

Solved Example: Multiple Linear Regression

Dataset: $n = 4$

Predictor variables = > Independent Variables: X_1 and X_2

Response variable => Dependent Variables: y

	X_1	X_2	y
	50	20	120
	62	25	150
	70	16	200
	75	11	215
Sum	257	72	685
Mean	64.25	18	171.25

Step 1: Prepare Regression Table

	X_1^2	X_2^2	$X_1 X_2$	$X_1 y$	$X_2 y$
	2500	400	1000	6000	2400
	3844	625	1550	9300	3750
	4900	256	1120	14000	3200
	5625	121	825	16125	2365
Σ	16869	1402	4495	45425	11715
Reg Σ	356.75	356.75	-131	1413.75	-615

Step 2: Calculate Regression Sums

$$\begin{aligned}\Sigma X_1^2 &= \Sigma X_1^2 - (\Sigma X_1)^2 / n = 356.75 \\ \Sigma X_2^2 &= \Sigma X_2^2 - (\Sigma X_2)^2 / n = 106 \\ \Sigma X_1 X_2 &= \Sigma X_1 X_2 - (\Sigma X_1 \Sigma X_2) / n = -131 \\ \Sigma X_1 y &= \Sigma X_1 y - (\Sigma X_1 \Sigma y) / n = 1413.75 \\ \Sigma X_2 y &= \Sigma X_2 y - (\Sigma X_2 \Sigma y) / n = -615\end{aligned}$$

Step 3: Calculate b_0 , b_1 , and b_2

$$\begin{aligned}b_1 &= [(\Sigma X_2^2)(\Sigma X_1 y) - (\Sigma X_1 X_2)(\Sigma X_2 y)] / [(\Sigma X_1^2)(\Sigma X_2^2) - (\Sigma X_1 X_2)^2] = 3.3548 \\ b_2 &= [(\Sigma X_1^2)(\Sigma X_2 y) - (\Sigma X_1 X_2)(\Sigma X_1 y)] / [(\Sigma X_1^2)(\Sigma X_2^2) - (\Sigma X_1 X_2)^2] = -1.6558 \\ b_0 &= \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2 = -14.4937\end{aligned}$$

Step 4: Place b_0 , b_1 , and b_2 in the estimated linear regression equation.

Estimated linear regression equation; \hat{y}

$$\begin{aligned}\hat{y} &= b_0 + b_1 X_1 + b_2 X_2 \\ \hat{y} &= -14.494 + 3.355 * X_1 + -1.656 * X_2\end{aligned}$$

Interpret a Multiple Linear Regression Equation

$b_0 = -14.494$ means => if $X_1 = X_2 = 0$, then $y = -14.494$

$b_1 = 3.355$ => 1 unit increase in X_1 is associated with a 3.355 unit increase in y ; if X_2 is constant.

$b_2 = -1.656$ => 1 unit increase in X_2 is associated with a -1.656 unit decrease in y ; if X_1 is constant.