SVD: Example 2

For symmetric matrices, eigenvectors for distinct eigenvalues are orthogonal

All eigenvalues of a real symmetric matrix are real

All eigenvalues of a positive semi-definite matrix are non-negative

A positive semi-definite matrix is a symmetric matrix with non-negative eigenvalues.

$$S = \begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix}$$

Ans: Calculate Eigenvalues

$$S - \lambda * I = 0$$

$$\begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} -5-\lambda & 2 \\ -7 & 4-\lambda \end{bmatrix} = 0$$

(-5-λ)*(4-λ)14 0

λ	=	2	λ	=	0
λ	=	-3	λ	=	0

Plug in these eigenvalues values to find eigenvectors

(S - λ * I) V = 0

Eigenvector for λ = 2

$$\begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} v1 \\ v2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & 2 \\ -7 & 2 \end{bmatrix} * \begin{bmatrix} 0.29 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Eigenvector for λ = -3

$$\begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix} - -3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} v1 \\ v2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 \\ -7 & 7 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

OR $$\begin{bmatrix} -2 & 2 \\ -7 & 7 \end{bmatrix} * \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The eigenvectors are orthogonal (and real):

Columns of U are eigenvectors of S; (Recall UU−1 =1)

$$U = \begin{bmatrix} 1 & 0.286 \\ 1 & 1 \end{bmatrix} \qquad U^{-1} = \begin{bmatrix} 1.4 & -0.4 \\ -1.4 & 1.4 \end{bmatrix}$$

Diagonal elements of Σ are eigenvalues of S

$$\Sigma = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$$

Reconstructed of S from U, U^{-1} and Σ

$$S = U\Sigma U^{-1}$$

$$S = \begin{bmatrix} 1 & 0.286 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix} * \begin{bmatrix} 1.4 & -0.4 \\ -1.4 & 1.4 \end{bmatrix}$$

$$S = \begin{bmatrix} -3 & 0.5714 \\ -3 & 2 \end{bmatrix} * \begin{bmatrix} 1.4 & -0.4 \\ -1.4 & 1.4 \end{bmatrix}$$

Reconstructed S

$$S = \begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix}$$