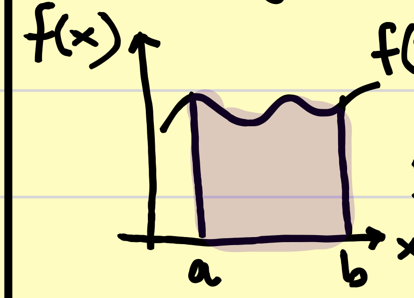
Wahrscheinlichkeitsfunktionen

W. : Wahrscheinlichkeit

Grundlage



$\square = \int_a^b f(x)dx$

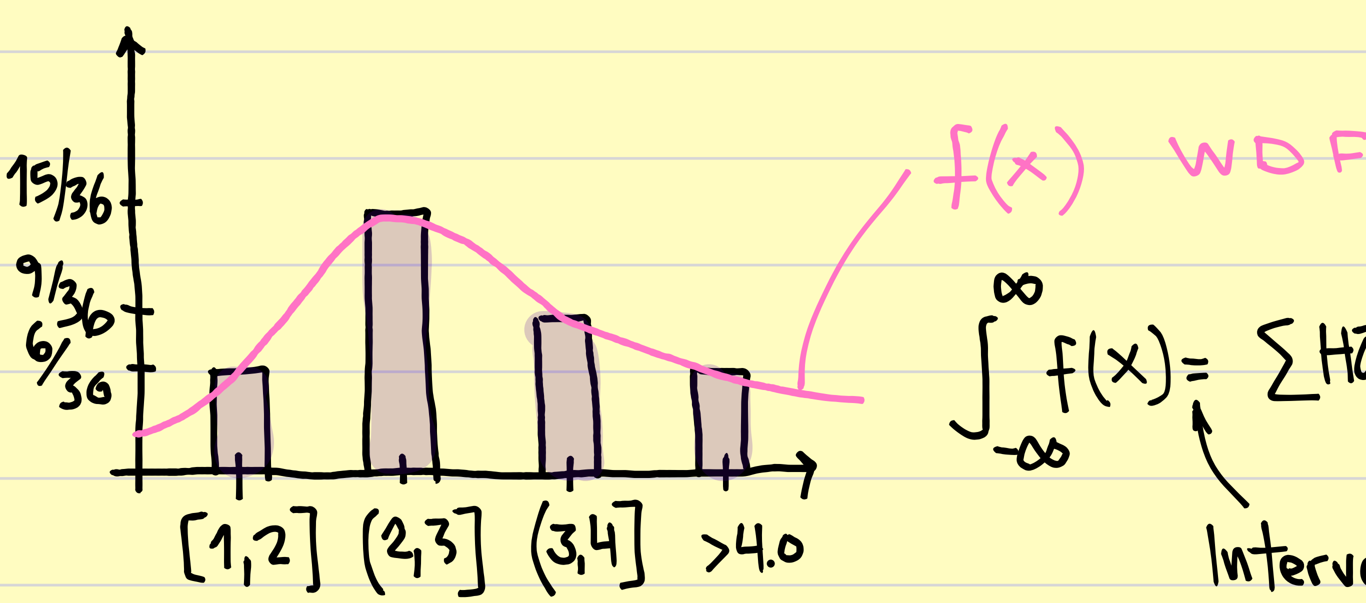
W. DICHTEFUNKTION (WDF)

Beispiel:

	X (Noten)	Häufigkeit vom Interval
>4.0	6	6/36
(3,4]	9	9/36
(2,3]	15	15/36
[1,2]	6	6/36
	$\sum = 36$	$\sum = 1$

Die Fläche unter der WDF ist immer 1.

$$\int_{-\infty}^{\infty} f(x) = 1$$

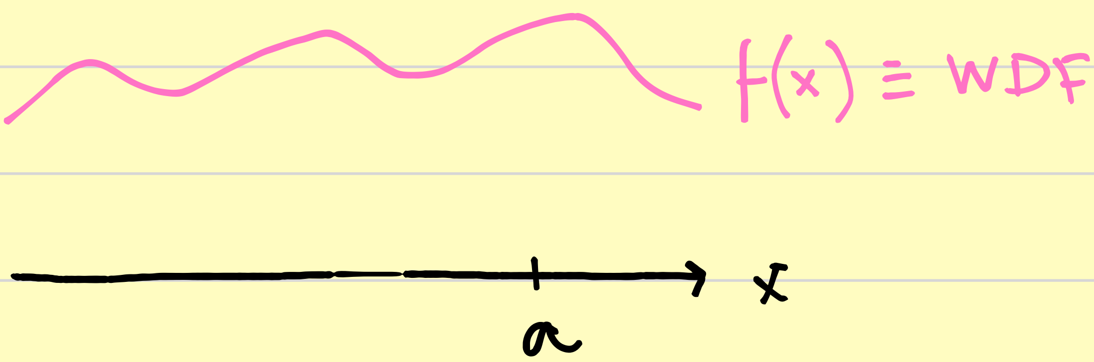


GRAPHISCH

$$\int_{-\infty}^{\infty} f(x) = \sum \text{Häufigkeiten} = 1$$

Intervallogik

W-Rechnung, angenommen die WDF ist bekannt.



