Statistical Hypothesis Testing [Parametric]

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Statistical Hypothesis Testing

- Need and Motivation
- Procedure in Hypotheses Testing
- Types of Hypothesis Testing
- Example of Hypotheses
- Error in Hypothesis Testing: Type I and II error
- Confidence interval
- Z test and 2 test for goodness of fit
- ANOVA (one way classification)

Learning Objectives

- Distinguish Parametric & Nonparametric Test Procedures
- Explain commonly used Nonparametric Test Procedures
- Perform Hypothesis Tests Using Parametric Procedures
- Perform Hypothesis Tests Using Nonparametric Procedures
- Know different Statistical Parameters

Parametric & Nonparametric Test Procedures

- Distinguish with Example

Parametric Tests	Nonparametric Tests
Depends of Probability Distribution	Distribution-free methods
Involve Population Parameters (Mean)	Do Not Involve Population Parameters (Example: Probability Distributions, Independence)
Have Stringent Assumptions (Normality)	Data Measured on Any Scale (Ratio or Interval, Ordinal or Nominal)
Examples: Z Test, t Test, x^2 Test, F test	Example: Wilcoxon Rank Sum Test
Parametric Test Example	Nonparametric Test Examle
t test	Sign Test
Z test	Wilcoxon Signed-Rank Test
x^2 Test	Wilcoxon rank sum test
F test (Analysis of variance)	Mann-Whitney-Wilcoxon Test
One Way ANOVA	Kruskal-Wallis H-Test
Linear correlation	Rank Correlation Test
	Runs test

Classification

- Statistical Analysis

Statistical Analysis	
Descriptive Statistics	**Inferential Statistics**
Graphical: Organizing and presenting the data Example: histogram, box plot, probability plot	Estimation: Estimate parameters of the pdf along with its confidence region
Numerical: Summarizing the sample set Example: mean, median, mode, range, quartile, variance, standard deviation	Hypotheses Testing: Making judgements about f(x) and its parameters

Need of (Motivation for) Hypothesis Testing

- Need of (Motivation for) Hypothesis Testing (Significance Testing)
  - Will an investment in MF yield > desired value?
  - Is incidence of diabetes > among male than female?
  - Are women > than male to change mobile service provider?
  - Has the efficiency of a pump < form its original value due to aging?
- Statistical tests are either parametric or non-parametric

Characteristics of Good Hypothesis

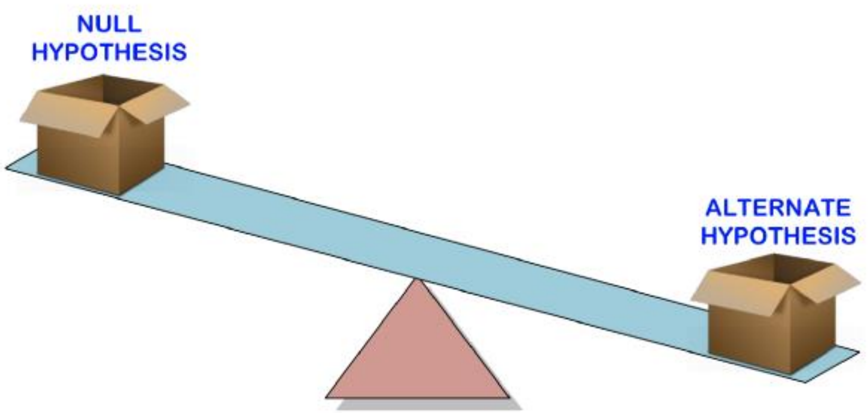
- A good Hypothesis must
  - Be clear, definite and stated in a simple manner all its concepts
  - Have concepts which have empirical basis
  - Be specific and precise while stating the relationship among variable
  - Be consistent with known fats
  - Be able to support or deny an existing theory
  - Have reasonable explanation to any problem in present state of knowledge

Basics of Hypothesis Testing

- Definition
  - Test to make decision from set of data by determining expected behavior
- Hypothesis are converted to a test of parameters
  - mean or variance of population or
  - difference in mean or variance of populations
- Hypothesis is a statement or postulate about parameters of distribution
- This statement is called as
  - Null Hypothesis(H_0)
    - Statement is true (wish to reject if sample provide sufficient evidence)
  - Alternative Hypothesis (H_a)
    - Statement is not true (wish to accept if H_0 is rejected)

Basics of Hypothesis Testing

- Null hypothesis (H_0) – Default (Claim)
  - The hypothesis will not be rejected unless the data provide convincing evidence that it is false
  - we are interested in disproving or, accumulating evidence for rejection
- Alternative or research hypothesis (H_a)
  - Accepted only if data provide convincing evidence of its truth



