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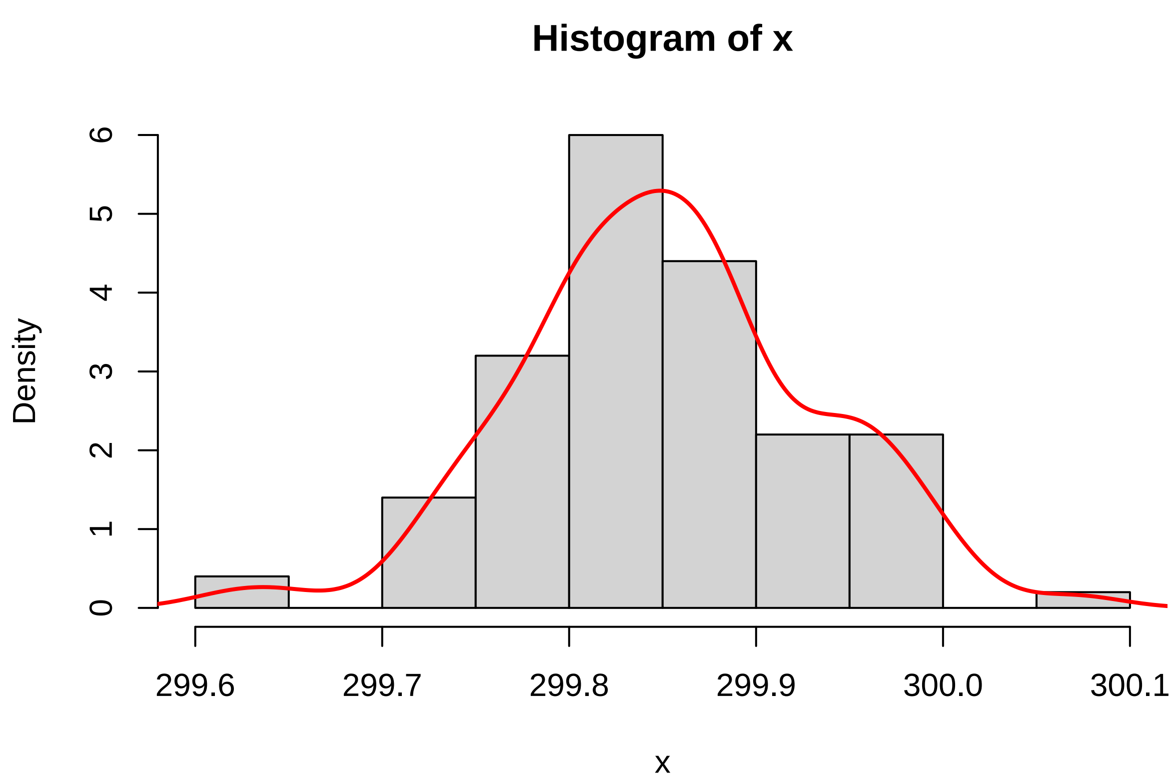
Chapter 11

Exercise 5

Recall Exercise 6.3 based on 100 measurements of the speed of light in air. In that chapter, we tested the data for normality. Use the same data to construct a density estimator that you feel gives the best visual display of the information provided by the data. What parameters did you choose? The data can be downloaded from http://www.itl.nist.gov/div898/strd/univ/data/Michelso.dat

```r
x <- as.numeric(read.delim2("Michelso.dat.txt")[[1]])
hist(x, freq = F, nclass = 10)
kde <- density(x, bw = "SJ")
lines(kde$x, kde$y, col = "red", lwd = 2)
```



Chapter 12

Exercise 1

Using robust regression, find the intercept and slope $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for each of the four data sets of Anscombe (1973) from p. 244. Plot the ordinary least-squares regression along with the rank regression estimator of slope. Contrast these with one of the other robust regression techniques. For which set does $\tilde{\beta}_1$ differ the most from its LS counterpart $\hat{\beta}_1 = 0.5$? Note that in the fourth set, 10 out of 11 Xs are equal, so one should use

