Stewardship of Mathematics: essential training for contributors to, and users of, the practice of mathematics

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Abstract

A *steward of the discipline* was originally defined as an individual to whom "we can entrust the vigor, quality, and integrity of the field" (p. 5), and more specifically, as "someone who will creatively generate new knowledge, critically conserve valuable and useful ideas, and responsibly transform those understandings through writing, teaching, and application" (Golde & Walker, 2006)[8]. Originally articulated for *doctoral education*, in 2019 the construct of stewardship was expanded so that it can also be applied to non-academic practitioners in any field, and can be initiated earlier than doctoral education (Rios et al. 2019)[18]. The 2019 formulation was the first to describe the knowledge, skills, and abilities (KSAs) of stewardship that can be taught, learned, and assessed across an entire higher education curriculum in any discipline. Even for those early in their training in mathematics, stewardly practice of mathematics can be introduced and practiced. Postsecondary and tertiary education in mathematics – for future mathematicians as well as those who will use math at work - can include curriculum-spanning training, and documented achievement in stewardship. Even before a formal ethical practice standard for mathematics is developed and deployed to help inculcate math students with a "tacit responsibility for the quality and integrity of their own work", higher education can begin to shape student attitudes towards stewardly professional identities. Learning objectives to accomplish this are described, to assist math instructors in facilitating the recognition and acceptance of responsibility for the quality and integrity of their own work and that of colleagues in the practice of mathematics.

1. Introduction

A *steward of the discipline* was originally defined as an individual to whom “we can entrust the vigor, quality, and integrity of the field” (p. 5), and more specifically, as “someone who will creatively generate new knowledge, critically conserve valuable and useful ideas, and responsibly transform those understandings through writing, teaching, and application” (Golde & Walker, 2006)[8]. In these definitions, “we” represents both the public and the discipline in which a steward is trained. These definitions were originally articulated for “scholars first and foremost” and *doctoral education*; in 2019 the construct of stewardship was expanded so that it can also be applied to non-academic practitioners in any field, and can be initiated earlier than doctoral education (Rios et al. 2019)[18]. Rios et al. (2019)[18] describe the knowledge, skills, and abilities (KSAs) of stewardship that can be taught, learned, and assessed across an entire higher education curriculum in any discipline. A critical motivating factor in the Rios et al. work was to contribute to achieving the following aspiration: “Upon entry into practice, all professionals assume at least a tacit responsibility for the quality and integrity of their own work and that of colleagues. They also take on a responsibility to the larger public for the standards of practice associated with the profession.” (Golde & Walker, 2006, p. 10)[8]. In order to promote this responsibility, and engage all pre-professionals in undertaking and perpetuating it, Golde & Walker and the Carnegie Initiative on the Doctorate introduced the concept of “the steward”. In his contribution to the Carnegie Initiative on the Doctorate, Bass (2006)[3] discussed doctoral education in mathematics as being primarily driven by the *disciplinary* considerations of mathematics, which fits with the original stewardship definition. However, Bass argues that modern (as of 2006) considerations of this field required augmenting the historical *disciplinary* perspective with a then-new emphasis on mathematics as a profession as well ([3]p. 102). Notably, Bass argued that the “steward of mathematics” is a concept that should encompass both the professional and the disciplinary attributes of practice. In the absence of mathematical ethical guidelines, stewardship is an existing vehicle for the introduction of the knowledge, skills, and abilities needed for ethical mathematical practice that is independent of the specifics of a code.

2. What is stewardship?

In order to be an effective steward, every (relevant) aspect of the definition must be demonstrated: “… creatively generate new knowledge, critically conserve valuable and useful ideas, and responsibly transform those understandings through writing, teaching, and application”. Through a cognitive task analysis (Clark et al. 2008)[5] applied to this definition, Rios et al. derived the following specific KSAs:

Requisite knowledge /situational awareness
Create and/or generate new methods/new knowledge
Critically evaluate extant knowledge
Conserve ideas (or not, if deemed rejectable & non-conservation is justified)
Responsibly write
Responsibly teach/mentor/model
Responsibly apply disciplinary knowledge
Responsibly communicate

Although the construct of a steward of the discipline was introduced in the 2001-2005 Carnegie Initiative on the Doctorate, Rios et al. (2019)[18] were the first to formalize the aspects of stewardship that can be taught, learned, and assessed across an entire higher education curriculum in any discipline. To promote stewardship of mathematics, Bass contemplates aspects of the practitioner *beyond* the context of working (doing research, teaching) with other practitioners and mathematicians in training. He asks, “In what ways does a mathematician function as a representative of the discipline in public arenas?” ([3] p. 111) The Mastery Rubric for Stewardship and its KSAs can map onto Bass’ disciplinary knowledge as well as teaching, service, and interdisciplinary and outward-facing aspects of the holder of a doctorate in mathematics. Importantly, four of the stewardship KSAs can be brought to bear throughout most mathematics courses:

Requisite knowledge /situational awareness
Critically evaluate extant knowledge
Responsibly apply disciplinary knowledge
Responsibly communicate

Assumptions and approximations are common elements of proofs and conceptual development in many, if not most, mathematical courses. This creates opportunities to leverage existing course material to direct student attention to the ramifications of: a) the limits of approximations; and b) the importance of assumptions being valid whenever the content is being applied, utilized, or adapted. The critical evaluation of extant knowledge is essential for scholarship, but it can also be helpful in engaging students' considerations of assumptions and approximations – how realistic these are in applications, and what might happen if assumptions are not met, or approximations do not hold. Similarly, responsible application of disciplinary knowledge must include recognition that assumptions and approximations must be plausible, otherwise the application of that knowledge (or those who rely on it) may fail. Recognizing the importance, and in some cases extent, of assumptions prior to applying disciplinary knowledge can help students to appreciate the importance of clear communication – both of the parameters of any given problem, but also of their solutions.

The other stewardship KSAs become more important for mathematics students as the progress in the major, and particularly if they choose to continue to graduate school.

The Mastery Rubric for Stewardship (Rios et al. 2019 [18]) describes the KSAs of stewardship, but it also concretely describes observable and reproducible, but flexible, performance level descriptions of each KSA at each developmental stage from the earliest ("novice") to independence ("journeyman"), and to the individual who is qualified by evidence to train others from novice through to independence and beyond, the "master" level, to use European guild terminology (Ogilvie 2014)[17]. Thus, not only does the Mastery Rubric for Stewardship describe a training trajectory that can be utilized by instructors or learners to begin, continue, or finalize their development into independent stewards of their discipline or profession, it also supports the achievement of the aspiration of universal capability for responsibility and integrity in the workplace articulated by Golde & Walker (2006)[8]. In fact, this was one of the motivating factors for the development of the Mastery Rubric for Stewardship.