Basics of Hypothesis: A Case

- To understand Hypothesis Testing (Significance testing)
  - What is Normal Distribution?
  - What is P-value?
  - What is Statistical Significance?
- To determine the statistical significance of our results p-value is used
  - i.e p-value is used to know if a claim is valid or not
- To test the validity of a claim (H_0) about a population using sample data
- H_a is believed to be true if H_0 is concluded to be untrue

Basics of Hypothesis: A Case

- Example:

  Suppose a pizza place claims their delivery times are 30 minutes or less on average but you think it's more than that. So you conduct a hypothesis test and randomly sample some delivery times to test the claim:

- Null hypothesis — The mean delivery time (MDT) is 30 minutes or less
- Alternative hypothesis — The mean delivery time is greater than 30 minutes
- Goal is to determine which claim
  - H_0 or H_a is better supported by the evidence found from sample data
- Want to test to see if there is a chance that MDT is greater than 30 minutes
  - In other words, we want to see if the pizza place lied to us somehow
- One-tailed test (our case): we care about if the MDT is greater than 30 minutes
- Two-tailed test : use if a MDT lower or equal to 30 minutes (even more preferable)

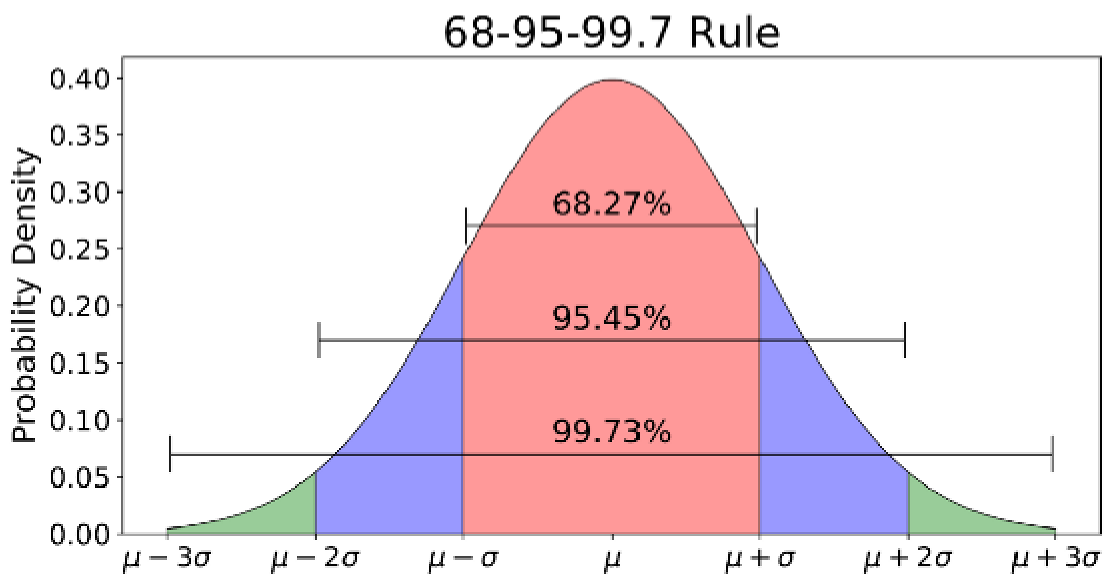
Basics of Hypothesis: A Case - What is Normal Distribution?

- Normal Distribution (PDF) with mean μ and standard deviation sigma σ
- Z-test can be to conduct our hypothesis testing
- For z-test we need to calculate Z-scores (to be used in our test-statistic)
- Z-score is the number of standard deviations from the mean a data point is
- Z-score can tell us where the overall data lies compared to average population
- In our case, each data point is the pizza delivery time that we collected