$$S_{ij} = (Y_j - Y_i)/(X_j - X_i + \epsilon)$$

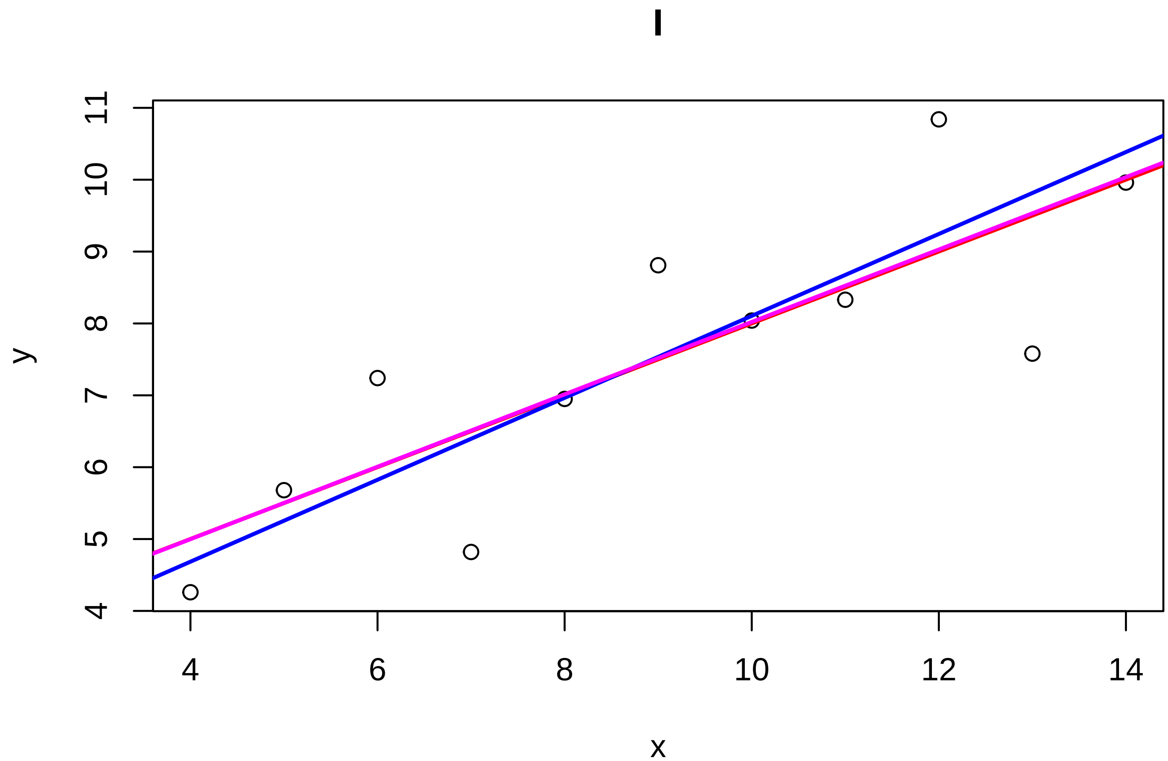
to avoid dividing by 0. After finding $\tilde{\beta}_0$ and $\tilde{\beta}_1$, are they different than $\hat{\beta}_0$ and $\hat{\beta}_1$? Is the hypothesis $H_0 : \beta_1 = 1/2$ rejected in a robust test against the alternative $H_1 : \beta_1 < 1/2$, for data set 3? Note here $\beta_{10} = 1/2$.

```r
library(robustbase)
library(MASS)
library(L1pack)
```

