Machine Learning

Learning with Regression and Trees

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Outline

- Learning with Regression and Trees
  - Learning with Regression
    - Simple Linear Regression
    - Multiple Linear Regression
    - Logistic Regression
  - Learning with Trees
    - Decision Trees
    - Constructing Decision Trees using Gini Index
    - Classification and Regression Trees (CART)

Types of Regression

- Regression models used to find the relationship between a DV and IV.
- Simple linear regression
  - To models the relationship between a DV and a single IV.
- Multiple linear regression
  - If you have more than one independent variable.
- Multiple Regression vs. Multivariate Regression
- Multiple Regression:
  - The influence of several IVs on a DV is examined.
  - One DV is taken into account to analyzed.
- Multivariate Regression:
  - Several regression models are calculated to allow conclusions to be drawn about several DV.
  - Several dependent variables are analyzed.

Simple Linear Regression

$$\hat{y} = b \cdot x + a$$

Multiple Linear Regression

$$\hat{y} = b_1 \cdot x_1 + b_2 \cdot x_2 + \ldots + b_k \cdot x_k + a$$

Logistic Regression

- Types of Logistic Regression
- Binomial Logistic Regression:
  - There can be only two possible types of the DVs, such as 0 or 1, Pass or Fail, etc.
- Multinomial Logistic regression:
  - There can be 3 or more possible unordered types of the DV, such as “cat”, “dogs”, or “sheep”
- Ordinal Logistic regression:
  - There can be 3 or more possible ordered types of DVs, such as “low”, “Medium”, or “High”.

Logistic Regression

- Logistic Regression
- Logistic regression is a supervised machine learning algorithm.
- It is extensively used in predictive modeling.
- It helps to predict the probability of an outcome, event, or observation.
- It is to predict the probability that an instance belongs to a given class or not.
- The model delivers a binary outcome limited to two possible outcomes: yes/no, 0/1, or true/false.
- It analyzes the relationship between one or more IVs and classifies data into discrete classes.
- It is used for binary classification tasks using sigmoid function.
- Sigmoid function takes input as IVs and produces a probability value between 0 and 1.
- Example
  - For two classes Class 0 and Class 1
  - if the value of the logistic function for an input > 0.5 (threshold value)
    - then it belongs to Class 1 otherwise it belongs to Class 0.
  - It is referred to as regression because it is the extension of linear regression
    - but is mainly used for classification problems.

Logistic Regression

- Logistic regression is commonly used in binary classification.
- In statistics, the logistic model (or logit model) is a statistical model.
- It models the log odds of an event as a linear combination of one or more IVs.
- In regression analysis,
  - logistic regression (or logit regression) estimates
    - the parameters of a logistic model (the coefficients in the linear or non linear combinations).
- In binary logistic regression
  - there is a single binary DV, coded by an indicator variable,
  - where the two values are labeled "0" and "1",
  - while the IVs can each be a binary variable (two classes) or a continuous variable (any real value).
- The function that converts log odds to probability is the logistic function, hence the name.
- The unit of measurement for the log odds scale is called a logit (logistic unit), hence the alternative names.

Logistic Regression

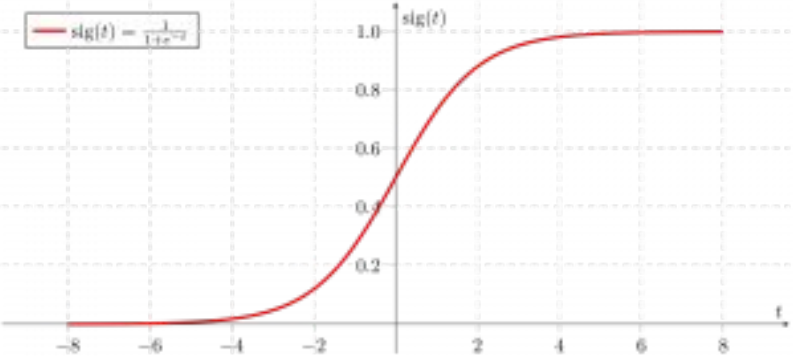
- Logistic regression predicts the output of a categorical dependent variable.
- Therefore, the outcome must be a categorical or discrete value.
- It can be either Yes or No, 0 or 1, true or False, etc.
  - But instead of giving the exact value as 0 and 1, it gives the probabilistic values which lie bet. 0 and 1.
- In Logistic regression, instead of fitting a regression line, we fit an "S" shaped logistic function,
  - which predicts two maximum values (0 or 1).