$$z = \frac{x - \mu}{\sigma}$$

$\mu =$ Mean

$\sigma =$ Standard Deviation



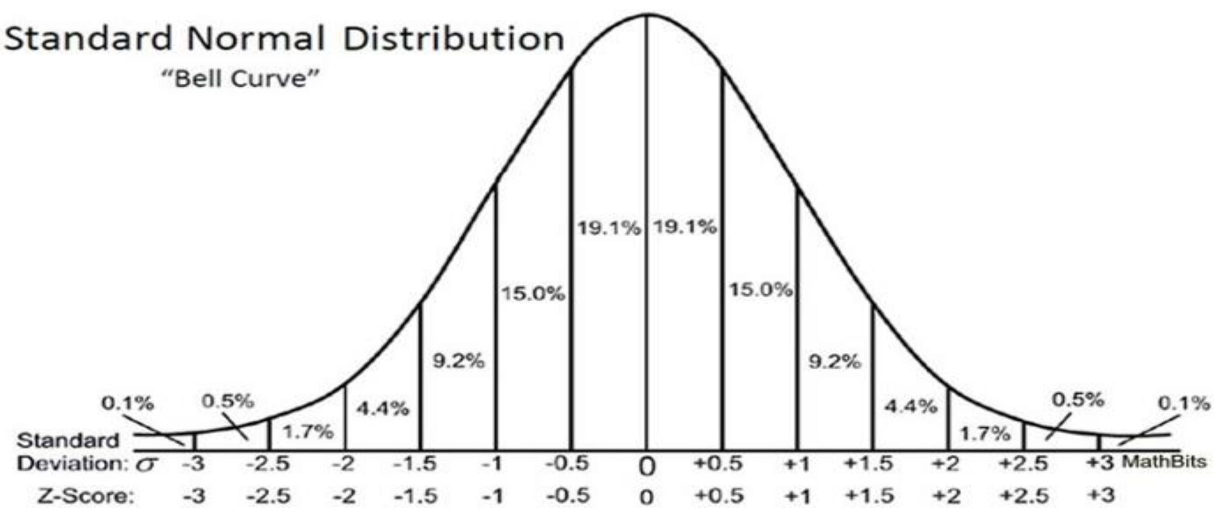
Basics of Hypothesis: A Case - What is P-value?

- Mr. Will Koehrsen: Higher or lower the Z-score, the more unlikely the result is to happen by chance and the more likely the result is meaningful
- But how high (or low) is considered as sufficiently convincing to quantify how meaningful our results are?
- This is where we need the last piece of item to solve the puzzle — **p-value**, and
- Check if our results are statistically significant based on the **significance level** (also known as **alpha α**) we set before we began our experiment

$$z = \frac{x - \mu}{\sigma}$$

$\mu =$ Mean

$\sigma =$ Standard Deviation



Basics of Hypothesis: A Case - What is P-value?

- p-value lead to our decisions for the hypothesis testing
- Recall example of randomly sampled some pizza delivery times and
- The goal is to check if the mean delivery time is greater than 30 minutes
- If final evidence supports the claim by pizza place (MDT is 30min or less), then
  - we will not reject the null hypothesis
- Otherwise, we will reject the null hypothesis

Basics of Hypothesis: A Case - What is P-value?

- The job of p-value is to answer the following question:
  - If I'm living in a world where the pizza delivery time is 30 minutes or less (null hypothesis is true), how surprising is my evidence in real life?
- P-value answers this question with a number — **probability**
- Lower the p-value, more surprising evidence is, the more ridiculous our H_0 looks
  - Ridiculous means we reject H_0 and choose our H_a instead
- If the p-value is lower than a predetermined significance level (**alpha α** - the threshold of being ridiculous), then
  - we reject the H_0
- After understanding the meaning of p-value means, let's apply it in our case

Basics of Hypothesis: A Case - What is P-value?

- Let's apply p-value meaningfully in pizza delivery times
- For collected some sampled delivery times
  - We perform the calculation and
  - Find that the MDT is longer by 10 minutes with a p-value of 0.03
- That means in a world where pizza delivery time is 30 min or less (H_0 is true),
  - There is a 3% chance we would see MDT is at least 10min longer due to random noise
- Lower the p-value, the more meaningful the result because it is less likely to be caused by noise

Basics of Hypothesis: A Case - What is P-value?

- Misinterpretation of p-value: It is 0.03 means that there is 3% (probability in %) that the result is due to chance — which is not true
- p-value of 0.03 is lower than the significance level of 0.05 (**alpha α**), and we can say that the result is statistically significant
- So what do we do?
  - At first, try to think of every possible way to make our initial belief (H_0) valid
  - Due to bad reviews from others and it often gave bad excuses that caused the late delivery
  - Even we ourselves feel ridiculous to justify for the pizza place anymore and hence, we decide to reject the H_0
  - Finally, we decision is to choose not to buy any pizza from that place again

- p-values

  Used as a tool to challenge our initial belief (H_0) when the result is statistically significant

Basics of Hypothesis: A Case - What is Statistical Significance?

- Finally we put everything together and test if the result is statistically significant
  - Having just the p-value is not enough
  - We need to set a threshold (**alpha α** — aka significance level)
  - **Alpha α** should always be set before an experiment to avoid bias
  - If the observed p-value is lower than **α**, then
    - We conclude that the result is statistically significant
  - Rule of thumb is to set **alpha α** = 0.05 or 0.01 (depends on application)
  - Assume that we set the alpha to be 0.05 before we began the experiment
  - Result is statistically significant since p-value of 0.03 is lower than **alpha α**