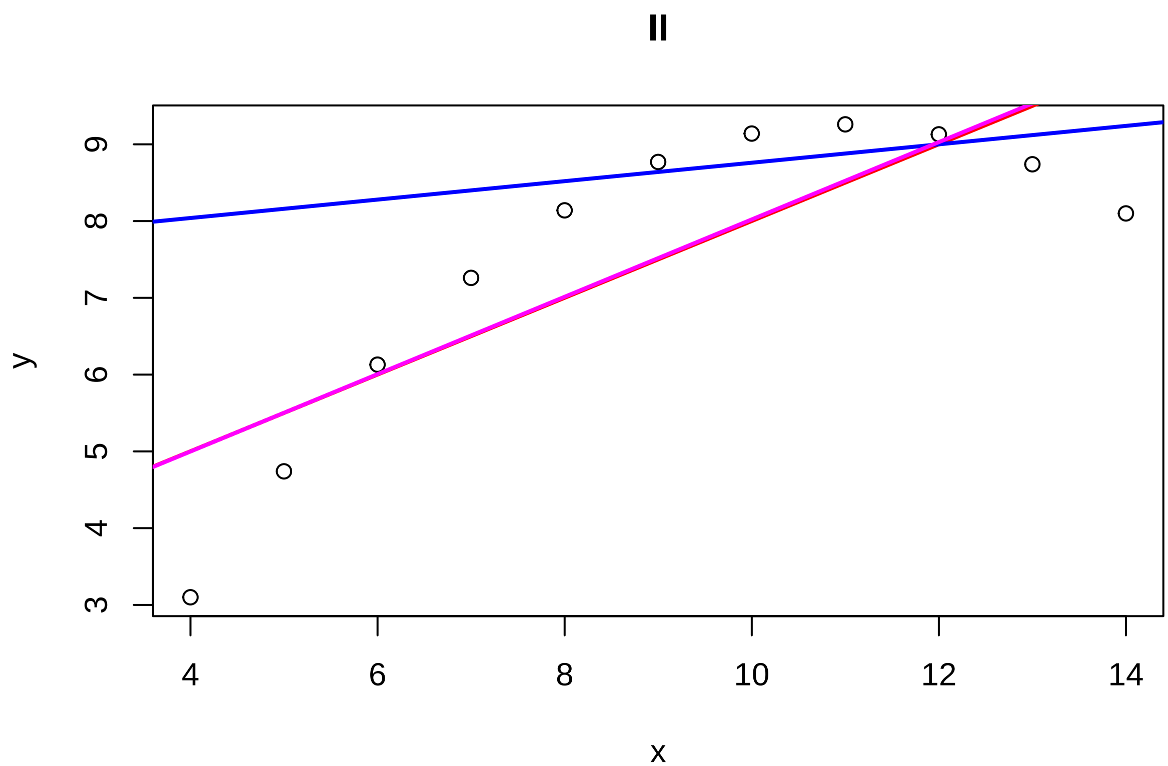
```
## Loading required package: fastmatrix
```

```r
data <- read.csv("anscombe.csv")
data$dataset <- factor(data$dataset)
data1 <- subset(data, subset = data$dataset == "I")
data2 <- subset(data, subset = data$dataset == "II")
data3 <- subset(data, subset = data$dataset == "III")
data4 <- subset(data, subset = data$dataset == "IV")
datasets <- list(data1, data2, data3, data4)
```

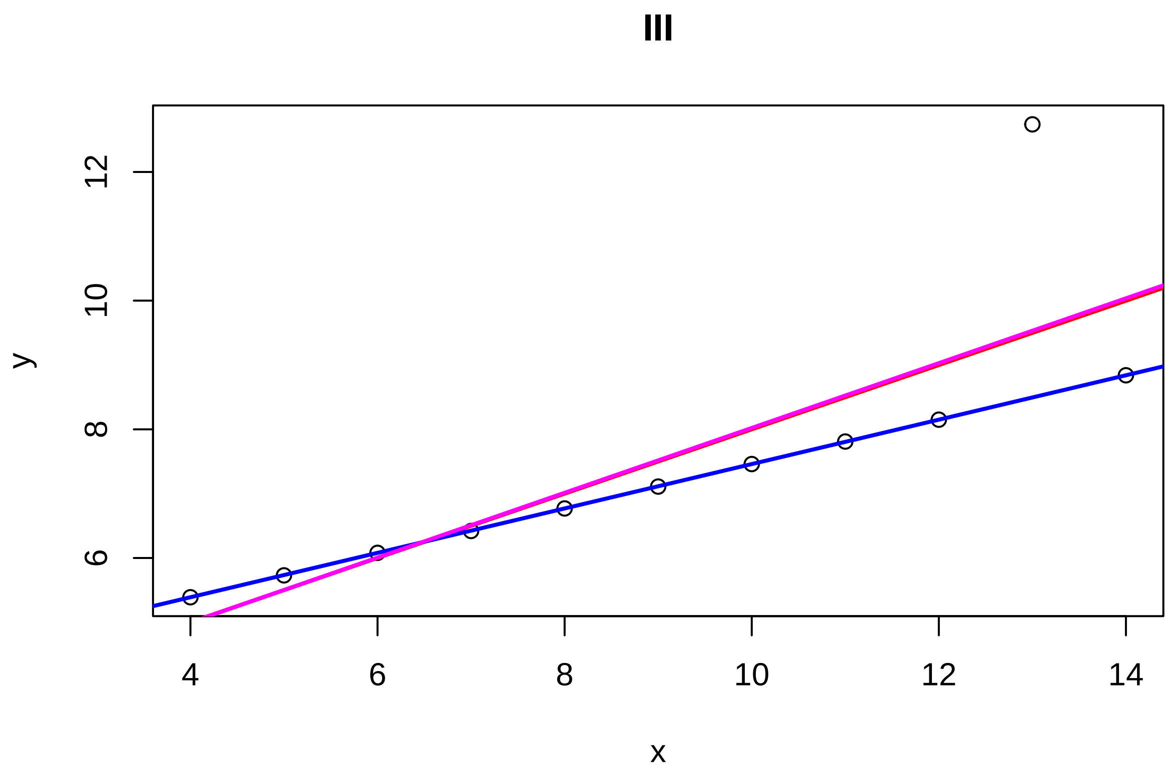
```r
for (dataset in datasets) {
    plot(y ~ x, data = dataset, main = dataset[[1]])
    lmfit <- lm(y ~ x, data = dataset)
    abline(coef = coefficients(lmfit), col = "red", lwd = 2)
    lmedfit <- lmsreg(y ~ x, data = dataset)
    abline(coef = coefficients(lmedfit), col = "blue", lwd = 2)
    robfit <- rlm(y ~ x, data = data1, method = "M", psi = psi.bisquare, init = coefficients(lmedfit))
    abline(coef = coefficients(robfit), col = "magenta", lwd = 2)

    cat("Coefficients for", dataset[[1]], ":")
    print(coefficients(lmfit))
    print(coefficients(lmedfit))
    print(coefficients(robfit))
}
```



