Machine Learning

Role of Log Odds in Logistic Regression

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Outline

- Learning with Regression and Trees
  - Learning with Regression
    - Simple Linear Regression
    - Multiple Linear Regression
    - Logistic Regression
  - Learning with Trees
    - Decision Trees
    - Constructing Decision Trees using Gini Index
    - Classification and Regression Trees (CART)

Types of Regression

- Regression models used to find the relationship between a DV and IV.
- Simple linear regression
  - To models the relationship between a DV and a single IV.
- Multiple linear regression
  - If you have more than one independent variable.
- Multiple Regression vs. Multivariate Regression
- Multiple Regression:
  - The influence of several IVs on a DV is examined.
  - One DV is taken into account to analyzed.
- Multivariate Regression:
  - Several regression models are calculated to allow conclusions to be drawn about several DV.
  - Several dependent variables are analyzed.

Simple Linear Regression

$$\hat{y} = b \cdot x + a$$

Multiple Linear Regression

$$\hat{y} = b_1 \cdot x_1 + b_2 \cdot x_2 + \ldots + b_k \cdot x_k + a$$

Logistic Regression

- Types of Logistic Regression
- Binomial Logistic Regression:
  - There can be only two possible types of the DVs, such as 0 or 1, Pass or Fail, etc.
- Multinomial Logistic regression:
  - There can be 3 or more possible unordered types of the DV, such as “cat”, “dogs”, or “sheep”
- Ordinal Logistic regression:
  - There can be 3 or more possible ordered types of DVs, such as “low”, “Medium”, or “High”.

Assumptions of Logistic Regression

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- Independent observations:
  - Each observation is independent of the other. meaning there is no correlation between any input variables.
- Binary dependent variables:
  - It takes the assumption that the DV must be binary or dichotomous, meaning it can take only two values.
  - For more than two categories SoftMax functions are used.
- Linearity relationship between independent variables and log odds:
  - The relationship between the IVs and the log odds of the DV should be linear.
- No outliers:
  - There should be no outliers in the dataset.
- Large sample size:
  - The sample size is sufficiently large.

Role of Log Odds in Logistic Regression

- Role of Log Odds in Logistic Regression
- Logistic Regression models the Log Odds of an event as a linear combination of one or more IVs.
- The function that converts Log Odds to probability is the logistic function, hence the name.
- The unit of measurement for the Log Odds scale is called a logit (logistic unit), hence the alternative names.
- There is need of linearity relationship between IVs and Log Odds;
  - That is the relationship between the IVs and the log odds of the DV should be linear.
- Log Odds is nothing but log of odds, i.e., log(odds).
- **Why Log odds?**
  - Log odds helps to convert the LR model from probability based to a likelihood based (log odds) model.
  - Both probability and log odds have their own set of properties,
    - however log odds makes interpreting the output easier.
    - Thus, using log odds is slightly more advantageous over probability.

Role of Log Odds in Logistic Regression

- Let us understand what odds are before understanding the logistic regression,

$$Odds\ Ratio = \frac{p}{1-p}$$

- **Odds:**
  - Simply put, odds are the chances of success divided by the chances of failure.
  - It is represented in the form of a ratio (as shown here in equation).
    - *Odds Ratio = p / (1-p)*; where, p -> success odds; 1-p -> failure odds
- In logistic regression, the odds of IV corresponding to a success is given by p =>

$$p = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

  - p -> odds of success;
  - β0, β1 -> assigned weights;
  - x -> independent variable;
- So, the odds of failure in this case will be given by (1- p) =>

$$1 - p = 1 - \frac{e^{(\beta_0+\beta_1 x)}}{1+e^{(\beta_0+\beta_1 x)}} = \frac{1}{1+e^{(\beta_0+\beta_1 x)}}$$

- Therefore, the odds ratio is defined as *p / (1-p) =>*
- *Now, we take the log of the odds ratio to get symmetricity in the results.*
- *Therefore, taking log on both sides gives ln[p/(1-p] =>*

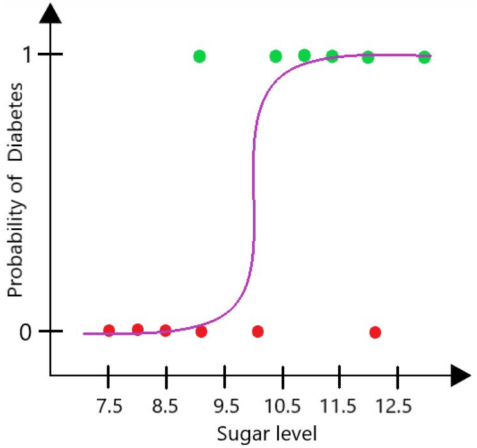
$$\frac{p}{1-p} = \frac{\frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}}{\frac{1}{1+e^{(\beta_0+\beta_1 x)}}} = e^{(\beta_0+\beta_1 x)}$$

