

HW5 - Lucas Fellmeth, Sven Bergmann

2023-11-16

1

The goal of this problem is to estimate the regression function of acceleration vs time for the `mcycle` data in the package `MASS`.

A

Show that the Nadaraya-Watson estimator can be expressed as $\hat{Y} = HY$. Find the “hat matrix” H explicitly.

We know that the Nadaraya-Watson estimator of $\hat{m}(x)$ is defined by

$$\hat{m}(x) = \frac{\sum_{i=1}^n K_h(X_i - x)Y_i}{\sum_{i=1}^n K_h(X_i - x)},$$

where

$$K_h(x) = \frac{1}{h}K\left(\frac{x}{h}\right)$$

with h as associated bandwidth.

B

For a reasonable range of bandwidths h , compute and plot the generalized cross validation measure $GCV(h)$ and find the optimal bandwidth.

```
library(MASS)
library(splines)
```

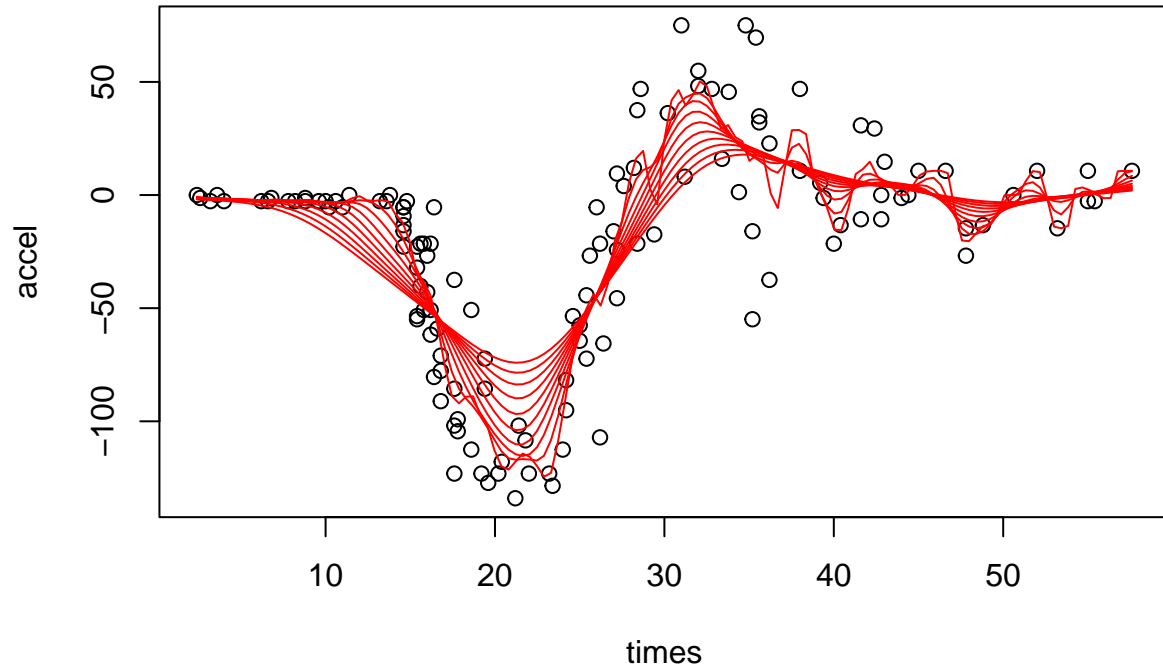
First, we implement the generalized cross validation measure $GCV(h)$ which is defined by

$$GCV(h) = \frac{1}{n} \sum_{i=1}^n \left[\frac{Y_i - \hat{m}_h(x)}{1 - \frac{trS(h)}{n}} \right]^2$$

