In [ ]:
```
# Initialize Otter
import otter
grader = otter.Notebook("q1.ipynb")
```

1 Qiskit Assignment 1

1.1 Practice with Pauli Gates and other Qiskit tools

Welcome to your second Qiskit assignment!

1.1.1 Learning Objectives

1. Build Pauli gates from Qiskit's U gate
2. Use the U gate to reverse a series of operations
3. Visualize rotations using the Bloch Sphere

NAME: REPLACE WITH YOUR NAME

STUDENT ID: REPLACE WITH YOUR STUDENT ID

In [1]:
```
# Import Qiskit and other needed packages
from qiskit import *
from qiskit.visualization import plot_histogram
from qiskit.quantum_info import Statevector
from qiskit.circuit.random import random_circuit
import numpy as np
import pprint
import random
```

Task 1 - Constructing Pauli Z gate from u Gate We can use Qiskit's U Gate to construct arbitrary quantum operations. Fill in the function below to return `qc_pauli_z`, a QuantumCircuit satisfying the following conditions: - it has 1 qubit, initialized to the parameter `initial_state` - it has 1 classical bit - it has a U gate with parameters which perform the same rotation as a Pauli Z gate - it does not use the built in Z gate - it performs a measurement following the rotation

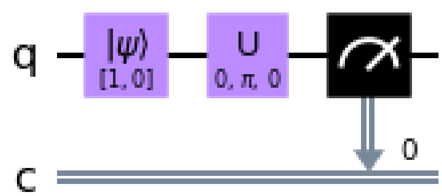
In [2]:
```
def qc_pauli_z(initial_state=[1,0]):
    # BEGIN SOLUTION
    theta = 0
    phi = np.pi
```

```
        lamda = 0

        qc = QuantumCircuit(1,1)
        qc.initialize(initial_state, 0)
        qc.u(theta, phi, lamda, 0)
        qc.measure(0,0)
        return qc
        # END SOLUTION
```

```
In [3]: qc_pauli_z().draw(output='mpl')
```

Out[3]:




```
In [ ]: grader.check("Task 1")
```

Task 2 (2A, 2B, 2C) - Unitary Inverse Puzzles We'll study the idea of uncomputation during our discussion of quantum algorithms. In general, we may find it helpful to return a qubit to its initial state.

This process is typically straightforward due to the properties of unitary gates. However, your task is to do so using only a single U gate. Complete the partial circuits below such that the measurements will yield a state equivalent to `initial_state` *up to a global phase.*

Task 2A

```
In [7]: def reverse_a(initial_state=[1,0]):
            qc = QuantumCircuit(1,1)
            qc.initialize(initial_state)
```

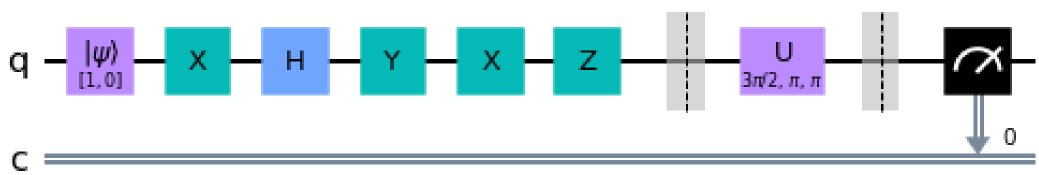
```
    qc.x(0)
    qc.h(0)
    qc.y(0)
    qc.x(0)
    qc.z(0)
    qc.barrier()

   # BEGIN SOLUTION
    # Can use commutation relation XY = iZ to reduce
    # Find parameters for U(?,?,?) = XH
    theta = 3*np.pi/2
    phi = np.pi
    lamda = np.pi
    qc.u(theta, phi, lamda, 0)
   # END SOLUTION

    qc.barrier()
    qc.measure(0,0)
    return qc
```

In [8]:
```
reverse_a().draw(output='mpl')
```

Out[8]:



In [ ]:
```
grader.check("Task 2A")
```

Task 2B

In [12]:
```
def reverse_b(initial_state=[1,0]):
    qc = QuantumCircuit(1,1)
    qc.initialize(initial_state)
```