Within the Mastery Rubric for Stewardship, the four stages of development are:
Novice: An early stage learner who does not "recognize that, or act as if, failures to act in a stewardly manner have ramifications beyond themselves. The novice stage represents the individual embedded in the acquisition of discipline-specific content. This could be an undergraduate declaring the major or an early-stage graduate student." The general description is "Has interest but limited experience in the discipline or profession, but is being introduced to the ideas and commitments that the Apprentice will build upon. Is discovering the importance of disciplinary and professional stewardship."
Apprentice: An individual who is actively engaged in study of the profession or discipline … developing the capacity to practice independently, but has not yet demonstrated ability qualification to do so." This individual is "learning the "tacit responsibility for the quality and integrity of their own work and that of colleagues", and the "responsibility to the larger public for the standards of practice associated with the profession" that Golde and Walker (2006) assume characterizes all those who enter "the workplace". The general description of the apprentice steward is "Actively engaged in study of the discipline and seeks opportunities to demonstrate and grow the KSAs. Developing the full range of Bloom's cognitive abilities, a greater awareness of their own limitations, and a commitment to professional and disciplinary stewardship."

Journeyman: "An independent scholar or practitioner—a steward of the discipline. … uniformly stewardly in their interactions with others in the disciplinary or professional community." The journeyman steward is generally described as "Demonstrates the KSAs and commitments of a steward of the discipline, including preserving disciplinary integrity. Is engaged in a disciplinary or professional community, and seeks additional opportunities to reinforce less-well developed skills."
Master: An individual whose qualifications to teach and mentor with respect to stewardship effectively are "recognized by evidence and consensus". The master steward is described as "An expert in the KSAs and someone to whom apprenticeship in stewardship can be entrusted. Formatively diagnoses and remediates the performance of KSAs, and develops and evaluates summative assessments for specific KSAs in support of stewardly development through the master level."

Considering the importance Bass (2006) [3] places on the incorporation of the development of the *professional* aspects of stewardship of mathematics into doctoral training programs, the possibility – and indeed necessity – of beginning to orient math students to these professional attributes *prior to the start of doctoral* e*ducation* is absolutely essential. While Golde & Walker introduce the construct of "the steward of the discipline", and Bass (2006) articulates that "the steward of mathematics" would embody both disciplinary and professional capacities, there is no real reason why "a strong sense of cultural awareness in students of the significance of their discipline in the larger worlds of science and society" ([3] p. 115) might not be initiated early in mathematics education.

Like every Mastery Rubric (Tractenberg, 2017)[21], the Mastery Rubric for Stewardship synthesizes KSAs with the articulated stages, and presents general performance-level descriptors of each KSA at each stage. The stages are recognizably distinct from one another, and performance of each KSA within each stage is concretely described using observable verbs from Bloom's taxonomy (Bloom et al. 1956[4]; see Kuh et al. 2016[11]). The Mastery Rubric for Stewardship, presented in Table 1, was designed and intended to promote the initiation of the development of stewardly attitudes among even the novices across disciplines. With trajectories that extend to the Master (who will be teaching and training new practitioners from the novice level up to the Master level), the Mastery Rubric for Stewardship is suitable for use in promoting stewardship and stewardly professional identities in students at all levels (not just those in doctoral programs).

3. **What is the difference between stewardship or *stewardly* practice of mathematics and *ethical* practice of mathematics?**

One core difference between stewardly and ethical practice in any profession, field, or discipline is the requirement for some structure that articulates what "ethical practice" actually -recognizably- is in order for practitioners to conform. The development of *ethical* practice standards is unusual historically – more typically, practice standards are articulated to clearly delineate for practitioners and consumers what is acknowledged to characterize the "expert" practitioner (see Tractenberg et al. 2015[22]). Professional practice standards (or "quality standards") are descriptions that are created, maintained, and endorsed by some accredited body and describe what "a skilled and experienced operator in the same type of industry or business sector as the Contractor would reasonably and ordinarily be expected to comply with"[1] (see Tractenberg et al. 2015[22]). Various national and international bodies have outlined "ethical guidelines" (American Mathematical Society, 1995) [1], furthering "the interests of mathematical scholarship and research"; a "code of ethics" (Mathematical Association of America, 2017)[12], which is specific to its employees (at work and organization-sponsored activities); and a "code of practice" (European Mathematical Society, 2010)[7] which comprises both "good practice and ethical behavior" specifically relating to mathematical research. Pertaining to mathematical scholarship, like the original conceptualization of stewardship, the European Mathematical Society code of practice tends to focus

[1] https://www.lawinsider.com/dictionary/work-practice-standard

attention, and limit responsibility, to doctoral level practitioners. Rios et al. intended to promote training in stewardship that can start well before the individual begins to practice, whereas ethical practice standards tend to pertain more specifically to those who enter (professional) practice.

The Mastery Rubric for Stewardship is excerpted in the next table. Three of the KSAs are presented with the performance-level descriptors that track development across the stages.

Table 1. The Mastery Rubric for Stewardship [18]

Developmental **stage**/ performance level of *stewardship* **Stewardship KSAs**	**Novice**	**Apprentice**	**Journeyman**	**Master**
Create and/or generate new methods/new knowledge	Has limited awareness of the knowledge and activities of the discipline, and limited exposure to the ethical issues involved in their creation and use. Learning that knowledge is *generated*; and that the creation of new methods or knowledge may have ramifications beyond the original intent.	Learning to create methods and knowledge in a manner that strengthens and advances the field and disciplinary community. Developing the ability to recognize when new methods or knowledge can be used for unethical ends, and how stewards of the discipline respond. Learning how to balance a commitment to strengthen and advance the discipline with advancing one's career.	Generates, and transparently communicates, new methods and knowledge to strengthen and advance the field. Considers how new ideas can be used for unethical ends, and models how to respond when such action occurs. Prioritizes the disciplinary community over metrics that devalue it. Challenges such metrics whenever possible.	Models, promotes, and teaches stewards to recognize and exhibit their responsibilities to the disciplinary community and society as they create and/or generate new methods and knowledge. Promotes transparency in the documentation of the new knowledge/methods to others in the disciplinary community and those outside it. Supports systems for professional assessment and developmental milestones for themselves, their mentees/trainees, and others in the community that are consistent with stewardly responsibilities.
Conserve ideas (or rejects ideas if non-conservation is justified)	Entering the field by learning about the fundamental ideas, thinkers, and accomplishments of the past.	Learning to conserve fundamental ideas of the field through engagement, application, relation, and extension, as well as qualification and critique.	Critically conserves the ideas that advance the field and preserve its integrity. Recognizes multiple perspectives, including cultural and	Trains stewards to recognize, understand, and critically evaluate the vetting that ideas in the field have/have had, including the influence of cultural and extra-disciplinary forces.