- Next: Basic steps for the whole experiment

Procedure in Hypotheses Testing

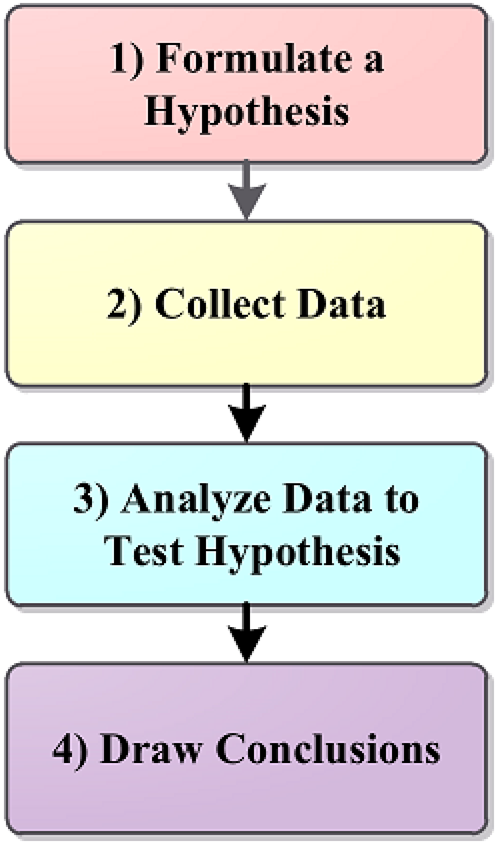
- Identify parameter (mean, variance, etc.) of interest to be tested
- State the null hypotheses H_0
- State the alternative hypotheses H_1
- Choose test criterion (threshold - alpha α)
- Find the Z-score associated with our alpha α level
- Choose kind of test need to perform (test-statistics)

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

  - e.g. (our case) Find the test statistic using this formula
- Compare the t-statistics
  - If the value of test statistic is less than the Z-score of alpha level (or p-value is less than alpha value), then
    - Reject the H_0. Otherwise, don't reject the H_0

Procedure in Hypotheses Testing

- No hypotheses test is perfect.
  - Observations are random
  - Sample is only available
- Test performance depends on
  - Extent of variability in data
  - Sample size (# of observations)
  - Test statistics (function of observations)
  - Threshold (test criterion)
- Example Next



Test of Hypothesis

- Test of Hypothesis
  - Test whether a population parameter is less than, equal to, or greater than a specified value
- Remember an inference without a measure of reliability is little more than a guess

Hypothesis Statement Example

- **Hypothesis Statement Example 1**
- If
  - We wanted to test the hypothesis that a coin is a fair
  - i.e. the coin lands on heads or tails with equal probability
- Then consider
  - $H_0 : p = 0.5$
  - $H_1 : p \neq 0.5$
  - where p is the probability of the coin landing on heads
- How to do it?

How to do Hypothesis Testing

- Hypothesis Statement Example 1: How to do it?
- It is tested by repeatedly tossing the coin and
- Recording the number of times that the coin landed on heads, and
- Testing H_0 using the Binomial distribution
- This is a one-sample test
- What about two-sample test?

How to do Hypothesis Testing

- Hypothesis Statement Example 2: Two-sample test
- Two coins in a two-sample test
- i.e. both coins land on heads with equal probability
- $H_0 : p1 = p2$ and $H_1 : p1 \neq p2$
  - where p1 & p2 are the prob. of coins 1 & 2 landing on heads
- Null hypothesis testing by repeatedly tossing both coins, and
- Recording # times that each coin landed on heads, and
- Obtaining the prob. that both values come from Bin.Dist. with equal

success probability $p = p1 = p2$

- Given the numbers of trials n1 and n2, respectively

How to do Hypothesis Testing

- Hypothesis Statement Example 2: Two-sample test
- Rejection of H_0depends on the statistical significance of test $P(H_0)$
- $P(H_0)$ is referred to as a p-value
- A result is considered significant if
  - it has been predicted to be highly unlikely to have occurred randomly by chance, given some threshold level
- This threshold is called significance level and denoted as alpha
- The significance level commonly set as alpha = 0.05
- It means there is only 5% probability that H_0 is correct
- If a p-value is found to be less than threshold(alpha),
  - then the result is considered as statistically significant
- Different significance levels may be selected depending on the application

Example of Hypotheses

- Example 3:
  - Researcher wants to find out if sex influences lang. development during childhood
  - He collected MLU values from a group of N year-old boys and N year-old girls
- Example of Hypotheses:
  - Sex does not influence development (i.e. MLU)
  - Sex influences development (i.e. MLU)
  - Girls have a higher MLU
  - Boys have a higher MLU

Example of Hypotheses

- Example 4:
  - Suppose building specifications for a city require that the average breaking strength of residential sewer pipe to be more than 2,400 lbs per foot of length.
  - To sell pipe in the city a manufacturer must demonstrate that its product meets the specifications.
- Example of Hypotheses:
  - the pipe does not meet specifications (H_0): $\mu \leq 2400$
  - the pipe meets specifications (H_1): $\mu > 2400$
- How can the city decide when enough evidence exists to conclude the pipe meets specifications?
  - When the sample mean convincingly indicates that the population mean exceeds 2,400 lbs per foot
- Decision is done using **Statistical Measures (**e.g. p-value**)**

