# HW5 - Lucas Fellmeth, Sven Bergmann

2023-11-16

## 1

The goal of this problem is to estimate the regression function of acceleration vs time for the `mcycle` data in the package `MASS`.

### A

Show that the Nadaraya-Watson estimator can be expressed as $\hat{Y} = HY$. Find the "hat matrix" $H$ explicitly.

We know that the Nadaraya-Watson estimator of $\hat{m}(x)$ is defined by

$$\hat{m}(x) = \frac{\sum_{i=1}^{n} K_h(X_i - x) Y_i}{\sum_{i=1}^{n} K_h(X_i - x)},$$

where

$$K_h(x) = \frac{1}{h} K\left(\frac{x}{h}\right)$$

with $h$ as associated bandwidth.

### B

For a reasonable range of bandwidths $h$, compute and plot the generalized cross validation measure $GCV(h)$ and find the optimal bandwidth.

```
library(MASS)
library(splines)
```

First, we implement the generalized cross validation measure $GCV(h)$ which is defined by

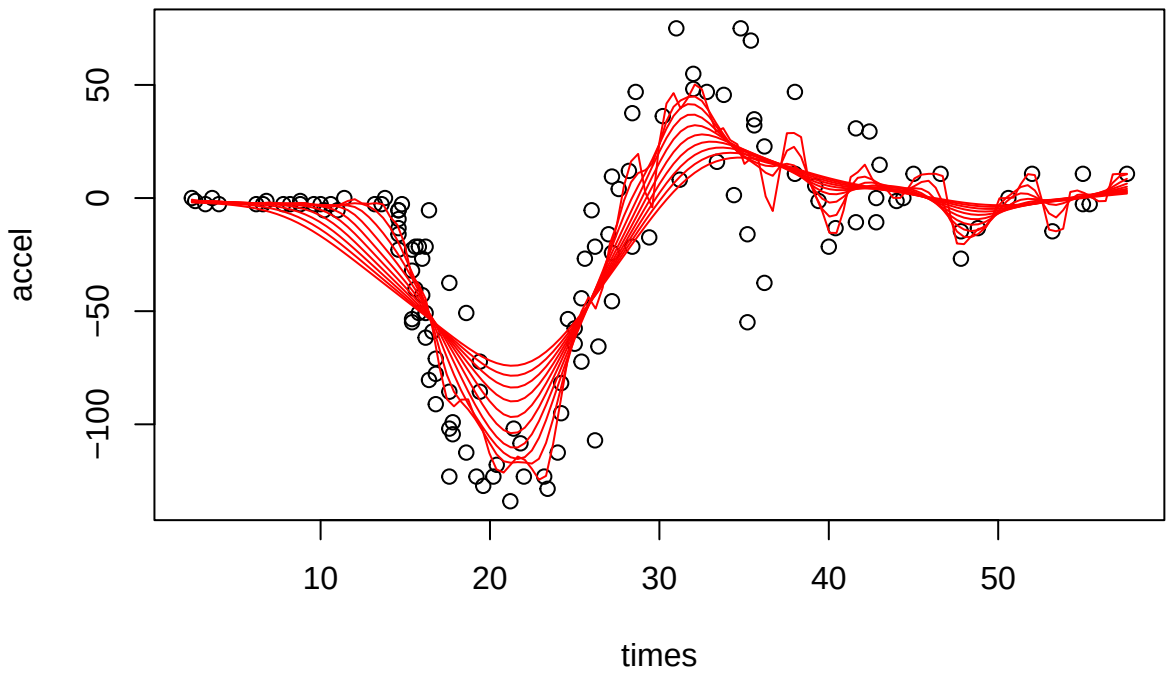
$$GCV(h) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{Y_i - \hat{m}_h(x)}{1 - \frac{trS(h)}{n}} \right]^2$$

where

```
GCV <- function(h) {

}
```

```
with(mcycle, {
    plot(times, accel)
    for (b in 1:10) {
        lines(ksmooth(times, accel, "normal", bandwidth = b), col = "red")
    }
})
```


accel
50
0
−50
−100
10
20
30
40
50
times