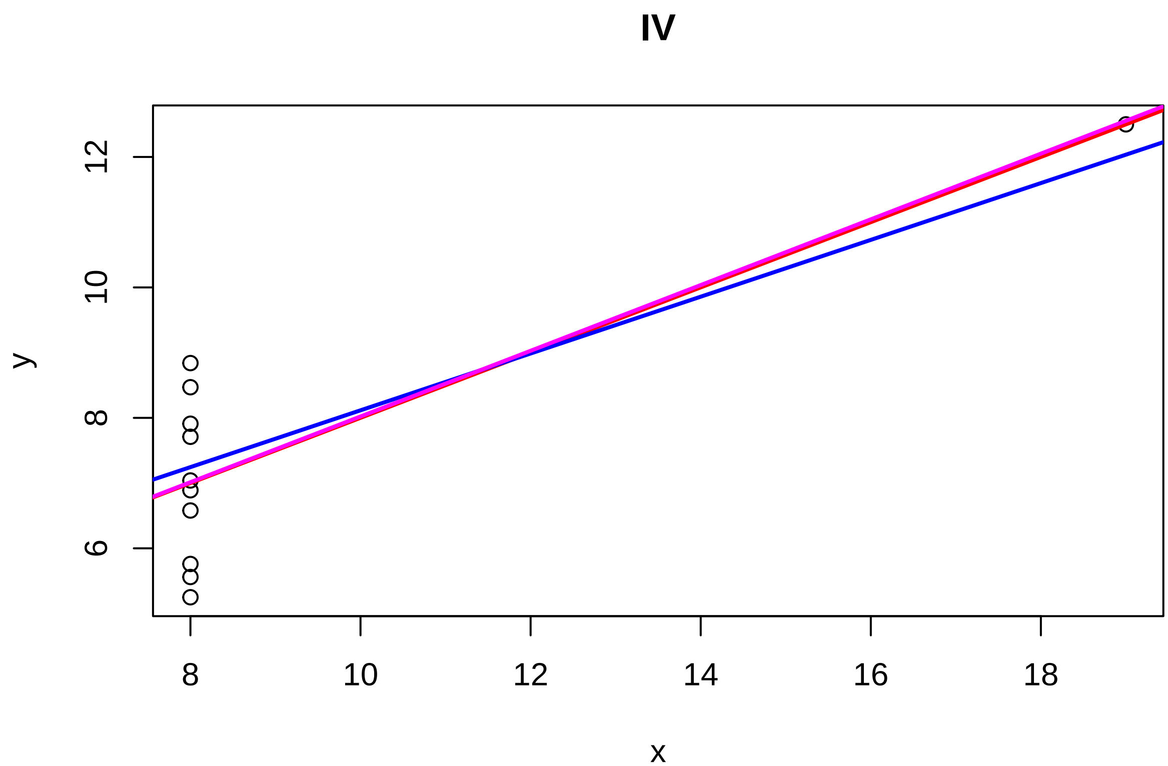

```
## Coefficients for 1 1 1 1 1 1 1 1 1 1 1 :(Intercept)           x
##   3.0000909   0.5000909
## (Intercept)           x
##       2.405       0.570
## (Intercept)           x
##   2.9836914   0.5037395
```




```
## Coefficients for 2 2 2 2 2 2 2 2 2 2 2 :(Intercept)           x
##    3.000909    0.500000
## (Intercept)           x
##        7.56        0.12
## (Intercept)           x
##   2.9837203   0.5037342
```