Example of Hypotheses

- Example 5: Website testing using hypothesis statement
  - if you had reason to believe that the color of your landing page might be having a detrimental effect (harmful or damaging effect) on conversions
- Our hypothesis statement could be:

  “Changing my landing page color from black to blue will have a statistically significant impact on conversions.”
- Once this hypothesis is established
  - We need to run your test to prove (or disprove) it
  - Including words “statistically significant” in the hypothesis statement is important
  - Since it means your sample sizes need to be adequate to analyze it as such

Error in Hypothesis Testing

- Statistical significance of test = $P(H_0)$ = p-value
- Significance level (α) = Rate of FP = type I errors = Size of the test
- Rate of FN (type II) error = β
- Power of a test = 1 - Beta
  - $\alpha = P(\text{reject } H_0 \mid H_0 \text{ is correct})$
  - $\beta = P(\text{do not reject } H_0 \mid H_0 \text{ is incorrect})$

Decision → Truth ↓	H_0 is not rejected	H_0 is rejected
H_0 is true	Correct Decision $Pr = 1 - \alpha$	Type I error $Pr = \alpha$
H_1 is true	Type II error $Pr = \beta$	Correct Decision $Pr = 1 - \beta$

Dataset, N = 20 {4Class, 5Item}		Actual Class (Condition Given) P			Actual Class (Condition Given) N	Performance Measures
Predicted Class (Outcome)	P	TP Correct Decision $Pr = 1 - \alpha$	2	1	FP Type I Errors $Pr = \alpha$	PPV = TP/(TP+FP) = 2 / (2+1) **0.400**
	N	FN Type II Errors $Pr = \beta$	3	14	TN Correct Decision $Pr = 1 - \beta$	NPV = TN/(TN+FN) = 3 / (3+14) **0.067**
Performance Measures		Sensitivity = TP/(TP+FN) = 2 / (2+3) **0.667**			Specificity = TN/(TN+FP) = 14 / (14+1) **0.824**	A=(TP+TN)/(TP+FP+TN+FN) = (2+14) / (2+1+3+14) **0.8**

Basics of Hypothesis Testing

- Size of a test (α) may be controlled by adjusting the significance level
- Power of a test (1-Beta) is determined by nature of particular statistical test used to test H_0
- Given that non-parametric tests tend to have lower power than parametric tests,
  - Non-parametric tests will have a greater tendency to fail to reject H_0 in cases where
    - H_0 is actually incorrect

Dataset, N = 20 {4Class, 5Item}		Actual Class (Condition Given) P			Actual Class (Condition Given) N	Performance Measures
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Types of Hypothesis Test

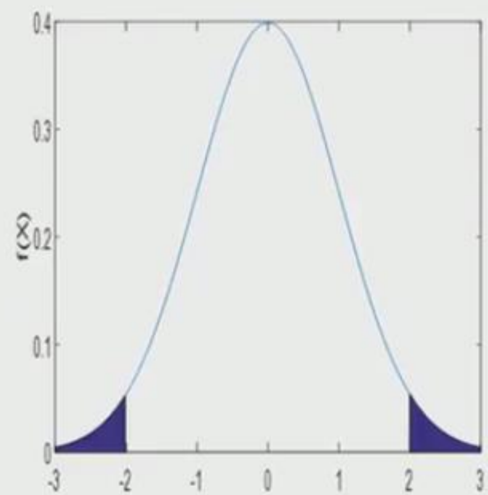
- Based on 2 different ways of computing significance level (p-value)
- One-tailed test: A two-tailed test considers any values at extremes of distribution
- Two-tailed test: A one-tailed test is being interested in detecting extreme outliers

- Two sided test

  $H_0 : \mu = 0$

  $H_1 : \mu \neq 0$

- Test statistic standard normal V z



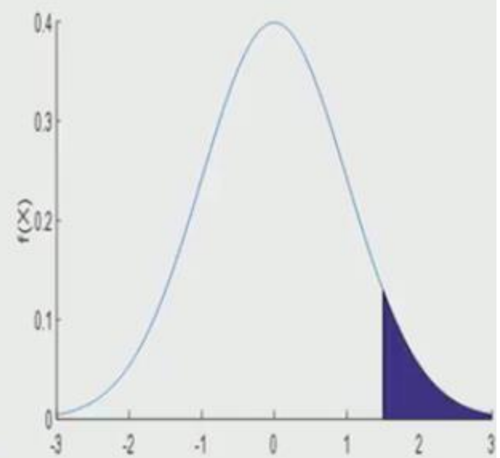
- Reject H_0 if $z \leq -2$ or $z \geq 2$

