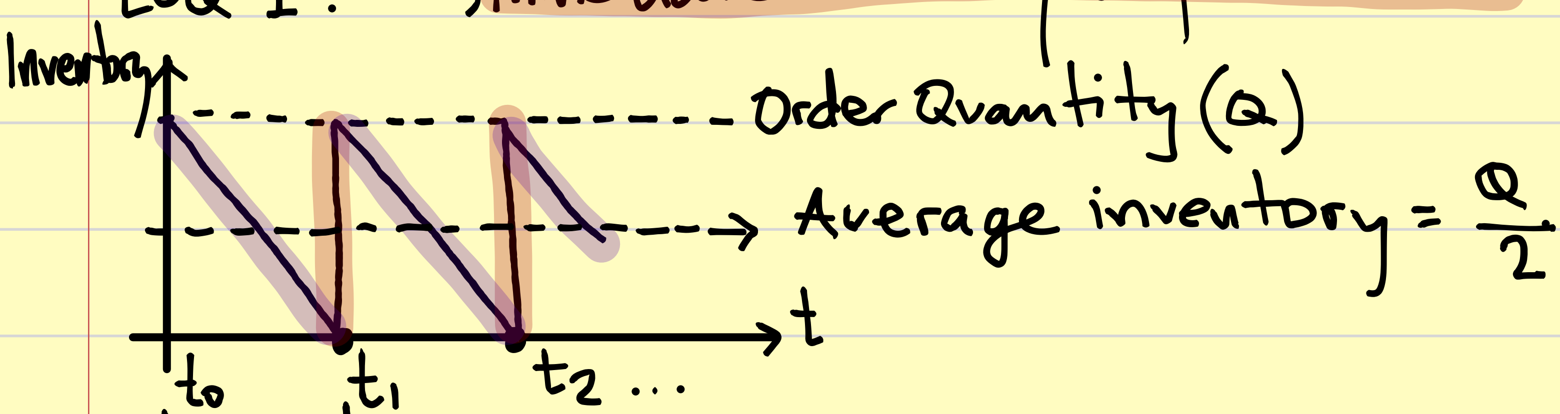
20231127_Supplier_Management_BEL4

Economic Order Quantity Model

Parameters:

- D. Demand [Parts/time]
- c. Cost per Unit [€/unit]
- A. Setup cost [€]
- h. Inventory holding cost [€/Part.time]

EOQ I. 1) Immediate delivery & production 2) Constant Demand



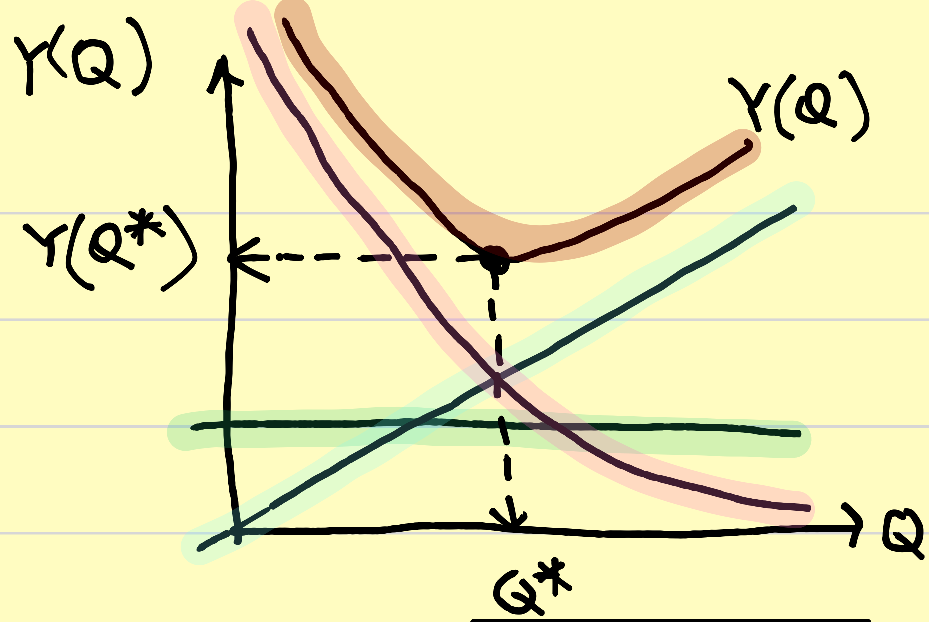
$$\frac{Q}{D} = \frac{[\text{Parts}]}{[\text{Parts/time}]} = [\text{time}]$$

Cost function $Y(Q)$ = cost of inventory + setup cost + Production cost =

Average Inventory × Inventory holding cost + setup cost × how often we do setup + Cost per unit × Units

$$\boxed{Y(Q) = \frac{Q}{2} \cdot h + A \cdot \frac{1}{Q/D} + c \cdot D}$$

- We want to know what is the EOQ, in other words, what is the MINIMUM of the function $Y(Q)$.