Developmental **stage**/ performance level of *stewardship* **Stewardship KSAs**	**Novice**	**Apprentice**	**Journeyman**	**Master**
	Attention is focused on remembering and understanding core (highly conserved) ideas; justifies neither their conservation nor rejection of ideas or arguments.	Learning to recognize processes by which ideas in the field are vetted and that re-evaluation and conservation are essential to the integrity of the field. Learning to describe and justify decisions about conservation or non-conservation. Learning how these decisions have shaped the history of the field and the ideas that are/have been conserved.	extra-disciplinary influences, in describing and justifying decisions of conservation or non-conservation of ideas, models, and views. Recognizes their role in shaping the field/ profession and its history.	Models, promotes, and teaches that conservation or non-conservation of ideas in the discipline or profession must be justified, and how to do so. Strives to instill in others, both in and outside of their own discipline, an understanding of the dynamics of the evolution of the field. Participates in the conservation, non-conservation, or rejection of ideas through teaching or training and enabling others to do so.
Responsibly write	Learning disciplinary writing standards with attention to the details of what must be recorded, how to construct written reports, and why responsible writing requires transparency.	Gaining greater proficiency in discipline-specific writing. Demonstrating increased sophistication in writing, including content, rhetoric and argumentation, and transparency and professional integrity.	Independently writes in the diversity of contexts and styles specific to the field, to generate, conserve, challenge, and reject field-specific knowledge and to engage others in and outside the field. Practices and promotes transparency in their	Trains stewards in the importance and execution of transparent, complete, and appropriate – responsible - writing within the field. May also train stewards in cross-disciplinary writing, and in writing for readers outside the discipline.

Developmental **stage**/ performance level of *stewardship* **Stewardship KSAs**	**Novice**	**Apprentice**	**Journeyman**	**Master**
			writing for the sake of the discipline and field.	
NOTE: * This cell encompasses the entire MR as well as the entire definition of the steward. The Mastery Rubric for Stewardship excerpts reprinted with permission from Rios et al. 2019 ([18]Table 1).				

In contrast to the lack of a universal mathematics practice standard, *stewardship* exists independent of disciplinary focus, and was demonstrated by Rios et al. (2019)[18], to have wide ranging relevance across a diverse set of disciplines (history, statistics, and neuroscience), and to be learnable and improvable across a career or educational trajectory. Thus, stewardship and stewardly mathematical practice can be taught and demonstrated (using the KSAs separately or together), beginning far earlier than doctoral level training and resulting in responsible practice by all who are trained in this paradigm. This can be initiated without a universally relevant ethical practice standard for mathematics (i.e., one that is not limited to employment or scholarly roles). Whether or not the intention is research or scholarship, stewardly mathematical practice is learnable and improvable.

Instructors can imagine that students taking one or a few courses in mathematics, but majoring in other fields, would be and remain at the "novice" stage with respect to stewardship of mathematics, whereas those who major in mathematics would be apprentices while undergraduates, aiming for "journeyman" status as graduate students (who engage in doctoral level work). As noted earlier, undergraduate mathematical instruction may focus on just four stewardship KSAs; if they do, then perhaps twice as much time and effort can be spent on those four rather than on all eight KSAs.

Another important consideration about stewardship is that teaching "ethics" or "integrity" can be daunting – and virtually impossible to document i.e., with replicable assignments that promote learning, and inform instructors about what learning has taken place. Assignments that meet principles of andragogy (and are appropriate for adult learning), are effective, and are appropriate for the preparation of future practitioners, are difficult to conceptualize. A typical approach is to use "interesting" or engaging cases for "discussion". One of the challenges for domains such as statistics, mathematics, computer science, and engineering, is that a great deal of material and instructional orientation for "ethical practice" or "ethical research" is actually derived from the more clinical or biomedical domains. As such, in institutions with a single "ethical research" training course for all incoming graduate students (for example), a great deal of time is focused on how to handle/work with human and animal research participants. For instructors who wish to "add ethical content" to a course where human and animal research participants are not a focus, these materials -while widely and easily available -are unlikely to promote the achievement of instructional objectives. They are also biased towards research and away from *how to practice mathematics ethically*.

A way to integrate all eight stewardship KSAs into a mathematics course is to focus on the approximations and assumptions that are needed in definitions, theorems (and corollaries, etc.), and proofs. Ensuring that students understand what the assumptions are, how far approximations truly extend, and the importance of articulating these is important to promoting a deep understanding of the definitions, theorems, and proofs. Emphasis on assumptions and approximations is authentic in all mathematics courses, and allows discussion or exploration of both the mathematical content and the potential impact of violations of assumptions, limits of approximations, and failures to disclose these on stakeholders like the public and the profession.

4. Crafting Learning Outcomes based on the Mastery Rubric for Stewardship

The following presents three examples of courses wherein the Mastery Rubric for Stewardship is utilized to generate actionable learning outcomes to integrate stewardship into an existing curriculum. These are: a stand-alone (new) course to orient learners, or the superimposition of orientation to the Mastery Rubric for Stewardship in an existing course (course 1); a stand-alone or integrating course intended to move students beyond simple orientation to stewardship or a capstone (course 2); and a longer-term independent study period where students' attention is on both the stewardship KSAs and also the learners' own self-assessment abilities. Each of the Nicholls [15] phases is discussed (and labeled) with respect to the type of course under consideration.

These materials are adapted from the forthcoming book describing Mastery Rubrics for curriculum development in higher education (Tractenberg, in preparation)[23], and follow Guidelines for the Development of Curriculum and Instruction (Tractenberg et al. 2021)[24]. In addition to following formal curriculum and instructional development guidelines, all learning outcomes are developed to be consistent with well-known criteria for learning outcomes drafting (e.g., Kuh et al. 2016[11]; Nilson, 2016[16]; National Institute for Learning Outcomes Assessment, 2016)[14].

Following the Nicholls (2000) paradigm (Figure 1), once learning outcomes, learning experiences, and assessments are organized, then content can be selected that will support learning outcomes, be consistent with learning experiences, and is aligned with the assessments.