- One sided test

  $H_0 : \mu = 0$

  $H_1 : \mu > 0$

- Test statistic standard normal V z



- Reject H_0 if $z \geq 1.5$

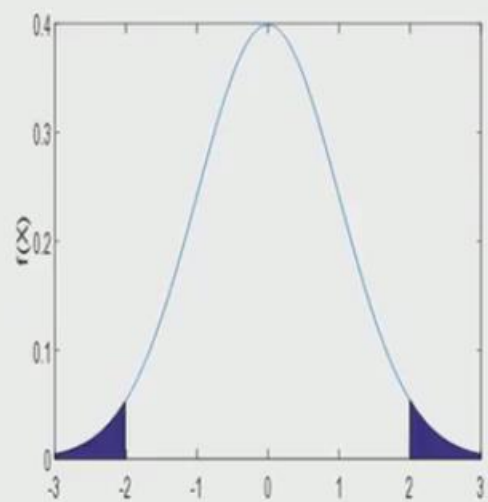
Types of Hypothesis Test

- Controlling Type Error 1 is minimized with selecting test criteria value
  - H_0 is specific
  - H_1 is clear

- Two sided test

$$H_0 : \mu = 0$$
$$H_1 : \mu \neq 0$$

- Test statistic standard normal V z



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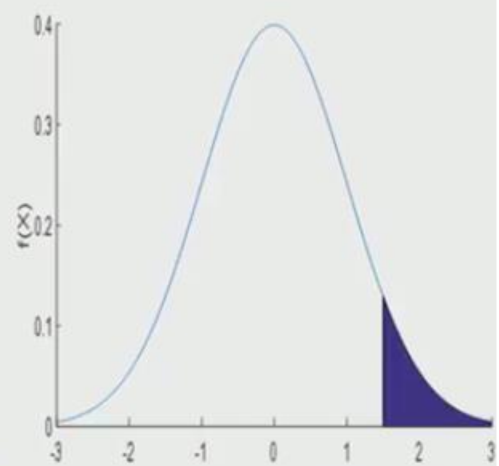
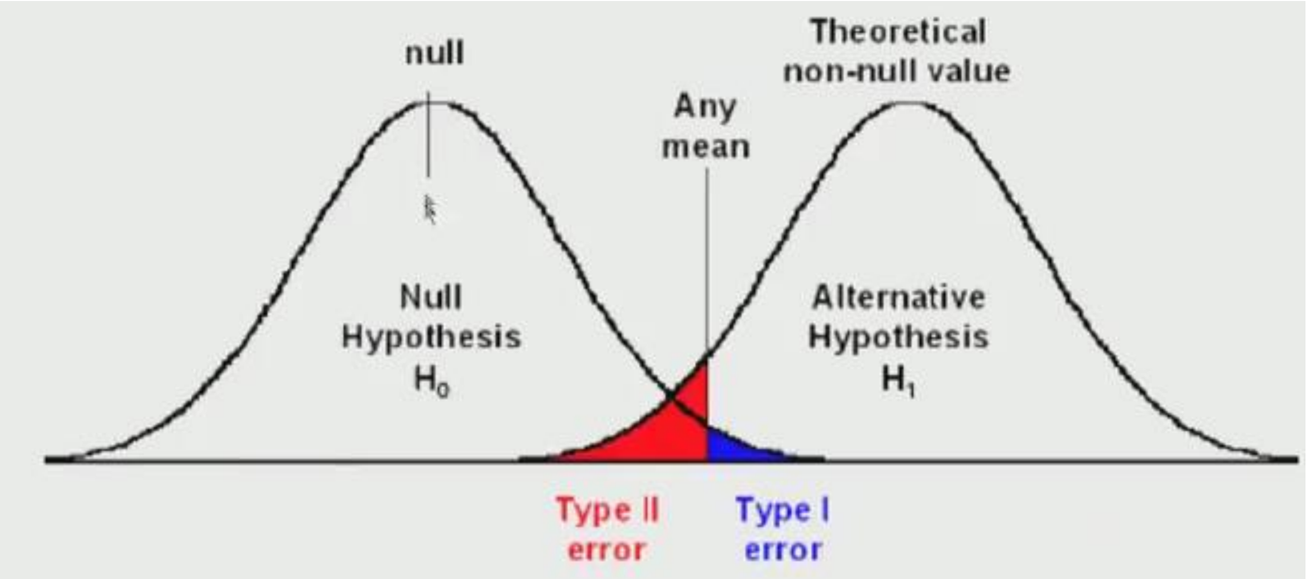


- Reject H_0 if $z \geq 1.5$

Error in Hypothesis Testing

- Type 1 and Type II error probability
  - Statistical Test Power= 1 – Beta = 1 – Type II error Prob.
  - Trade-off: If decrease(Type 1 error prob.) Then increase(type II error prob.)

