

# Machine Learning From Data HW1

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## Exercise 1.3

**a**

We know that  $y(t), \mathbf{x}(t)$  is misclassified. This tells us, by definition, that  $y(t) \neq \text{sign}(\mathbf{w}^T \mathbf{x}(t))$ . So if the signs are not matching, the result will be a negative number. This confirms that  $y(t)\mathbf{w}^T(t)\mathbf{x}(t) < 0$ .

**b**

We need to show that  $y(t)\mathbf{w}^T(t+1)\mathbf{x}(t) > y(t)\mathbf{w}^T(t)\mathbf{x}(t)$ . As shown in the last problem,  $y(t)\mathbf{w}^T(t)\mathbf{x}(t) < 0$ . The learning algorithm, which by definition is always correct, gives us  $y(t)\mathbf{w}^T(t+1)\mathbf{x}(t)$ . Since  $y(t)\mathbf{w}^T(t+1)\mathbf{x}(t)$  is correct,  $y(t)$  and  $\mathbf{w}^T(t+1)\mathbf{x}(t)$  are the same sign. This means  $y(t)\mathbf{w}^T(t+1)\mathbf{x}(t)$  is positive, which concludes that  $y(t)\mathbf{w}^T(t+1)\mathbf{x}(t) > y(t)\mathbf{w}^T(t)\mathbf{x}(t)$ .

**c**

We can argue that the move from  $\mathbf{w}(t)$  to  $\mathbf{w}(t+1)$  is a move in the 'right direction' by keeping in mind what  $y(t)$  is.  $y(t)$  is the correct output of the data inputs  $\mathbf{x}(t)$ . Since what we're doing is adding a vector to  $\mathbf{w}(t)$ , we just need to know if that vector is the 'right direction'. If the correct result, given by  $y(t)$ , is +1, the new vector  $\mathbf{w}(t+1)$  will be made by adding a positive vector. If the correct result, given by  $y(t)$ , is -1, the new vector  $\mathbf{w}(t+1)$  will be made by adding a negative vector.

## Exercise 1.5

**a**

Learning approach. This is because you don't know the age that someone needs the test. Even though you know what the test searches for, the test wasn't designed for a certain age group.

**b**

Design approach. This is because it is known exactly what a prime and non-prime are. These specifications have already been given to you.

**c**

Learning approach. This is because there is no clear specification for what charges are going to be frauds.

**d**

Design approach. The specifications for gravity, air resistance, etc, are all already defined.

**e**

Leaning approach. There are no clear specifications that define when to turn a light green or red. This must be defined through learning.

## Exercise 1.6

**a**

Supervised (Reinforcement Possible). This is similar the one of the first examples from class regarding movies on Netflix. The input data,  $x$ , is all the information about the book. The output data,  $y$ , is whether or not the movie is recommended. This could also be reinforcement learning if the output is now this: one part for whether or not the book is recommended, and another part if the user actually clicked on the recommended book

**b**

Reinforcement. This is similar to the backgammon example. The learning in this case would be picking a reasonable move, and then reporting how it went. Our data would be the previous moves and state of the game, our output would be a move, and our result would be if we won the game in the end.

**c**

Unsupervised. There are no defined groups or categories for the movies. Our task is to just separate them. This is very similar to the coin cluster example. Our data is a bunch of details about the movie, with no outputs.

**d**

Unsupervised. In this example, we don't have a defined goal or specifications to reach. The machine can listen to music, but there is no explicit meaning or 'correct answer' behind it. Our data is a mass of songs and notes.

**e**

Supervised. This is very similar to the problem used in class. Our data set is all the information about the bank customer,  $x$ , and whether or not the bank made money on the bank customer,  $y$ .

## Exercise 1.7

**a**

The  $g$  picked is the function that always returns a black dot.

$$f_1 = 0$$

$$f_2 = 1$$

$$f_3 = 1$$

$$f_4 = 2$$

$$f_5 = 1$$

$$f_6 = 2$$

$$f_7 = 2$$

$$f_8 = 3$$

**b**

The  $g$  picked is the function that always returns a white dot.