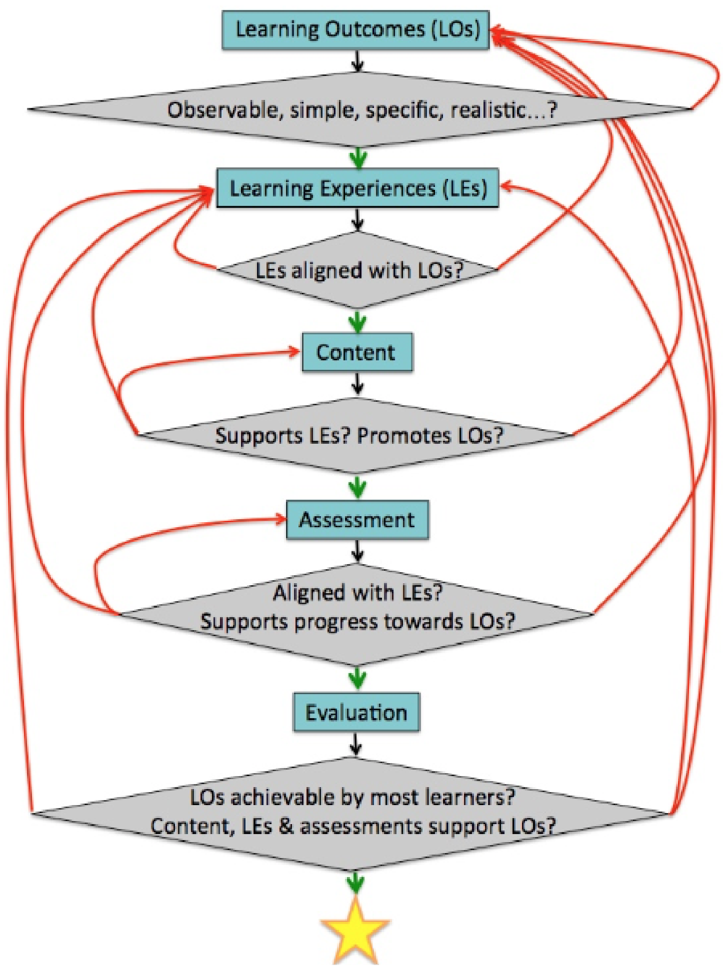

Figure 1. Nicholls' (2002)[15] five phases of curriculum and instructional development. Reprinted with permission from Tractenberg et al. 2021[24].

<u>Course 1</u>. Orienting students towards stewardship

Learning outcomes for a course wherein it is intended to orient the student to the Mastery Rubric for Stewardship must focus on what the students will do with the "new information" that the Mastery Rubric for Stewardship provides. This can be superimposed on an existing course, or be a new stand-alone course.

Course 1. Learning outcomes

Orientation to the Mastery Rubric for Stewardship involves – or, may primarily focus on – "requisite knowledge" (KSA 1) –which may also include professional or ethical practice guidelines for one or more disciplines or professions that are relevant for individual students or programs. For example, any existing guidelines for ethical mathematical practice, or the ones for statistics and data science (American Statistical Association, 2022) or computing (Association of Computing Machinery, 2018), can be utilized. An orienting course can incorporate many diverse examples and the diversity of emphases (on illegal vs. legal or ethical/unethical conduct) of the guidance that such documents can provide. The *teaching* objectives of the orienting course are to promote the creation of a new schema or mindset for the student – that stewardship comprises a learnable, improvable set of knowledge, skills, and abilities; and that they can develop, and self-assess this development, of the KSAs of stewardship in progressive – manageable- ways. Whether students will become mathematicians (as a profession) or will be using mathematics in their jobs, a stewardly mindset and approach to mathematical practice can be initiated. Examples of learning outcomes include:

After the course, students will be able to:

1. Rank order the KSAs of stewardship according to the student's perception of their importance to competent and ethical practice (of mathematics or whatever their major/discipline is).
2. Describe how a given mathematics course contributes to their sense of professional identity, compare the contribution of such a course to this sense with and without the construct of stewardship.
3. Relate stewardship KSAs to the conceptual content of the course, e.g., proofs, definitions, theorems (or whatever is most relevant for their major and the role of the mathematics courses in their major).
4. Justify how many mathematics courses must be taken before a student owes a duty of stewardship for the discipline of mathematics.
5. Explain the relationships between the Mastery Rubric for Stewardship and the overall curriculum in their degree/program.
6. Describe 2-4 differences between a transcript record of their participation in this course and a self-assessment of their performance/capabilities/achievements/plans using the Mastery Rubric for Stewardship.
7. Carry out and explain a self-assessment on each of the stewardship KSAs at the end of the course, and compare themselves with where they were at the start of the course.

Course 1. Learning experiences

Orienting students to the fact that the stewardship KSAs are learnable and improvable, and that the overall construct can in fact help them formulate a professional identity, can increase student engagement with the orienting course. Discussion, role playing, think-pair-share, and other active learning experiences are essential in an orienting course, particularly because the orientation is to a new way of thinking about learning, and about self-assessing that learning, with which learners may have no or limited experience. Depending on the context and the availability of resources, instructors can try a lot of different learning experiences with the stewardship orientation course – but the key is to make it as active as possible, and lecture as little as possible. If there is some specific learning experience that must be used (e.g., if the course must be delivered asynchronously for some reason, or via hybrid online/face to face), the learning outcomes may need to be refined so they are compatible with the learning experiences. Tying stewardship KSAs to an understanding of assumptions and approximations -and impacts of failures of assumptions and approximations - within mathematics courses from a stand-alone course can be streamlined depending on the point in the curriculum where the course is offered. For later-in-major courses, fewer students may be enrolled so peer-to-peer discussion, role-playing, and other learning experiences can be utilized. If the course is intended for early in the curriculum, there will need to be more explicit activities to learn and practice assessing the impacts of deviations from assumptions/approximations on stewardly practice of mathematics.

Course 1. Assessments

Assessments must be aligned with the learning outcomes. Short answer and essay assessments are likely to be the most supportive of the learning outcomes outlined above; however, assessments should also be supportive of the learning experiences that are chosen. If the learning experiences are primarily utilizing written material, or video-based interactions, then assessments could be done similarly.

Course 1. Content

The content of a course for orienting students to the construct of stewardship or to the Mastery Rubric for Stewardship can vary widely, particularly if students come from – or identify with – different disciplines. In the KSA on requisite knowledge, there is an emphasis on "practice standards". However, there are no such standards universally accepted for "mathematical practice (as of December 2021); and not all students will know what "profession" they will want to practice. Therefore, it might be helpful to have students identify *any* two practice standards – e.g., one that seems closest to what they intend to pursue as a career, and one that they think would be a good "general guide" -or, one that they would think would <u>not</u> be useful as a general guide given their target career. Comparing and contrasting these is an excellent way to engage with the idea of community-based standards, practice standards, and the self-determination of professional identity. If diverse scientific (or other) disciplines are brought in as examples (or, students are encouraged to identify, and then compare and contrast diverse disciplines using the Mastery Rubric for Stewardship), the content can be very diverse indeed. The diversity of knowledge, skills, and abilities that are required for successful scientific practice in the modern world can serve to highlight different career options as well as different disciplinary silos and intersections. Note that diversity in content may have implications for assessment; e.g., essays that specifically require comparison and contrast are more complex and might need to be longer than simple descriptive ones. New learning outcomes that are aligned with more complex essays may also be needed.