$$\left.\frac{dY(Q)}{d(Q)}\right|_{Q=Q^*} = 0 \longrightarrow \frac{h}{2} - \frac{AD}{Q^{*2}} = 0 \longrightarrow \boxed{Q^*_I = \sqrt{\frac{2AD}{h}}}$$

$$Y(Q^*_I) = \frac{h}{2}\cdot Q^*_I + \frac{AD}{Q^*_I} + \cancelto{0}{cD} = \frac{h}{2}\cdot\sqrt{\frac{2AD}{h}} + AD\cdot\sqrt{\frac{h}{AD}} =$$

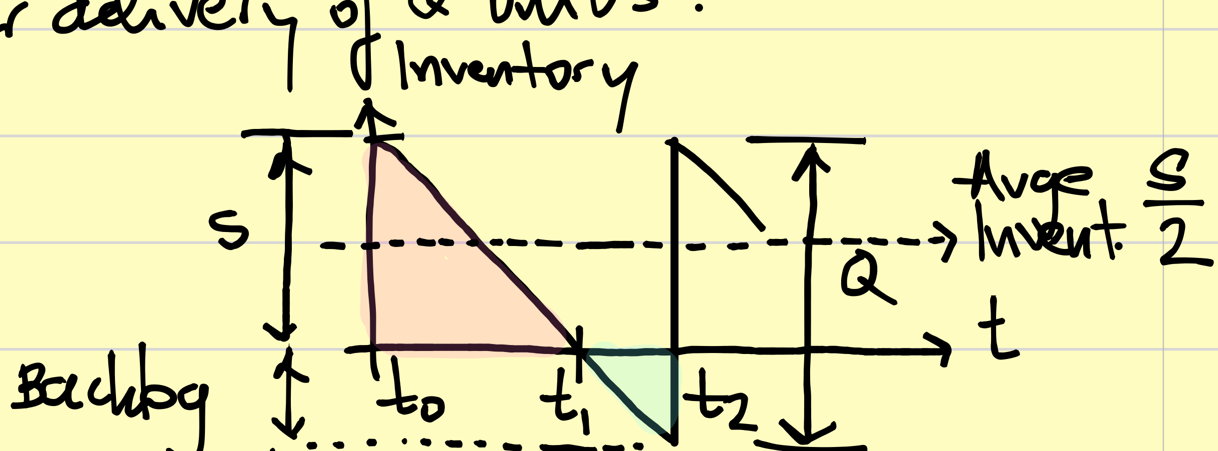
$$\boxed{Y(Q^*_I) = \sqrt{2ADh}}$$

EOQ II. Supplier Model

1) We accept "stock-out" in which supplier has a backlog
2) Production & delivery are inmediate (like EOQ_I)
3) Demand is constant (like EOQ_I)

Additional parameters:

p. cost for not supplied orders [€/unit]
S. Inventory after delivery of Q units.
Backlog = Q − S



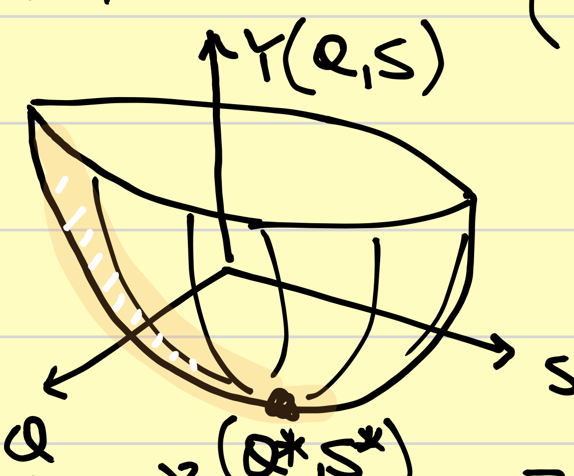
$t_1 = \frac{S}{D}$, $t_2 = \frac{Q}{D}$

$$Y(Q,S) = \text{Inventory holding cost} + \text{Backlog Cost} + \text{Setup Cost} + \text{Production Cost}$$

$$= h \cdot \frac{S}{2} \cdot \frac{S}{D} \cdot \frac{D}{Q} + p \cdot \frac{(Q-S)}{2} \cdot \frac{Q-S}{D} \cdot \frac{D}{Q} +$$

