\frac{T_1+T_2}{\cancel{2}\sqrt{T_1T_2}}} < 1{,}1 \longrightarrow \boxed{KK_R < \frac{1{,}1(T_1+T_2)}{\sqrt{T_1T_2}}}$$

Anstiegszeit $t_R = \frac{1}{\omega_E} = \boxed{\sqrt{T_1T_2} < 2}$