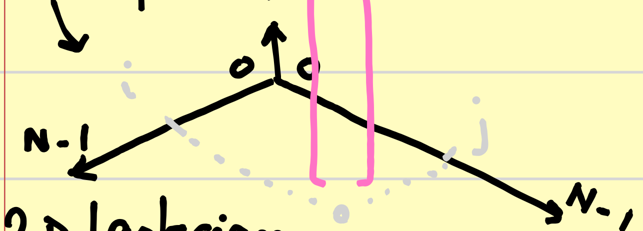
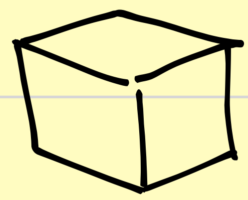
20240404_Business_Intelligence_WIN6

Netzwerk
(nicht Euklydisch)



2D Laplacian

Übung

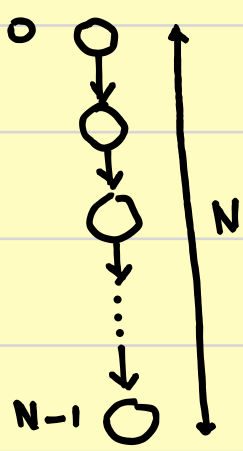


$\mathbb{R}^3$
(Euklydisch)

Business Network Structures (BNS) quantify & compare

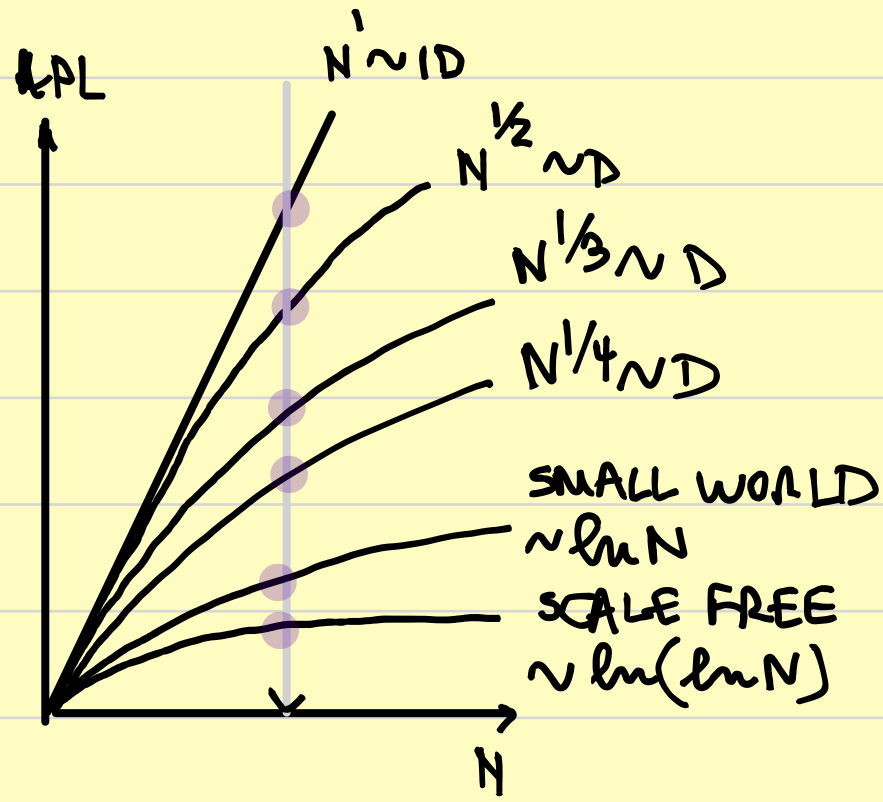
1 DIMENSIONAL BNS.