Course 1. Evaluation

Evaluation of a course designed to orient students to the Mastery Rubric for Stewardship would follow directly from the learning outcomes. Within the course, specific assignments or activities may be identified as contributing towards, or demonstrating the achievement of, specific learning outcomes. The use of formative feedback or assessments (the essays in the example given here) can also generate information about the effectiveness of specific assignments or activities; particularly with respect to whether and how these assignments and activities contribute towards, or allow demonstration of the achievement of, specific learning outcomes. The strongest and weakest learning outcomes could be identified this way – and, ideas of how to strengthen the weakest learning outcomes – either to modify them because the level was set too high; or to modify the assessment if it simply wasn't well aligned, would be generated. The course's effectiveness could be summarized by describing the % of completers who met each learning outcome. Creating an "optional" learning outcome, focusing on a plan to continue to develop the stewardship KSAs, and following up with students to determine if any of them achieved this plan, would generate concrete evidence of the sustainability of the learning the orienting course provided. The features of this orientation-to-stewardship course can be superimposed on existing courses, by choosing one problem per week to focus attention on assumptions and approximations, and the potential for impact on stakeholders – including the profession – when these are not met (assumptions) or are exceeded (approximations).

Course 2. Integrating stewardship beyond an orientation

An integration-of-the-Mastery Rubric for Stewardship course would go beyond just orientation to the construct of stewardship for more advanced students. Evidence that students are in fact integrating the stewardship KSAs into their thoughts and work as they progress through a curriculum or major might be focused on either movement towards the Apprentice level on relevant KSAs, or possibly by more extensive development on one KSA (e.g., one they've focused on since last considering their evidence and the Mastery Rubric) than on any of the others. In some cases, an integrating course like this could be, or be part of, a capstone – where students pull together (or create) evidence of performance at the target KSA levels that represents their individualized synthesis of what they have learned – about their discipline but also about professional practice in the modern world. More advanced students within a mathematics major could begin to consider the varieties of decisions they make depending on the content and its application/applicability. The impact of mathematical practice on various stakeholders – which, in a stewardship model, must include the profession itself – can be discussed.

Course 2. Learning outcomes

Learning outcomes for a course intended to promote the *integration* of the stewardship KSAs into their habits of mind must focus on what the students can do, or have done, with the "new information", and – to represent the otherwise totally abstract construct of "integration", - articulation of what they may intend to do later. The *teaching* objectives of the integrating course are to reinforce the new schema about being/becoming a steward, and self-assessment in the stewardship KSAs, for the student. Examples of learning outcomes for the integrating course, include:

After the course, students will:

1. Describe how the KSAs of the Mastery Rubric for Stewardship have been featured in their courses up to this point of their program/major;
2. Discuss the relevance of stewardship KSAs in "real life", i.e., when applying their acquired mathematical knowledge outside of schoolwork.
3. Rank order the stewardship KSAs according to the student's perception of their importance to competent and ethical practice (in their domain, mathematics or otherwise).
4. Explain the relationships between the Mastery Rubric for Stewardship and the overall curriculum in their degree/program.

5. Describe 2-4 differences between a transcript record of their participation in this course and their self assessment of their performance/capabilities/achievements/plans on the Mastery Rubric for Stewardship.
6. Effectively self-assess: identify strengths and weaknesses in their own performance of any given KSA and identify the level at which the performance is best characterized.
7. Explain their own self assessment on each KSA at the end of the course, and compare their end-of-course performance with where they were at the start of the course.
8. Make and self-evaluate/revise a plan for improving their performance on at least two KSAs to a more advanced stage over the next (course, year, or other reasonable period).

Students who are positioned to integrate the stewardship KSAs into their thinking will be more advanced than those who were beginning to orient their thinking towards it. The 7th learning outcome is specifically included as an exercise to evaluate (self-assess) the student's evidence that they are not only "on the board", but to begin to concretely describe themselves in terms of changes in their performance levels and capabilities on all relevant KSAs. This level of self-assessment and reflection will require relatively high Bloom's level cognitive skills, and the teaching and practice in this integration course will require practice with feedback on self assessment (learning outcome #8).

Note that these learning outcomes reflect the intention of getting students to engage with the stewardship KSAs as they relate to themselves more explicitly than in the orientation course. The authenticity (and thereby, utility) of the Mastery Rubric for Stewardship will not have changed since the orientation course, but the student's awareness of what about the Mastery Rubric for Stewardship KSAs is actually authentic may have changed over time. By engaging with the Mastery Rubric for Stewardship in these ways, the student continues to articulate for themselves the relevance of the Mastery Rubric for Stewardship KSAs for their work and thinking. Although "integration" is not observable, learning outcomes like the eight listed here can support the desired integration by employing concrete and observable tasks at the appropriate Bloom's levels. Like Course 1, the features of this integrating-stewardship course can be superimposed on existing courses. Alternatively, capstone courses can incorporate stewardship elements, although these would be most successful if there was also an orienting-to-stewardship course or course content woven throughout the program.

Course 2. Learning experiences

Depending on the program in which both an orienting and an integrating course would fit, there may be fewer students enrolling in an integrating course than the orienting one (this was our experience with PhD students in the MR-ER courses). This means that learning experiences can be even more learner-centered and individualized. The deeper level of engagement with the Mastery Rubric for Stewardship KSAs – as indicated by the learning outcomes – will need to be reflected in each class meeting – or the one-on-one meetings if that is convenient/efficient. A focus on peer-to-peer instruction, and peer evaluation, may be important for time management but these approaches, while requiring intensive instructional effort up front and focused feedback and monitoring in early experiences, can quickly become efficient tools that reinforce learning as well as keeping the instruction learning and learner centered. Note also that teaching your students to teach effectively adds more content to the course – not everyone will need to be a teacher/have presentation skills! So, to utilize peer-to-peer teaching effectively, learning experiences that focus on teaching and practice with feedback for the peer activities are needed – and this must be accommodated in the overall course.

