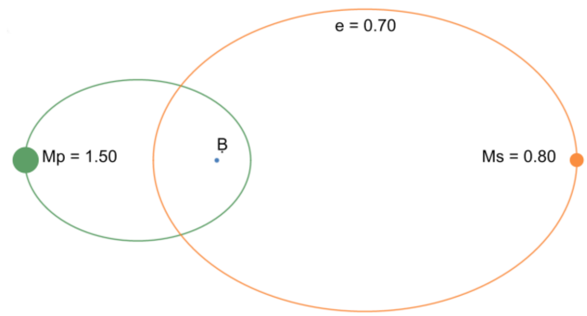
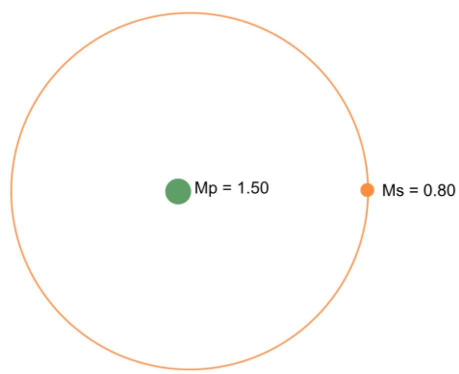
Planemo Two-Body Relationships

Barycenter Locus

When any two bodies are in a binary orbital relationship, they both technically orbit around the barycenter of their combined masses:



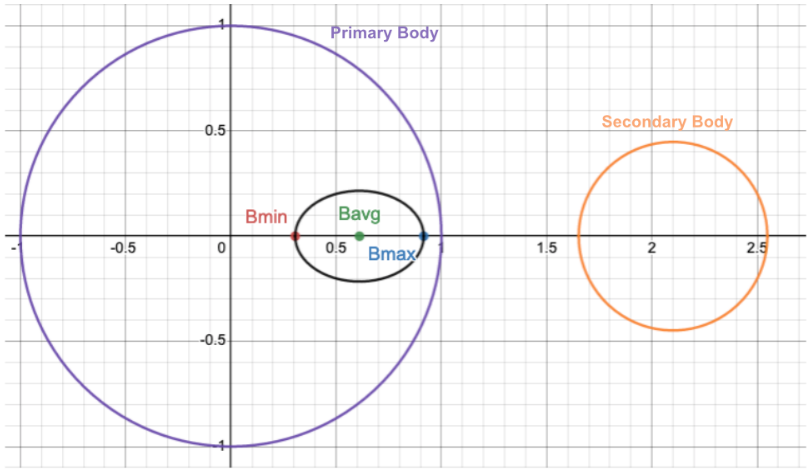
However, the Primary *can* be considered stationary, and the Secondary can be considered to be orbiting the Primary (this is the *relative orbit* of the Secondary, discussed above):



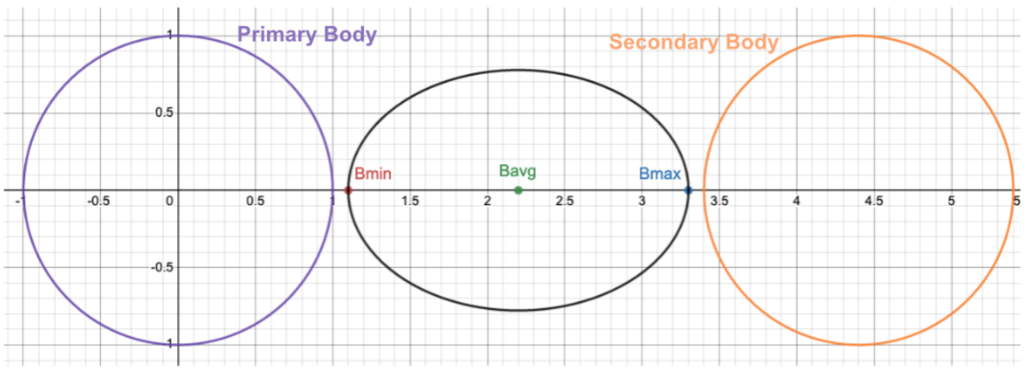
In this case, the *barycenter* (Ḅ) can be thought of as *orbiting relative to the center-of-mass of the Primary*, on an elliptical path with the same eccentricity as the system orbital eccentricity.

