Logistische Prozeße mit Laplace-Transformation.

1. Diagramm.

$$u_e(t)=D \rightarrow \boxed{} \rightarrow u_a(t)?$$

- Ausschlußquote von α %. $\quad K=1+\alpha$
- Reaktionszeit von „a" Zeiteinheiten

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} F(s)=\frac{K}{s+a}$$

$$\mathcal{L}(u_e(t))=\mathcal{L}(D)=\frac{D}{s}$$

$$\frac{D}{s} \rightarrow \boxed{\frac{K}{s+a}} \rightarrow u_a(s) \longrightarrow u_a(s)=\frac{D}{s}\cdot\frac{K}{s+a}$$

$$u_a(s)=\frac{D}{s}\cdot\frac{K}{s+a}=\frac{A}{s}+\frac{B}{s+a}=\frac{A(s+a)+Bs}{s(s+a)}$$

Beispiel:

$$\frac{3}{5}\cdot\frac{4}{7}=\frac{A}{5}+\frac{B}{7}=\frac{7\cdot A+5\cdot B}{5\cdot 7} \rightarrow 3\cdot 4=7A+5B$$

$$D\cdot K=A(s+a)+Bs$$

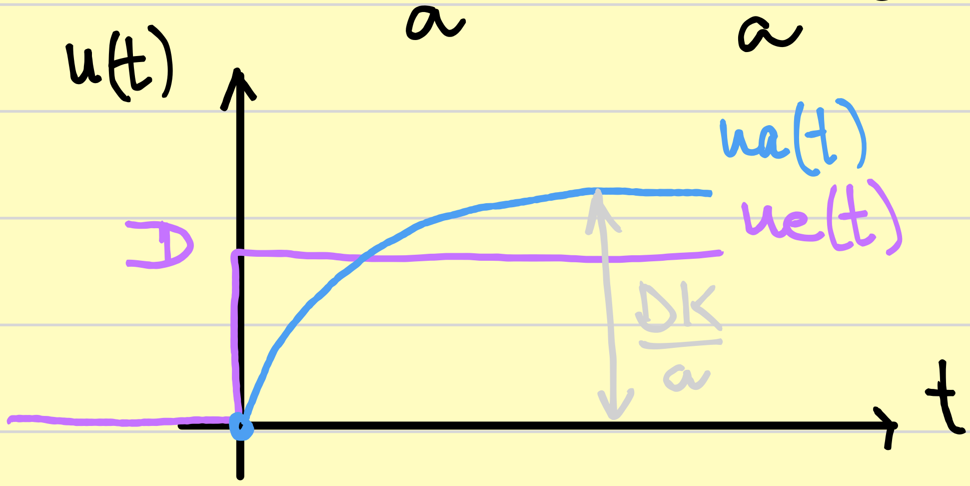
$$s^*=0 \rightarrow D\cdot K=A(0+a)+B\cdot 0 \rightarrow A=\frac{D\cdot K}{a}$$

$$s^*=-a \rightarrow D\cdot K=A(-a+a)+B(-a) \rightarrow B=-\frac{DK}{a}$$

$$u_a(s) = \frac{A}{s} + \frac{B}{s+a} = \frac{D\cdot K}{a}\cdot\frac{1}{s} - \frac{DK}{a}\cdot\frac{1}{s+a}$$

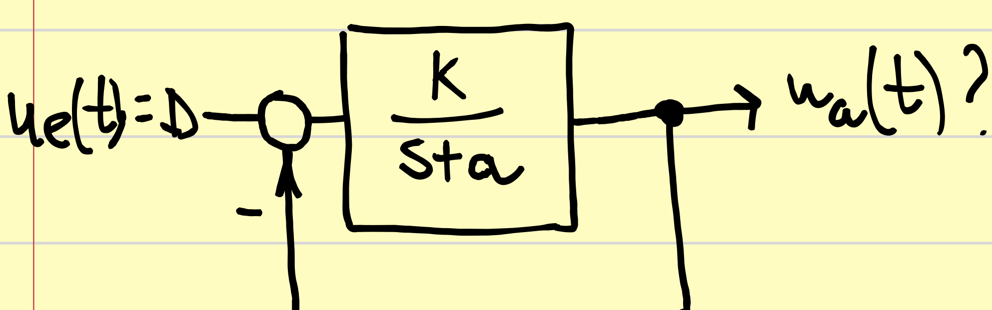
$$\mathcal{L}^{-1}(u_a(s)) = u_a(t) = \frac{DK}{a}\cdot\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \frac{DK}{a}\mathcal{L}^{-1}\left(\frac{1}{s+a}\right) \rightarrow$$