Course 2. Assessments

Assessments must be aligned with the learning outcomes. "Grading Rubrics" (Stevens & Levi, 2005)[20] can be constructed and shared with learners, in order to encourage their specific self-assessment. Instructors' attention is then on both the typical stewardship-oriented learning outcomes and additional self-assessment learning outcomes -which must be specified and shared with learners if these rubrics are to be utilized. The rubrics provide structure for all students – even though their content may vary widely – and enables consistent formative feedback from the instructors. This also allows peer-feedback and other peer-to-peer discussions and assessments. The use of task-specific grading rubrics will also enable consistent formative feedback from the instructors.

Course 2. Content

The content of a course for orienting students to the Mastery Rubric for Stewardship can vary widely, particularly if students come from – or identify with – different disciplines. In the KSA on requisite knowledge, there is an emphasis on "practice standards" – and not all students will know what "profession" they will want to practice. It might be helpful to have students identify two such standards – one that seems closest to what they intend to pursue as a career, and one that they think would be a good "general guide". Comparing and contrasting these is an excellent way to engage with the idea of community based standards, practice standards, and the self-determination of professional identity. If diverse scientific (or other) disciplines are brought in as examples (or, students are encouraged to identify, and then compare and contrast diverse disciplines using the Mastery Rubric for Stewardship), the content can be very diverse indeed. The diversity of knowledge, skills, and abilities that are required for successful scientific practice in the modern world can serve to highlight different career options as well as different disciplinary silos and intersections.

Course 2. Evaluation

Evaluation of a course designed to get students to integrate the Mastery Rubric for Stewardship KSAs into their habits of mind will follow directly from the specific learning outcomes that are utilized. Grades that are given on assessments will also generate information about the alignment of assignments and specific learning outcomes. The strongest and weakest learning outcomes could be identified this way – and, ideas of how to strengthen the least-supported or least-often achieved learning outcomes – either to modify them because the level was set too high; or to modify the assessment if it simply wasn't well aligned, would be generated. Course effectiveness could be summarized by describing the % of completers who met each learning outcome. The inclusion of an additional learning outcome that involves a plan to continue to develop the Mastery Rubric for Stewardship KSAs, and following up with students to determine if any of them achieved this plan, would generate concrete evidence of the sustainability of the learning the integrating course provided.

<u>Course 3</u>. Documenting Achievement of KSA-specific Learning Objectives

Stewardship courses with the learning outcomes listed in either of the other two courses should generate evidence that students are in fact oriented to the Mastery Rubric for Stewardship and that it can support their learning going forward (as self-directed learners, if need be). In addition to learning about the stewardship KSAs, students also need to develop the schema to understand the evidence they have, or would need, in order to convince you (or any other independent evaluator) that they are indeed performing a given stewardship KSA at their target level. Courses that involve the Mastery Rubric for Stewardship could therefore be an important way to also orient students to the ideas – and importance – of self assessment and metacognition. The next table lists learning outcomes that could be used as challenges for learners to grow – and document growth in – each of the stewardship KSAs. In a capstone year, these learning outcomes could be used as “independent-study” challenges, or as peer-to-peer or peer-evaluated activities. Table 2 presents example learning outcomes that could be targeted for each – or a subset of – the stewardship KSAs over a capstone year or other independent study, longer term instructional opportunity.

Table 2. Learning outcomes by stewardship KSA for Course 3, a year-long program [23].

At the end of this (capstone/challenge year), stewards-in-training will be able to:							
Requisite knowledge/ situational awareness	*Create and/or generate new methods/ new knowledge*	*Critically evaluate extant and emerging ideas*	*Conserve ideas (or not, if deemed rejectable & non-conservation is justified)*	*Responsibly write (disciplinary scholarship)*	*Responsibly teach/ mentor/ model (formally and informally)*	*Responsibly apply the knowledge and principles of the discipline*	*Responsibly communicate (outside of scholarly venues)*
Summarize given ethical code(s) or rules. Match a profession with an applicable code or set of standards. Identify shared objectives from diverse disciplines as reflected in their ethical code.	List the levels of support/ strength of evidence for characterizing information as “what is known” in a field. Identify topics and concepts that have greater or lesser potential to impact stakeholders.	Differentiate *claims* from evidence in arguments, including proofs. Explain the impact of failures of assumptions or approxima-tions for applications of core concepts. (general, not applied to the discipline at this stage.)	Recognize fundamental ideas, thinkers, and accomplish-ments that support modern practice in the field. Discuss examples of conserved and rejected ideas, and discuss the merits and problems caused when new ideas replace older ones.	Describe disciplinary writing standards. Compare the standards of mathematical writing with another discipline. List standards details, including what must be recorded and how to construct written reports. Explain why “responsible writing” requires transparency.	Describe, and recognize features of, responsible teaching and mentoring. Even if not teaching or mentoring, discuss the stewardly rationale for responsibly teaching and mentoring. Identify key stakeholders in responsible teaching and mentoring.	Describe features and examples of ethical *and not-ethical* application of knowledge and ideas in the discipline. Discuss the importance of communication in the responsible application of domain knowledge.	Differentiate communication that is /is not discipline-specific and appropriate for the audience. Explain how communication that is not appropriate for the audience is not stewardly. Identify benefits and harms to stakeholders that arise from stewardly communication.

These learning outcomes could be tailored for more-applied or more-theoretical mathematical applications or jobs. They can also be adapted to encourage math students to contemplate how stewardship of the mathematics profession by a mathematics practitioner might differ (or not!)

from stewardship of a different discipline or profession where mathematics practice is important, but not central. learning experiences, assessments, and content would all be dependent on both these learning outcomes and the specific role the course is intended to fill in the learners' curriculum or educational purpose. Evaluation of a year long course or program with these learning outcomes would be similar to what has been described. The Nicholls model supports systematic evaluation of instruction (learning experiences) and learning outcomes, as well as identification of opportunities for new/different content and assessments.

5. How are learning outcomes based on stewardship moving instruction in mathematics practice towards supporting ethical mathematical practice?

The foregoing has argued in favor of instructors' integrating stewardship into mathematics (higher) education, supporting this argument with both structure for reproducible teaching with concrete, evaluable, and actionable learning outcomes and the rationale that stewardship can be taught effectively since a universal set of ethical guidelines for mathematical practice do not yet exist. One KSA of the Mastery Rubric for Stewardship relates to teaching, mentoring, and modeling of ethical mathematical practice. While undergraduates might not have or get experience with these roles, they can still consider the responsibilities to teach, mentor, and model in a stewardly way. Those who do not continue to graduate programs in mathematics can still maintain a stewardly mentoring or modeling style when they go on to apply their mathematical knowledge in their jobs and roles.