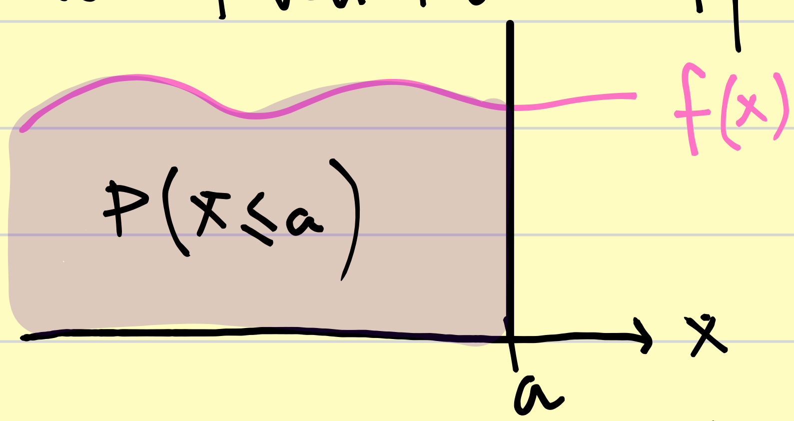
Was ist die W dafür, dass X kleiner/gleich „a" ist?

$$\int_{-\infty}^{a} f(x)\,dx = F(a) = P(\bar{x} \leq a) \equiv$$ W. dafür, dass $x \leq a$ ist.

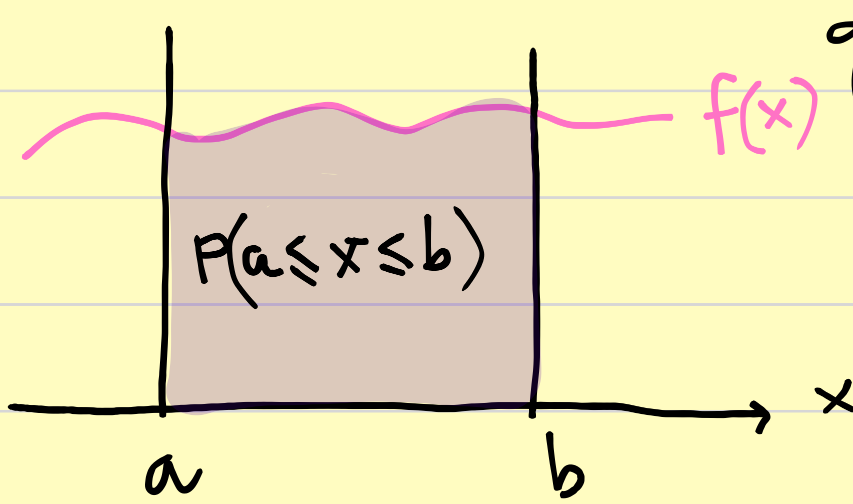
Kummulative Funktion

(*)

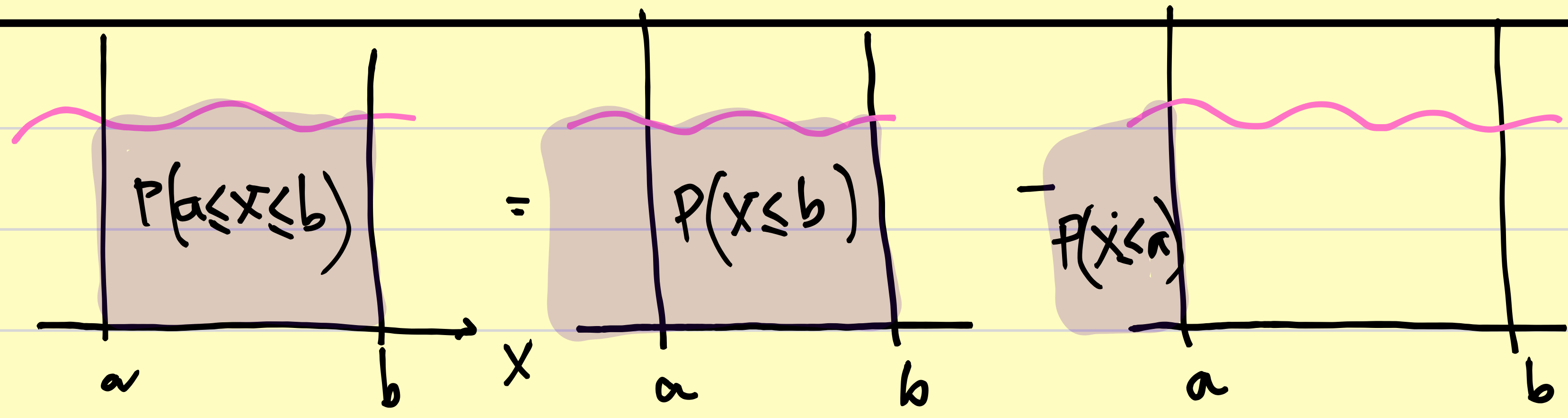
(*) „P" kommt von Probability = Wahrscheinlichkeit



Was ist die W- dafür, dass X kleiner/gleich „b" UND größer/gleich „a" ?



