Machine Learning

Classification using Bayesian Belief Networks

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Outline

- Classification
  - Bayesian Belief Networks
  - Hidden Markov Models
  - Support Vector Machine
    - Maximum Margin Linear Separators
    - Quadratic Programming solution to finding maximum margin separators
    - Kernels for learning non-linear functions
  - Classification using k Nearest Neighbour Algorithm

Bayesian Belief Networks

- Naive Bayes Classifier OR Naive Bayesian Classifier
  - It is useful for text classification and spam filtering
- Bayesian Network OR Bayesian Belief Networks OR Belief Networks or Bayes Nets
  - It is useful in medical diagnosis, bioinformatics, and NLP

Naive Bayesian Classifier

- **Naive Bayesian Classifier**
- It is based on Bayes' theorem with independence assumptions between predictors.
- It requires less data to get a good result in many cases.
- A Naive Bayesian model is easy to build,
  - with no complicated iterative parameter estimation which
  - makes it particularly useful for very large datasets.
- Despite its simplicity, the Naive Bayesian classifier often does surprisingly well and
- It is widely used because it often outperforms more sophisticated classification methods.
- Use it if you're only interested in solving a prediction task: use Naive Bayes.
- Algorithm:
  - Bayes theorem provides a way of calculating the posterior probability, P(c|x), from P(c), P(x), and P(x|c).
  - Naive Bayes classifier assume that the effect of the value of a predictor (x) on a given class (c) is independent of the values of other predictors.
  - This assumption is called class conditional independence.

Naive Bayesian Classifier

- P(c|x) is posterior prob. of class (target) given predictor (attribute).
- P(c) is the prior probability of class.
- P(x|c) is the likelihood which is the prob. of predictor given class.
- P(x) is the prior probability of predictor.

Likelihood

Class Prior Probability

$$P(c \mid x) = \frac{P(x \mid c)\,P(c)}{P(x)}$$

Posterior Probability

Predictor Prior Probability

$$P(c \mid \mathrm{X}) = P(x_1 \mid c) \times P(x_2 \mid c) \times \cdots \times P(x_n \mid c) \times P(c)$$

Naive Bayesian Classifier

- Example:
- The posterior prob. can be calculated by first, constructing a frequency table for each attribute against target.
- Then, transforming the frequency tables to likelihood tables and
- Finally use the Naive Bayesian equn to calculate the posterior probability for each class.
- The class with the highest posterior probability is the outcome of prediction.

$$P(x \mid c) = P(Sunny \mid Yes) = 3/9 = 0.33$$

Frequency Table		Play Golf	
		Yes	No
Outlook	Sunny	3	2
	Overcast	4	0
	Rainy	2	3

Likelihood Table		Play Golf		
		Yes	No	
Outlook	Sunny	3/9	2/5	5/14
	Overcast	4/9	0/5	4/14
	Rainy	2/9	3/5	5/14
		9/14	5/14	

$$P(x) = P(Sunny) = 5/14 = 0.36$$

$$P(c) = P(Yes) = 9/14 = 0.64$$

Posterior Probability: $P(c \mid x) = P(Yes \mid Sunny) = 0.33 \times 0.64 \div 0.36 = 0.60$

Naive Bayesian Classifier

- The zero-frequency problem
  - Add 1 to the count for every attribute value-class combination (Laplace estimator) when an attribute value (Outlook=Overcast) doesn't occur with every class value (Play Golf=no).
- Numerical Predictors
  - Numerical variables need to be transformed to their categorical counterparts (binning) before constructing their frequency tables.
  - Other option is using the distribution of the numerical variable to have a good guess of the frequency.
  - For example, one common practice is to assume normal distributions for numerical variables.
  - The PDF for the normal distribution is defined by two parameters (mean and standard deviation).

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Mean

$$\sigma = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2\right]^{0.5}$$

Standard deviation

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Normal distribution

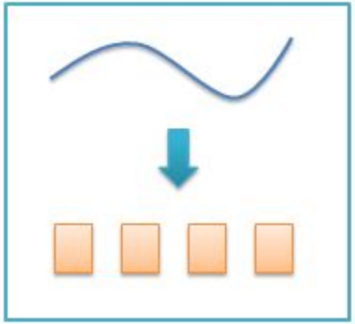
		Humidity	Mean	StDev
Play Golf	yes	86 96 80 65 70 80 70 90 75	79.1	10.2
	no	85 90 70 95 91	86.2	9.7

$$P(\text{humidity} = 74 \mid \text{play} = \text{yes}) = \frac{1}{\sqrt{2\pi}(10.2)} e^{-\frac{(74-79.1)^2}{2(10.2)^2}} = 0.0344$$

$$P(\text{humidity} = 74 \mid \text{play} = \text{no}) = \frac{1}{\sqrt{2\pi}(9.7)} e^{-\frac{(74-86.2)^2}{2(9.7)^2}} = 0.0187$$

Naive Bayesian Classifier

- **Binning**
- Binning or discretization is the process of transforming numerical variables into categorical counterparts.
- An example is to bin values for Age into categories such as 20-39, 40-59, and 60-79.
- Numerical variables are usually discretized in the modeling methods based on frequency tables
  - e.g., decision trees
- Moreover, binning may improve accuracy of the predictive models by reducing the noise or non-linearity.
- Finally, binning allows easy identification of outliers, invalid and missing values of numerical variables.
- There are two types of binning, unsupervised and supervised.