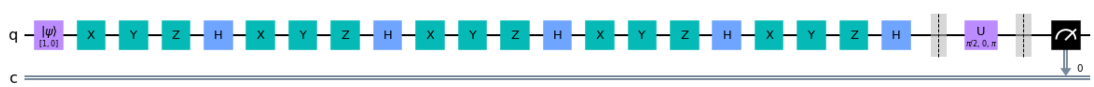
```
    for i in range(5):
        qc.x(0)
        qc.y(0)
        qc.z(0)
        qc.h(0)
    qc.barrier()


    # BEGIN SOLUTION
    # Use commutation relation YX = -iZ
    # so that (HZYX)^5=(-iHZZ)^5=(-iH)^5=-iH
    # Hence to measure state within a global we use
    # parameters for the U gate such that U == H
    theta = np.pi/2
    phi = 0
    lamda = np.pi
    qc.u(theta, phi, lamda, 0)
    # END SOLUTION

    qc.barrier()
    qc.measure(0,0)
    return qc
```

```
In [13]: reverse_b().draw(output='mpl')
```

Out[13]:



```
In [ ]: grader.check("Task 2B")
```

Task 2C **Hint**: The P gate generalizes rotation about the Z-axis to an arbitrary angle ϕ, where $P(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$

```
In [17]: def reverse_c(initial_state=[1,0]):
    qc = QuantumCircuit(1,1)
    qc.initialize(initial_state)
```

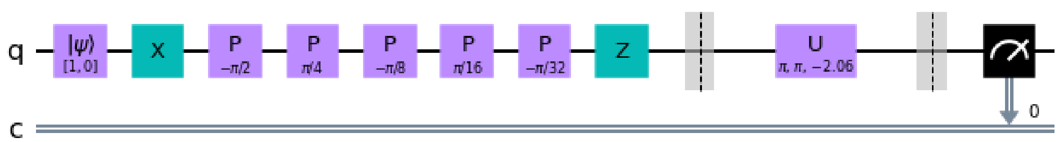
```
    qc.x(0)
    for i in range(1,6):
        qc.p((-1)**(i)*np.pi/(2**i), 0)
    qc.z(0)
    qc.barrier()

    # BEGIN SOLUTION
    # Trick: combine P gates and Z into a single P gate
    # Note that Z == P(pi)
    # Find parameters for U(?,?,?) = (P(-11pi/32)*X)
    theta = np.pi
    phi = np.pi
    lamda = -21*np.pi/32
    qc.u(theta, phi, lamda, 0)
    # END SOLUTION

    qc.barrier()
    qc.measure(0,0)
    return qc
```

In [18]:
```
reverse_c().draw(output='mpl')
```

Out[18]:



In [ ]:
```
grader.check("Task 2C")
```

Task 3 (3A, 3B, 3C) - Using Rotation to Obtain Probabilities

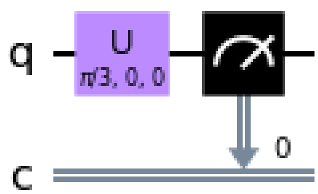
Task 3A Fill in the function below to return `qc_rot_a`, a single-qubit QuantumCircuit satisfying the following conditions: - it performs a measurement to a single classical bit - Pr(seeing $|0\rangle$ on measurement) = `0.75` - your circuit only uses gates from the following list: X, Y, Z, P, H, U

Plot your results using a histogram to verify your solution over `1024` trials.

In [22]:
```
def qc_rot_a():
    # BEGIN SOLUTION
    # Find wave amplitudes for ket zero and ket one
    # Use amplitudes to determine the proportion of rotation needed
    qc = QuantumCircuit(1,1)
    theta = np.pi/3
    phi = 0
    lamda = 0
    qc.u(theta, phi, lamda, 0)
    qc.measure(0,0)
    # END SOLUTION
    return qc
```