$$\rightarrow u_a(t) = \frac{DK}{a}\cdot 1 - \frac{DK}{a}\cdot e^{-at} \rightarrow \boxed{u_a(t) = \frac{DK}{a}\left(1-e^{-at}\right)}$$



· wenn $\frac{K}{a} > 1$ ist, dann ist der Kunde immer versorgt.

2. Diagramm



$$u_a(s) = \frac{D}{s}\cdot\frac{\frac{K}{s+a}}{1+\frac{K}{s+a}} = \frac{D}{s}\cdot\frac{K}{s+(a+K)} = \frac{A}{s} + \frac{B}{s+(a+K)} \overset{(*)}{=} \frac{A(s+(a+K))+BS}{s\cdot(s+(a+K))}$$

$$D\cdot K = A(s+(a+K)) + BS$$

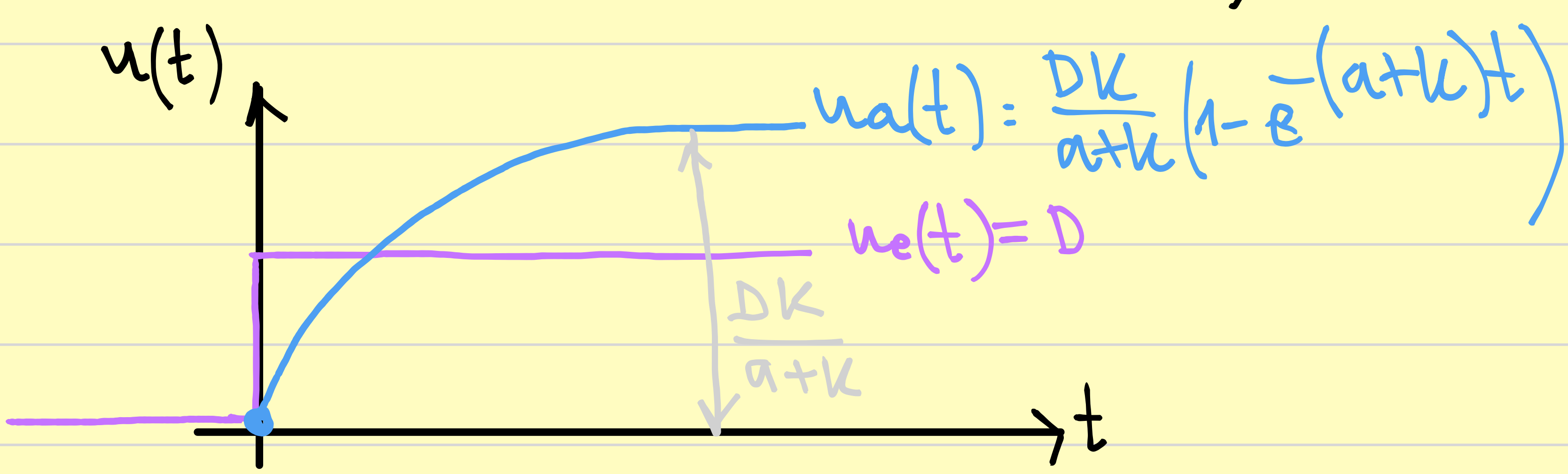
$$\overset{*}{s=0} \rightarrow D\cdot K = A(0+(a+K)) + B\cdot 0 \rightarrow A = \frac{DK}{a+K}$$

$$s^* = -a-k \rightarrow D\cdot k = A\cdot\left(-a-k+(a+k)\right) + B(-a-k)$$

$$\rightarrow B = \frac{D\cdot k}{-a-k} = -\frac{Dk}{a+k}$$

$$u_a(s) = \frac{A}{s} + \frac{B}{s+(a+k)} = \frac{DK}{a+k}\cdot\frac{1}{s} - \frac{DK}{a+k}\cdot\frac{1}{s+(a+k)}$$

$$\rightarrow \mathcal{L}^{-1}(u_a(s)) = u_a(t) = \frac{DK}{a+k}\left(1-e^{-(a+k)t}\right)$$



