SVD: Example 2

For symmetric matrices, eigenvectors for distinct eigenvalues are orthogonal

All eigenvalues of a real symmetric matrix are real

All eigenvalues of a positive semi-definite matrix are non-negative

A positive semi-definite matrix is a symmetric matrix with non-negative eigenvalues.

$$S = \begin{array}{c|c} -5 & 2 \\ \hline -7 & 4 \end{array}$$

Ans: Calculate Eigenvalues

$$\begin{vmatrix}
-5-\lambda & 2 \\
-7 & 4-\lambda
\end{vmatrix} = 0$$

0

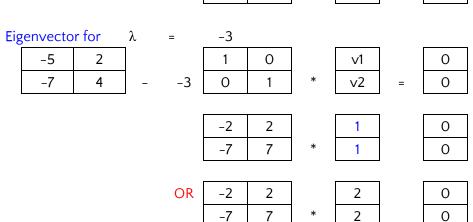
0

$$\lambda = 2 \qquad \lambda = 0$$

$$\lambda = -3 \qquad \lambda = 0$$

Plug in these eigenvalues values to find eigenvectors

$$(S - \lambda * I) V = 0$$



The eigenvectors are orthogonal (and real):

Columns of U are eigenvectors of S; (Recall UU-1 =1)

$$U = \begin{array}{|c|c|c|c|c|}\hline 1 & 0.286 \\\hline 1 & 1 \\\hline \end{array}$$

$$U^{-1} = \begin{array}{c|c} 1.4 & -0.4 \\ \hline -1.4 & 1.4 \end{array}$$

Diagonal elements of $\boldsymbol{\Sigma}$ are eigenvalues of \boldsymbol{S}

$$\Sigma = \begin{array}{|c|c|c|c|c|} \hline -3 & 0 \\ \hline 0 & 2 \\ \hline \end{array}$$

Reconstructed of S from U, $U^{\text{--}1}$ and Σ

$$S = U\Sigma U^{-1}$$

$$S = \begin{bmatrix} -3 & 0.5714 \\ -3 & 2 \end{bmatrix}$$

Reconstructed S