where

```
GCV <- function(h) {
  Y <- ksmooth(times, accel, "normal", bandwidth = h)$y
}

with(mcycle, {
  plot(times, accel)
  for (b in 1:10) {
    lines(ksmooth(times, accel, "normal", bandwidth = b), col = "red")
    print(ksmooth(times, accel, "normal", bandwidth = b)$y)
  }
})
```



```
## [1] -0.70495048 -1.30182894 -1.62130427 -1.42989820 -1.89470370
## [6] -2.38300000 -2.60163110 -2.69223988 -2.65625186 -2.51403550
## [11] -2.21601342 -1.95433355 -2.36341764 -2.65556605 -2.47814173
## [16] -2.15836893 -2.19533718 -2.83200622 -3.48751470 -3.80901757
## [21] -3.71694540 -2.79939057 -1.31910637 -0.51174650 -2.32494780
## [26] -2.57111882 -2.26209966 -2.76815997 -8.38246059 -11.84024803
## [31] -19.96018390 -34.23526477 -38.06944014 -44.47991303 -57.98630113
## [36] -75.35104435 -88.85463017 -92.05646597 -89.12090924 -88.95373587
## [41] -97.20609296 -101.97150785 -112.72125251 -119.90380777 -121.34810693
## [46] -117.81854256 -114.24300741 -115.70716811 -119.67771404 -124.50130928
## [51] -122.81008815 -108.65308564 -90.71225935 -72.27876824 -58.71128904
## [56] -50.17316288 -43.86602035 -48.86918196 -39.53253631 -20.29490460
## [61] -11.65159367 5.39974064 16.19019760 19.51786200 2.53502445
## [66] -4.11507406 24.55942654 41.77748745 46.38756062 39.55434834
## [71] 44.15214435 50.20328927 48.05829556 38.18426768 29.34427331
## [76] 30.89911554 25.60559574 23.75739748 15.08090288 17.75674699
## [81] 16.52186670 0.72543662 -5.73846606 11.80019116 28.66657541
## [86] 28.75472009 26.97265756 8.67480179 1.22225397 -5.96247549
## [91] -15.35444984 -15.43674010 -5.40067660 9.27221156 11.44763952
## [96] 14.75632451 8.66642369 4.50306673 4.12841641 0.08253330
## [101] 0.06669964 3.79717118 8.67884026 10.34404980 10.70000000
## [106] 10.70000000 9.71912129 -6.72220473 -20.09141177 -20.38373161
## [111] -16.96028293 -13.62776235 -13.28580304 -8.57368863 -0.09992732
## [116] 0.00000000 0.19557887 6.08869986 10.57260435 10.15045417
## [121] -0.99364457 -13.95105605 -14.68001301 -13.58800367 2.68122709
## [126] 3.34404970 2.21225158 0.30483996 -1.40035607 -1.31074789
```

## [131]	10.50623705	10.70000000	10.70000000		
## [1]	-1.0465961	-1.2335440	-1.4033316	-1.5443843	-1.6756737
## [6]	-1.8409424	-2.1144490	-2.3751650	-2.3966408	-2.3327393
## [11]	-2.2773453	-2.2716263	-2.3294240	-2.3857832	-2.3963781
## [16]	-2.4370225	-2.6192288	-2.9475003	-3.2647868	-3.4081540
## [21]	-3.3207650	-3.0264825	-2.6054336	-2.2848443	-2.5691090
## [26]	-3.6887143	-5.6676458	-8.7716292	-13.0822032	-18.6157078
## [31]	-25.2685458	-32.3609696	-39.3502481	-46.8301129	-56.1038859
## [36]	-67.3489218	-78.1324916	-85.6287993	-89.9653388	-93.4306946
## [41]	-97.6319792	-102.6366116	-108.1004452	-113.2045881	-116.3270560
## [46]	-116.9769611	-116.7010495	-116.8948445	-117.3618145	-115.2289821
## [51]	-108.2277989	