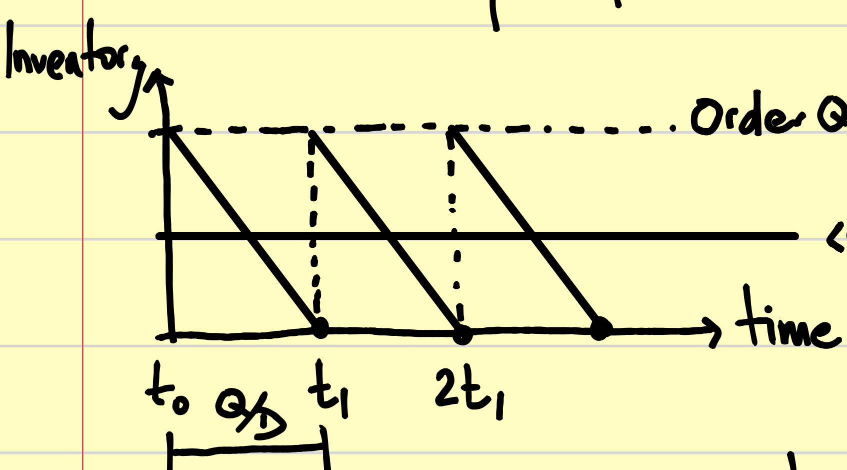
20211110_Supplier_Management_MBW7

Review Economic Order Quantity
- EOQ I
- EOQ II (Supplier Model - Backlog)
- EOQ III (Manufacturing Model)

Parameter:
- D: Demand [Parts/time]
- C: Cost per Unit [€/Part]
- A: Setup cost [€]
- h: Inventory Holding Cost [€/Part·time]

EOQ I. Economic Order Quantity I

· Immediate delivery and production. · Constant Demand.



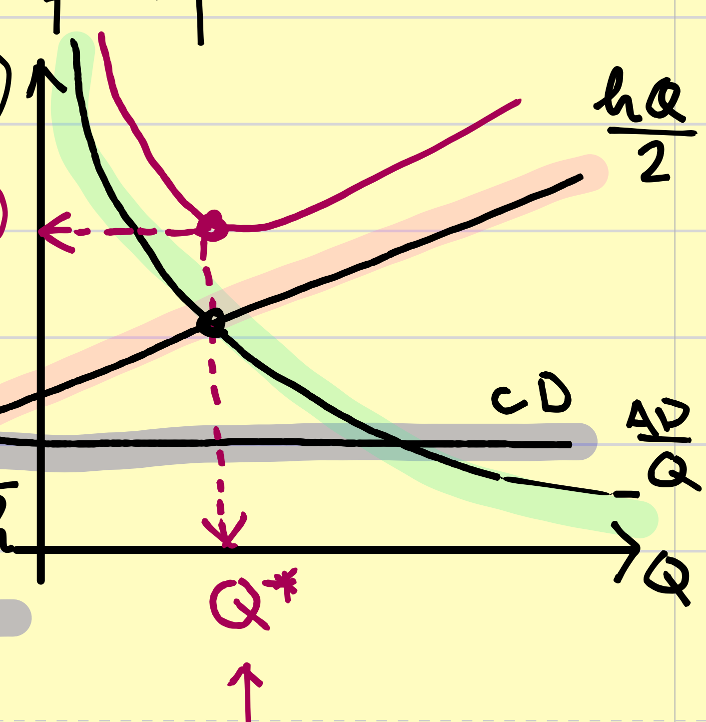
Cost function $Y(Q)$ = cost of Inventory + Setup Cost + Production Cost =

$$= \underbrace{\frac{Q}{2}}_{\text{Average Inventory}} \cdot \underbrace{h}_{\text{Holding Cost per Unit}} + \underbrace{A}_{\text{Setup Cost}} \cdot \underbrace{\frac{1}{Q/D}}_{\text{Frequency of Setup}} + \underbrace{c \cdot D}_{\text{Production Cost}}$$

· $\left.\frac{dY(Q)}{dQ}\right|_{Q=Q^*} = 0$

$$\frac{h}{2} - \frac{AD}{Q^{*2}} = 0 \rightarrow Q^*_I = \sqrt{\frac{2AD}{h}}$$

$$Y(Q^*_I) = \frac{hQ^*}{2} + \frac{AD}{Q^*} + \cancel{cD} = \frac{h}{2} \cdot \sqrt{\frac{2AD}{h}} + \frac{AD}{\sqrt{\frac{2AD}{h}}} = \sqrt{2ADh}$$



- $\frac{hQ^*}{2} = \frac{AD}{Q^*} \rightarrow \frac{hQ^{*2}}{2} = AD \rightarrow Q^* = \sqrt{\frac{2AD}{h}}$ Economic Order Quantity

$\boxed{\text{EOQ II. Supplier Model}}$.

