HW4 - Lucas Fellmeth, Sven Bergmann

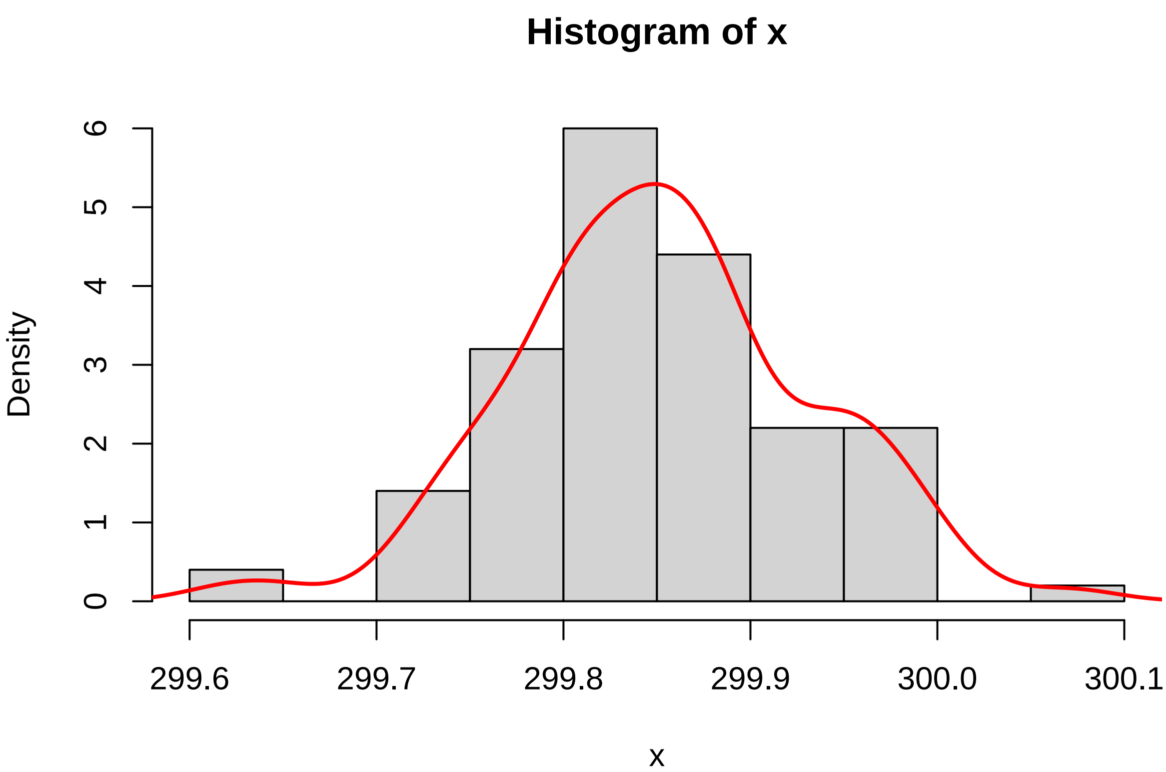
2023-11-01

Chapter 11

Exercise 5

Recall Exercise 6.3 based on 100 measurements of the speed of light in air. In that chapter, we tested the data for normality. Use the same data to construct a density estimator that you feel gives the best visual display of the information provided by the data. What parameters did you choose? The data can be downloaded from http://www.itl.nist.gov/div898/strd/univ/data/Michelso.dat

```
x <- as.numeric(read.delim2("Michelso.dat.txt")[[1]])
hist(x, freq = F, nclass = 10)
kde <- density(x, bw = "SJ")
lines(kde$x, kde$y, col = "red", lwd = 2)
```



We chose 'nclass=10' and 'bw="SJ"' for the kernel density estimator. 'SJ' chooses the bandwidth automatically, we felt like this gives a good visual display of the data.