Decision → / Truth ↓	H_0 is not rejected	H_0 is rejected
H_0 is true	Correct Decision $Pr = 1 - \alpha$	Type I error $Pr = \alpha$
H_1 is true	Type II error $Pr = \beta$	Correct Decision $Pr = 1 - \beta$



Type 1 and Type 2 Error

- **Type 1 and Type 2 error**
  - Type 1 error: we reject a true null hypothesis
  - Type 2 error: We accept a false null hypothesis
- p-value indicates the probability of making a type 1 error
- p-value does not say anything about making a type 2 error!

Model-> Prediction	True (Null Hypothesis)	False (Null Hypothesis)
$P > 0.05$	Correct	Error [Type 2]
$P < 0.05$	Error [Type 1]	Correct

Dataset, N = 20 {4Class, 5Item}		Actual Class (Condition Given) P			N	Performance Measures
Predicted Class (Outcome)	P	TP Correct Decision Pr = $1 - \alpha$	2	1	FP Type I Errors Pr = α	PPV = TP/(TP+FP) = 2 / (2+1) **0.400**
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Parametric & Nonparametric Test Procedures

- Statistical tests are either parametric or non-parametric(distribution-free hypothesis tests)

Parametric Tests	Non-parametric Tests
Depends of Probability Distribution	Distribution-free methods
Require more conditions to be satisfied	Require fewer conditions than their parametric counterparts
Involve Population Parameters (Mean)	Do Not Involve Population Parameters
Use Mean, P	Use median
Have Stringent Assumptions (Normality)	Data Measured on Any Scale (Ratio or Interval, Ordinal or Nominal)
Parametric Test Example:	Nonparametric Test Example:
• t test	• Sign Test
• Z test	• Wilcoxon Signed-Rank Test
• x^2 Test	• Wilcoxon rank sum test
• F test (Analysis of variance)	• Mann-Whitney-Wilcoxon Test
• One Way ANOVA	• Kruskal-Wallis H-Test
• Linear correlation	• Rank Correlation Test
	• Runs test

Parametric Hypothesis Tests

- t test
- Z test
- x^2 Test
- F test (Analysis of variance)
- One Way ANOVA
- Linear correlation

T Test

- **Compares the mean of your sample data to a known value**
- We might want to know how sample mean compares to the population mean
- We should run a one sample t test when
  - We don’t know the population std.dev. or you have a small sample size
- Assumptions: Data should meet these requirements for the test to be valid
  - Data is independent
  - Data is collected randomly
  - Data is approximately normally distributed
- T score is used in t-tests can be positive or negative

T Score

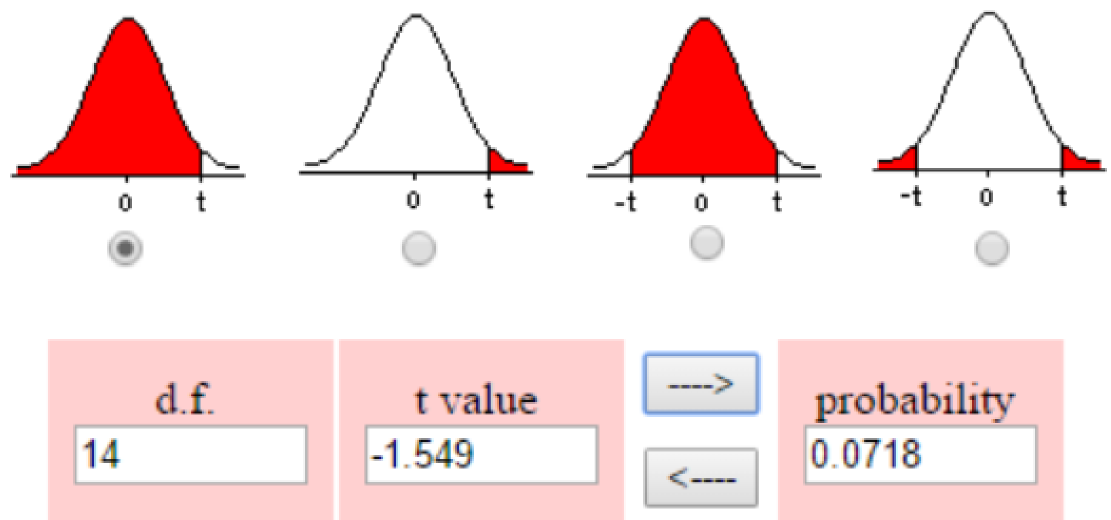
- A t-score is one form of a standardized test statistic (other is the z-score)
- It enables to take an individual score and transform it into a standardized form > one which helps you to compare scores $$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$
- It is used when we don't know the std.dev and we have a small sample (under 30)
  - $\bar{x}$ = sample mean
  - μ_0 = population mean
  - s = sample standard deviation
  - n = sample size
- If you have only one item in sample, the sqrt in the denominator becomes √1
  - This means the formula becomes: $$t = \frac{\bar{x} - \mu}{s}$$
- i.e the larger the t score, larger the difference is between the groups you are testing