Hierarchical Pure

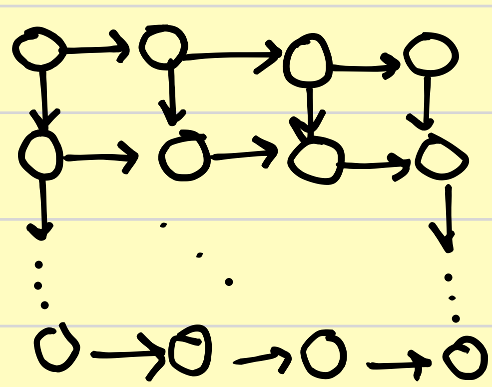


N Nodes

$N-1$: Number of steps to go through the whole hierarchy



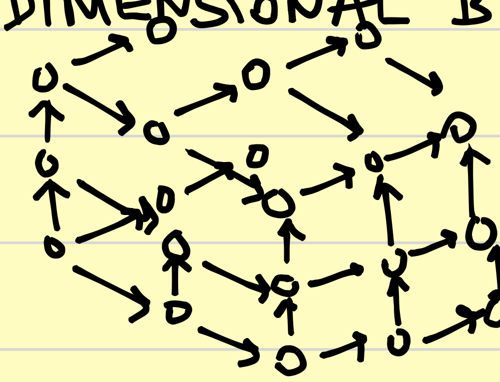
2 DIMENSIONAL BNS.



N. Nodes

$\sqrt{N}$. Schritte to go from one node to another.

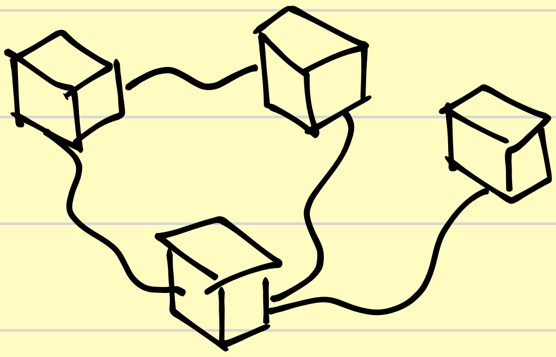
3 DIMENSIONAL BNS



N: Nodes

$\sqrt[3]{N}$: Steps to go from one node to another

4 DIMENSIONAL BNS



N : Nodes

$N^{1/4}$: Steps to go from one node to another

Mathematical formalization of complex network dynamics

Description of spreading Phenomena within complex networks.

2 Assumptions:

1. Individual nodes can be in two different states
   (S) susceptible. not yet infected
   (I) infected.

2. Each individual can infect anyone else.
   (HOMOGENEOUS MIXING HYPOTHESIS)

I SUSCEPTIBLE · INFECTIOUS · SUSCEPTIBLE (SIS) Model

We consider a behavioural pattern like a "disease" that spreads in a population distributed in a network and we allow for individuals to "forget" the behavioural pattern and abandon it.

Parameter: I.1. $i(t)$: the fraction of infected individuals compared to the total.

$$i(t) = \frac{\#\text{ infected }(t)}{\#\text{ total }(t)}$$

I.2. β : rate of infection: speed at which individuals get infected.

I.3. $\langle k \rangle$: average number of neighbours an individual has in the network.
average network degree.

I.4. $\mu \cdot i(t)$: rate of forgetfulness: speed at which individuals forget

The differential equation that describes the dynamics of infection in this network is:

$$\frac{di(t)}{dt} = \text{SPEED of CONTAGIOUSNESS} - \text{SPEED OF FORGEFULNESS}$$

$$= \beta \langle k \rangle \, i(t) \left(1 - i(t)\right) - \mu \, i(t)$$

Infection Rate

How well individuals are connected

Fraction of infected individuals

Fraction of not infected individuals

Forgetfulness Rate.

$$\ldots \quad i(t) = \left[1 - \frac{\mu}{\beta\langle k\rangle}\right] \cdot C \cdot \frac{e^{(\beta\langle k\rangle - \mu)t}}{1 + C e^{(\beta\langle k\rangle - \mu)t}}$$

$$C = \frac{i(t=0)}{1 - i(t=0)\frac{\mu}{\beta\langle k\rangle}}$$

what happens when $t \to \infty$?