Chapter 12

Exercise 1

Using robust regression, find the intercept and slope $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for each of the four data sets of Anscombe (1973) from p. 244. Plot the ordinary least-squares regression along with the rank regression estimator of slope. Contrast these with one of the other robust regression techniques. For which set does $\tilde{\beta}_1$ differ the most from its LS counterpart $\hat{\beta}_1 = 0.5$? Note that in the fourth set, 10 out of 11 Xs are equal, so one should use

$$S_{ij} = (Y_j - Y_i)/(X_j - X_i + \epsilon)$$

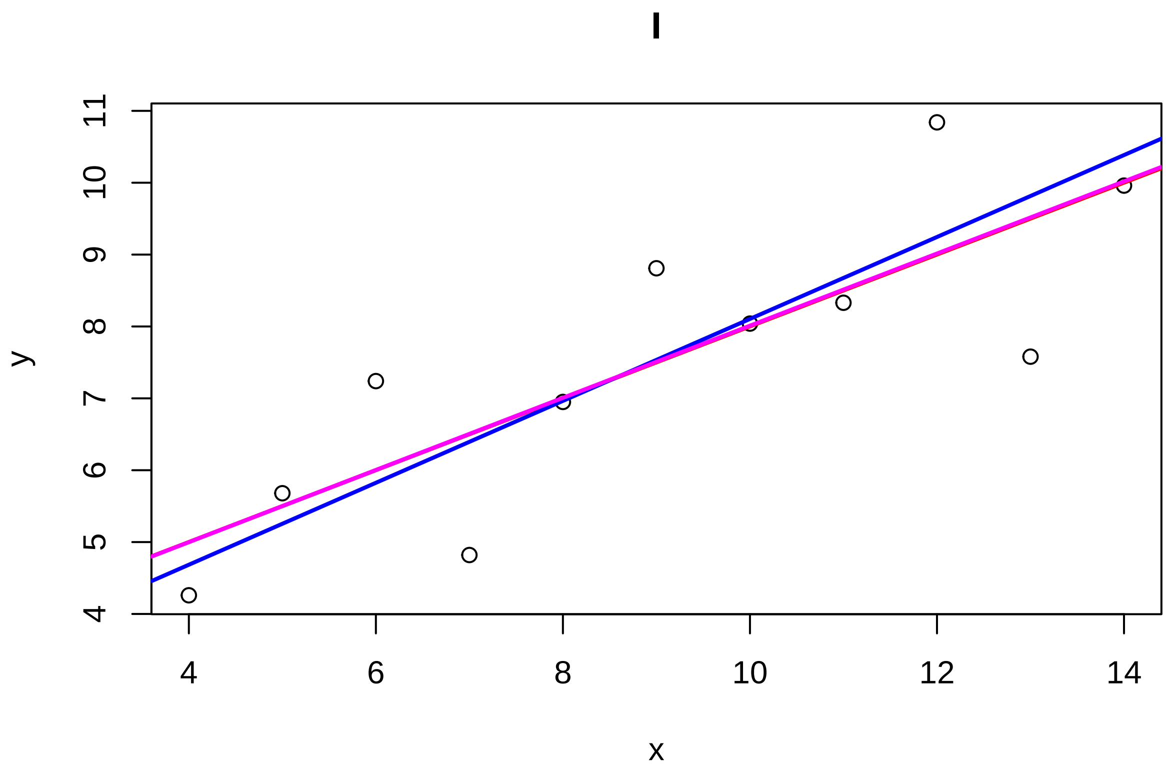
to avoid dividing by 0. After finding $\tilde{\beta}_0$ and $\tilde{\beta}_1$, are they different than $\hat{\beta}_0$ and $\hat{\beta}_1$? Is the hypothesis $H_0 : \beta_1 = 1/2$ rejected in a robust test against the alternative $H_1 : \beta_1 < 1/2$, for data set 3? Note here $\beta_{10} = 1/2$.

```
library(robustbase)
library(MASS)
library(L1pack)
```