-97.5971276	-85.1676335	-72.6271789	-61.7533016
## [56]	-53.6206482	-47.6371784	-41.6867094	-33.6597668	-23.3426473
## [61]	-11.9538150	-1.1283867	6.9839431	11.0551858	12.3074648
## [66]	15.2257624	23.8332952	34.0461566	40.3233679	43.1537545
## [71]	44.6074780	44.9085972	42.9081097	38.4207452	33.1373074
## [76]	28.3646487	24.0811348	20.5255357	18.0037339	16.0439489
## [81]	13.8327694	11.1874180	9.8161121	13.8320041	20.9456356
## [86]	22.6324545	18.1335078	9.7584258	1.2544832	-4.9089062
## [91]	-8.1830706	-7.4488111	-1.8531091	5.1079466	8.9486356
## [96]	9.4590255	8.2274808	6.4192080	4.5380991	3.0561489
## [101]	2.7184205	3.6391070	5.2951777	7.0997015	7.6397487
## [106]	4.5622764	-2.0099560	-9.4212575	-14.7681010	-17.1342263
## [111]	-17.2302179	-15.8947686	-13.1234531	-8.1618804	-2.8922112
## [116]	0.2391298	2.5498479	4.8648914	5.7024392	3.3628278
## [121]	-1.6772579	-6.7632967	-8.5906390	-5.3364413	-0.6546415
## [126]	1.3858531	1.6753131	1.4385478	1.3464395	2.7160142
## [131]	6.7803945	9.7138671	10.5106605		
## [1]	-1.17940766	-1.27800640	-1.37929834	-1.48763364	-1.61450050
## [6]	-1.77655718	-1.97269936	-2.15200239	-2.25831920	-2.30033983
## [11]	-2.31589875	-2.33217959	-2.36239147	-2.41470834	-2.49744320
## [16]	-2.61652499	-2.76703761	-2.92560129	-3.05649489	-3.13125518
## [21]	-3.13537309	-3.12467770	-3.26409978	-3.97750510	-5.72148538
## [26]	-8.42379624	-11.73608293	-15.49231469	-19.68669809	-24.33242825
## [31]	-29.40045149	-34.89152930	-40.85634668	-47.40574931	-54.61078368
## [36]	-62.37830105	-70.35546916	-78.02188584	-84.92886460	-90.93178706
## [41]	-96.22405434	-101.02728546	-105.47701994	-109.41076027	-112.53040323
## [46]	-114.55791049	-115.32654509	-114.52192648	-111.55418957	-105.97759109
## [51]	-98.18329348	-89.16958164	-79.86943809	-70.89522306	-62.54254624
## [56]	-54.82722316	-47.48303334	-40.09426607	-32.30548474	-24.08052754
## [61]	-15.75057987	-7.84159739	-0.81406719	5.31425988	11.00286654
## [66]	17.07047006	23.88448031	30.72189609	36.30466731	39.90139832
## [71]	41.51445140	41.28607124	39.38931322	36.05261256	31.81418038
## [76]	27.48008418	23.70239379	20.71201785	18.41183191	16.61641845
## [81]	15.21778941	14.28473993	14.03780216	14.55675073	15.14779327
## [86]	14.43062659	11.67931881	7.50600286	3.12591707	-0.32427066
## [91]	-2.01952048	-1.59760987	0.57597331	3.37933968	5.62022565
## [96]	6.74924265	6.86279469	6.32656754	5.51413256	4.75816035
## [101]	4.30478678	4.23471370	4.34714287	4.04164417	2.44585430
## [106]	-0.83013409	-5.03545742	-8.99596327	-12.01693138	-13.90486211
## [111]	-14.64200975	-14.15919887	-12.30878500	-9.11172213	-5.16439252
## [116]	-1.53982165	0.96095574	2.07298450	1.85003105	0.55772682
## [121]	-1.22982419	-2.67848075	-3.04969188	-2.29568088	-1.04098786
## [126]	0.09164032	0.90473586	1.52613860	2.23255705	3.33780922