$$\int_{a}^{b} f(x)\,dx = P(a \leq \bar{x} \leq b)$$



$$P(a \le X \le b) = P(X \le b) - P(X \le a)$$

$$\int_a^b f(x)\,dx = \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx$$

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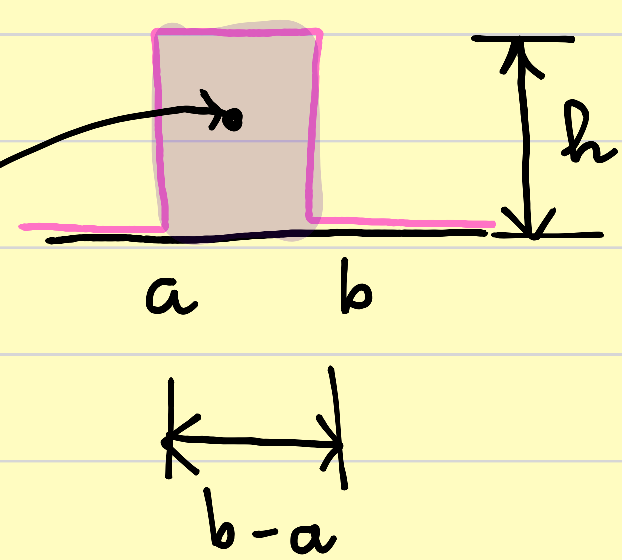
1. UNIFORM VERTEILUNG / WDF

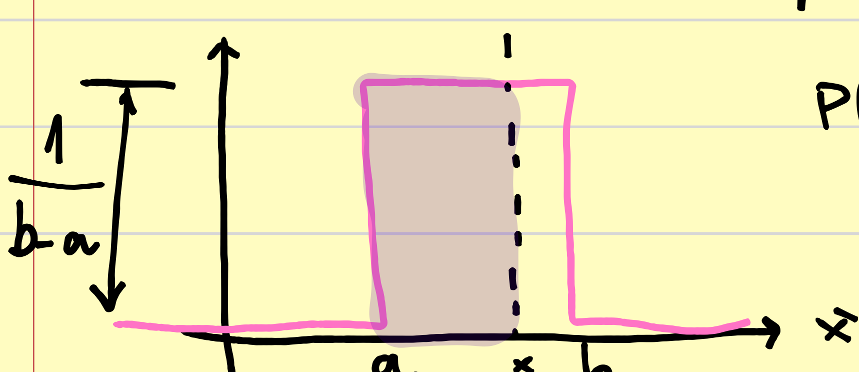
$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{sonst} \end{cases}$$



$$\text{Fläche} = 1 = (b-a)h \rightarrow h = \frac{1}{b-a}$$

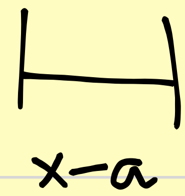
$$\boxed{M_1 = \frac{a+b}{2}} \qquad \boxed{\sqrt{m_2} = \frac{b-a}{\sqrt{12}}}$$

Was ist die W. dafür, dass die Variable $X \le x$ ist?