$$f_1 = 3$$

$$f_2 = 2$$

$$f_3 = 2$$

$$f_4 = 1$$

$$f_5 = 2$$

$$f_6 = 1$$

$$f_7 = 1$$

$$f_8 = 0$$

**c**

$$f_1 = 2$$

$$f_2 = 3$$

$$f_3 = 1$$

$$f_4 = 1$$

$$f_5 = 1$$

$$f_6 = 2$$

$$f_7 = 0$$

$$f_8 = 1$$

**d**

$$f_1 = 1$$

$$f_2 = 0$$

$$f_3 = 2$$

$$f_4 = 2$$

$$f_5 = 2$$

$$f_6 = 1$$

$$f_7 = 3$$

$$f_8 = 2$$

## Problem 1.1

$$P[\text{pick BB bag}] = 0.5$$

$$P[\text{pick BW bag}] = 0.5$$

$$P[\text{pick black first}|\text{pick BB bag}] = 1.0$$

$$P[\text{pick black first}|\text{pick BW bag}] = 0.5$$

$$\begin{aligned} &P[\text{pick black first \& pick BB bag}] \\ &= P[\text{pick black first}|\text{pick BB bag}] * P[\text{pick BB bag}] \\ &= 1.0 * 0.5 \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} &P[\text{pick black first \& pick BW bag}] \\ &= P[\text{pick black first}|\text{pick BW bag}] * P[\text{pick BW bag}] \\ &= 0.5 * 0.5 \\ &= 0.25 \end{aligned}$$

$$P[\text{pick black first}] = 0.75$$

$$\begin{aligned} &P[\text{pick BB bag}|\text{pick black first}] \\ &= \frac{P[\text{pick black first \& pick BB bag}]}{P[\text{pick black first}]} \\ &= \frac{0.5}{0.75} \\ &= 0.666 \end{aligned}$$

## Problem 1.2

**a**

To generate an equation in the form  $x_2 = ax_1 + b$ :

$$\mathbf{w}^T \mathbf{x} = \sum_{i=0}^2 w_i x_i = w_0 x_0 + w_1 x_1 + w_2 x_2 = 0$$

Because  $x_0$  is 1, this becomes:

$$w_1 x_1 + w_2 x_2 = -w_0$$

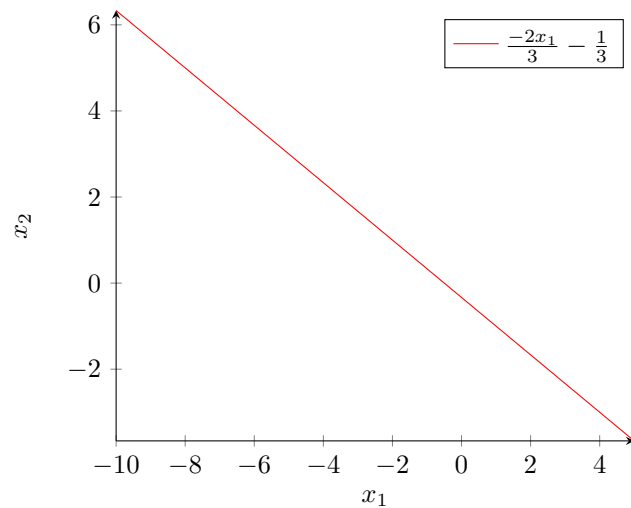
$$w_2 x_2 = -w_0 - w_1 x_1$$

$$x_2 = \frac{-w_1}{w_2} x_1 - \frac{w_0}{w_2}$$

**b**

There are two cases, one where  $\mathbf{w}^T$  is  $[1 \ 2 \ 3]$  and one where  $\mathbf{w}^T$  is  $[-1 \ -2 \ -3]$ .

They both result in the same line, shown below.

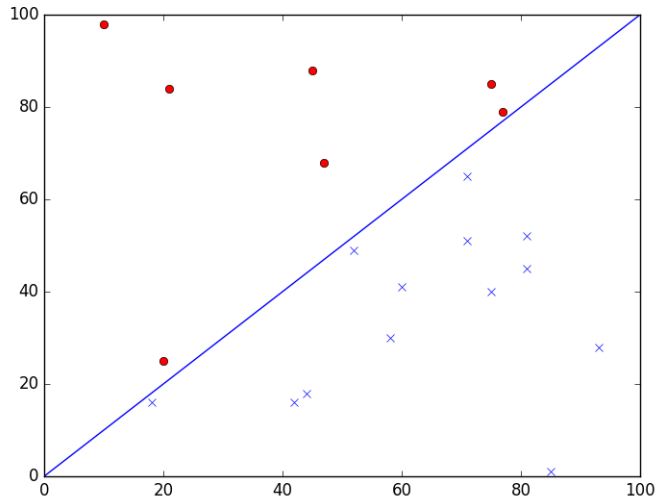


In the first  $\mathbf{w}^T$ , the correct region is above the line. In the second  $\mathbf{w}^T$ , the correct region is below the line.