· wenn $\frac{k}{a+k} > 1$, der Kundenbedarf ist immer gedeckt

Beispiel (*)

$$\frac{3}{x}\cdot\frac{4}{x-3} = \frac{A}{x} + \frac{B}{x-3} = \frac{A(x-3)+Bx}{x(x-3)} \longrightarrow 3\cdot 4 = A(x-3)+Bx$$

$$x=0 \rightarrow 3\cdot 4 = A(0-3)+B\cdot 0 \rightarrow 12 = A(-3) \rightarrow A=-4$$

$$x=3 \rightarrow 3\cdot 4 = A(3-3)+B\cdot 3 \rightarrow 12 = B\cdot 3 \rightarrow B=4$$

$$\frac{3}{x}\cdot\frac{4}{x-3} = \frac{-4}{x} + \frac{4}{x-3}$$

3. Diagramm

$$u_e(t)=D \longrightarrow \boxed{\frac{K_1}{s+a}} \longrightarrow \boxed{\frac{K_2}{s+b}} \longrightarrow u_a(t)?$$

$$\left.\begin{aligned} u_e(s) &= \frac{D}{s} \\ F(s) &= \frac{K_1}{s+a}\cdot\frac{K_2}{s+b} \end{aligned}\right\} \; u_a(s) = \frac{D}{s}\cdot\frac{K_1}{s+a}\cdot\frac{K_2}{s+b}$$

$$= \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b}$$

$$u_a(s) = \frac{A}{s}\cdot\frac{(s+a)(s+b)}{(s+a)(s+b)} + \frac{B}{s+a}\cdot\frac{s(s+b)}{s(s+b)} + \frac{C}{s+b}\cdot\frac{s(s+a)}{s(s+a)} =$$

$$= \frac{A(s+a)(s+b) + Bs(s+b) + Cs(s+a)}{s(s+a)(s+b)} = \frac{DK_1K_2}{s(s+a)(s+b)}$$