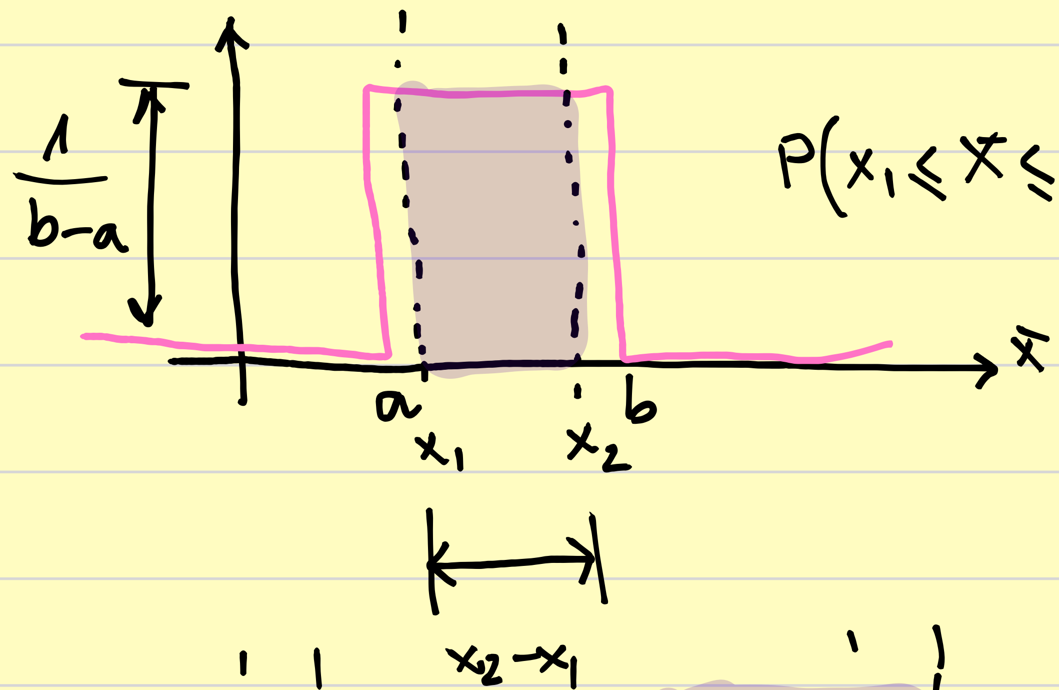


$$P(X \le x) = \int_{-\infty}^{x} f(x)\,dx = \int_{a}^{x} f(x)\,dx =$$

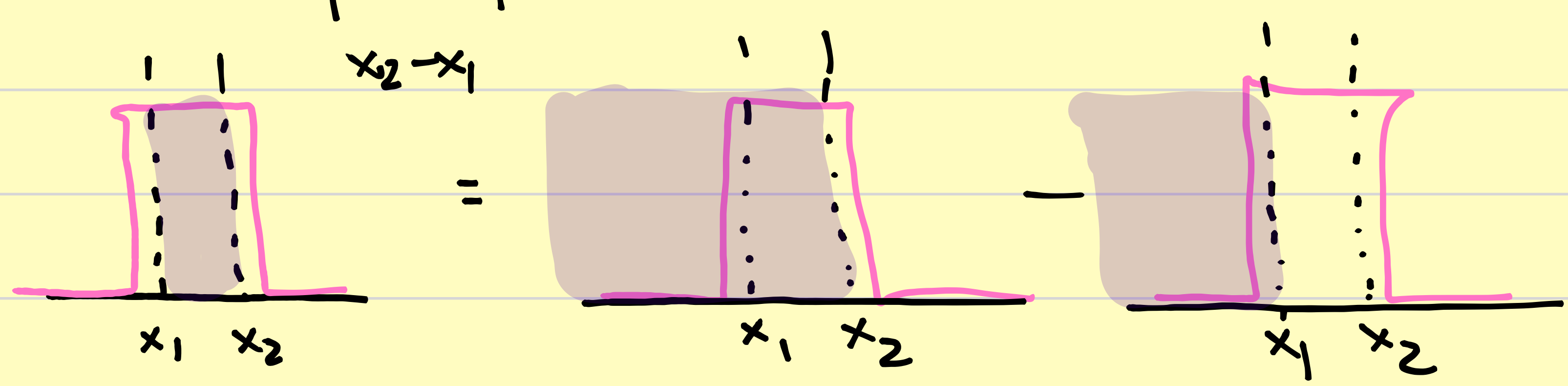
$$P(X \le x) = (x-a)\cdot\frac{1}{b-a} = \frac{x-a}{b-a}$$



Was ist die W. dafür, dass die Variable $X \leq x_2$ und $X \geq x_1$ ist?



$$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b-a}$$



$$P(x_1 \leq X \leq x_2) = P(X \leq x_2) - P(X \leq x_1)$$

$$\frac{x_2 - a}{b-a} - \frac{x_1 - a}{b-a}$$

$$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b-a}$$

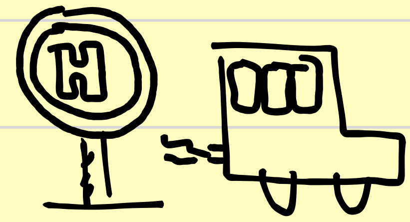
Übung: Die Wartezeit an einer Bushaltestelle ist Uniformverteilt.
$X \sim U(1, 12)$ min.

a) Was ist die WDF?

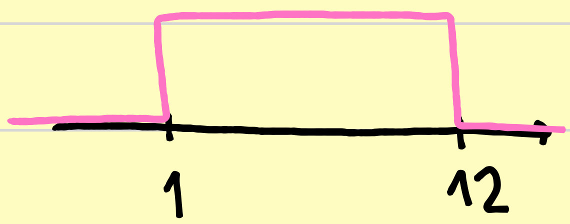
b) Was ist die W. dafür, dass die Wartezeit $X \leq 8$ min ist?

c) Was ist die W. dafür, dass die Wartezeit $X > 4$ min ist?

d) Was ist M_1 und $\sqrt{m_2}$?

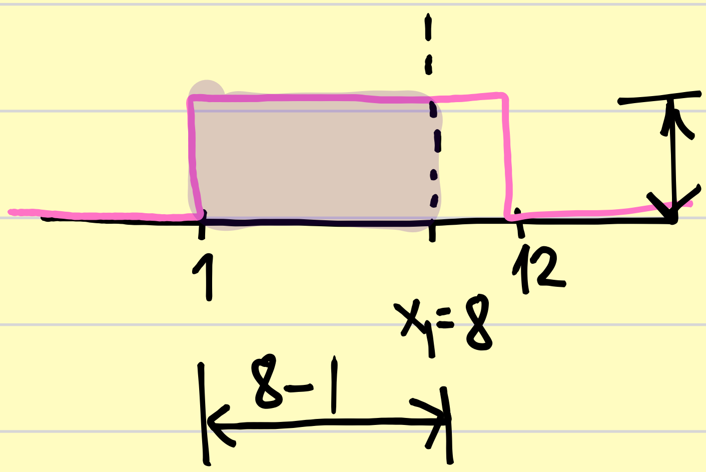


a) WDF

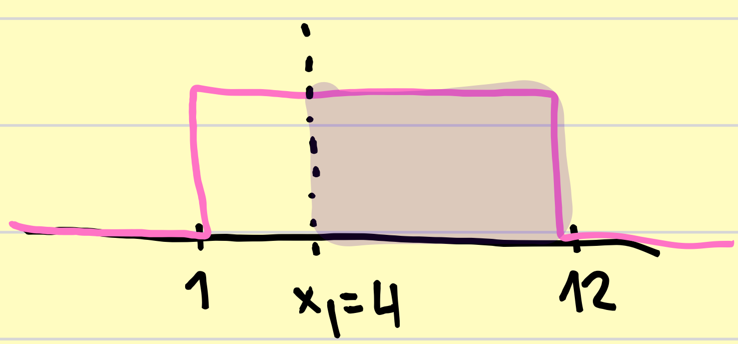


$$h = \frac{1}{12-1} = \frac{1}{11} \qquad f(x) = \begin{cases} \frac{1}{11} & 1 \leq x \leq 12 \\ 0 & \text{rest} \end{cases}$$