## [131]	4.98947238	6.90012957	8.51884077		
## [1]	-1.26093258	-1.33380184	-1.41664846	-1.51382104	-1.63012244
## [6]	-1.76634669	-1.91392463	-2.05525223	-2.17165561	-2.25784865
## [11]	-2.32161288	-2.37655514	-2.43490212	-2.50433448	-2.58682752
## [16]	-2.68192886	-2.78103859	-2.88535689	-3.00005175	-3.16219185
## [21]	-3.46140859	-4.09462087	-5.31757090	-7.41152760	-10.31017963
## [26]	-13.74573300	-17.40795310	-21.16698903	-25.02014715	-29.03411184
## [31]	-33.27343131	-37.80262136	-42.67638795	-47.93707426	-53.60020268
## [36]	-59.64857086	-66.01158544	-72.55699978	-79.10180955	-85.43890592
## [41]	-91.37883718	-96.77626486	-101.50838813	-105.45040770	-108.43753088
## [46]	-110.15254544	-110.24568069	-108.36316587	-104.37913853	-98.55225617
## [51]	-91.47596205	-83.80577182	-76.04766213	-68.46430729	-61.13888266
## [56]	-54.02168225	-47.03976416	-40.10048697	-33.16704801	-26.24254429
## [61]	-19.37285891	-12.61202586	-5.96854663	0.61458797	7.24217363
## [66]	13.92881875	20.51812611	26.57274629	31.54401073	34.99985531
## [71]	36.74986982	36.83622620	35.49867597	33.09128480	30.09723280
## [76]	26.97315681	24.08085773	21.60288534	19.58324835	17.98585100
## [81]	16.74475898	15.78664836	15.00386359	14.20733377	13.14337996
## [86]	11.56265761	9.43116393	6.97771380	4.63125757	2.81204744
## [91]	1.80739306	1.68862872	2.28700128	3.27983665	4.29373268
## [96]	5.07785578	5.51969916	5.63066497	5.49276015	5.19510821
## [101]	4.78743748	4.24417932	3.43015155	2.14118459	0.22160397
## [106]	-2.26688641	-5.01288959	-7.60186183	-9.70899923	-11.13619090
## [111]	-11.79386719	-11.61105156	-10.56270051	-8.72782772	-6.37820336
## [116]	-3.97367873	-2.00815165	-0.79243911	-0.35080095	-0.48192661
## [121]	-0.86888758	-1.19953112	-1.27612450	-1.04684020	-0.58474738
## [126]	0.01670427	0.68726821	1.40703311	2.19918774	3.09977560
## [131]	4.12645461	5.25007699	6.39306062		
## [1]	-1.34242872	-1.40995392	-1.48756469	-1.57730495	-1.67937258
## [6]	-1.79226211	-1.91128195	-2.02958009	-2.14040077	-2.23978103
## [11]	-2.32740439	-2.40616821	-2.48347349	-2.55889262	-2.64952698
## [16]	-2.75606770	-2.90307321	-3.12752729	-3.48238012	-4.08995534
## [21]	-5.09070911	-6.63119608	-8.82594964	-11.63974921	-14.90499286
## [26]	-18.40577898	-21.97852804	-25.54762200	-29.12128629	-32.73929193
## [31]	-36.46191607	-40.34471709	-44.43800569	-48.78449583	-53.40985382
## [36]	-58.32671363	-63.52359439	-68.96200238	-74.56751622	-80.22692133
## [41]	-85.78558963	-91.04996688	-95.77902487	-99.70635040	-102.52325183
## [46]	-103.90313041	-103.58808610	-101.47730374	-97.64730615	-92.44565349
## [51]	-86.31595657	-79.69249343	-72.91106565	-66.17967524	-59.58047659
## [56]	-53.13677125	-46.83038413	-40.61792116	-34.46313501	-28.33230628
## [61]	-22.19347402	-16.00664426	-9.73849864	-3.36881792	3.07574983
## [66]	9.49670084	15.66946228	21.28102501	25.97529680	29.42379482
## [71]	31.49646317	32.19698232	31.74448705	30.42937399	28.58081001
## [76]	26.50665045	24.42067631	22.46893040	20.72165347	19.18123330
## [81]	17.82048526	16.57340713	15.35756624	14.07887700	12.65596772
## [86]	11.05576427	9.32343549	7.58170426	6.00521013	4.74896143
## [91]	3.91789648	3.51815060	3.49099647	3.71958510	4.07608200
## [96]	4.44322769	4.73297411	4.89352070	4.88958155	4.70007288
## [101]	4.29184046	3.62485459	2.65009242	1.33161313	-0.30973043
## [106]	-2.18854776	-4.14635543	-5.99016266	-7.54180714	-8.66479750
## [111]	-9.27605453	-9.33605110	-8.85527757	-7.89576600	-6.59795133
## [116]	-5.15025954	-3.76974934	-2.63205603	-1.82007257	-1.31535118
## [121]	-1.02819274	-0.84238479	-0.66108403	-0.42159635	-0.09718597
## [126]	0.31663768	0.81115458	1.37889239	2.00764706	2.69634508

```