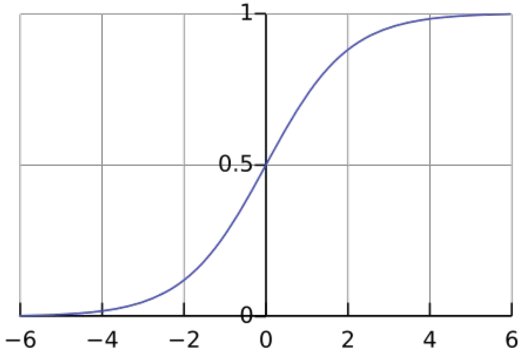
Logistic Regression

- Assumptions of Logistic Regression
- Independent observations:
  - Each observation is independent of the other. meaning there is no correlation between any input variables.
- Binary dependent variables:
  - It takes the assumption that the DV must be binary or dichotomous, meaning it can take only two values.
  - For more than two categories SoftMax functions are used.
- Linearity relationship between independent variables and log odds:
  - The relationship between the IVs and the log odds of the DV should be linear.
- No outliers:
  - There should be no outliers in the dataset.
- Large sample size:
  - The sample size is sufficiently large.

Logistic Regression

- Understanding Sigmoid Function (the core of logistic regression)
- The sigmoid function is a mathematical function used to map the predicted values to probabilities.
- It maps any real value into another value within a range of 0 and 1.
- The value of the logistic regression must be between 0 and 1,
  - which cannot go beyond this limit, so it forms a curve like the “S” form.
  - The S-form curve is called the Sigmoid function or the logistic function.
- In logistic regression, we use the concept of the threshold value,
  - which defines the probability of either 0 or 1.
  - Such as values above the threshold value tends to 1, and a value below the threshold values tends to 0.
- Standard logistic function where; L = 1; k = 1; x0 = 0

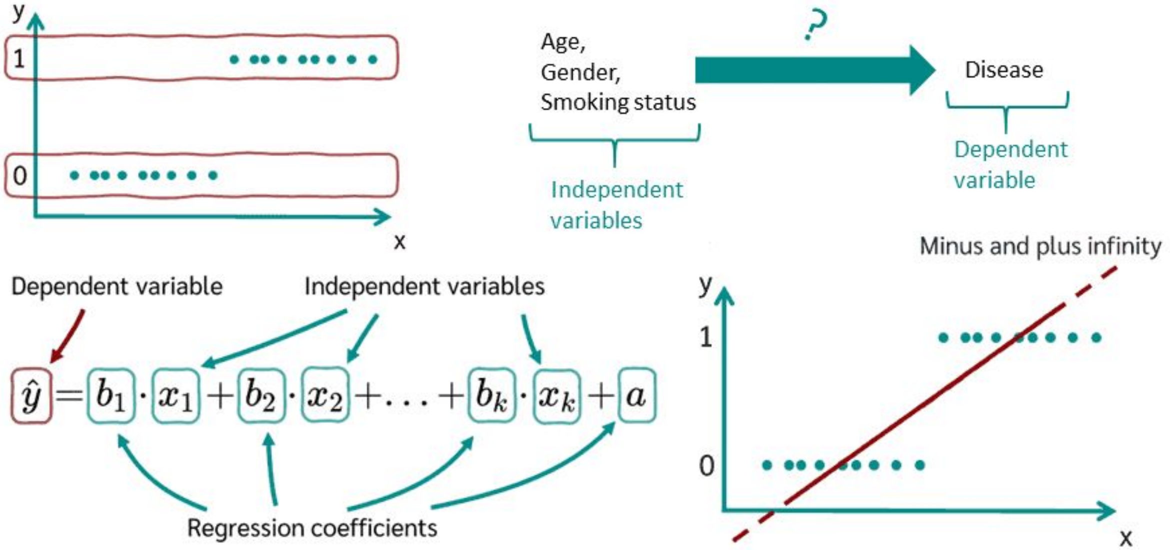




Logistic Regression

- Understanding Sigmoid Function (the core of logistic regression)
- The Figure sigmoid function converts the continuous variable data into the probability i.e. between 0 and 1.