b) $$P(X \leq 8 \text{ min}) = (8-1) \cdot \frac{1}{11} = \frac{7}{11}$$



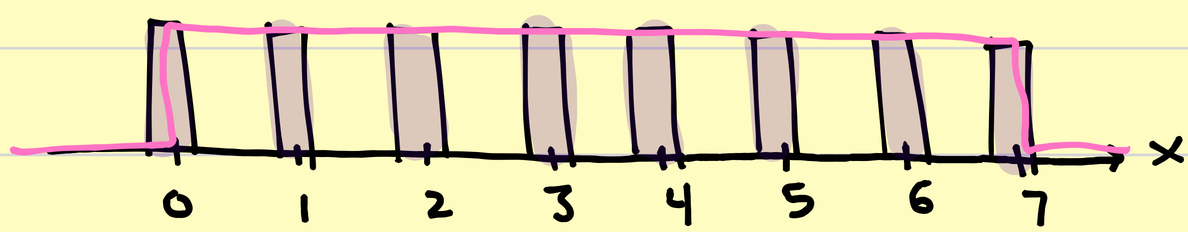
c) $$P(X > 4 \text{ min}) = 1 - P(X \leq 4) = 1 - \frac{4-1}{11} = \frac{8}{11}$$



d) $$M_1 = \frac{a+b}{2} = \frac{12+1}{2} = 6'5$$

$$\sqrt{m_2} = \frac{b-a}{\sqrt{12}} = \frac{12-1}{\sqrt{12}} = 3'175$$

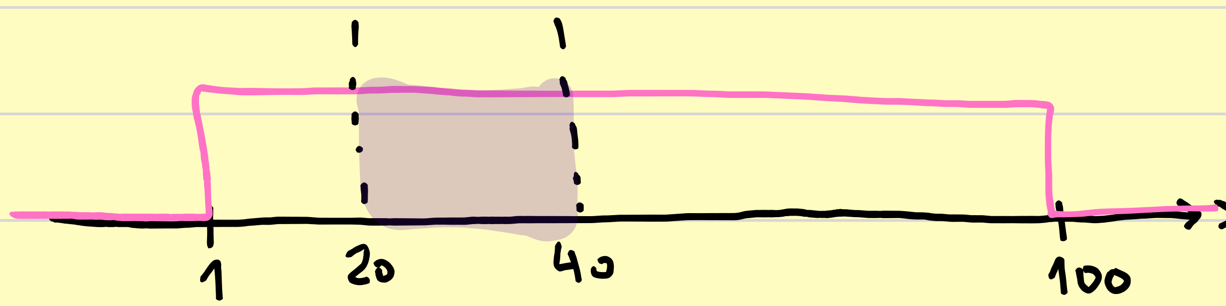
Übung 1.1. 20240318



$$P(X = x_1) = \frac{1}{8}$$

Einen bestimmten Preis x_1

Übung 1.2 20240318.



$$P(20 \le X \le 40) = (40-20)\cdot\frac{1}{100} = \frac{20}{100} = 0'2$$

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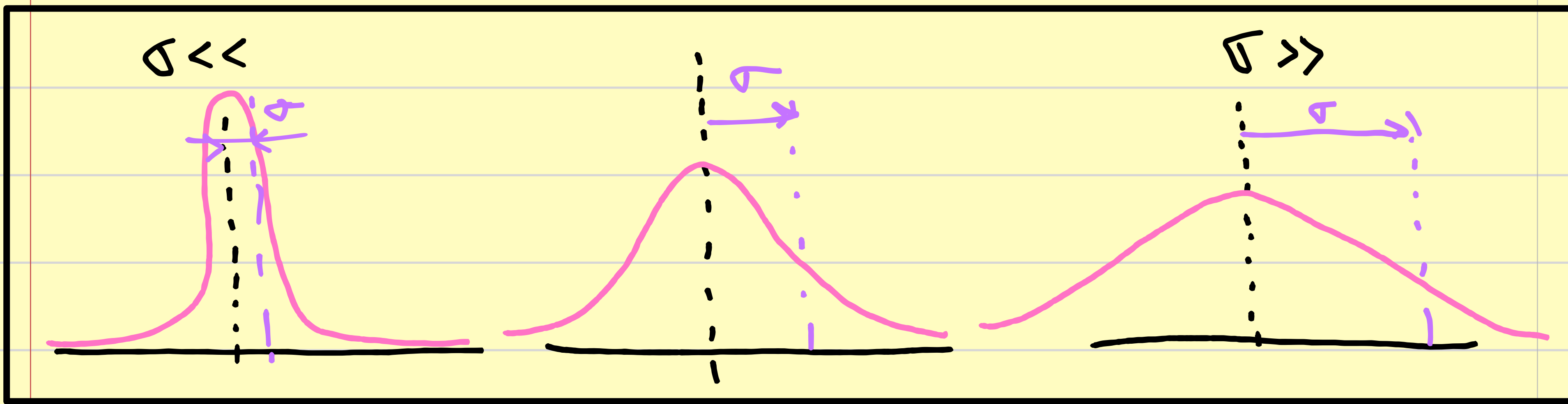
2 NORMALVERTEILUNG