Holding Cost Per Unit; Average Inventory; Time to consume Inventory; How often; backlog cost per unit; average backlog; time backlog $(t_2 - t_1)$; how often

$$+ \frac{AD}{Q} + cD$$



$$\left.\frac{\partial Y(Q,S)}{\partial Q}\right|_{Q^*,S^*} = 0 \rightarrow \frac{\partial}{\partial Q}\left[\frac{hS^2}{2Q} + \frac{p(Q-S)^2}{2Q} + \frac{AD}{Q} + cD\right]_{Q^*,S^*} = 0$$

$$\left.\frac{\partial Y(Q,S)}{\partial S}\right|_{Q^*,S^*} = 0 \rightarrow \frac{\partial}{\partial S}\left[\frac{hS^2}{2Q} + \frac{p(Q-S)^2}{2Q} + \frac{AD}{Q} + cD\right]_{Q^*,S^*} = 0$$

$$-\frac{hS}{Q^{*2}} + \frac{p(Q^*-S^*)}{Q^*} - \frac{p(Q^*-S^*)^2}{2Q^{*2}} - \frac{AD}{Q^{*2}} = 0$$

$$\frac{hS^*}{Q^*} - \frac{p(Q^*-S^*)}{Q^*} = 0$$

$\rightarrow \dots \rightarrow$

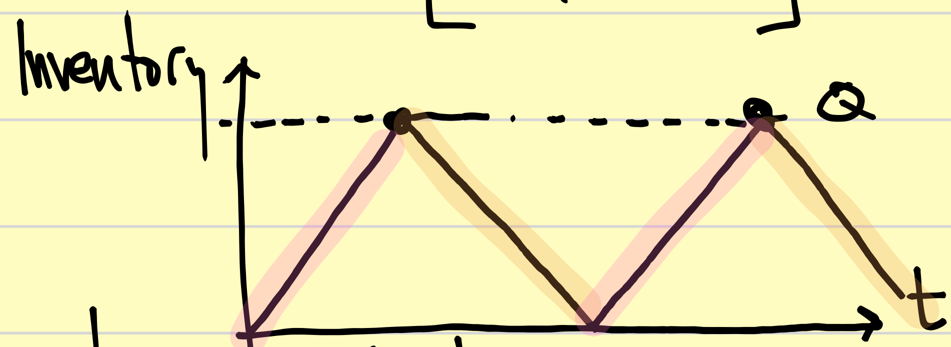
$$\rightarrow \boxed{\begin{aligned} Q^*_{II} &= \sqrt{\frac{2AD}{h}} \cdot \sqrt{\frac{p+h}{p}} = Q^*_I \sqrt{\frac{p+h}{p}} > Q^*_I \\ S^*_{II} &= \sqrt{\frac{2AD}{h}} \sqrt{\frac{p}{p+h}} = Q^*_I \sqrt{\frac{p}{p+h}} < Q^*_I \end{aligned}}$$

for $p \gg h \rightarrow Q_{II}^* = S_{II}^* = Q_I^*$

EOQ III (Manufacturing model) - without Backlog

1) No backlog
2) Constant demand (like EOQ I)
3) Production & Delivery NOT immediate

Parameter = K = Production rate [part/time]



$$Y(Q) = \underbrace{\text{Inventory}}_{} + \text{cost} + \text{cost} =$$

Holding cost — Inventory; setup cost; Production cost

$$= h \cdot \underbrace{\frac{Q}{2}\left(1 - \frac{D}{K}\right)}_{\text{Average Inventory}} + \frac{AD}{Q} + cD$$

Minimum : $\left.\frac{dY}{dQ}\right|_{Q^*} = 0 \rightarrow \frac{h}{2}\left(1 - \frac{D}{K}\right) - \frac{AD}{Q^{*2}} = 0 \rightarrow$

$$\boxed{Q_{III}^* = \sqrt{\frac{2AD}{h\left(1 - \frac{D}{K}\right)}}}$$

Notice when production is infinite fast $K \to \infty$, then we have immediate production, so EOQ_I.

$$Y(Q^*_{III}) = \frac{h Q^*_{III}}{2}\left(1-\frac{D}{K}\right) + \frac{AD}{Q^*_{III}} + \cancel{cD}^{\,0} =$$

$$= \frac{h}{2}\cdot\left(1-\frac{D}{K}\right)\cdot\sqrt{\frac{2AD}{h\left(1-\frac{D}{K}\right)}} + AD\cdot\frac{1}{\sqrt{\frac{2AD}{h\left(1-\frac{D}{K}\right)}}} =$$

$$= \sqrt{\frac{2AD\,h^2\left(1-\frac{D}{K}\right)}{2^2 h}} + \sqrt{\frac{ADh\left(1-\frac{D}{K}\right)}{2}} =$$

$$\boxed{Y(Q^*_{III}) = \sqrt{2ADh\left(1-\frac{D}{K}\right)}}$$

$$K \to \infty \to Y(Q^*_{III}) = \sqrt{2ADh} = Y(Q^*_I)$$