$$\lim_{t\to\infty} i(t) = \lim_{t\to\infty} \left[1 - \frac{\mu}{\beta\langle k\rangle}\right] \cdot C \cdot \frac{e^{(\beta\langle k\rangle - \mu)t}}{1 + C e^{(\beta\langle k\rangle - \mu)t}} =$$

$$= 1 - \frac{\mu}{\beta\langle k\rangle} = 1 - \frac{1}{R_0}$$

$R_0 =$ Reproductive Number $= \dfrac{\beta\langle k\rangle}{\mu} =$ Number of people being infected by an individual in a period of time.

$$\boxed{R_0 = \frac{\beta\langle k\rangle}{\mu}}$$

$\beta\langle k\rangle \leftarrow$ Infection Rate β * connectivity $\langle k\rangle$

$\mu \leftarrow$ Forgetfulness

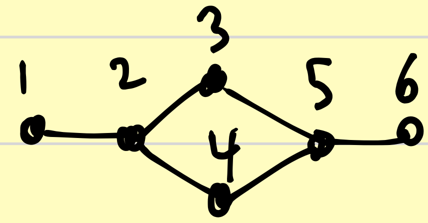
$R_0 > 1$: $\beta\langle k\rangle > \mu$: we get infected faster that we get cured : spread of virus

$R_0 < 1$: $\beta\langle k\rangle < \mu$: we get cured faster than we get infected : virus dies out

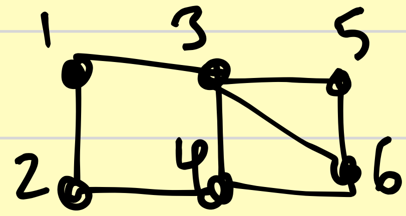
Beispiel:

$\beta = 0{,}1$

$\mu = 0{,}2$



N_1 $\quad \langle k_1\rangle = \dfrac{1+3+2+2+3+1}{6}$



N_2 $\quad \langle k_2\rangle = \dfrac{2+2+4+3+2+3}{6}$

Explanation:

- If $R_0 \uparrow\uparrow$ because $\beta \uparrow\uparrow \to$ Infection rate is high

  ACTION : bring $\langle K\rangle$ down!

- if $R_0 \uparrow\uparrow$ because $\langle k \rangle \uparrow\uparrow$ → People are connected to each other intensively

ACTION : bring β down!

- if $R_0 \uparrow\uparrow$ because $\mu \downarrow\downarrow$ → People recover slowly

ACTION : develop vaccine!

II We allow individuals to transmit the disease ONLY to those they are in contact with. This is a more realistic approach than SIS.

Parameter: II.1. i_k : Fraction of nodes with degree "k" that are infected among all all others with degree "k".

$$i_k = \frac{\#\text{ infected nodes with degree } k}{\#\text{ nodes with degree } k}$$

II.2. Θ_k : Fraction of infected neighbours from a susceptible node with degree k.

$$\frac{di_k}{dt} = \beta \cdot i_k(1-i_k) \cdot \Theta_k - \mu \cdot i_k$$

The condition for global spread of the behavioural pattern is given when the time to achieve e^{-1} fraction of the population infected is described by τ (characteristic time):

$$\boxed{\tau = \frac{\langle k \rangle}{\beta \langle k^2 \rangle - \mu \langle k \rangle}}$$

$\langle k^2 \rangle \equiv$ heterogenity $\equiv$ standard deviation of the degree distribution!

$\tau > 0$: condition for global spread is met.

This means that τ exists.

This means $\beta \langle k^2 \rangle - \mu \langle k \rangle > 0$

This condition is equivalent to:

$$\lambda = \frac{\beta}{\mu} > \frac{\langle k \rangle}{\langle k^2 \rangle} = \lambda_c$$

This depends on the virus

These depend on the structure of the network

This means: high λ means $\rightarrow$ $\beta \uparrow$ (high infec. rate) $\rightarrow$ $\mu \downarrow$ (low cureness rate)

Special case: for scale free networks $\langle k^2 \rangle = \infty \rightarrow \lambda_c = 0$

$\rightarrow$