$M_1 = \mu \quad \sqrt{m_2} = \sigma$

$$f(x) = \frac{1}{\sqrt{2\pi}\sqrt{m_2}} \cdot e^{-\frac{(x-M_1)^2}{2m_2}} = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} = N(\mu,\sigma)$$



z-Transformation

MITTELWERT

$$X \sim N(\mu, \sigma) \longrightarrow Z \sim N(0, 1)$$

STANDARD ABWEICHUNG

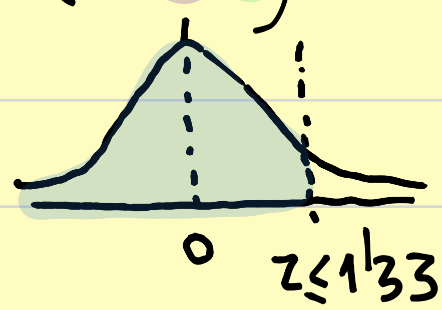
$$Z = \frac{X-\mu}{\sigma}$$ NORMIERTE NORMALVERTEILUNG

Verteilungstabellen

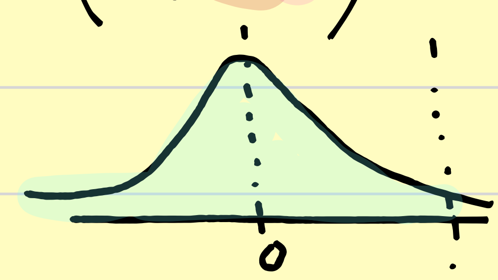
Standardnormalverteilung

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

$P(Z \leq 1'33) = 0'9082$



$P(Z \leq 1'97) = 0'9756$

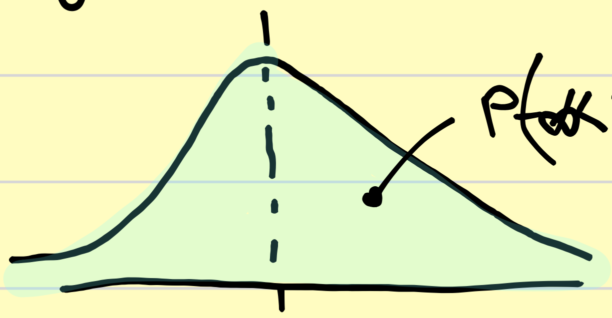


$P(Z \leq -0'87) =$

$1 - P(Z \leq 0'87) =$

$1 - 0'8078 = 0'1922$

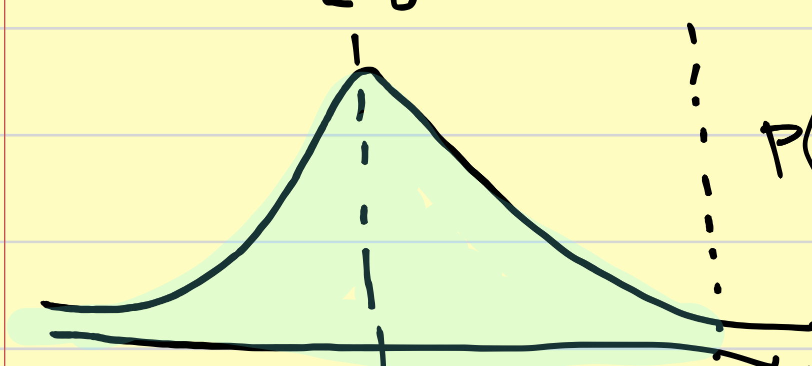
Frage. M



$P(-\infty < X \leq \infty) = 1$

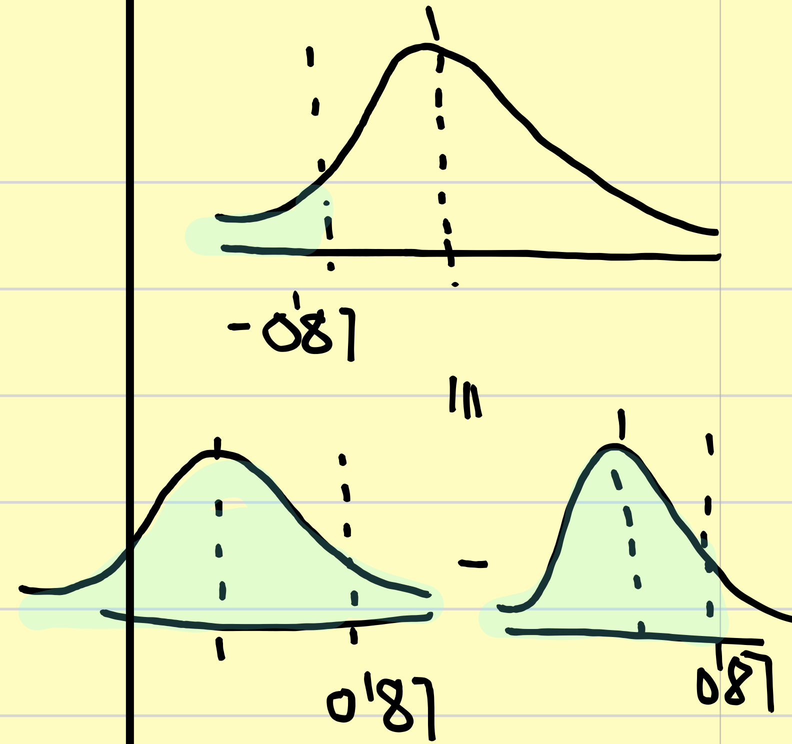
$P(Z \leq 0) = 0'5$

$z = 0$

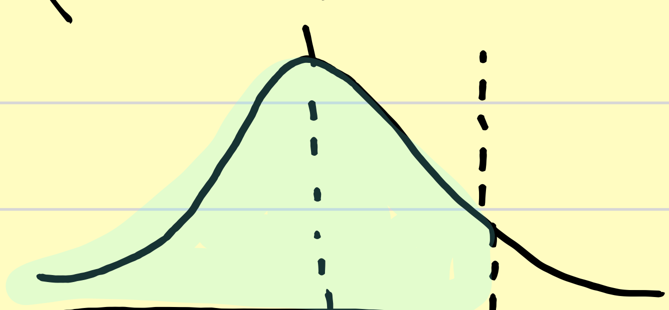


$P(Z \leq 3'89) = 0'9999 \simeq 1$

$z = 3'89$



$P(Z \leq 2'3) = 0'9893$



$P(Z \leq -1) = P(Z > 1) =$

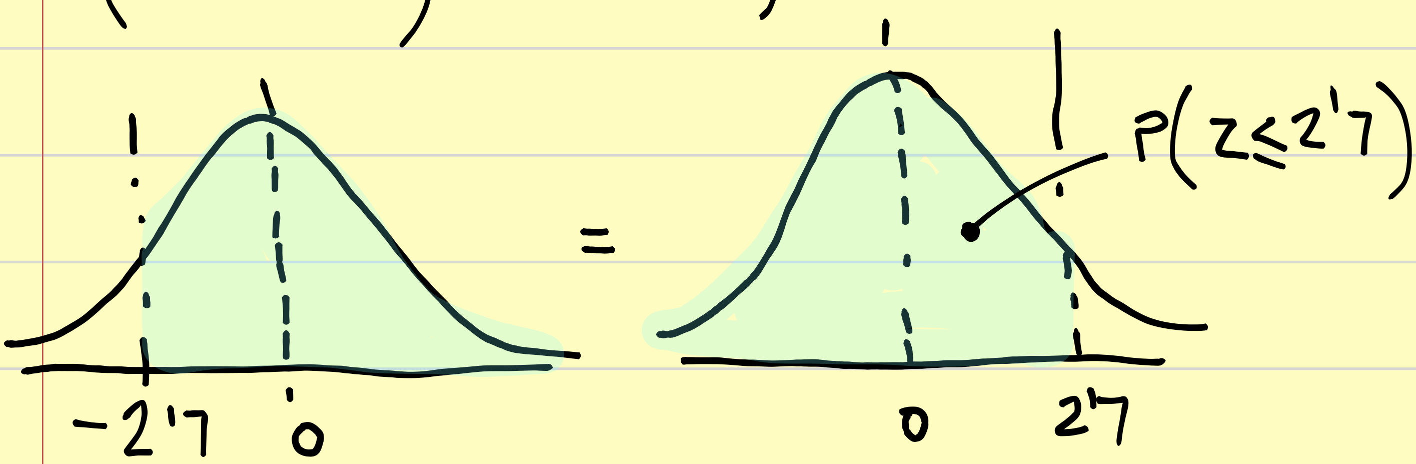
$= 1 - P(Z \leq 1) =$

$= 1 - 0'8413 = 0'1587$

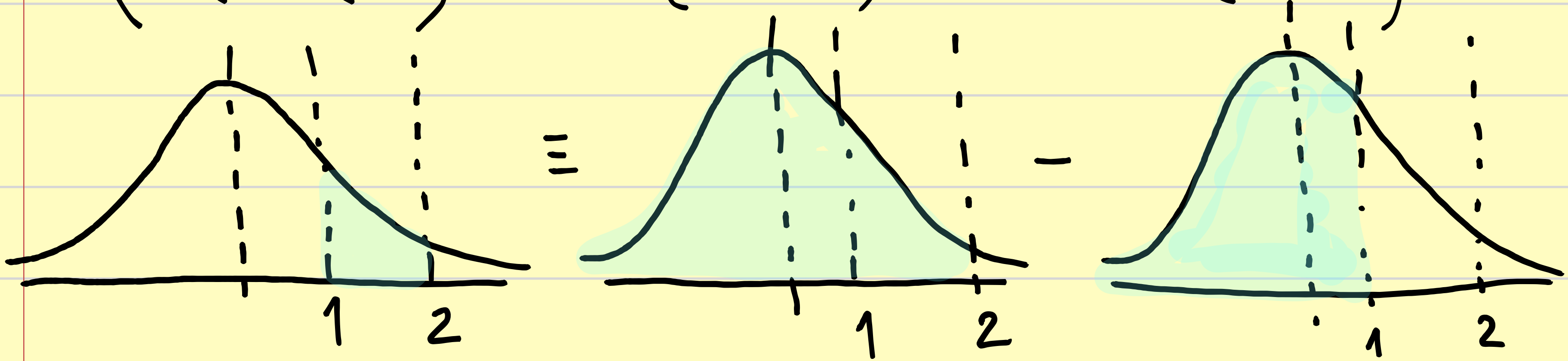
$P(Z > 1)$

$P(Z \leq -1)$

$P(Z > -2'7) = P(Z \leq 2'7) = 0'9965$