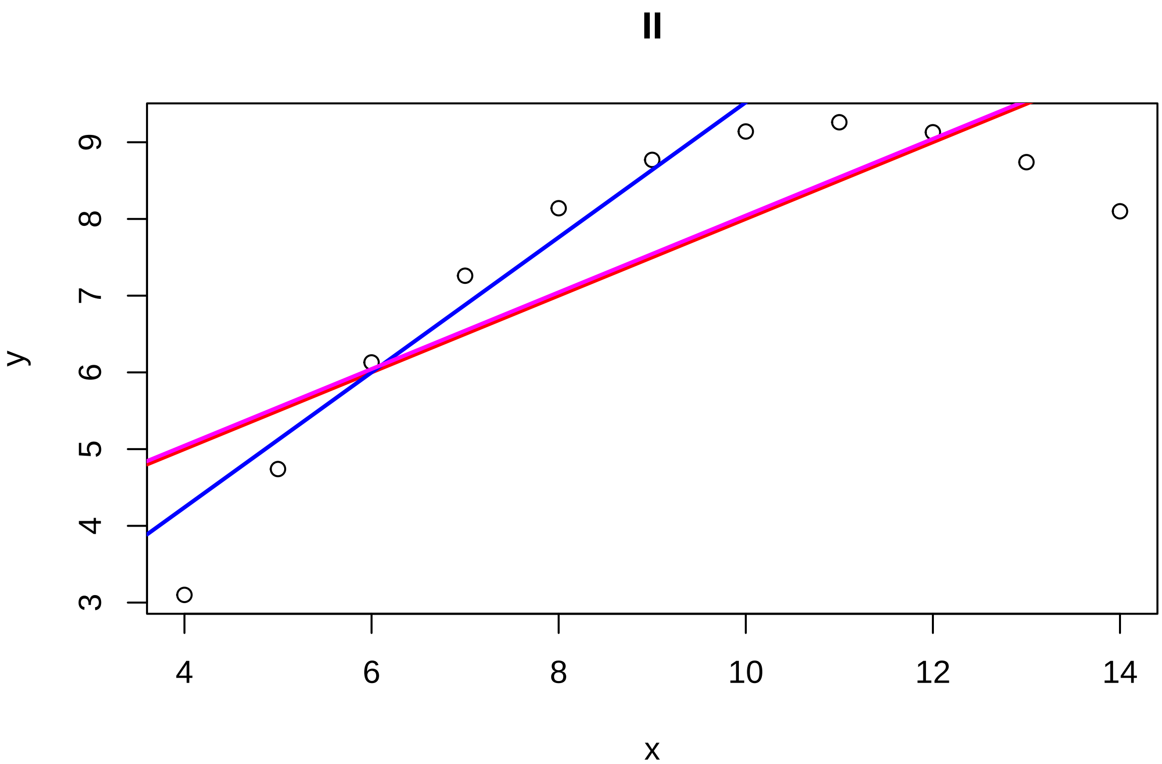
```
## Loading required package: fastmatrix
```

```
data <- read.csv("anscombe.csv")
data$dataset <- factor(data$dataset)
data1 <- subset(data, subset = data$dataset == "I")
data2 <- subset(data, subset = data$dataset == "II")
data3 <- subset(data, subset = data$dataset == "III")
data4 <- subset(data, subset = data$dataset == "IV")
datasets <- list(data1, data2, data3, data4)
```