Table 3 outlines how courses based on the Mastery Rubric for Stewardship, and one based on a professional ethics code (now or in future), can be used to meet National Academy of Engineering criteria (National Academy of Engineering, 2013) [13] for ethical training goals (Kalichman, 2013: p. 11)[10].

Table 3. How characteristics of ethical training goals can be accomplished with courses based on ethical practice standards **or** learning objectives derived from the Mastery Rubric for Stewardship

NAE criterion	**Professional Ethics Code for semester course content**	**Stewardship KSAs & developmental trajectory**
1. Goal should represent something relevant to the ethical or responsible conduct of research or practice.	Ensures that at least one code of professional conduct will be introduced and discussed. (however, as of June 2022, these would be focused on scholarship following AMS, or MAA employment following MAA). No development over a career is included in the existing codes/guidelines.	Professional and disciplinary stewardship are relevant to ethical practice [18], and can be developed up to a specific target level. Once ethical practice standards exist, learning outcomes about stewardship can be augmented with ethical practice learning outcomes.
	Gap is glaring: no or rare integration of *ethical mathematical practice* in programs of study; low relevance of research-oriented	

2. Goal should identify and address some concrete deficiency.	codes for *non-research* programs and practitioners. Engineering ethics courses and available materials do include professional codes/conduct, but may not be accessible or relevant across mathematics courses. Stakeholders in engineering practice and applied mathematics may be so different as to be unrecognizable. stewardship addresses this gap, as it is immediately recognizable for researchers in, and users of, mathematics.	
3. Achievement of the goal should be independent of other (possibly related) goals.	A course may meet an institutional “responsible conduct of research training requirement”, without introducing a relevant code of conduct. Not-specific material may fail to orient students towards ethical practice. Stewardship is not orthogonal to ethical practice.	Stewardship of the profession and discipline [5] can be an achievable target of a mathematics curriculum, but would also be useful training for graduates who choose any profession.
4. Goal should be actually and observably amenable to an active intervention.	Both faculty and students can use the Mastery Rubric for Stewardship developmental trajectory to ensure that adequate opportunities are given/taken in order to compile evidence of the requisite level of performance on every domain and for every stewardship KSA. All performance level descriptors can support observable and assessable learning objectives- by the instructor and by the students themselves.	
5. Achievement of the goal should be documented/ documentable with either quantitative or qualitative <as appropriate> outcomes.	Assignments (or capstone) could require the students’ self assessment with justification of their assessment (and faculty assessment of each student), but existing codes/guidelines are specific to *research*. They are less relevant to math students and practitioners who do not do/do not want to do research.	Assignments (or capstone) should require the students’ self assessment with justification of their assessment (and faculty assessment of each student), and engaging in self-assessment through stewardship can start *before* ethical mathematical practice standards exist; early in an academic career; and be useful over time, across disciplines, and separately from research.
6. Achievement of the goal should result in a change that is detectible and meaningful.	Students learning mathematical scholarship who participate in a course structured with a mathematics code of ethics would be able to demonstrate their knowledge of, and compliance with, such codes.	All math students at any level who participate in a course structured with learning objectives based on the Mastery Rubric for Stewardship would be able to demonstrate their knowledge of, and compliance with, the definition of professional <u>or</u> disciplinary stewardship [5].

7. Goal should be feasible.	The learning objectives outlined here can be feasibly integrated into existing courses; new stand-alone courses can also be created to support the accomplishment of stewardship-oriented learning objectives. Training faculty to encourage, elicit, and provide formative feedback on stewardship as required by the matrix is one challenge; another is getting faculty to accept this challenge.

The learning outcomes articulated for the three courses above would support courses that meet the NAE criteria for effective ethics instruction. Additionally, Diamond (2008: p.16 [6]) suggests a set of general questions, which have been adapted for the evaluation of instruction in stewardship:

1. Are there a clearly defined mission and a vision for training in stewardship?
2. Are the educational (professional development) goals of each training opportunity consistent with the stated priorities of the a) funder, and b) end user (instructors, departments)?
3. Has the end user described how they intend to evaluate the extent to which their professional/training development goals are achieved?
4. If relevant for the end user, do they have a plan to evaluate and document educational effectiveness of the stewardship training?
5. Are both qualitative and quantitative evidence of student achievement systematically collected by the end user?
6. For instructors who use or share stewardship training materials, is student achievement determined by how well a student meets the articulated learning goals of the course?
7. Does every student have the opportunity – and recognize their responsibility – to reach the target level of achievement for the stewardship training?

Either the seven NAE objectives for ethics training goals, or Diamond's questions, or both could be utilized to guide the development of stewardship instruction, or to document efforts to implement this instruction. These tools can be used in an instructor's teaching portfolio or to promote discussions within a program in support of implementing new training in stewardship. Successful integration of stewardship into mathematics courses would result in greater self-identification of mathematics practitioners as stewardly, including instructors.

6. Discussion

Currently, professional ethics are not taught in math courses/degree programs – because there is no set of universal practice standards, among other reasons (it is not typically taught in statistics and data science programs, either, and these domains do have the American Statistical Association's Ethical Guidelines for Statistical Practice (American Statistical Association 2022) [2] available and relevant). A coherent conceptualization of ethical mathematical practice simply does not exist – nor is there any emphasis on these critical concepts in math courses that are taught by other disciplines (e.g., biology, psychology, policy, business, economics, etc.). Rios et al. (2019)[18] specifically pointed out that "There is no KSA for "ethical practice" because the entire stewardship model implicitly reflects a virtue ethics approach to professional conduct and identity. The focus in the Mastery Rubric for Stewardship is on taking, and demonstrating, responsibility in the dimensions of stewardly practice, enabling ethical practice even if there are no/no specific ethical practice guidelines available." ([18] p. 21 of 27). While this is an implicit reflection, everything in - and derived from – the Mastery Rubric for Stewardship is intended to be concrete, observable, and developmental. STEM practitioners have a responsibility to consider the ethical, legal, and social implications (ELSI) of their work. What is *infrequently* discussed is that those who utilize, and base decisions on, the outputs of those STEM disciplines *also* have a responsibility to consider the harms and benefits that may accrue from a reliance on STEM outputs. Rios et al. sought specifically to support the integration of stewardship into higher education curricula – STEM fields included. The learning outcomes presented here can be used to accomplish this aim with mathematics education.