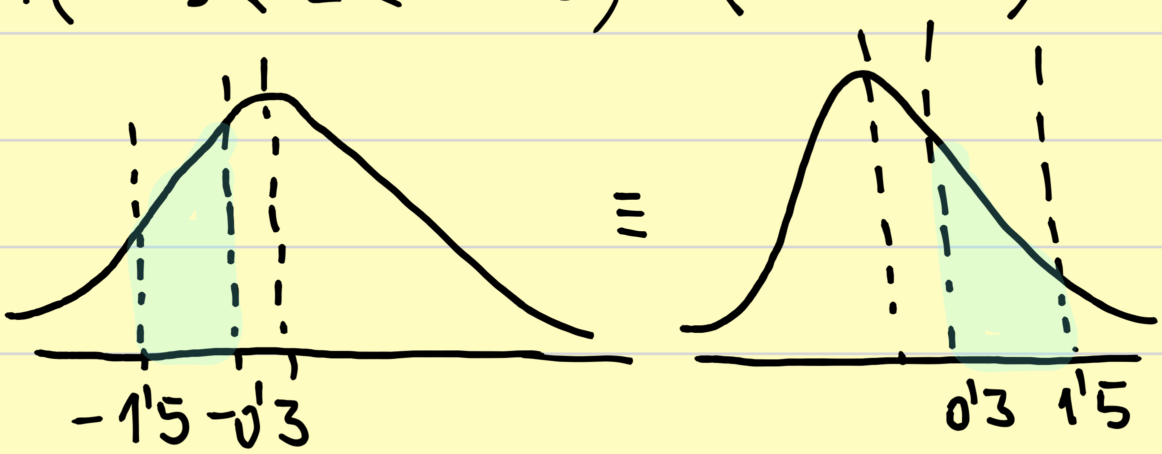


$P(1 \leq Z \leq 2) = P(Z \leq 2) - P(Z \leq 1)$



$= 0'9772 - 0'8413 = 0'1359$

$P(-1'5 \leq Z \leq -0'3) = P(0'3 \leq Z \leq 1'5) = P(Z \leq 1'5) - P(Z \leq 0'3)$



NORMIERUNG

$$z_i = \frac{x_i - M_1}{\sqrt{m_2}}$$

$x_1 = 5$ $\quad x_4 = 8$

$x_2 = 6$ $\quad x_5 = 9$

$x_3 = 7$ $\quad x_6 = 10$

$$M_1 = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{6}\left(5+6+7+8+9+10\right) = 7'5$$

$$m_2 = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - M_1\right)^2 = \frac{1}{6}\Big[\left(5-7'5\right)^2+\left(6-7'5\right)^2+\left(7-7'5\right)^2+ \left(8-7'5\right)^2+\left(9-7'5\right)^2+\left(10-7'5\right)^2\Big] =$$

$$z_1 = \frac{5-7'5}{\sqrt{m_2}} \qquad z_4 = \frac{8-7'5}{\sqrt{m_2}}$$

$$z_2 = \frac{6-7'5}{\sqrt{m_2}} \qquad z_5 = \frac{9-7'5}{\sqrt{m_2}}$$

$$z_3 = \frac{7-7'5}{\sqrt{m_2}} \qquad z_6 = \frac{10-7'5}{\sqrt{m_2}}$$

Mit diesen NORMIERTEN Daten darf ich die Z-Tabelle nutzen.

Übung 2.1.

$$X \sim N\left(175, 8\right)$$

$\mu \equiv M_1$

$$P\left(X \geq 183\right) = 1 - P\left(X < 183\right) = 1 - P\left(\frac{X-175}{8} < \frac{183-175}{8}\right)$$

$\sqrt{m_2} \equiv \sigma$