## [131] 3.43452691 4.21228775 5.01105038
## [1] -1.4328255 -1.4982977 -1.5721048 -1.6545144 -1.7451017 -1.8424913
## [7] -1.9441098 -2.0468223 -2.1499369 -2.2479456 -2.3478959 -2.4498991
## [13] -2.5642648 -2.7011139 -2.8886521 -3.1444584 -3.5481690 -4.1319129
## [19] -4.9995919 -6.2383702 -7.9340819 -10.1110675 -12.7491547 -15.7412618
## [25] -18.9633451 -22.2904671 -25.6397642 -28.9682529 -32.2790188 -35.5995756
## [31] -38.9637324 -42.4159956 -45.9928161 -49.7288383 -53.6525417 -57.7817207
## [37] -62.1208985 -66.6590617 -71.3559870 -76.1403241 -80.9045692 -85.4744731
## [43] -89.6305437 -93.0991988 -95.5720601 -96.7717199 -96.4959394 -94.6994088
## [49] -91.4901667 -87.1370019 -81.9529158 -76.2791469 -70.3551872 -64.3788911
## [55] -58.4481361 -52.6111527 -46.8691852 -41.2047578 -35.5798257 -29.9566122
## [61] -24.2959082 -18.5586711 -12.7229574 -6.8017446 -0.8389025 5.0528948
## [67] 10.6989788 15.8766032 20.3273895 23.8412640 26.2941094 27.6865809
## [73] 28.1317009 27.8050249 26.9262719 25.6943806 24.2770727 22.7937166
## [79] 21.3111227 19.8697527 18.4637766 17.0862649 15.7031329 14.2918700
## [85] 12.8375799 11.3511894 9.8701997 8.4560028 7.1812415 6.1183014
## [91] 5.3030142 4.7479479 4.4243983 4.2822978 4.2630957 4.3042925
## [97] 4.3439376 4.3291829 4.2159436 3.9592890 3.5316485 2.8904074
## [103] 2.0330131 0.9564586 -0.3092378 -1.7005829 -3.1289150 -4.4912767
## [109] -5.6776291 -6.6030108 -7.2044383 -7.4457936 -7.3293904 -6.8889379
## [115] -6.1891223 -5.3197156 -4.3835170 -3.4749821 -2.6591998 -1.9754046
## [121] -1.4193493 -0.9662500 -0.5790551 -0.2211917 0.1361166 0.5137584
## [127] 0.9225053 1.3667795 1.8489119 2.3653608 2.9156882 3.4842838
## [133] 4.0701700
## [1] -1.52807410 -1.59175534 -1.66173202 -1.73750158 -1.81866755
## [6] -1.90778410 -1.99856083 -2.10338815 -2.21357434 -2.33944580
## [11] -2.48905320 -2.66241800 -2.89685211 -3.20997855 -3.63425929
## [16] -4.21405139 -5.01267683 -6.07901703 -7.46597924 -9.21039547
## [21] -11.31525896 -13.76101244 -16.48161652 -19.40212032 -22.43883379
## [26] -25.52832357 -28.62507637 -31.70757535 -34.77635091 -37.84320939
## [31] -40.93156216 -44.06765348 -47.27781680 -50.58840997 -54.01938262
## [36] -57.58259970 -61.28405759 -65.11313827 -69.03856177 -73.00724844
## [41] -76.92493558 -80.66233577 -84.03856161 -86.84730898 -88.85294505
## [46] -89.84980494 -89.68865159 -88.32562675 -85.81922014 -82.33482060
## [51] -78.08769341 -73.31010303 -68.20814770 -62.93966306 -57.61937800
## [56] -52.29421992 -47.00459719 -41.74433066 -36.49872365 -31.24238524
## [61] -25.94917628 -20.60040369 -15.18708771 -9.73870799 -4.29456878
## [66] 1.05341166 6.18082328 10.92380164 15.13116005 18.64573553
## [71] 21.38167383 23.30276467 24.45066611 24.91125270 24.81564460
## [76] 24.29638388 23.46986224 22.44222844 21.27418236 20.02736122
## [81] 18.72889529 17.39345802 16.03707177 14.66041931 13.27666312
## [86] 11.90923263 10.58266448 9.33290733 8.19687092 7.20683884
## [91] 6.38135829 5.72333075 5.22191528 4.85380121 4.58342622
## [96] 4.37531648 4.18558237 3.98031412 3.72021405 3.37572397
## [101] 2.91947091 2.33664193 1.61772712 0.76720434 -0.19036873
## [106] -1.22185145 -2.27371887 -3.29082170 -4.20981086 -4.97501607
## [111] -5.54315482 -5.87567132 -5.96717166 -5.82702964 -5.48184875
## [116] -4.97465564 -4.35990698 -3.69153679 -3.01696815 -2.37331123
## [121] -1.77871768 -1.24331398 -0.76001990 -0.31900506 0.09242489
## [126] 0.48762660 0.87763614 1.27033557 1.67259557 2.08655841
## [131] 2.51321745 2.95137477 3.40033364
## [1] -1.62425870 -1.68824866 -1.75523284 -1.83439908 -1.92201457
## [6] -2.02353057 -2.13876557 -2.28015211 -2.45129955 -2.66360741
## [11] -2.92740708 -3.27338532 -3.72170240 -4.31021314 -5.06239209

```

```

## [16] -6.02611847 -7.22401560 -8.68877584 -10.43096858 -12.44920784
## [21] -14.71428119 -17.19117090 -19.83507741 -22.58883106 -25.41172539
## [26] -28.26206843 -31.11805848 -33.96749685 -36.80872817 -39.65019467
## [31] -42.50317925 -45.38301427 -48.30671850 -51.28930953 -54.33991734
## [36] -57.46617939 -60.66625085 -63.92451870 -67.21234898 -70.47769963
## [41] -73.64635249 -76.61362624 -79.25731254 -81.42235374 -82.96133020
## [46] -83.72627695 -83.61961662 -82.59330799 -80.67177511 -77.93459920
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