There tends to be an academic-setting bias in STEM ethics education; for example, the focal activities are "conduct of research" (i.e., training is focused on "the responsible conduct of research" and not "the responsible application of disciplinary knowledge"). Moreover, "scientific misconduct" is specifically defined to be "fabrication, falsification, and plagiarism in proposing, performing, or reviewing research, or in reporting research results" (Steneck 2007, p. 20 [19]; (45 CFR 93.103)). The KSAs of stewardship go beyond these constructs; for example, the steward <u>responsibly applies</u> disciplinary knowledge: i.e., s/he "Applies the KSAs [inherent in a discipline] in a way that preserves and advances the field by demonstrating integrity, transparency, and respect in interactions within and outside of the profession or discipline". This individual also: "exercises professional practice standards [where they exist] and recognizes situations in which stewardship should be modeled and/or applied with respect to themselves and others, and to interactions within and outside of the profession or discipline"; "generates, and transparently communicates, new methods and knowledge to strengthen and advance the field"; and "considers how new ideas can be used for unethical ends, and models how to respond when such action occurs." (Rios et al., 2019 Table 1 [18]). Thus, more than simply arguing against research misconduct, the KSAs of stewardship can feasibly support responsible mathematics practice at all levels -and in both research and applied settings. Efforts are under way to create community-based ethical guidelines for mathematical practice. Rios et al. analyzed the alignment of stewardship KSAs with the ethical practice standards of the disciplines of history, statistics, and neuroscience and found considerable overlap between these KSAs and all of the disciplinary standards. Thus, a focus on a stewardly approach to mathematical practice can be implemented early in mathematics education, and will not conflict with the ethical practice standards that are eventually developed. Instead, these future guidelines will only serve to more clearly define what "responsible mathematical practice" looks like for stewards of mathematics.

Where ethical guidelines or professional practice standards exist, they can promote observable professional behavior, and trust in their work. The Rios et al. model of stewardship accommodates -*but does not require* -ethical practice standards as a core knowledge-based component of stewardship, but goes far beyond this particular type of knowledge. With or without ethical practice guidelines, instructors can initiate *apprenticeship* in **stewardship**, and strive to get all mathematics practitioners to the independent (*journeyman*) performance level of stewardship. A cohort of competent mentor stewards is essential to this effort, and the Mastery Rubric for Stewardship provides concrete and observable outcomes for qualifying, and documenting one's qualification, to teach, mentor, and assess others in their development of stewardship.

7. Conclusions

> *I hold every man a debtor to his profession; from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves, by way of amends, to be a help and ornament thereunto. This is performed, in some degree, by the honest and liberal practice of a profession; where men shall carry a respect not to descend into any course that is corrupt and unworthy thereof, and preserve themselves free from the abuses wherewith the same profession is noted to be infected: but much more is this performed, if a man be able to visit and strengthen the roots and foundation of the science itself; thereby not only gracing it in reputation and dignity, but also amplifying it in profession and substance.*
> — Sir Francis Bacon
> Opening sentences of *Preface, Maxims of Law* (1596), in *The Works of Francis Bacon: Law tracts. Maxims of the Law (1803), Vol. 4, p. 10*

These sentiments from Sir Francis Bacon in 1596 (expanded beyond just men!) reflect the current interest in stewardship for professions as well as academic disciplines. For future mathematicians as well as those who will use math at work, education in mathematics can accommodate curriculum-spanning training, and documented achievement, in the relevant knowledge, skills, and abilities (KSAs) that are required for stewardly use of - as well as contributions to – mathematics. That is, stewardly aspects of mathematical practice should be taught to all who will use math, not solely to those who will contribute to the mathematical knowledge base through their scholarship. Integrating stewardship teaching and learning into all courses, for those who complete training in the discipline (i.e., majors, minors and those in professional and trade training programs) and those who will utilize outputs from and/or manage practitioners in a STEM discipline, will move the STEM fields towards the aspiration that *all* professionals recognize their responsibility, and are able, to maintain "the quality and integrity of their own work and that of colleagues". Mathematics students who complete their disciplinary training with an explicit focus on this responsibility - because it has been coherently integrated into a developmental trajectory within their curriculum – will be inculcated to promote and value stewardly practice however they engage with STEM fields. Those who will utilize STEM outputs, or become managers in STEM fields, could then legitimately be expected to bear their responsibility for "the quality and integrity of their own work and that of colleagues". Whether a community-based standard for ethical mathematical practice or workplace-specific policies for ethical practice are available, initiating training in stewardship can prepare students to engage with such standards and policies. If both, or neither are available, then training stewards of mathematics can "confer a strong sense of cultural awareness in students of the significance of their discipline in the larger worlds of science and society, and of the expectation that they will serve as emissaries of their discipline in the outside world." (Bass 2006, pages 115-6)[5].

In Rios et al. 2019, recognizable stages of performance of these KSAs were also described, together with performance level descriptors of each KSA at each stage. These concrete KSAs of stewardship can be taught, assessed, and practiced throughout the curriculum and career. Rios et al. based this work on/synthesized well-established theoretical and empirical results from cognitive and educational sciences in order to support the integration of stewardship by different instructors, as well as by the self-directed learner, throughout a curriculum or learning path. The learning sciences/evidence -based developmental trajectories for the KSAs of stewardship can be leveraged to teach, provide practice in, and document achievement of, stewardly practice among professionals in STEM fields as well as among those whose professions utilize or depend on the outputs of STEM fields. Although it originated with doctoral education (2005) and was only expanded for college level, curriculum spanning programs in 2019, the ideas of stewardship can be introduced in high school as well.

The Mastery Rubric for Stewardship (Rios et al 2019)[18] describes the developmental trajectory supporting disciplinary, professional, or vocational stewardship, to promote: a) the formal integration of the stewardship model into PhD training; b) the formalization of master-level stewardship to support evaluable mentorship that can then foster a commitment to stewardship in new students and mentees; and expansion of the model of stewardship to include profession and vocation.

"**Professional identity formation means becoming aware of … what values and interests shape decision-making**." (Trede, 2012, p. 163)[25]

Recent work in the development of professional identity suggested that "(S)tudents could learn more from their experiences if they were more explicitly guided to look out for certain aspects of professionalism and given further opportunities to discuss and critique their observations and experiences." (Grace & Trede, 2011 p. 12)[9]. Trede & McEwen (2012) [26] also argue that the development of professional identity is supported by instruction and practice in justifying professional decisions: "…students need to learn to articulate the reasons behind their actions." (Trede & McEwen, 2012 p. 10)[26].

Whether or not mathematical ethical practice standards exist, or are perceived to be generalizable for all practitioners, stewardship is an existing vehicle for the introduction of the knowledge, skills, and abilities needed for ethical mathematical practice that is independent of a code should one be identified or developed. However, should such a code or policy be available, the KSAs of stewardship would accommodate it as part of "requisite knowledge". Stewardship is general, and can be applied *right now* to promote responsibility among potential, developing, and expert mathematics practitioners, as well as those who may study mathematics for its applications rather than for research and scholarship.

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