In [23]:
```
qc_rot_a().draw(output='mpl')
```

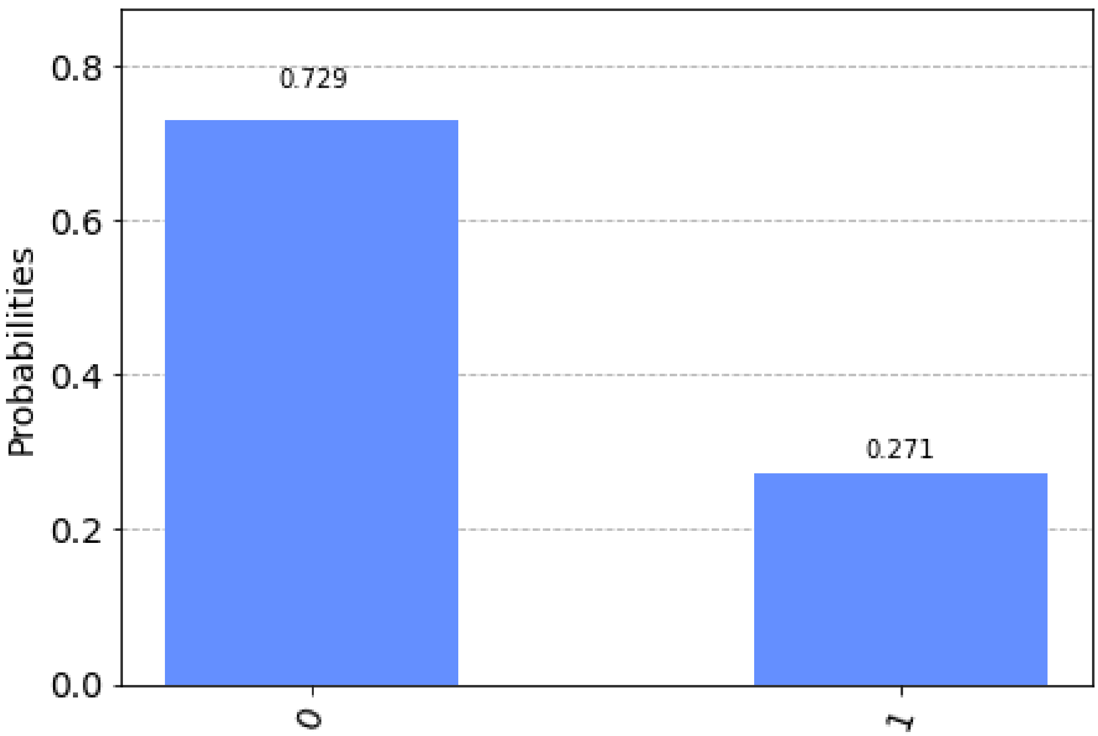
Out[23]:



In [24]:
```
# Plot your results in this cell!

# BEGIN SOLUTION
qc = qc_rot_a()
qasm_sim = BasicAer.get_backend("qasm_simulator")
job = execute(qc, qasm_sim)
counts = job.result().get_counts()
plot_histogram(counts)
# END SOLUTION
```

Out[24]:



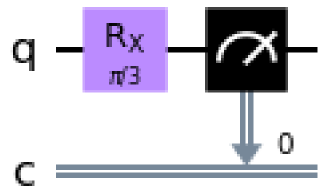
Task 3B - Rotation Operator Gates Again, fill in the function below to return `qc_rot_b`, a single-qubit QuantumCircuit satisfying the following conditions: - it performs a measurement to a single classical bit - Pr(seeing $|0\rangle$ on measurement) = `0.75` - your circuit only uses gates from the following list: RX, RY, RZ

Plot your results using a histogram to verify your solution over `1024` trials.

```
In [25]: def qc_rot_b():
             # BEGIN SOLUTION
             # Using the provided link, it can be seen that RX(pi) = -iX
             # From this info and part A, we need to use RX(pi/3) to get the desired state
             qc = QuantumCircuit(1,1)
             qc.rx(np.pi/3, 0)
             qc.measure(0,0)
             # END SOLUTION
             return qc
```

```
In [26]: qc_rot_b().draw(output='mpl')
```

