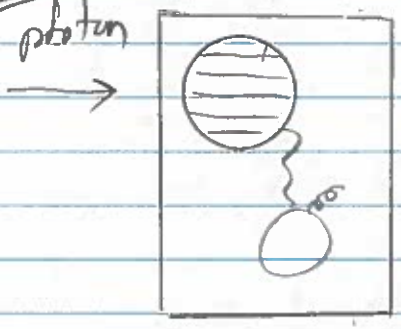
CMU version You get a box, empty →

OR Bomb!

photon



if horizontal |0> photon goes through
else bomb explodes if |1>

Box effectively measures in {|0>, |1>}

Which box do you have? Empty or not
Classically — nothing you can do cleverly
send |0> no info, empty or no explode
|1> bomb will explode if it's there

Idea

Send in $1/\sqrt{2}(|0\rangle + |1\rangle) = |+\rangle$

Measure photon in +/- basis (See 87.1)

1) Empty? |+> in, |+> out, see |+> 100% prob
2) Bomb

$|+\rangle \longrightarrow \equiv$ → 50% |0> ≡
↘ 50% |1> ⇒ Boom

if |0> makes it thru, measure
|+> 50%
|-> 50% — bomb must be good

25% good bomb
25% inconclusive
50% explode

} Better than classic!

Input State	Box	Output State	Measure
$\|+\rangle$		$\|+\rangle$	$\|+\rangle$ always
$\|+\rangle$		$\|1\rangle$	Explode!
		$\|0\rangle$	$\|+\rangle$ 1/2 time $\|-\rangle$ 1/2 time

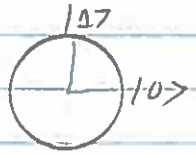
No Bomb

Always see $|+\rangle$
No conclusion
Presence of bomb related to prob of its existence

Bomb

1/2 time	Explode	
1/2		See $\|+\rangle$ Inconclusive
1/2		See $\|-\rangle$ Bomb there No explosion

Better version



Start with $|0\rangle$ and rotate slightly

Rotate by ϵ $\begin{bmatrix} \cos\epsilon & -\sin\epsilon \\ \sin\epsilon & \cos\epsilon \end{bmatrix}$

↑ where $|0\rangle$ goes ↑ where $|1\rangle$ goes

R_ϵ rotate by ϵ $|0\rangle$ counterclockwise

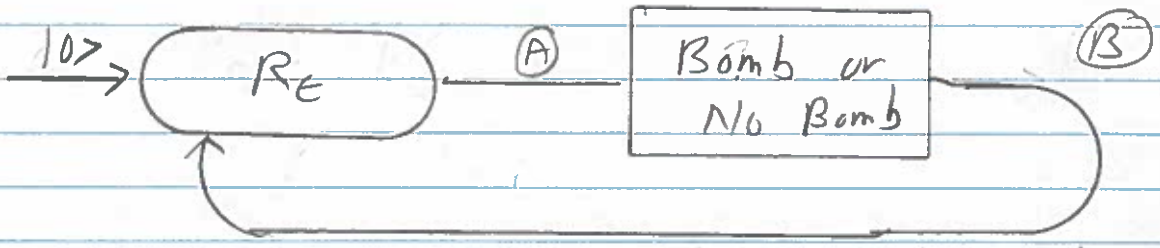
Send $|L\rangle$ to box

No Bomb – state still angle ϵ

Bomb? state either $|0\rangle$ or $|1\rangle$

Prob $|0\rangle$ $\cos^2\epsilon$ close to 1

Prob $|1\rangle$ $\sin^2\epsilon \sim \epsilon^2$ small



Pick large n, say 100
→ $\epsilon = 90°/n$ or $\pi/2n$

Run the loop above n times

At (B) we see either

(A) if Box empty
|0> if Bomb there, no explode

Choose n as large as you like
so ϵ is as small as you like

After n iterations, at (B)
No bomb, particle is at 90°
→ measures |1>

Bomb, particle at |0> measures |0>

Claim the bomb is arbitrarily unlikely to explode

Why?

Bomb present

$P[\text{Bell} \mid 0\rangle] = \cos^2\epsilon$ close to 1

$P[\text{explode}] = \sin^2\epsilon$

In radians $\sin\epsilon \leq \epsilon$ small ϵ

$$\sin^2\epsilon \leq \epsilon^2$$

ϵ is $1/1000$, ϵ^2 is $1/1,000,000$

Very low chance of explosion

$$Pr[\text{explode}] = \underbrace{\epsilon^2 + \epsilon^2 + \ldots + \epsilon^2}_{n}$$

$$= n\epsilon^2 \qquad \epsilon = \pi/2n$$

$$= \frac{n\pi^2}{4n^2} = \frac{\pi^2}{4n} \leq \frac{2.5}{n}$$

Can be made arbitrarily small