$$\sigma = \frac{1}{1+e^{-x}}$$



Logistic Regression

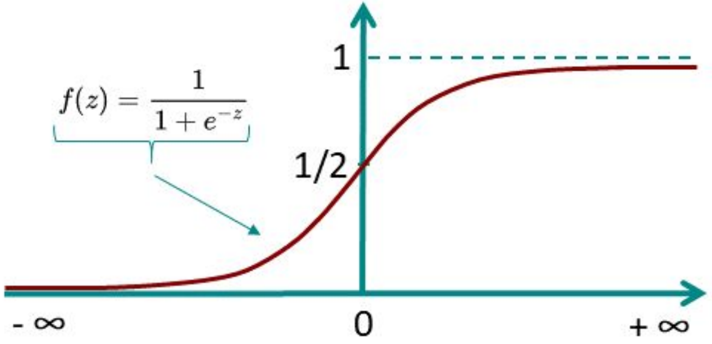
$$\sigma = \frac{1}{1+e^{-x}}$$

- In linear regression
  - IVs (e.g., age and gender) are used to estimate the specific value of the DV (e.g., body weight).
- In logistic regression
  - DV is dichotomous variables (0 or 1) and
  - the probability of the occurrence of value 1 (characteristic present) is estimated.
- Perform logistic regression:
  - To find out which variables have an influence on a disease.
  - To examine effect of age, gender and smoking or not for detecting a particular disease.
    - In this case, 0 could stand for not diseased and 1 for diseased.
  - To estimate the probability of occurrence and not the value of the variable itself.
  - To do this, it is necessary to restrict the value range for the prediction to the range bet. 0 and 1 (see figure).
  - To ensure that only values between 0 and 1 are possible, the logistic function f is used:
    - $f(x) = 1 / (1 + e^{-x})$

Logistic Regression

$\sigma = \frac{1}{1+e^{-x}}$

- Logistic function
- The logistic model is based on the logical function.
- The special thing about the logistic function is that for values between minus and plus infinity,
  - it always assumes only values between 0 and 1.
- To ensure that only values between 0 and 1 are possible, the logistic function f is used: f(z) = 1 / (1 + e^{-z})



$$\hat{y} = \boxed{b_1 \cdot x_1 + b_2 \cdot x_2 + \ldots + b_k \cdot x_k + a}$$

$$f(z) = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{-\boxed{(b_1 \cdot x_1 + \ldots + b_k \cdot x_k + a)}}}$$

Logistic Regression

- To ensure that only values between 0 and 1 are possible, the logistic function f is used: f(z) = 1 / (1 + e^{-z})
- Diseased = Yes = 1 i.e P(y=1)
- Diseased = No = 0 i.e P(y=0)
- The probability that for given values of the independent variable the dichotomous dependent variable y is 0 or 1 is given by:

$$P(y=1|x_1,\ldots,x_n)=\frac{1}{1+e^{-(b_1\cdot x_1+\ldots+b_k\cdot x_k+a)}}$$

$$P(y=0|x_1,\ldots,x_n)=1-\frac{1}{1+e^{-(b_1\cdot x_1+\ldots+b_k\cdot x_k+a)}}$$

- To calculate the probability of a person being sick or not using the logistic regression:
  - The model parameters b1, b2, b3 and a must first be determined.
  - Once these have been determined, the equation for the example above is:

$$P(\text{Diseased})=\frac{1}{1+e^{-(b_1\text{Age}+b_2\text{Gender}+b_3\text{Smoking status}+a)}}$$

$$\sigma=\frac{1}{1+e^{-x}}$$

Logistic Regression

- Example 1
  - Use a logistic regression with one explanatory variable and two categories to answer the question:
  - Dataset: A group of 5 person with disease yes/no for age, gender and smoking status.
  - Question: How does the age, gender and smoking status affect the probability of having a particular disease?