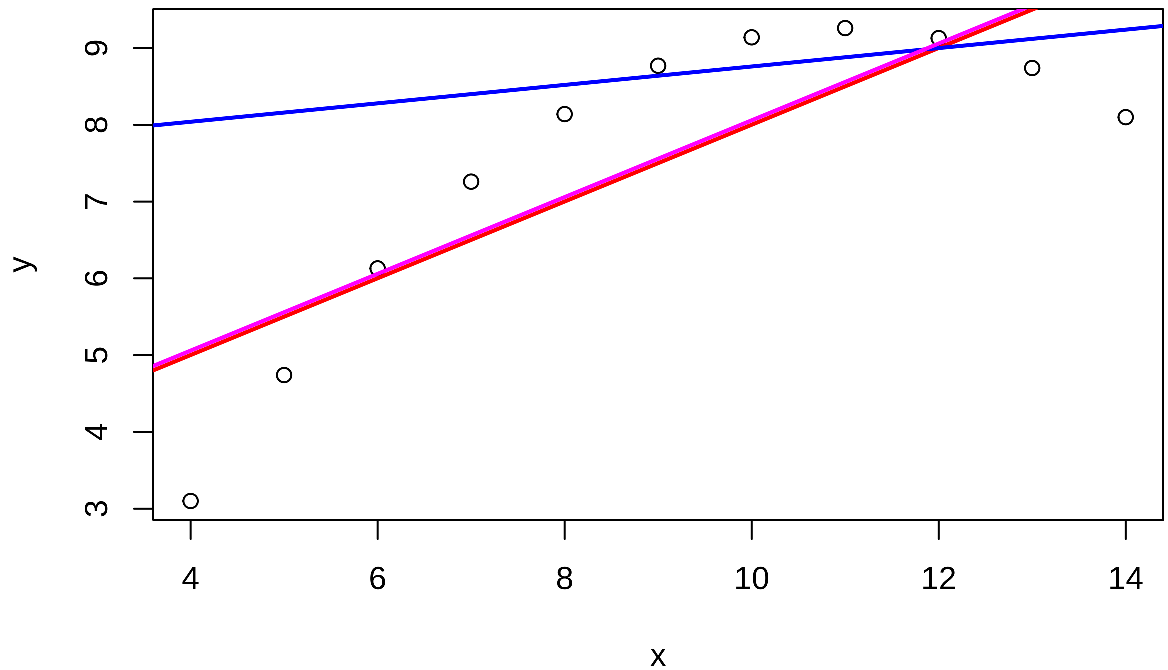

```
## Coefficients for 3 3 3 3 3 3 3 3 3 3 3 :(Intercept)           x
##   3.0024545   0.4997273
## (Intercept)           x
##       4.010       0.345
## (Intercept)           x
##   2.9837249   0.5037332
```




```
## Coefficients for 4 4 4 4 4 4 4 4 4 4 4 :(Intercept)           x
##   3.0017273   0.4999091
## (Intercept)           x
##   3.7613636   0.4354545
## (Intercept)           x
##   2.9837352   0.5037315
```

For dataset 1 $\tilde{\beta}_1$ differs the at most 0.070 from the LS counterpart $\hat{\beta}_1 = 0.5$. This difference is attained with the least median of squares estimator.

```
plot(y ~ x, data = data2)
lmfit <- lm(y ~ x, data = data2)
abline(coef = coefficients(lmfit), col = "red", lwd = 2)
lmedfit <- lmsreg(y ~ x, data = data2)
abline(coef = coefficients(lmedfit), col = "blue", lwd = 2)
robfit <- rlm(y ~ x, data = data2, method = "M", psi = psi.bisquare, init = coefficients(lmedfit))
abline(coef = coefficients(robfit), col = "magenta", lwd = 2)
```




```
coefficients(lmfit)
```

```
## (Intercept)           x
##    3.000909    0.500000
```

```
coefficients(lmedfit)
```