$$= 1 - P\left(Z < 1\right) = 1 - 0'8413 = 0'1587 = 15'87\,\%$$

$Z = \frac{X-175}{8}$ WIR HABEN ES NORMIERT!

③ BINOMIALVERTEILUNG

3 Konditionen:

a) das Experiment unterliegt „N" unabhängige Versuche

b) Jeder Versuch hat NUR ZWEI Ausgänge (Erfolg - Miserfolg)

c) Die W. vom Erfolg „p" ist konstant.

$$P(X \leq x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$n \equiv$ Anzahl Versuche

$x \equiv$ Anzahl Erfolge

$p \equiv$ W. Erfolg

$M_1 = n \cdot p$

$\sqrt{m_2} = \sqrt{np(1-p)}$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} = \frac{n(n-1)(n-2)\cdots 1}{[x(x-1)(x-2)\cdots 1][(n-x-1)(n-x-2)\cdots 1]}$$

Beispiel: Die W. dafür, dass ein Techniker das Projekt erfolgreich beendet ist 80%. Wenn X eine Variabel ist, welche beschreibt $X \equiv$ Anzahl Techniker welcher aus einer Gruppe von 10 Technicker das Projekt erfolgreich beenden. Ermitteln Sie M_1 und $\sqrt{m_2}$ eines erfolgreichen Projektabschlusses.

Konditionen

a) $N = 10$ ✓

b) Erfolg/Miserfolg ✓

c) $p = 0'8$ konstant ✓

$M_1 = n \cdot p = 10 \cdot 0'8 = 8$

$\sqrt{M_2} = \sqrt{n \cdot p(1-p)} = \sqrt{10 \cdot 0'8 \cdot (1-0'8)} = 1'26$

Wenn „n" sehr groß wird, tendiert die Binomial-
verteilung zu einer Normalverteilung.