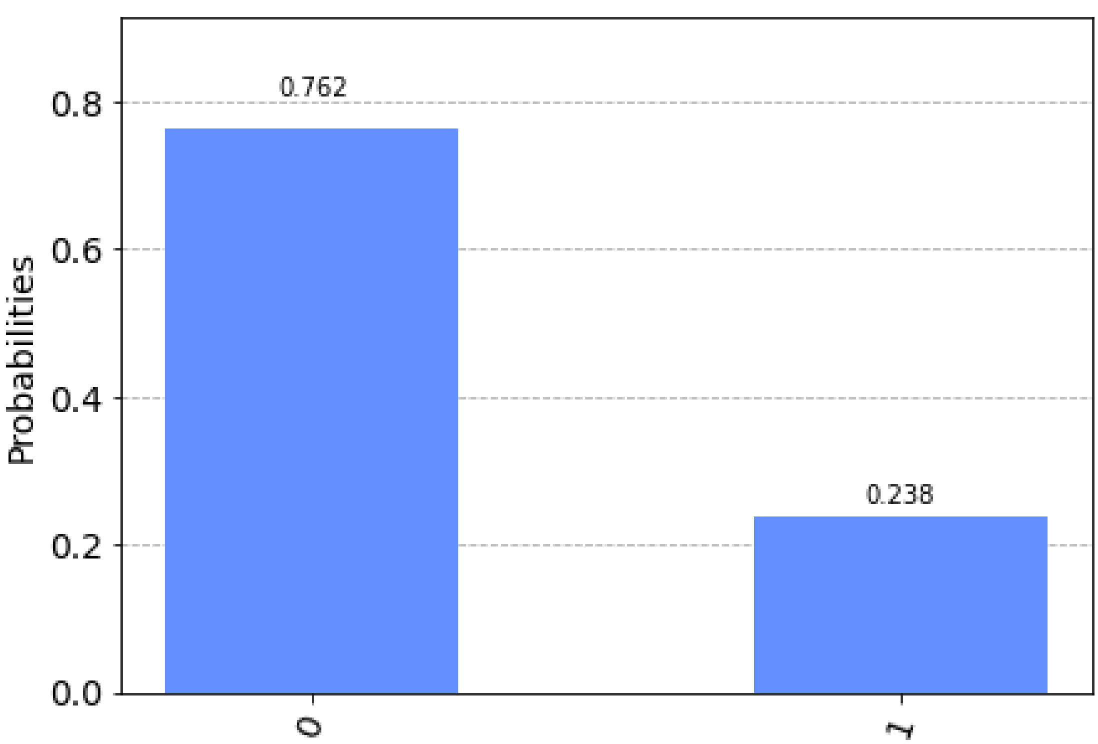
Out[26]:



In [27]:
```
# Plot your results in this cell!

# BEGIN SOLUTION
qasm_sim = BasicAer.get_backend("qasm_simulator")
job = execute(qc_rot_b(), qasm_sim)
counts = job.result().get_counts()
plot_histogram(counts)
# END SOLUTION
```

Out[27]:



Task 3C Suppose we apply a Z gate to your circuit from task 3B just before measuring. How will the probability of measuring $|0\rangle$ change from that of the original circuit? Will measurement on the modified circuit yield a state equivalent to the original circuit up to a global phase?

Type your answer here, replacing this text.

The probability of seeing $|0\rangle$ won't change since Z is just a rotation about the z-axis. However, it's not equivalent up to a global phase since you can't pull out a factor.

Task 4 - P vs. RZ In tasks 2C, the P gate was introduced. In lecture and task 3B, you've seen the RZ gate. Both are related to Z-axis rotations, but what utility is there to having each? Is there a mathematical relationship between them? If so, describe it.

Feel free to use any resources for your research, including `lecture 6: A single qubit`.

Type your answer here, replacing this text.