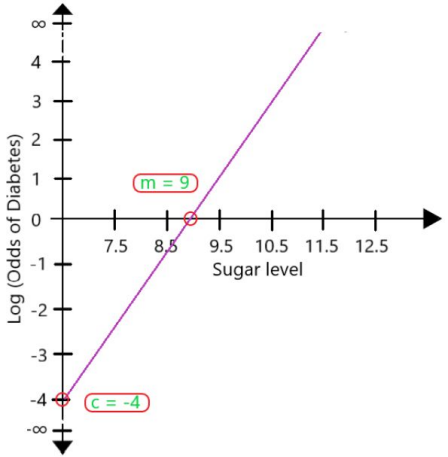
  - *which is the general equation of logistic regression.*
- *Now, in the logistic model, L.H.S contains the log of odds ratio*
  - *that is given by the R.H.S involving a linear combination of weights and IVs.*

$$\ln(\frac{p}{1-p}) = \beta_0 + \beta_1 x$$

Role of Log Odds in Logistic Regression

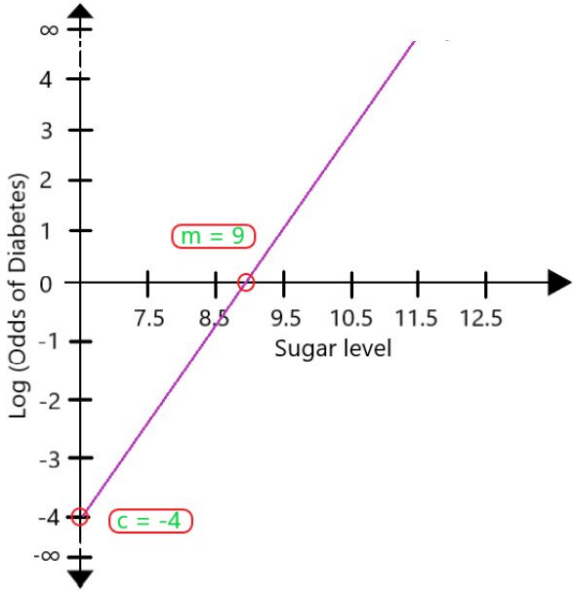
- **1. Example**: (Problem: with Probability based output in Logistic Regression.)
  - Build a LR model to determine
    - the prob. of a person suffering from diabetes based on their sugar level.
  - Observe the plot for this shown in the Figure:
  - The problem remains that o/p of model is only binary based on this plot.
  - To tackle this problem,
    - we use the concept of log odds present in logistic regression.
- 2. **Example** to solve the above problem: (Solution: Transforming Output)
  - We convert the probability-based output to log odds based output as:
    - P(diabetes) = log (odds of diabetes) =log_e [p/(1-p)]
    - Assume random values of p and see how the y-axis is transformed.





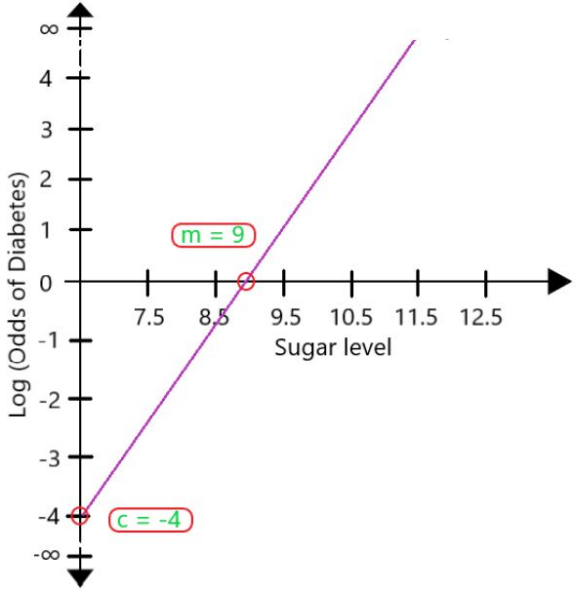
Role of Log Odds in Logistic Regression

- P(diabetes) = log (odds of diabetes) =$\log_e$ [p/(1-p)]
- Assume random values of p and see how the y-axis is transformed.
- 1. Boundary values;
  - at p=1 => $\log_e(p/(1-p)) = \log_e(1/0) = \log_e(1) - \log_e(0) = 0-(-) =$
  - at p=0 => $\log_e(p/(1-p)) = \log_e(0/1) = \log_e(0) - \log_e(1) = (-)-0 = -$
  - So, the domain of y axis is: (-∞, ∞)
- 2. Middle value;
  - at p=0.5 => $\log_e(p/(1-p)) = \log_e(0.5/0.5) = \log_e(1) = 0$
  - So, at p = 0.5 -> log (odds) = y = 0.
- 3. At random values;
  - at p=0.75 => $\log_e(p/(1-p)) = \log_e(0.75/0.25) = \log_e(3) = 1.09$
  - at p=0.3 => $\log_e(p/(1-p)) = \log_e(0.3/0.7) = \log_e(3) - \log_e(7) = -0.84$
  - So, at p > 0.5 -> we get value of log(odds) in range (0, ∞)
  - and at p < 0.5 -> we get value of log(odds) in range (-∞, 0)
- If we map these values onto a transformed plot,
  - it would look like as shown in this Figure.



Role of Log Odds in Logistic Regression

- Based on the value of slope (m) and intercept (c),
  - we can easily interpret the model and get non-binary deterministic output.
  - This is power of log odds in Logistic Regression.
- Log odds commonly known as Logit function is used in Logistic Regression models
  - when we are looking non-binary output.
- This is how logistic regression is able to work as both a regression as well as classification model.



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Thank You.