Bayesian Belief Networks

- Bayesian Network
- A Naive Bayes classifier is a simple model that describes particular class of Bayesian network
  - where all of the features are class-conditionally independent.
- Because of this, there are certain problems that Naive Bayes cannot solve XOR problem.
- Example: XOR
  - You have a learning problem with binary features x1 and x2 and a target variable y = x1 XOR x2.
  - In a Naive Bayes classifier, x1 and x2 must be treated independently
    - so you would compute things like "The probability that y = 1 given that x1 = 1"
    - hopefully you can see that this isn't helpful,
    - because x1 = 1 doesn't make y = 1 any more or less likely.
    - Since a Bayesian network does not assume independence, it would be able to solve such a problem.
- Bayesian Network Models
  - It model relationships between features in a very general way.
  - If you know what these relationships are, or have enough data to derive them,
  - then it may be appropriate to use a Bayesian Network.

Bayesian Belief Networks

- Bayesian Network OR Bayesian Belief Networks OR Belief Networks or Bayes Nets
- It is a type of probabilistic graphical model.
- It represents the relationship between variables in the form of a directed acyclic graph (DAG)
- It is used for reasoning, learning and decision making.
- It can handle uncertainty.
- It can incorporate prior knowledge into the model.
- It is useful in medical diagnosis, bioinformatics, and NLP

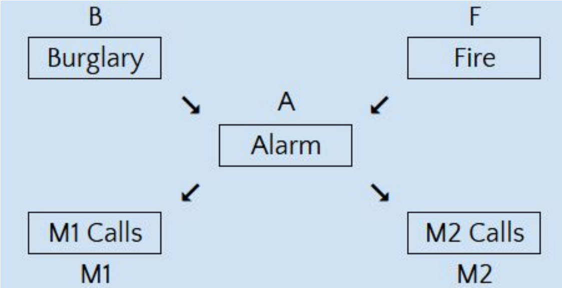
Bayesian Belief Networks

- Construction of a Bayesian network
- Let us assume that the problem can be defined in terms of n random variables.
- Steps for the construction of a Bayesian network:
  - 1. Choose an ordering of variables X1, … ,Xn
  - 2. For i = 1 to n
    - add Xi to the network
    - select parents from X1, … ,Xi−1 such that
    - P (Xi | Parents(Xi)) = P (Xi | X1, … Xi−1)
  - This choice of parents guarantees:
    - P (X1, … ,Xn) = πi =1P (Xi | X1, … , Xi−1) (chain rule)
    - = πi =1P (Xi | Parents(Xi)) (by construction)

$$P(x_1,\ldots,x_n) = \prod_{i=1}^{n} P(x_i \mid Parents(X_i))$$

Bayesian Belief Networks

- Step 1:
- 1.1 Determine what the propositional (random) variables should be.
- 1.2 Determine causal (or another type of influence) relationships and develop the topology of the network.
- Variables are
  - Burglary, Earthquake, Alarm, M1 Calls, M2 Calls
- The Network topology reflecting the “causal” knowledge is as follows:
  - A burglar can set the alarm off
  - An earthquake can set the alarm off
  - The alarm can cause Mary to call
  - The alarm can cause John to call
- The resulting Topology of the Bayesian Network is shown here:



Bayesian Belief Networks

- Step 2: To specify a conditional probability table or CPT for each node.
- Burglary B:
  - P (B=T) = 0.001 (‘B’ is TRUE i.e burglary has occurred)
  - P (B=F) = 0.999 (‘B’ is FALSE i.e burglary has not occurred)
- Fire F:
  - P (F=T) = 0.002 (‘F’ is TRUE i.e fire has occurred)
  - P (F=F) = 0.998 (‘F’ is FALSE i.e fire has not occurred)

Alarm A		P(A|B,E)	
B	**F**	**P (A=T)**	**P (A=F)**
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	**0.999**

Alarm A node can be TRUE or FALSE (i.e may have rung or may not have rung).
It has two parent nodes burglary B and fire F which can be TRUE or FALSE
i.e it may have occurred or may not have occurred depending upon diff. conditions.