For binary planemo-mass pairs, especially those in which the Primary is significantly more massive and larger in radius than the Secondary, the barycenter often falls internal to the physical volume of the Primary and never migrates beyond it. One criterion that often is cited is that for a pair to be considered a double-*planet*, their barycenter *must consistently be located external to the physical volume of the Primary body*, otherwise the system is a planet-and-moon pairing. I find this criterion to be too restrictive; rather, this is one of *three* possible configurations.

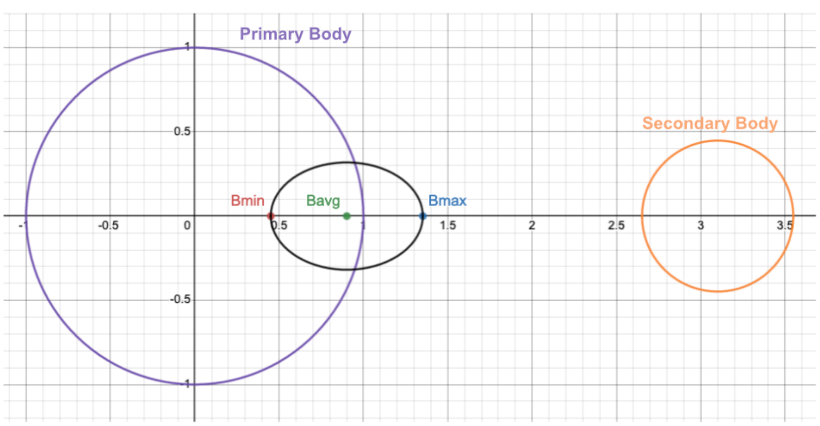
1. Either the orbit of the barycenter is entirely internal to the physical volume of the Primary:



2. Or the orbit of the barycenter is entirely external to the physical volume of the Primary:



3. Or the orbit of the barycenter migrates between being interior-to and exterior-to the physical volume of the Primary (whether B_{avg} is interior-to *or* exterior-to the physical volume of the Primary):



The equations below are used to determine which situation prevails, based upon the masses of the two bodies, their average separation, and the system eccentricity.

Masses, Eccentricity, Barycenter

$$M_S = M + m$$

$$B_{min} = r\frac{m(1-e)}{M_S} = 2B_{avg} - B_{max} = 2\frac{B_{max}}{1+e} - B_{max} = B_{avg}(1-e)$$

$$B_{avg} = r\frac{m}{M_S} = \frac{B_{min}}{1-e} = \frac{B_{max}}{1+e}$$

$$B_{max} = r\frac{m(1+e)}{M_S} = 2B_{avg} - B_{min} = 2\frac{B_{min}}{1-e} - B_{min} = B_{avg}(1+e)$$

$$e \gg \frac{1}{B_{avg}} - 1; \qquad e = 1 - \frac{B_{min}}{B_{avg}} = \frac{B_{max}}{B_{avg}} - 1$$

$$r = B_{max}\frac{M_S}{m(1+e)} = B_{min}\frac{M_S}{m(1-e)} = B_{avg}\frac{M_S}{m}$$

$$M = r\frac{m(1+e)}{B_{max}} - m = r\frac{m(1-e)}{B_{min}} - m = r\frac{m}{B_{avg}} - m$$

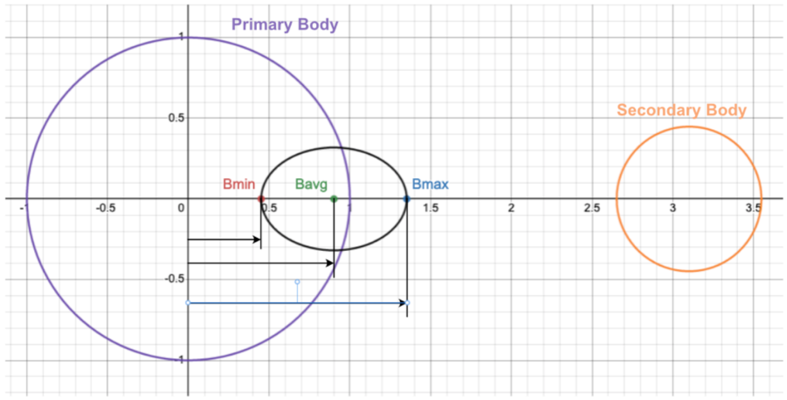
$$m = \frac{MB_{max}}{r(1+e) - B_{max}} = \frac{MB_{min}}{r(1-e) - B_{min}} = \frac{MB_{avg}}{r - B_{avg}}$$

r = the average separation between the two bodies*
B_{min} = min. distance of barycenter from the C.O.M. of the *Primary*
B_{avg} = avg. distance of barycenter from the C.O.M. of the *Primary*
B_{max} = max. distance of barycenter from the C.O.M. the *Primary*
e = the system eccentricity
M = mass of the Primary body
m = mass of the Secondary body (in the same units as M)
M_S = M + m