$$A(s+a)(s+b) + Bs(s+b) + Cs(s+a) = DK_1K_2$$

$$s^* = 0 \rightarrow A(0+a)(0+b) + B\cdot 0(0+b) + C\cdot 0(0+a) = DK_1K_2$$

$$\rightarrow Aab = DK_1K_2 \rightarrow A = \frac{DK_1K_2}{ab}$$

$$s^* = -a \rightarrow A(-a+a)(-a+b) + B(-a)(-a+b) + C(-a)(-a+a) =$$

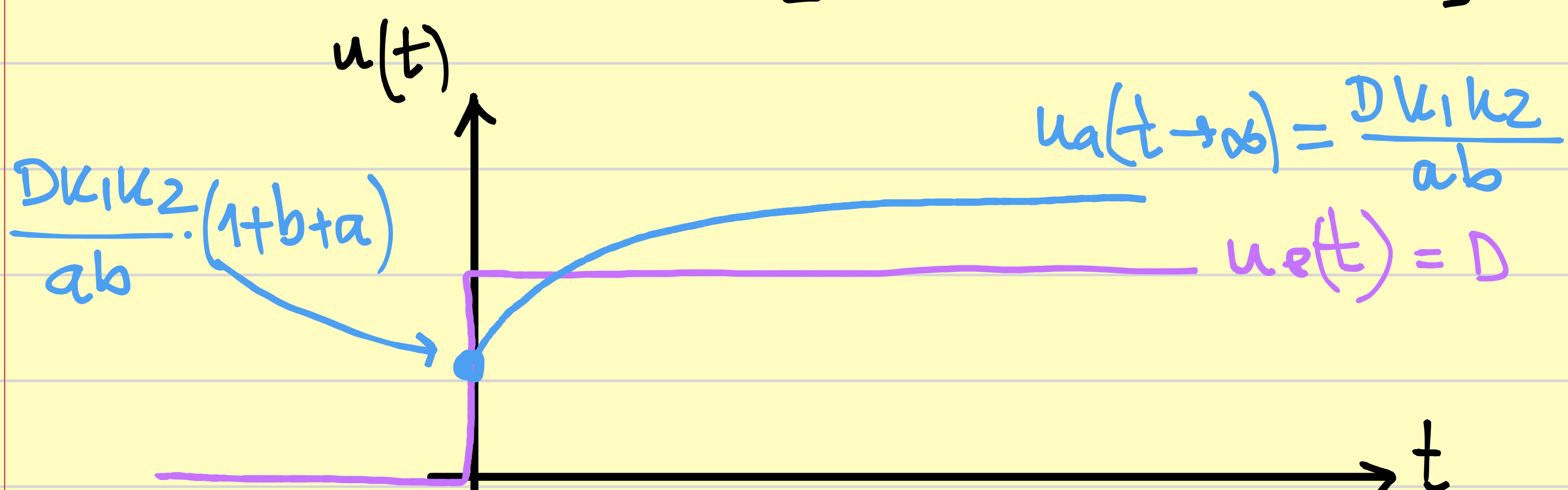
$$= DK_1K_2$$

$$\rightarrow B(-a)(-a+b) = DK_1K_2 \rightarrow B = \frac{DK_1K_2}{a(b-a)}$$

$$s^* = -b \to A(-b+a)(-b+b) + B\cdot(-b)(-b+b) + C(-b)(-b+a)$$
$$= Dk_1k_2$$
$$\to C = \frac{Dk_1k_2}{b(b-a)}$$

$$u_a(s) = \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b} = \frac{Dk_1k_2}{ab}\cdot\frac{1}{s} + \frac{Dk_1k_2}{a(b-a)}\cdot\frac{1}{s+a} +$$
$$+ \frac{Dk_1k_2}{b(b-a)}\cdot\frac{1}{s+b}$$

$$\mathcal{L}(u_a(s)) = u_a(t) = \frac{Dk_1k_2}{ab} + \frac{Dk_1k_2}{a(b-a)}\,e^{-at} + \frac{Dk_1k_2}{b(b-a)}\cdot e^{-bt}$$
$$= \frac{Dk_1k_2}{ab}\left[1 + b\,e^{-at} + a\,e^{-bt}\right]$$



4. Diagramm

$$\left.\begin{aligned} u_e(s) &= \mathcal{L}\left\{u_e(t)\right\} = \frac{D}{s} \\ F(s) &= \frac{k_1}{s+a} + \frac{k_2}{s+b} \end{aligned}\right\} u_a(s) = \frac{D}{s}\cdot\left[\frac{k_1}{s+a} + \frac{k_2}{s+b}\right] =$$

$$= \frac{D}{s}\cdot\left[\frac{k_1(s+b)+k_2(s+a)}{(s+a)(s+b)}\right]$$

$$= \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b}$$

$$D\cdot\left[k_1(s+b)+k_2(s+a)\right] = A(s+a)(s+b) + Bs(s+b) + Cs(s+a)$$

$$s^* = 0 \longrightarrow D\left[k_1 b + k_2 a\right] = A\cdot ab \rightarrow A = \frac{D\left[k_1 b + k_2 a\right]}{ab}$$

$$s^* = -a \longrightarrow D\left[k_1(b-a)\right] = B(-a)(b-a) \rightarrow B = \frac{-Dk_1}{a}$$

