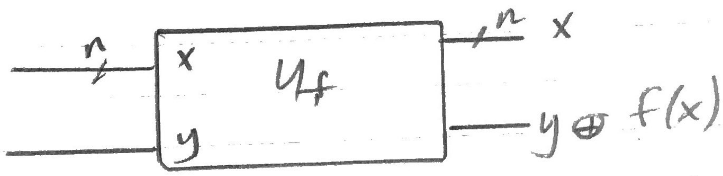


We cannot realize U_f as unitary matrix

$$\begin{array}{cc} & 2^n \\ 2^n & \left(\qquad \qquad \right) \end{array}$$

Takes $\Omega(2^n)$ space
blows our time bound   IBM-Q allows at most 4x4

We need to realize U_f using circuit elements

But how?

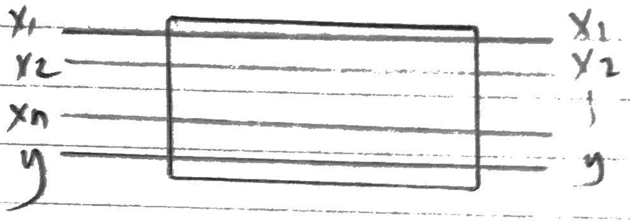
1) $\forall x \; f(x) = 0$

x_1	--	x_n	y
0	—	0	0
0	—	0	1
0	—	1	0
0	—	1	1
	⋮		

x_1	--	x_n	$y \oplus f(x)$ $y \oplus 0 = y$
0		0	0
0		0	1
0		1	0
0	—	1	1
		⋮	

U_f is Identity –
no circuitry needed

☆ If U_f behaves properly for all basis states then it works for superpositions as well

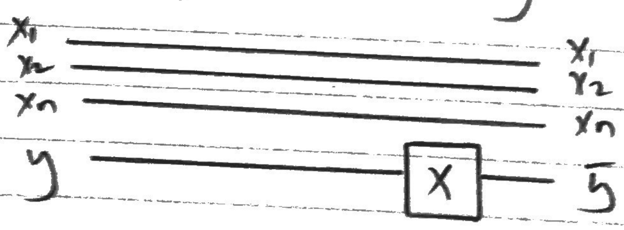


2) $\forall x \quad f(x) = \underline{1}$

x_1	—	x_n	y
0	—	0	0
0	—	0	1
⋮			

x_1	--	x_n	$\bar{y}$
0	—	0	1
0	—	0	0
⋮			

We simply invert y



3) $f(x)$ balanced

$\frac{1}{2}$ inputs $\to 0$
$\frac{1}{2}$ $\to 1$

Many possible f's, here's one
$f(x) = x_n$

x_1 –	x_n	y
0	0	0
0	0	1
0	1	0
0	1	1

x_1 – –	x_n	$y \oplus f(x)$
0	0	0
0	0	1
0	0	1
0	0	0

How do we create this effect in a circuit

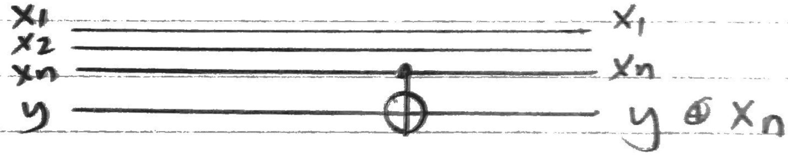
x_n	y
0	0
0	1
1	0
1	1

x_n	$y \oplus x_n$
0	0
0	1
1	1
1	0

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

CNOT!

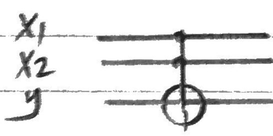
So we can do



We expected entanglement, without which there is no quantum advantage

How about a faulty U_f — neither constant nor balanced

Try CCNOT



x_1	x_2	y
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

x_1	x_2	CNOT(x_1, x_2, y)
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	1
1	1	0

So look at that circuit as U_f

x_1	x_2	y
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

x_1	x_2	$y \oplus f(x)$	$f(x)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	1	1
1	1	0	1

AHA $f(x)$ is neither balanced nor constant

1) What amplitude is present on $|00\rangle$ as $x_1 x_2$ on output now? So what's the probability of measuring $|00\rangle$?
2) What does emulation yield?
3) Run on IBMQ?