* Expressed in terms of the radius of the Primary; this means that the values of B_{min}, B_{avg}, and B_{max} are also expressed in terms of the radius of the Primary body.

While these are similar to the equations for binary stars given above, here they are formulated such that the *barycenter* is treated as the variable value, measured in distance from the gravitational center-of-mass of the Primary body in the binary system.

Note that B_{avg} ***is*** the barycenter (Ḅ) of the system; while the eccentricity value *does* affect the locations of B_{min} and B_{max} (their distance from B_{avg} and how far apart they are from one another), it has *no impact* upon the location of B_{avg}. B_{avg} is affected *only* by the mass ratio of the two bodies and their average separation (which are interrelated quantities). The farther apart the Primary and the Secondary orbit and/or the closer the Primary and Secondary are in mass, the more likely that B_{avg} will lie external to the physical volume of the Primary.



Thus, we can say that there exist four configurations of binary planemos. In ascending order of their distance of B_{avg} from the Primary's center-of-mass:

Class	Configuration	Barycenter Orbit
Class 1	$B_{min} < B_{avg} < B_{max} \leq r$	Interior-Bound
Class 2	$B_{min} < B_{avg} \leq r < B_{max}$	Interior-Migratory
Class 3	$B_{min} \leq r < B_{avg} < B_{max}$	Exterior-Migratory
Class 4	$r \leq B_{min} < B_{avg} < B_{max}$	Exterior-Bound

Thus, the "barycenter locus" can either be "interior" (when $B_{avg} < r$) or "exterior" (when $B_{avg} \geq r$). I call the varying location of the barycenter between B_{min} and B_{max} the "barycenter motility", and it can be either of two states: *bound* or *migratory*. There is a "quick test" to determine whether the barycenter ever migrates exterior-to the physical volume of the Primary:

$$\frac{1}{1+e} < B_{avg} < \frac{1}{1-e}$$

If both halves of the relation evaluate to TRUE, then the barycenter migrates between interior-to and exterior-to the physical volume of the Primary[1]. If the left half evaluates to TRUE and the right half evaluates to FALSE, then the barycenter is consistently exterior-to the physical volume of the Primary. If the left half evaluates to FALSE and the right half evaluates to TRUE, then the barycenter is consistently interior-to the physical volume of the Primary. (Because of the nature of the relationship between the barycenter, the masses of the bodies, and the eccentricity of the orbits, it will never happen that both sides evaluate to FALSE at the same time.)
This touches on a relation shown in the set above:

$$e \gg \frac{1}{B_{avg}} - 1$$

[1] It is still necessary to check the B_{min} value to determine whether the barycenter is interior-migratory or exterior-migratory.

This relation reveals the *minimum eccentricity* necessary to cause both halves of the complete relation to evaluate to TRUE. For the Earth-and-Moon, this value is:

$$e \gg \frac{1}{B_{avg}} - 1 \gg \frac{1}{0.7331} - 1 \gg 1.364 - 1 \gg 0.364$$

Thus, any orbital eccentricity for the Earth-Moon system greater-than 0.364 will result in the barycenter migrating between interior-to and exterior-to the physical volume of the Earth. An eccentricity of e = 0.364 is 6.63 times greater than the actual eccentricity of e = 0.0549.

The Mass Ratio

It is also worth noting that *any* natural object in orbit around a more massive companion can technically be considered a "moon"[2] of the more massive body. Thus, *any* planemo in orbit around a more massive planemo can (should?) technically be classified as a "major moon" (see below for a more in-depth description of major moons).

So, an Earth-mass planemo orbiting a Jupiter-mass planemo could be classed as a major moon of the more massive body, even though it would be a full planet were it to be orbiting its star independently.[3] This leaves us with the question of how to determine whether-or-not a binary planemo pair should be considered a double-planet or a planet-moon pairing.