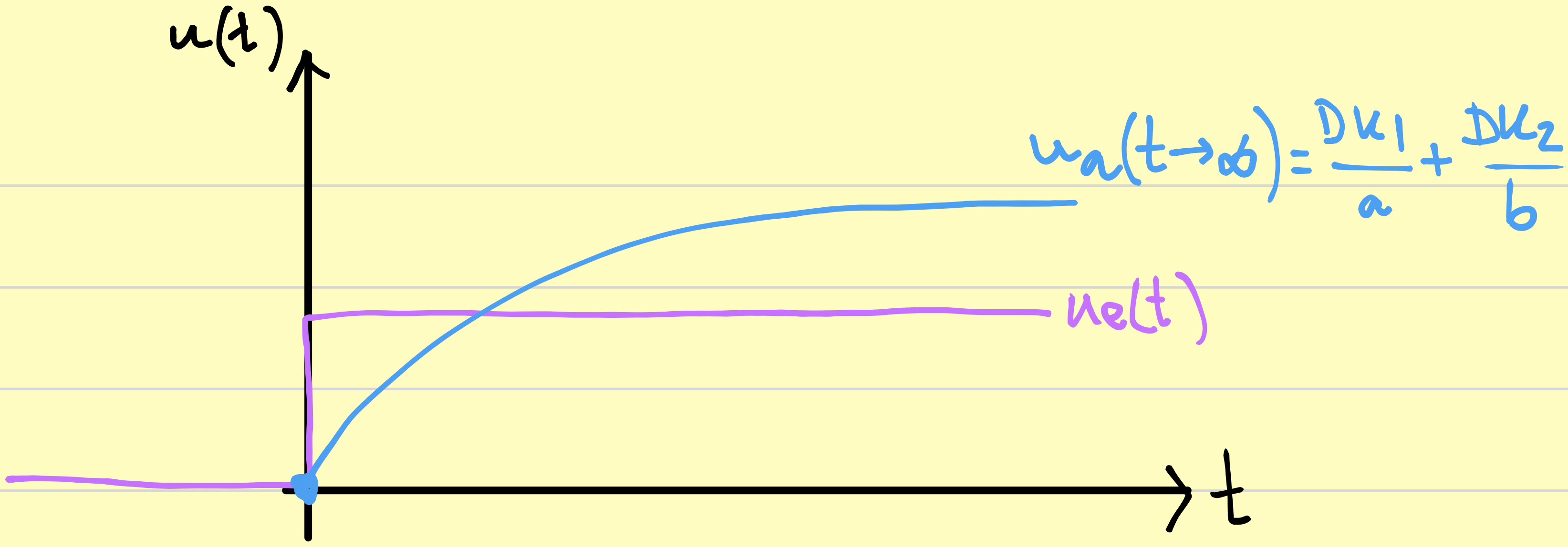
$$s^* = -b \longrightarrow D\left[k_2(a-b)\right] = C(-b)(a-b) \rightarrow C = \frac{-Dk_2}{b}$$

$$u_a(s) = \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b} = \left[\frac{Dk_1}{a} + \frac{Dk_2}{b}\right]\frac{1}{s} - \frac{Dk_1}{a}\cdot\frac{1}{s+a}$$

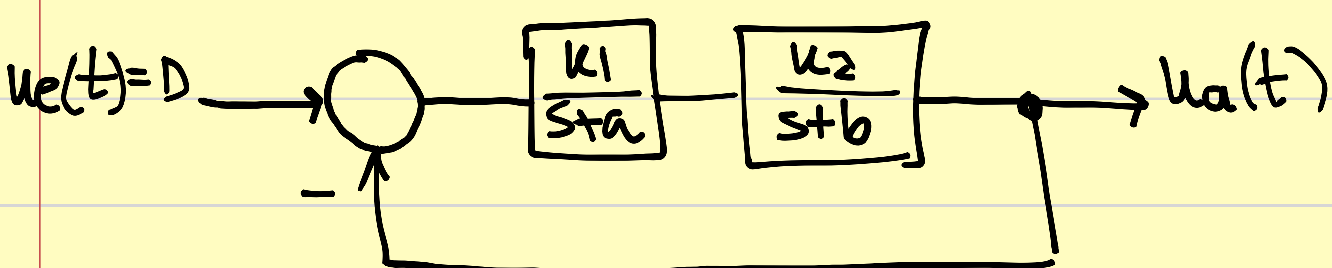
$$- \frac{Dk_2}{b}\cdot\frac{1}{s+b}$$

$$\mathcal{L}^{-1}\left(u_a(s)\right) = u_a(t) = \frac{Dk_1}{a} + \frac{Dk_2}{b} - \frac{Dk_1}{a}\cdot e^{-at} - \frac{Dk_2}{b}\cdot e^{-bt} =$$

$$= \frac{Dk_1}{a}\left(1-e^{-at}\right) + \frac{Dk_2}{b}\left(1-e^{-bt}\right)$$