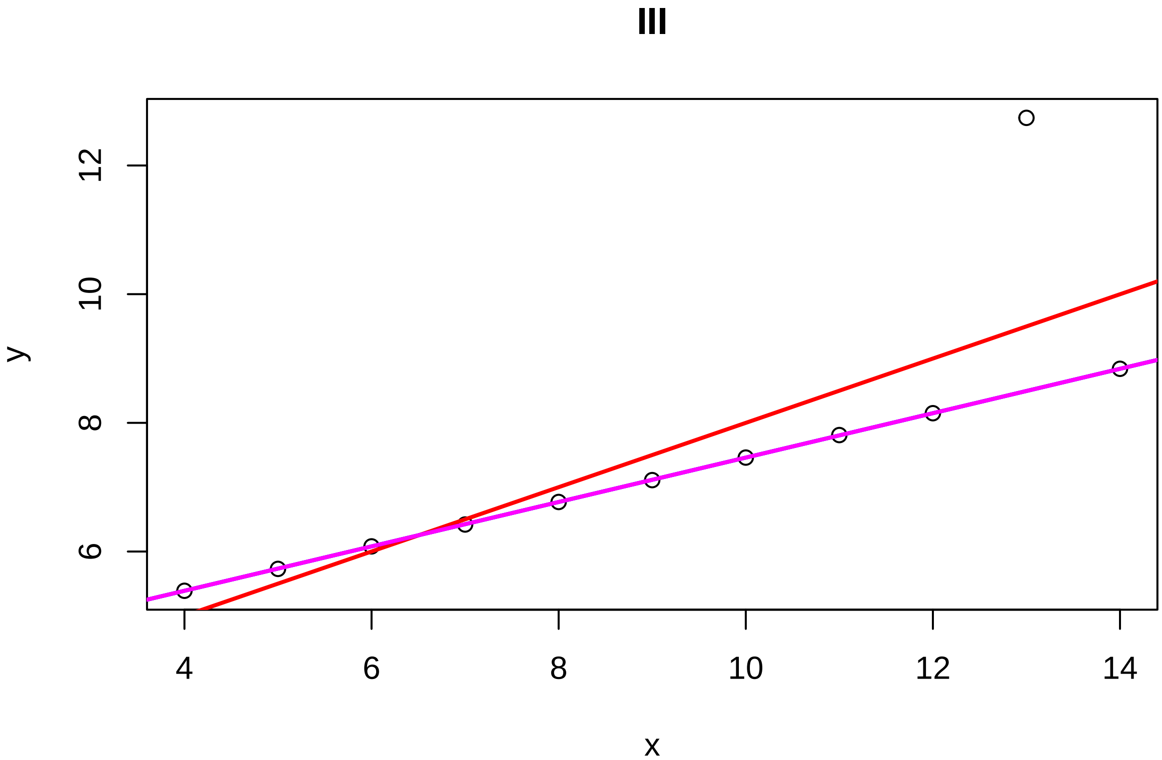
```
for (dataset in datasets) {
    plot(y ~ x, data = dataset, main = dataset[[1]])
    lmfit <- lm(y ~ x, data = dataset)
    abline(coef = coefficients(lmfit), col = "red", lwd = 2)
    lmedfit <- lmsreg(y ~ x, data = dataset)
    abline(coef = coefficients(lmedfit), col = "blue", lwd = 2)
    robfit <- rlm(y ~ x, data = dataset, method = "MM", psi = psi.bisquare, init = coefficients(lmedfit)
    abline(coef = coefficients(robfit), col = "magenta", lwd = 2)
    cat("Coefficients for dataset", dataset[[1]][1], ":\n")
    print(coefficients(lmfit))
    print(coefficients(lmedfit))
    print(coefficients(robfit))
}
```