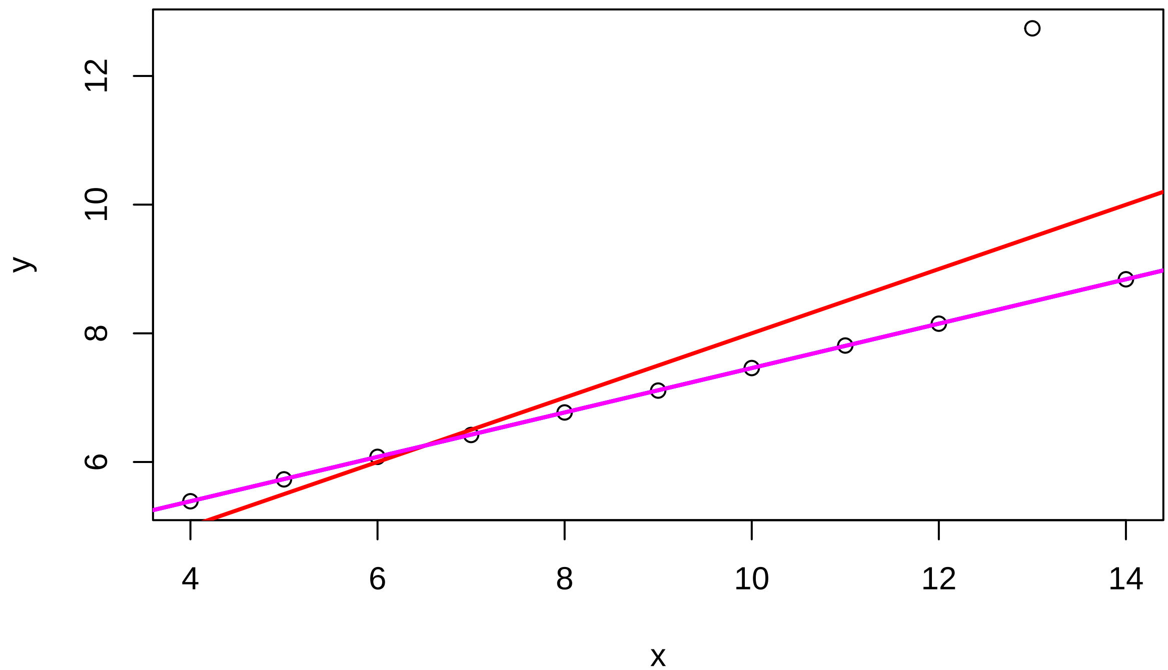
```
## (Intercept)           x
##        7.56        0.12
```

```
coefficients(robfit)
```

```
## (Intercept)           x
##   3.0589785   0.4999953
```

For dataset 2 $\tilde{\beta}_1$ differs at most 0.38 from the LS counterpart $\hat{\beta}_1 = 0.5$. This difference is attained with the least median of squares estimator.

```
plot(y ~ x, data = data3)
lmfit <- lm(y ~ x, data = data3)
abline(coef = coefficients(lmfit), col = "red", lwd = 2)
lmedfit <- lmsreg(y ~ x, data = data3)
abline(coef = coefficients(lmedfit), col = "blue", lwd = 2)
robfit <- rlm(y ~ x, data = data3, method = "MM", psi = psi.bisquare, init = coefficients(lmedfit))
abline(coef = coefficients(robfit), col = "magenta", lwd = 2)
```




```r
coefficients(lmfit)
```

```
## (Intercept)           x
##   3.0024545   0.4997273
```

```r
coefficients(lmedfit)
```