5. Diagramm



$$U_e(s) = \mathcal{L}\left(u_e(t)\right) = \frac{D}{s}$$

$$F(s) = \frac{\frac{k_1k_2}{(s+a)(s+b)}}{1+\frac{k_1k_2}{(s+a)(s+b)}} = \frac{k_1k_2}{(s+a)(s+b)+k_1k_2}$$

$$U_a(s) = \frac{D}{s}\cdot\frac{k_1k_2}{(s+a)(s+b)+k_1k_2} = \frac{D}{s}\cdot\frac{k_1k_2}{s^2+s(a+b)+(ab+k_1k_2)}$$

$$s^2+s(a+b)+(ab+k_1k_2) = 0 \longrightarrow s^* = \frac{-(a+b)\pm\sqrt{(a+b)^2-4(ab+k_1k_2)}}{2}$$

$$= \begin{cases} s_1^* = \dfrac{-(a+b)+\sqrt{(a+b)^2-4(ab+k_1k_2)}}{2} \\ s_2^* = \dfrac{-(a+b)-\sqrt{(a+b)^2-4(ab+k_1k_2)}}{2} \end{cases}$$

$$(s-s_1^*)(s-s_2^*) = 0$$

$$u_a(s) = \frac{D}{s} \cdot \frac{k_1 k_2}{\left(s - \frac{-(a+b)+\sqrt{(a+b)^2-4(ab+k_1k_2)}}{2}\right)\cdot\left(s - \frac{-(a+b)-\sqrt{(a+b)^2-4(ab+k_1k_2)}}{2}\right)}$$

$$= \frac{A}{s} + \frac{B}{s-s_1^*} + \frac{C}{s-s_2^*}$$

$\boxed{a=3\ ;\ b=0\ ;\ k_1=k_2=1{,}1\ ;\ D=1}$ Beispiel

$$u_a(s) = \frac{1}{s} \cdot \frac{1{,}21}{\left(s - \frac{-3+2{,}03}{2}\right)\left(s - \frac{-3-2{,}03}{2}\right)} =$$

$$= \frac{1}{s} \cdot \frac{1{,}21}{(s+0{,}48)(s+2{,}515)} = \frac{A}{s} + \frac{B}{s+0{,}48} + \frac{C}{s+2{,}515}$$

$$1{,}21 = A(s+0{,}48)(s+2{,}515) + Bs(s+2{,}515) + Cs(s+0{,}48)$$

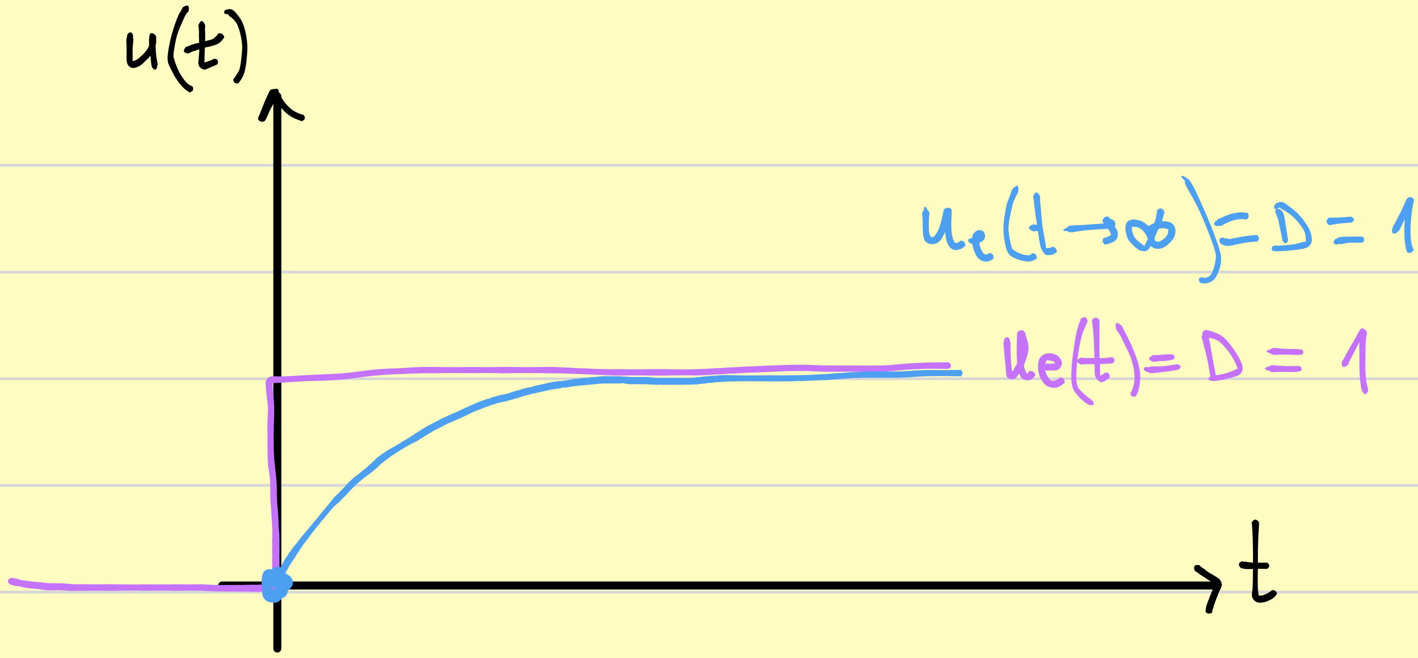
$$s^* = 0 \longrightarrow 1{,}21 = A \cdot 0{,}48 \cdot 2{,}515 \longrightarrow A = 1$$