Sr No	Age	Gender	Smoking	Disease
1	25	0	1	1
2	37	1	0	0
3	40	0	0	0
4	49	0	1	1
5	55	1	1	1

  - Answer:
    - Disease = Yes = 1 i.e P(y=1) and Disease = No = 0 i.e P(y=0)
    - $f(z) = P(Disease) = 1 / (1 + e^{-(b0+b1*Age+b2*Gender+b3*Smoking)})$
    - Logistic Regression Solved Example 1 [PDF]

Logistic Regression

- Logistic Regression Solved Example 1 [PDF]

Answer: IV is Independent variable and DV is Dependent variable

	x1	x2	x3	y	e =	2.71828
Sr No	**Age (x1)**	**Gender (x2)**	**Smoking (x3)**	**Disease (y)**	**Calcuated Y (Fitted Value)**	**Prediction**
1	25	0	1	1	0.9947	1
2	37	1	0	0	0.0034	0
3	40	0	0	0	0.0184	0
4	49	0	1	1	0.9263	1
5	55	1	1	1	0.4518	0

Determine Predicted value of y:

Disease = Yes = 1 i.e P(y=1) and Disease = No = 0 i.e P(y=0)

$$f(z) = P(Disease) = 1 / (1 + e^{-(b0+b1*Age+b2*Gender+b3*Smoking)})$$

C' Matrix	Predicted 1	Predicted 0
Actual 1	TP	FN
Actual 0	FP	TN

C' Matrix	**Predicted 1**	**Predicted 0**	**Total (N)**
Actual 1	2	1	3
Actual 0	0	2	2
Total (N)	2	3	**5**

P, Precision or PPV =	TP / (TP+FP)=	1.000
NPV =	TN / (TN+FN)=	0.667
False Omission Rate (FOR) =	1 - NPV=	0.333
R, Recall (Sensitivity) or TPR =	TP / (TP+FN)=	0.667
Specificity or NPV or TNR =	TN / (TN+FP)=	1.000
False Positive Rate (FPR) =	FP / (FP+TN)=	0.000
False Negative Rate (FNR) =	FN / (FN+TP)=	0.333
Accuracy =	(TP+TN)/(TP+TN+FP+FN)=	0.800
F1 score =	2 * P * R / (P + R)=	0.8

Logistic Regression

- Example 2
  - Use a logistic regression with one explanatory variable and two categories to answer the question:
  - Dataset: A group of 20 students spends between 0 and 6 hours studying for an exam.
  - Question: How does the # hours spent studying affect the probability of the student passing the exam?
  - Answer:
    - Result = Pass = 1 i.e P(y=1) and Result = Fail = 0 i.e P(y=0)
    - f(z) = P(Result) = 1 / (1 + $e^{-(b0+b1*Slept+b2*Study)}$)
    - Logistic Regression Solved Example 2 [PDF]

Hours (x_k)	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.25	4.50	4.75	5.00	5.50
Pass (y_k)	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	1

Logistic Regression

- Logistic Regression Solved Example 2 [PDF]

Answer: IV is Independent variable and DV is Dependent variable

e = 2.71828

Sr No	Slept (x1) (IV)	Study (x2) (IV)	Result (y) (DV)	Calcuated Y (Fitted Value)	Prediction
1	7.23	5.55	0	0.0101	0
2	8.12	5.12	1	0.0023	0
3	9.23	6.23	0	0.0018	0
4	5.12	8.12	1	0.6342	1
5	9.23	6.23	0	0.0018	0
6	3.55	9.55	1	0.9798	1
7	4.32	8.32	1	0.8472	1

Determine Predicted value of y: Result = Pass = 1 i.e P(y=1) and Result = Fail = 0 i.e P(y=0)

$f(z) = P(Result) = 1 / (1 + e^{-(b0+b1*Slept+b2*Study)})$

C' Matrix	Predicted 1	Predicted 0
Actual 1	TP	FN
Actual 0	FP	TN

C' Matrix	Predicted 1	Predicted 0	Total (N)
Actual 1	3	1	4
Actual 0	0	3	3
Total (N)	3	4	7

P, Precision or PPV =	TP / (TP+FP)=	1.000
NPV =	TN / (TN+FN)=	0.750
False Omission Rate (FOR) =	1 - NPV=	0.250
R, Recall (Sensitivity) or TPR =	TP / (TP+FN)=	0.750
Specificity or NPV or TNR =	TN / (TN+FP)=	1.000
False Positive Rate (FPR) =	FP / (FP+TN)=	0.000
False Negative Rate (FNR) =	FN / (FN+TP)=	0.250
Accuracy =	(TP+TN)/(TP+TN+FP+FN)=	0.857
F1 score =	2 * P * R / (P + R)=	0.8571428571

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Thank You.