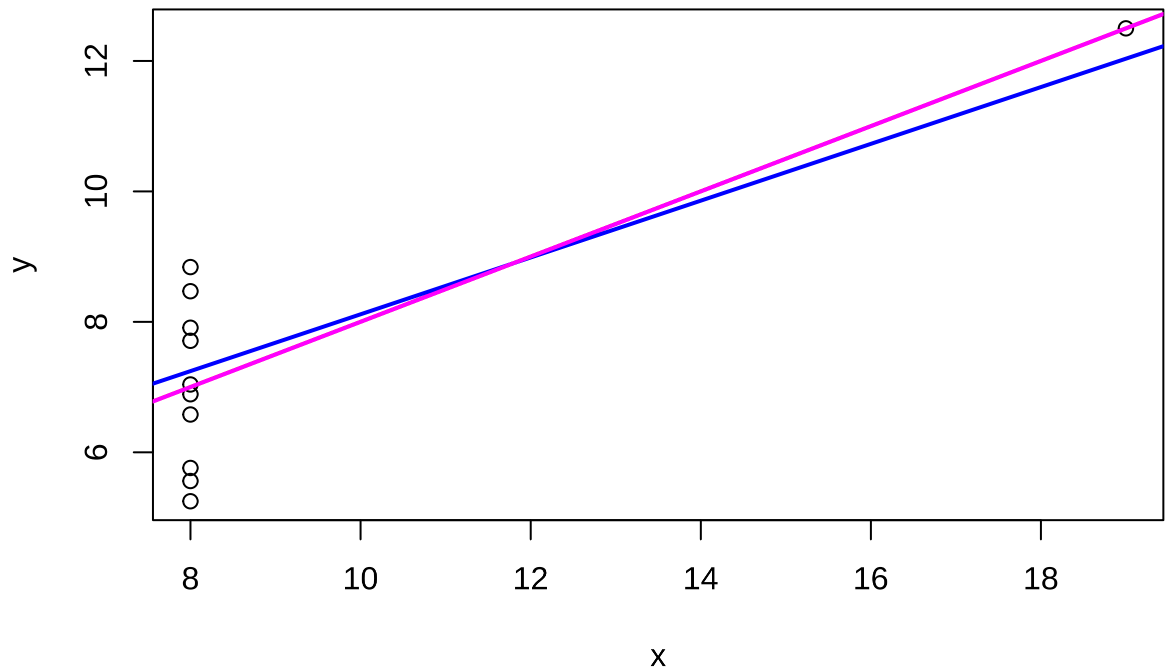
```
## (Intercept)           x
##       4.010       0.345
```

```r
coefficients(robfit)
```

```
## (Intercept)           x
##       4.010       0.345
```

For dataset 3 $\tilde{\beta}_1$ differs at most 0.155 from the LS counterpart $\hat{\beta}_1 = 0.5$. This difference is attained with the least median of squares estimator & the M estimator using Tukey's bisquare function.

```r
plot(y ~ x, data = data4)
lmfit <- lm(y ~ x, data = data4)
abline(coef = coefficients(lmfit), col = "red", lwd = 2)
lmedfit <- lmsreg(y ~ x, data = data4)
abline(coef = coefficients(lmedfit), col = "blue", lwd = 2)
robfit <- rlm(y ~ x, data = data4, method = "M", psi = psi.bisquare, init = coefficients(lmedfit))
abline(coef = coefficients(robfit), col = "magenta", lwd = 2)
```




```r
lmfit$coefficients
```

```
## (Intercept)           x
##   3.0017273   0.4999091
```

```r
lmedfit$coefficients
```

```
## (Intercept)           x
##   3.7613636   0.4354545
```

```r
robfit$coefficients
```

```
## (Intercept)           x
##   3.0005408   0.4999715
```

For dataset 4 $\tilde{\beta}_1$ differs at most 0.10 from the LS counterpart $\hat{\beta}_1 = 0.5$. This difference is attained with the least median of squares estimator.

Dataset 3 has the highest difference between $\tilde{\beta}_1$ and $\hat{\beta}_1 = 0.5$ with a difference of 0.155.

$\tilde{\beta}_0$ and $\tilde{\beta}_1$ are different than $\hat{\beta}_0$ and $\hat{\beta}_1$ for two of the three estimators in dataset 3.

Is the hypothesis $H_0 : \beta_1 = 1/2$ rejected in a robust test against the alternative $H_1 : \beta_1 < 1/2$, for data set 3?

```r
n <- length(data3$x)
x <- data3$x
y <- data3$y
beta_1 <- 0.5
beta_10 <- 0.5
U <- y - beta_10 * x
rho_hat <- beta_1 * sqrt(sum(rank(x)^2 - mean(rank(x))))/sqrt(sum(rank(U)^2 - mean(rank(U))))
p <- 1 - pnorm(rho_hat * sqrt(n - 1))
p
```

```
## [1] 0.05692315
```

We accept the hypothesis $H_0 : \beta_1 = 1/2$ since $p > 0.05$