$$s^* = -0{,}48 \rightarrow 1{,}21 = B(-0{,}48)(2{,}035) \rightarrow B = -1{,}239$$

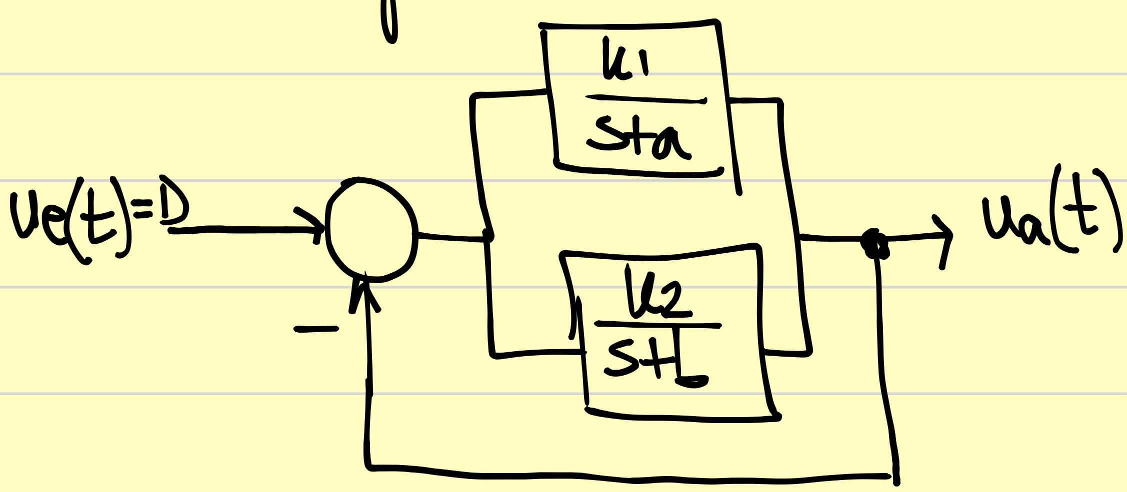
$$s^* = -2{,}515 \rightarrow 1{,}21 = C(-2{,}515)(-2{,}035) \rightarrow C = 0{,}195$$

$$u_a(s) = \frac{1}{s} - \frac{1{,}239}{s+0{,}48} + \frac{0{,}195}{s+2{,}515}$$

$$u_a(t) = 1 - 1{,}239\, e^{-0{,}48t} + 0{,}195\, e^{-2{,}515t}$$



6. Diagramm.



$$U_e(s) = \frac{D}{s}$$

$$F(s) = \frac{\frac{k_1}{s+a} + \frac{k_2}{s+b}}{1 + \frac{k_1}{s+a} + \frac{k_2}{s+b}} = \frac{k_1(s+b) + k_2(s+a)}{(s+a)(s+b) + k_1(s+b) + k_2(s+a)} =$$

$$= \frac{k_1(s+b) + k_2(s+a)}{s^2 + s\left((a+b) + k_1 + k_2\right) + (ab + k_1 b + k_2 a)}$$

$$\boxed{Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}}$$