T Score

- Example
  - A law school claims it's graduates earn an average of $300 per hour.
  - A sample of 15 graduates is selected and found to have a mean salary of $280 with a sample standard deviation of $50.
  - Assuming the school's claim is true, what is the probability that the mean salary of graduates will be no more than $280?
- Step 1: Plug the information into the formula and solve:
  - $\bar{x}$ = sample mean = 280
  - μ_0 = population mean = 300
  - s = sample standard deviation = 50
  - n = sample size = 15
  - $t = (280 - 300)/ (50/\sqrt{15}) = -20 / 12.909945 = -1.549$
- Step 2: Subtract 1 from the sample size to get the degrees of freedom:
  - 15 – 1 = 14
  - Degrees of freedom lets you know which form of the t distribution to use
  - There are many, but you can solve these problems without knowing that fact!

Example of the T Score Formula

- Step 3: Use a calculator to find the probability using your degrees of freedom (8)
  - You have several options, including the TI-83
  - Here's the result from that calculator
  - Radio button selected under the left tail, as we're looking for a result that's no more than $280:



- The probability is 0.0718, or 7.18%

One Sample T Test: Example

- **Sample question:**
  - **A company wants to improve sales**
  - **Past sales data indicate that the average sale was Rs100 per transaction**
  - **After training the sales force, recent sales data (taken from a sample of 25 salesmen) indicates an average sale of Rs130, with a std.dev of Rs15**
  - **Did the training work?**
  - **Test your hypothesis at a 5% alpha level**

One Sample T Test: Example 1 Steps

- Step 1:
  - Write H_0 statement
  - The accepted hypothesis is that there is no difference in sales, so: H_0: μ = Rs100
- Step 2:
  - Write H_a
  - This is the one you're testing
  - We think that there *is* a difference (that mean sales increased), so: H_a: μ > Rs100
- Step 3:
  - Identify the following pieces of info you'll need to calculate the test statistic
  - The question should give/reveal us these items:
    - **Sample mean**($\bar{x}$): This is given in the question as Rs130
    - **Population mean**(μ): Given as Rs100 (from past data)
    - **Sample standard deviation**(s) = Rs15
    - **Number of observations**(n) = 25

One Sample T Test: Example 1 Steps

- Step 4: Insert the items from above into the t score formula $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
  - **t-value,** t = (130 – 100) / ((15 / √(25))
  - **t-value,** t = (30 / 3) = 10
- Step 5: Find the t-table value
  - We need two values to find t-table value
    - α : 0.05
    - Degree of Freedom (DoF) = # items in the sample (n) minus 1 = 25 – 1 = 24
  - Look up 24 DoF in the left column and 0.05 in the top row form Table = 1.711
  - 1.711 is our one-tailed critical t-value
  - What this critical value means
    - We would expect most values to fall under 1.711
    - If our calculated t-value (from Step 4) falls within this range, H_0 is likely true

One Sample T Test: Example 1 Steps

- Step 6: Compare Step 4 to Step 5
  - Step 4 value **does not** fall into the range calculated in Step 5, so we can reject H_0
  - Value of 10 falls into the rejection region (the left tail)
  - In other words:
    - It is highly likely that the mean sale is greater
    - The sales training was probably a success

Z score

- z-score is also known as a standard score
- it can be placed on a normal distribution curve
- Definition
  - a measure of how many std.dev below or above the μ a raw score is
  - a z-score is # μ from the mean a data point is
- Z-scores range from -3 std. dev. up to +3 std. dev. of Normal Distribution curve
- $z = (x - \mu) / \sigma$

$$z_i = \frac{x_i - \bar{x}}{s}$$

- **Example**

Z score: One Sample

$$z_i = \frac{x_i - \bar{x}}{s}$$

- For example
  - Let's say you have a test score of 190
  - The test has a mean (μ) of 150 and a standard deviation (σ) of 25
- Assuming a normal distribution, your z score would be:
- z = (x – μ) / σ
- z = 190 – 150 / 25 = 1.6
- z score tells how many standard deviations from the mean our score is
- z score is 1.6 standard deviations *above* the mean

Z test

- For solved examples:
  - goo.gl/f5rC77

x^2 test

- For solved examples:
  - goo.gl/f5rC77

One-Way ANOVA test

- For solved examples:
  - goo.gl/f5rC77

Summary

- For solved examples:
  - goo.gl/f5rC77

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Thank You.