There is (big surprise), no official position on this question (other than the stricture on the location of the barycenter, which I have already rejected as too restrictive). I propose that a mass-ratio ($^M/_m$) limitation is the most direct and convenient test, and I further propose that a mass-ratio of $^M/_m$ = 100 : 1 is appropriate. With a mass ratio of 100 : 1, regardless of the system eccentricity, the Secondary must orbit at a distance of 101 times the radius of the Primary for B_{avg} to lie *exactly* on the radius of the Primary. With an extreme system eccentricity of e = 0.7, a Secondary of 0.010 Primary masses would have to orbit at almost 60 times the radius of the Primary (see below) for B_{max} to calculate to exactly the radius of the Primary.

Thus, if the Primary is a full planet and its mass is M ≥ 100 times more than the mass of the Secondary, the two are *automatically considered a planet-moon pair*, and the location and motility of the barycenter are irrelevant.

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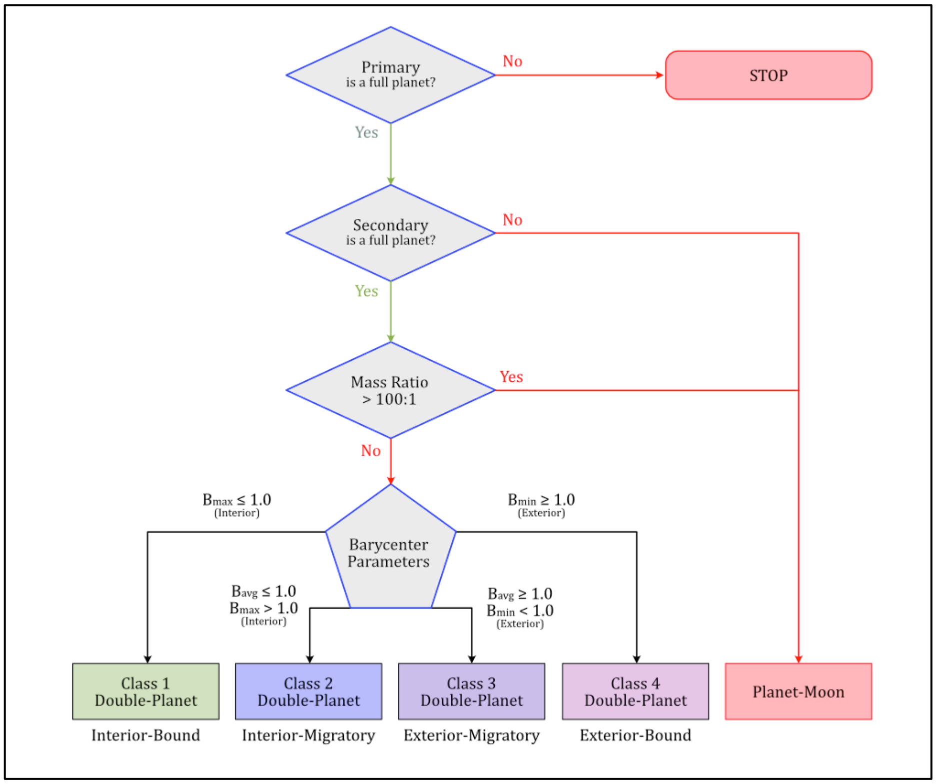
[2] In the sense of "natural satellite".

[3] Two excellent examples are the Forest Moon of Endor and Yavin 4 in the *Star Wars* universe. In our own universe, Ganymede, Titan, Callisto, Io, Luna, Europa, Io, and Triton are all examples, as well.



Note that Earth's Moon, at M = 0.0123 t, is slightly more than $^1/_{100}$th the mass of the Earth, and currently orbits at just over sixty Earth-radii (r = 60.336), but the Earth-Moon system eccentricity is quite small at e = 0.0549. Thus, for the Earth-Moon, B_{min}, B_{avg}, and B_{max} fall at 0.693, 0.733, and 0.773 Earth-radii, respectively. Thus, by our definition, the Earth-and-Moon qualify as a Class 1 double-planet, with an interior-bound barycenter motility.

Note also, that the *largest* mass the Primary can have if the Secondary is the smallest full planet mass (m = 0.005 t) is M = 0.050, just less than the mass of Mercury (0.055 t), otherwise the two become a planet-moon pairing.