

## HW 5

**Problem 1 (8.4)** Suppose  $X$  is a discrete RV with mass function

$$\begin{aligned}P(X = 0) &= \frac{2}{3}\theta \\P(X = 1) &= \frac{1}{3}\theta \\P(X = 2) &= \frac{2}{3}(1 - \theta) \\P(X = 3) &= \frac{1}{3}(1 - \theta),\end{aligned}$$

where  $0 \leq \theta \leq 1$ . The following 10 independent observations were taken from this distribution:  $(3, 0, 2, 1, 3, 2, 1, 0, 2, 1)$ .

- (a) Find the likelihood for  $\theta$ .
- (b) Assume the prior  $\theta \sim \text{Unif}[0, 1]$  and find the posterior distribution for  $\theta$ .

**Problem 2 (8.21)** Suppose that  $X_1, \dots, X_n$  are IID with density  $f(x) = e^{-(x-\theta)}\mathbb{I}\{x \geq \theta\}$ . Find the method of moments estimate of  $\theta$ .

**Problem 3 (8.21)** As in the previous problem, suppose that  $X_1, \dots, X_n$  are IID with density  $f(x) = e^{-(x-\theta)}\mathbb{I}\{x \geq \theta\}$ . Suppose we observe the data  $(x_1, x_2, x_3) = (1, 2, -1)$ . Draw a sketch of the likelihood function and find the MLE of  $\theta$ .