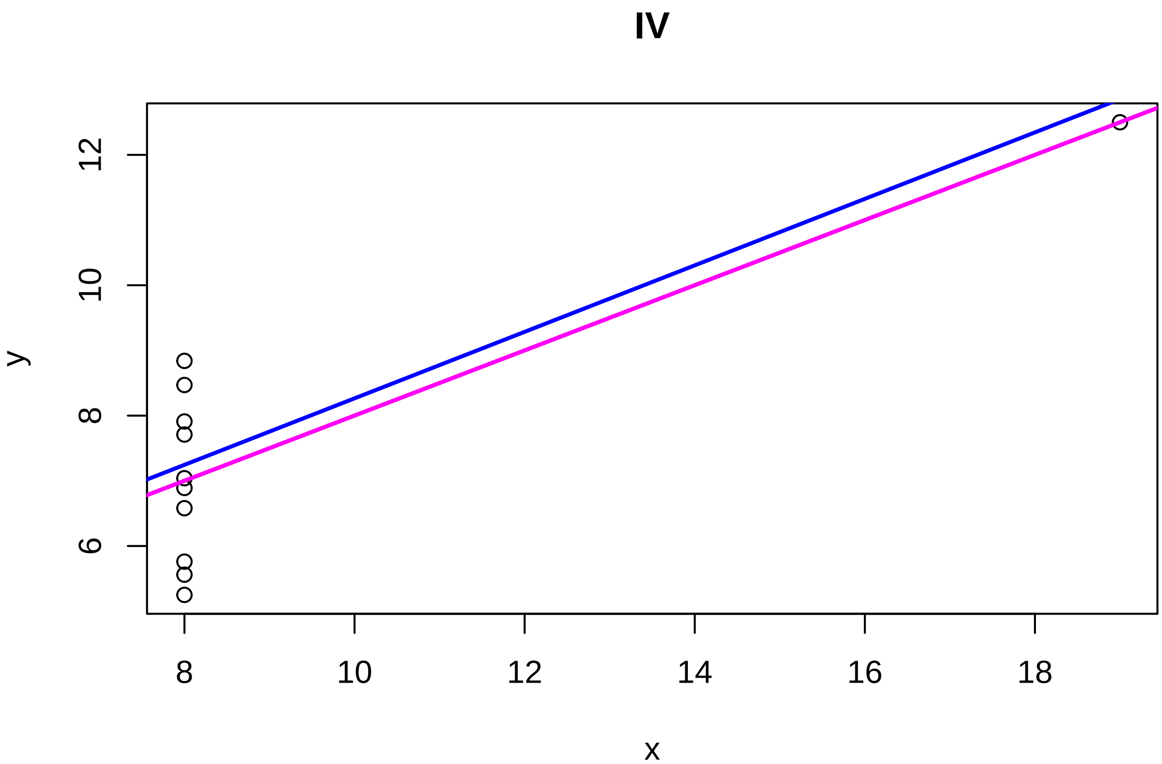

```
## Coefficients for dataset 1 :
## (Intercept)           x
##   3.0000909   0.5000909
## (Intercept)           x
##       2.405       0.570
## (Intercept)           x
##    2.992274    0.501876
```




```
## Coefficients for dataset 2 :
## (Intercept)           x
##    3.000909    0.500000
## (Intercept)           x
##        0.72        0.88
## (Intercept)           x
##   3.0435347   0.4999972
```




```
## Coefficients for dataset 3 :
## (Intercept)           x
##   3.0024545   0.4997273
## (Intercept)           x
##       4.010       0.345
## (Intercept)           x
##       4.010       0.345
```




```
## Coefficients for dataset 4 :
## (Intercept)           x
##   3.0017273   0.4999091
## (Intercept)           x
##       3.165       0.510
## (Intercept)           x
##   3.0008522   0.4999551
```

For dataset 1 $\tilde{\beta}_1$ differs the at most 0.070 from the LS counterpart $\hat{\beta}_1 = 0.5$. This difference is attained with the least median of squares estimator.

For dataset 2 $\tilde{\beta}_1$ differs at most 0.38 from the LS counterpart $\hat{\beta}_1 = 0.5$. This difference is attained with the least median of squares estimator.

For dataset 3 $\tilde{\beta}_1$ differs at most 0.155 from the LS counterpart $\hat{\beta}_1 = 0.5$. This difference is attained with the least median of squares estimator & the M estimator using Tukey's bisquare function.

For dataset 4 $\tilde{\beta}_1$ differs at most 0.10 from the LS counterpart $\hat{\beta}_1 = 0.5$. This difference is attained with the least median of squares estimator.

Dataset 3 has the highest difference between $\tilde{\beta}_1$ and $\hat{\beta}_1 = 0.5$ with a difference of 0.155.

$\tilde{\beta}_0$ and $\tilde{\beta}_1$ are different than $\hat{\beta}_0$ and $\hat{\beta}_1$ for two of the three estimators in dataset 3.

Is the hypothesis $H_0 : \beta_1 = 1/2$ rejected in a robust test against the alternative $H_1 : \beta_1 < 1/2$, for data set 3?

```r
n <- length(data3$x)
x <- data3$x
y <- data3$y
beta_1 <- 0.5
beta_10 <- 0.5
U <- y - beta_10 * x
rho_hat <- beta_1 * sqrt(sum(rank(x)^2 - mean(rank(x))))/sqrt(sum(rank(U)^2 - mean(rank(U))))
```

```r
p <- 1 - pnorm(rho_hat * sqrt(n - 1))
p
```

```
## [1] 0.05692315
```

We accept the hypothesis $H_0 : \beta_1 = 1/2$ since $p > 0.05$