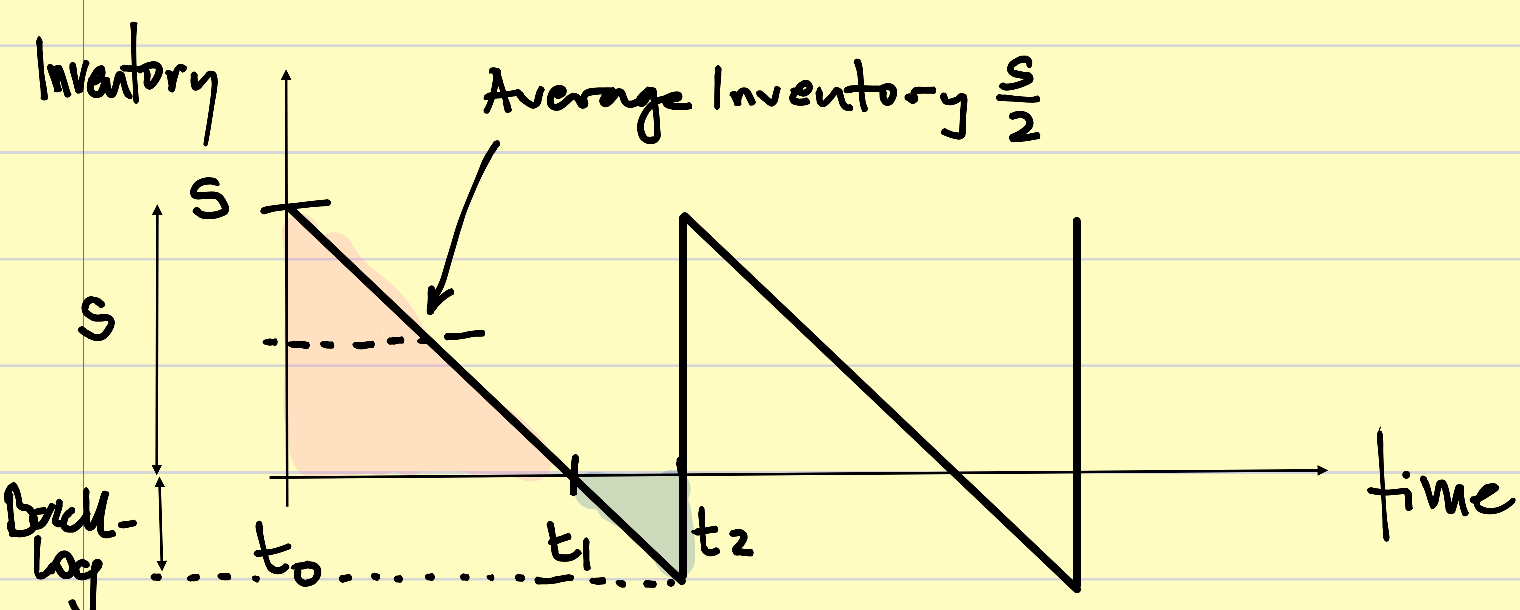
Asumptions:
- We accept a "stock-out" in which the supplier has a backlog of orders.
- Production and delivery are inmediate
- Demand is constant.