Person M1	P(M1|A)	
A	**P (M1=T)**	**P (M1=F)**
T	**0.95**	0.05
F	0.05	0.95

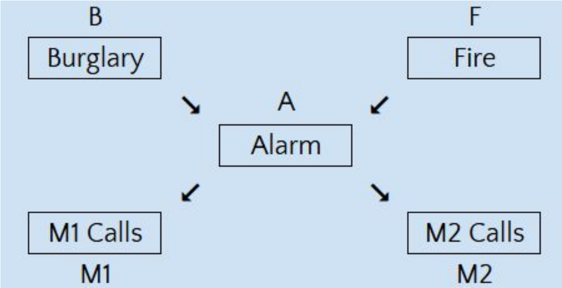
Person M1 node can be TRUE or FALSE (i.e may have called person XYZ or not).
It has a parent node, the alarm A, which can be TRUE or FALSE;
That is it may have rung or may not have rung, upon burglary B or fire F.

Person M2	P(M2|A)	
A	**P (M2=T)**	**P (M2=F)**
T	**0.8**	0.2
F	0.01	0.99

Person M2 node can be TRUE or FALSE (i.e may have called person XYZ or not).
It has a parent node, the alarm A, which can be TRUE or FALSE;
That is it may have rung or may not have rung, upon burglary B or fire F.

Bayesian Belief Networks

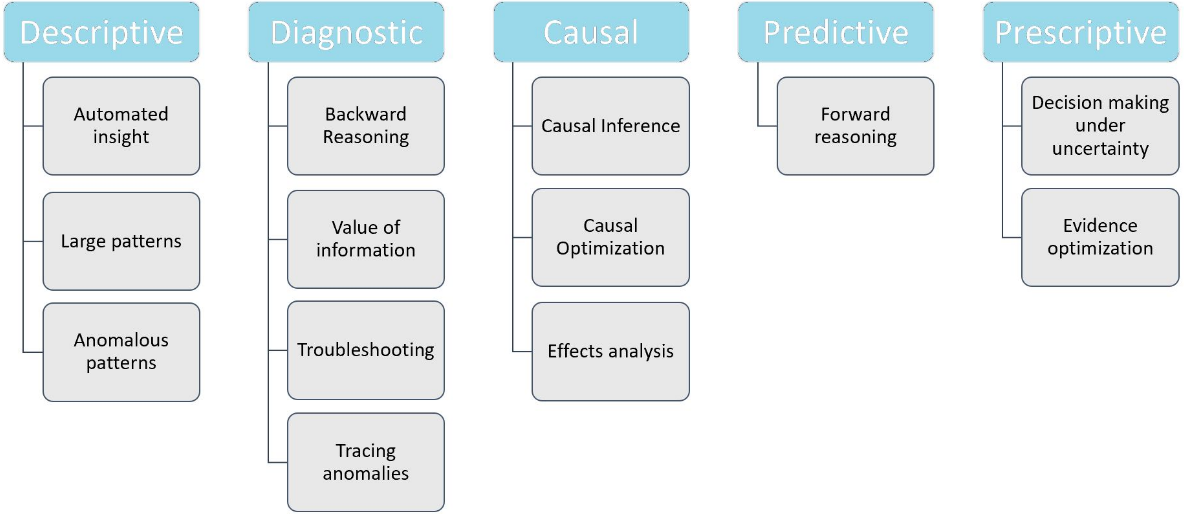
- Example: Calculating Conditional Probability of Events using Bayesian Belief Networks
- Question:
  - Find the probability that M1 is TRUE (M1 has called XYZ), M2 is TRUE (M2 has called XYZ)
  - when the alarm ‘A’ rang, but no burglary ‘B’ and fire ‘F’ has occurred.
  - i.e Find the probability => P (M1, M2, A, ~B, ~F)
  - M1, M2 & A are TRUE events and ~B & ~F are FALSE events
- Solution PDF



$$P(x_1,\ldots,x_n) = \prod_{i=1}^{n} P(x_i \mid Parents(X_i))$$

Bayesian Belief Networks

- Capabilities in terms of the analytics disciplines
  - Descriptive analytics
  - Diagnostic analytics
  - Causal AI.
  - Predictive analytics
  - Prescriptive analytics



Bayesian Belief Networks

- Applications of Bayesian Belief Networks
- Medical Diagnosis:
  - To model relationships between symptoms, diseases, and risk factors.
- Risk Assessment and Decision-Making:
  - To assess risks by modeling dependencies among factors such as
    - market volatility, economic indicators, and credit scores.
- Machine Learning and Data Mining:
  - To discover patterns to predict outcomes, such as fraud detection in banking,
  - by analyzing dependencies between variables.

Bayesian Belief Networks

- **Advantages of Bayesian Belief Networks**
- Handling Uncertainty:
  - To handle uncertainty by updating probabilities dynamically when new evidence becomes available.
- Flexibility and Scalability:
  - To scale to larger networks through incremental updates.
- Incorporating Expert Knowledge:
  - To integrate expert knowledge with data-driven models.

- **Challenges and Limitations**
- Computational Complexity:
  - Computationally expensive as the number of variables and dependencies increases.
- Scaling Issues:
  - Flexible but scaling to very large networks introduces challenges.
- Defining Accurate Priors:
  - Assigning accurate prior probabilities is crucial for the network's reliability.

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Thank You.