Source: https://quantum-computing.ibm.com/composer/docs/iqx/operations_glossary#phase-gate

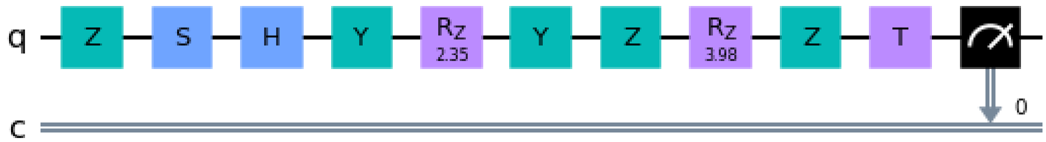
The P gate is equivalent up to a global phase with RZ. The P gate applies a phase to $|1\rangle$ of $e^{i\theta}$. Up to a global phase of $e^{i\theta/2}$, it is equivalent to $RZ(\theta)$

Task 5 (5A, 5B) - Transpiling Circuits When you submit a job to IBM, the quantum computer will most likely run a different circuit than you built. This is because the quantum computer can only do a very limited set of operations relative to the number of unitary gates. For IBM devices, the transpile step reduces all single-qubit operations to I, X, SX, and RZ (source).

Task 5A Choose one of the IBM backends (see the first assignment for a refresher on this). Use the transpile method to optimize the given circuit for the backend. Draw the transpiled circuit.

In [28]:
```
qc_rand = random_circuit(1, 10, measure=True, seed=6)
qc_rand.draw(output='mpl')
```

Out[28]:

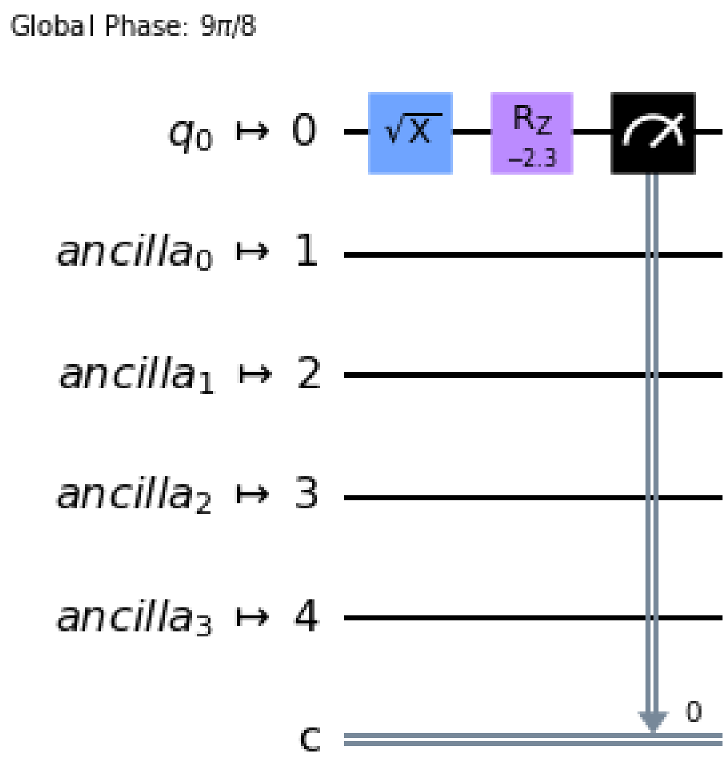



```
In [29]: IBMQ.load_account()
         # BEGIN SOLUTION
         provider = IBMQ.get_provider(hub='ibm-q')
         for backend in provider.backends():
             status = backend.status().to_dict()
             if status['operational'] and status['status_msg']=='active':
                 if 'simulator' not in status['backend_name']:
                     print(status['backend_name'])

         transpile(qc_rand, provider.get_backend('ibmq_lima')).draw(output='mpl')
         # END SOLUTION
```

```
ibmq_armonk
ibmq_bogota
ibmq_lima
ibmq_belem
ibmq_quito
ibmq_manila
```

Out[29]:



Task 5B Which gates from {I, X, SX, RZ}, and how many of each, are used in the transpiled circuit?

Type your answer here, replacing this text.

SX and RZ are used. There is one SX gate used and one RZ gate used.

1.2 Conclusion

Next week: the EV bomb!

To double-check your work, the cell below will rerun all of the autograder tests.

In [ ]:
```
grader.check_all()
```

1.3 Submission

Make sure you have run all cells in your notebook in order before running the cell below, so that all images/graphs appear in the output. The cell below will generate a zip file for you to submit. **Please save before exporting!**

In [ ]:
```
# Save your notebook first, then run this cell to export your submission.
grader.export(pdf=False)
```