Parameters:

p. Cost for not supplied Orders [€/Unit]

S. Inventory after Delivery $Q - S \equiv$ Backlog.

Q: Order Quantity



$t_1 = \frac{S}{D}$ $t_2 = \frac{Q}{D}$

$Y(Q,S) =$ Inventory Holding Cost $+$ Backlog Cost $+$ Setup Cost $+$ Production Cost $=$

$$= h \cdot \frac{S}{2} \cdot \frac{S}{D} \cdot \frac{D}{Q} + \frac{p(Q-S)}{2} \cdot \frac{Q-S}{D} \cdot \frac{D}{Q} + \frac{AD}{Q} + cD$$

holding cost per unit — average inventory — time consume inventory $[t_1]$ — how often — backlog cost per unit — average backlog — time of backlog $[t_2 - t_1]$ — how often

The optimal Q^* [Economic Order Quantity] & the optimal S^*:

$$\left.\frac{\partial Y(Q,S)}{\partial Q}\right|_{Q^*,S^*} = 0 \rightarrow \frac{\partial}{\partial Q}\left[\frac{hS^2}{2Q} + \frac{p(Q-S)^2}{2Q} + \frac{AD}{Q} + cD\right]_{Q^*,S^*} = 0$$

$$\left.\frac{\partial Y(Q,S)}{\partial S}\right|_{Q^*,S^*} = 0 \rightarrow \frac{\partial}{\partial S}\left[\frac{hS^2}{2Q} + \frac{p(Q-S)^2}{2Q} + \frac{AD}{Q} + cD\right]_{Q^*,S^*} = 0$$

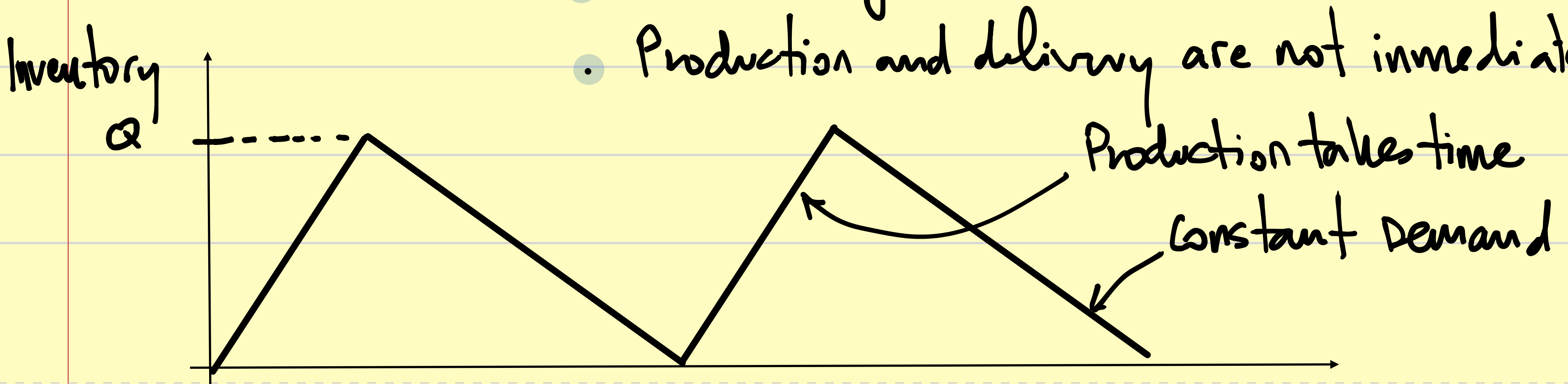
$$\rightarrow \left\{\begin{array}{l} \dfrac{-hS^{*2}}{Q^{*2}} + \dfrac{p(Q^*-S^*)}{Q^*} - \dfrac{p(Q^*-S^*)^2}{2Q^{*2}} - \dfrac{AD}{Q^{*2}} = 0 \\ \dfrac{hS^*}{Q^*} - \dfrac{p(Q^*-S^*)}{Q^*} = 0 \end{array}\right\} \rightarrow \dots \rightarrow$$

$$\rightarrow \left\{\begin{array}{l} S^* = \sqrt{\dfrac{2AD}{h}} \cdot \sqrt{\dfrac{p}{p+h}} \\ Q^* = \sqrt{\dfrac{2AD}{h}} \cdot \sqrt{\dfrac{p+h}{p}} \end{array}\right\} \rightarrow \boxed{\begin{array}{l} S^* = Q^*_I \cdot \sqrt{\dfrac{p}{p+h}} \\ Q^*_{II} = Q^*_I \sqrt{\dfrac{p+h}{p}} \end{array}} > Q^*_I$$

EOQ III (Manufacturing Model) - without Backlog.

