

Inventory Control Strategies:

1. Single-Period Inventory Model (Newsvendor Model)
2. Inventory Review Systems.

1. SPIM .

Def. The SPIM is designed for managing inventory in situations where there is only one opportunity to sell the product before it either perishes or becomes obsolete. The goal of the SPIM is to find the optimal order quantity that balances the risk of stockouts with the cost of overstocking.

Examples: Newspapers, perishable goods, seasonal products

We need to focus on two types of costs :

1) Underage cost (Shortage cost , c_u). This cost is incurred when you run out of stock and cannot meet demand. It includes the lost profit and penalties or customer dissatisfaction.

2) Overage cost (Excess cost , c_o). This cost is incurred when you order too much and are left with unsold inventory. It includes holding costs, disposal costs, or obsolescence cost.

UNDERAGE COST [C_u]: $C_u = \text{Selling Price} - \text{Cost per Unit}$

This represents the profit you lose for each unit of demand that you cannot fulfill.

OVERTAGE cost [C_o]: $C_o = \text{Cost per Unit} - \text{Salvage Value}$

This represents the loss you incur for each unsold unit accounting for salvage value (e.g. resale, recycling, or markdown)

CRITICAL RATIO (CR): To balance these two costs, we use the CR, which determines the optimal service level. The CR is the ratio of the C_u to the total cost:

$$CR = \frac{C_u}{\text{Total Cost}} = \frac{C_u}{C_u + C_o}$$

The CR helps decide how much of the demand distribution we want to cover: it provides the probability that the optimal order quantity will meet demand.

→ $C_o = 0 \rightarrow CR = 1 \rightarrow 100\% \text{ to meet demand}$

$C_o \gg C_u \rightarrow CR \rightarrow 0 \rightarrow 0\% \text{ to meet demand}$

How do we find the optimal order quantity Q^* ?

Q^* is determined by the critical ratio. Once CR is calculated, it corresponds to a Z-Score from the normal distribution, which helps find the point of the demand where we should place our order.

If the demand follows a normal distribution with known (μ) and (σ) , the optimal quantity is given by:

$$Q^* = \mu + Z \cdot \sigma$$

Z-Score corresponding to the critical ratio.

$$CR = 0.87 \rightarrow Z = 1.13$$

$$\mu = 2$$

$$\sigma = 1.5$$

$$Q^* = 2 + 1.13 \cdot 1.5 = 3.695$$

Verteilungstabellen

Standardnormalverteilung

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8467	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8685	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

problem. You manage the inventory for a bakery that sells fresh croissants each day. The selling price is 3€/unit. The cost of production is 1€/unit. Unsold units can

be sold to a recycler for 0'20€/unit. Daily demand follows a normal distribution $\mu = 200$ units and $\sigma = 30$ units. Calculate Q^* .

$$1. C_u = \text{Selling price} - \text{Cost per unit} = 2\text{€/unit}$$

$$2. C_o = \text{Cost per Unit} - \text{Salvage value} = 0'8\text{€/unit}$$

$$3. C_R = \frac{C_u}{C_o + C_u} = \frac{2}{2+8} = 0'714$$

$$4. Z\text{-Score} = 0'57$$

$$5. Q^* = \mu + Z \cdot \sigma = \\ = 200 + 0'57 \cdot 30 = 218 \text{ units}$$

We should order 218 units each day to balance the risk of stockouts with the cost of overstocking.

Verteilungstabellen

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2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

2. Inventory Review Systems

The next challenge is to manage inventory over a longer time horizon, where products have a continuous or repeated selling period. Two main systems are used to monitor inventory & decide when/how much to order:

1) Continuous Review System (Q, R)

2) Periodic Review System (T, S)

2.1) Continuous Review System (Q,R) [CRS]

In a CRS, the inventory is reviewed continuously, and an order of a fixed quantity (Q) is placed as soon as the inventory reaches a reorder point (R).
This is based on the EOQ calculations (see lecture, 20240130).

2.2) Periodic Review System (T,S) [PRS]

In a PRS, inventory levels are reviewed at fixed intervals (e.g. weekly, ...), and an order is placed to bring inventory up to a target level. Unlike the CRS, where you place an order as soon as the reorder point is reached, here you review & replenish the stock only at the end of the review period.

With PRS we need more stock than with CRS because PRS does not immediately react to demand fluctuations.

- Review Period (T): Time interval b/w. inventory revs
- Order-up-to-level (S): The inventory level to which stock is replenished at each review.
$$S = D[L+T] + SS$$

The amount of order

$$Q = S - I$$

- I is the current inventory level.

Problem. You manage the inventory of laptops with following characteristics:

- Average daily demand. 10 units.
- lead time. 5 days.
- Review Period. 20 days.
- Safety Stock. 50 units.

1. Calculate S.

The Order-up-to-level S must cover the demand during both lead time & review period:

$$S = [D \cdot (L + T)] + SS$$

$\frac{\text{Units}}{\text{Time}}$ lead Time Review Period Safety Stock

$$\cdot S = \left[10 \frac{\text{Units}}{\text{day}} \cdot (20 + 5) \right] + 50 = 300 \text{ units}$$

2. Determine Q

$$Q = S - I = 300 - 120 = 180 \text{ units}$$

2.3. Comparison: CRS & PRS

feature	CRS [Q,R]	PRS [T,S]
• Monitoring	Continuous (real-time)	Periodic (intervals)
• Order Trigger	When inventory reaches Reorder Point.	At the end of review period
• Order Quantity	EOQ	Variable
• Safety Stock	lower (inventory is closely monitored)	higher (reviews are less frequent)
• Complexity	Higher (constant tracking)	lower (simpler to manage)
• Risk of Stockout	lower (constant monitoring)	higher (run out b/w revs)

Conclusion:

1. S P i M . ideal for managing products with short life-cycles or perishable goods .
2. Inventory Review Systems , either CRS or PRS based on different monitoring and ordering approaches , suited for various operational contexts .