## Problem 1.4

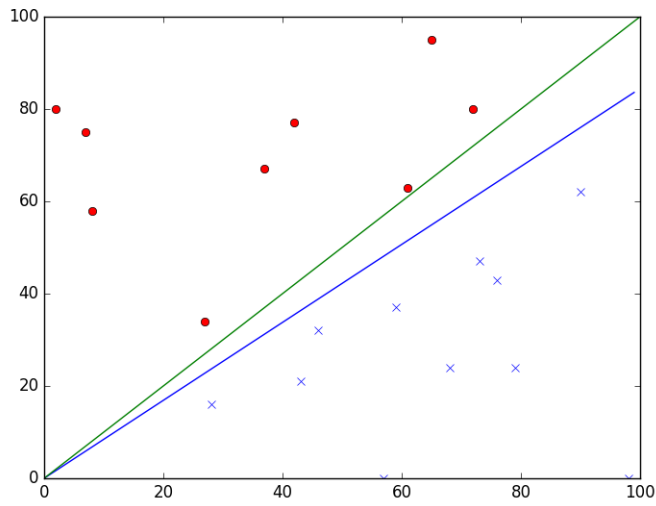
**a**

The  $f$  is the blue line



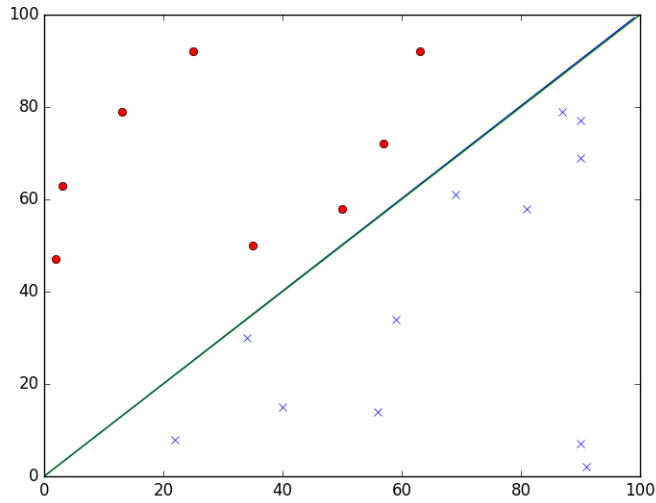
**b**

The  $f$  is the green line. The  $g$  that the algorithm found is the blue line. The algorithm found the  $g$  in 4 iterations. Although they are both correctly labeling the data, the  $g$  is clearly off from  $f$



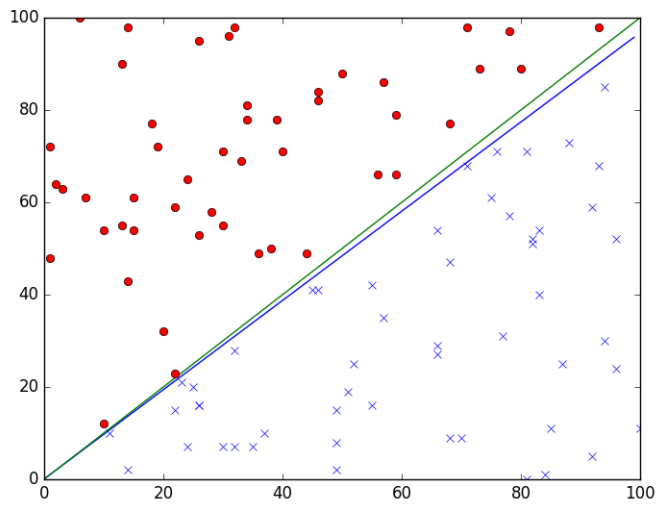
**c**

The  $f$  is the green line. The  $g$  that the algorithm found is the blue line. The algorithm found the  $g$  in 46 iterations. This time, the algorithm took much longer, but got a much closer result.



**d**

The  $f$  is the green line. The  $g$  that the algorithm found is the blue line. The algorithm found the  $g$  in 21 iterations. Surprisingly, this took fewer iterations than part c.



**e**

The  $f$  is the green line. The  $g$  that the algorithm found is the blue line. The algorithm found the  $g$  in 38 iterations. The amount of iterations doesn't seem

to increase with the amount of data points.