Parameter
K = Production rate

Assumptions:
- Constant demand.
- No backlog.
- Production and delivery are not immediate.



time

$$Y(Q) = \text{Holding cost Inventory} + \text{setup cost} + \text{Production cost} =$$

$$= h \cdot \frac{Q}{2} \cdot \left(1 - \frac{D}{K}\right) + \frac{AD}{Q} + cD$$

Minimum is found when Holding cost Inventory = Setup Cost : Q^*

$$\frac{hQ^*}{2} \cdot \left(1 - \frac{D}{K}\right) = \frac{AD}{Q^*} \rightarrow h Q^{*2}\left(1 - \frac{D}{k}\right) = 2AD \rightarrow$$

$$Q^*_{III} = \sqrt{\frac{2AD}{h\left(1 - \frac{D}{K}\right)}}$$

Notice that when the production rate is infinite (inmediate production); we get the EOQ_I Model:

$$K \to \infty : \quad Q^*_I = \sqrt{\frac{2AD}{h}}$$

$$Y(Q^*_{III}) = \frac{h \cdot Q^*_{III}}{2}\left(1 - \frac{D}{K}\right) + \frac{AD}{Q^*_{III}} + \cancel{cD} \rightarrow \emptyset =$$

$$= \frac{h}{2} \cdot \left(1 - \frac{D}{K}\right) \cdot \sqrt{\frac{2AD}{h\left(1 - \frac{D}{k}\right)}} + AD \cdot \frac{1}{\sqrt{\frac{2AD}{h\left(1 - \frac{D}{k}\right)}}} =$$

$$= \frac{h}{2}\sqrt{\frac{2AD\left(1 - \frac{D}{k}\right)^2}{h\left(1 - \frac{D}{k}\right)}} + \sqrt{\frac{A^2D^2 \cdot h\left(1 - \frac{D}{k}\right)}{2AD}} =$$

$$= \sqrt{\frac{2AD\,h^2\left(1-\frac{D}{k}\right)}{2^2 h}} + \sqrt{\frac{AD\,h\left(1-\frac{D}{k}\right)}{2}} =$$

$$Y(Q^*_{III}) = \sqrt{2ADh\left(1-\frac{D}{k}\right)}$$

$$k \to \infty: \quad Y(Q^*_I) = \sqrt{2ADh}$$

Optimum number of Production setups:

$$\frac{D}{Q^*_{III}} = \frac{D}{\sqrt{\frac{2AD}{h\left(1-\frac{D}{k}\right)}}} = \sqrt{\frac{hD(1-D/k)}{2A}}$$

Optimum length of Production setup:

$$t_1^* = \frac{Q^*}{k} = \sqrt{\frac{2AD}{h\left(1-\frac{D}{k}\right)}} \cdot \frac{1}{k} = \sqrt{\frac{2AD}{k \cdot h(k-D)}}$$

$k \to \infty$: EOQ Model $\to t_1^* = 0$ (Immediate Production).

Ex. A company must deliver 10000 Products per day. When the company starts production, they can produce 25000 Units/day. The cost to hold one Product unit on inventory per Year is 50€. The setup cost are 1800€ per setup. How often should the company organize the setup? Please use EOQ_{II}.

Ex. A company produces a product for several clients. They experience flat demand of 2500 units/Year. The production rate is 10000 Units/year. The setup cost is 50 €/run. The production cost is 2€/Unit. The holding cost is 50 €/Unit per Year and has 30% annual interest rate. Determine the optimum order quantity and optimal size of production run. EOQ_{III}.

Ex. The demand for water @ office is 600 litres / Week. The setup cost for placing an order to replenish inventory is 25€. The order is delivered by a supplier who charges 0'1 €/liter as transport cost. This cost increases the cost of water to 1'25€/liter. The holding cost of water (lost of freshness) is 2'6€/liter. Determine how often the office should order water, and what should be the economic order quantity.

Ex. The yearly demand of a product is 7200 Units. The holding cost "h" of inventory is 1500 €/Unit Year. The backlog cost "p" are 2000 €/Unit Year. Find the optimum Order Quantity (Q^*) and optimal cost $Y(Q^*)$ following EOQ III. What is the optimum Backlog? $(Q^* - S^*)$ Find the best Ordering Frequency $\left(\frac{Q^*}{D}\right)$

Setup cost: 1000€ ; $c \sim \phi$.

Factory Physics

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