LINEAR PROGRAMMING

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LINEAR PROGRAMMING

- A Linear Programming Problem LPP.
- LP formulation.
- LP format.
- Feasible set FS.
- Fundamental theorem of LP.
- A method for solving LPP.
- Shadow prices.

LINEAR PROGRAMMING -LP

- A method for solving optimization problems
- A linear function is to be max/min –
  - Eg. Cost, profit, distance, weight, …
- Mathematic models involve
  - Systems of linear inequalities
  - → Systems of linear equations
  - → Matrices

FURNITURE MANUFACTURING PROBLEM - FMP

- Product Type : Chairs & Sofas
- Operations : Carpentry, Finishing, Upholstery

	Chairs	Sofas	Available time (h/d)
Carpentry	6h	3h	96
Finishing	1h	1h	18
Upholstery	2h	6h	72
Profit	$80	$70	

- How many Chairs & Sofas produced each day to maximized the profit

FMP – MATHEMATIC FORMULATION

- x , y : No. Chairs , Sofas produced
- Objective function : Max profit

  $$\text{Max } Z = 80x + 70y$$

- Constrains : Available time
  - Carpentry : $6x + 3y \leq 96$
  - Finishing : $1x + 1y \leq 18$
  - Upholstery: $2x + 6y \leq 72$
  - Not negative: $x \geq 0$, $y \geq 0$

LP FORMAT

$$\text{Max} \qquad Z = 80x + 70y$$

Subject to:

$$6x + 3y \le 96$$

$$1x + 1y \le 18$$

$$2x + 6y \le 72$$

$$x \ge 0 \;,\; y \ge 0$$

FEASIBLE SET

- Feasible set – FS
  - A set
  - containing all points that satisfy all the restrictions

Ex: FME

$6x + 3y \leq 96$

$1x + 1y \leq 18$

$2x + 6y \leq 72$

$x \geq 0$, $y \geq 0$

- A LPP –
  - To find the point/points in the FS
  - at which the value of the objective function is either maximized or minimized.

GRAPH THE FEASIBLE SET

- Put the inequality in standard form
- Graph the straight line corresponding to each inequality
- Determine the side of the line belonging to the graph of each inequality.

  Cross out the other side.

  The remaining region is the FS

GRAPH THE FEASIBLE SET

$6x + 3y \leq 96$

$1x + 1y \leq 18$

$2x + 6y \leq 72$

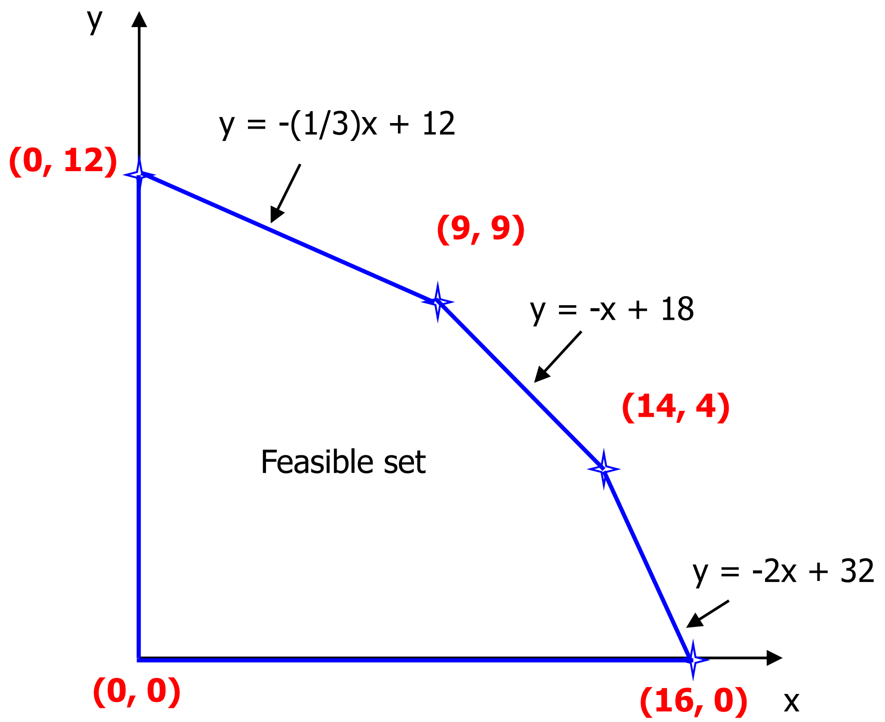
$x \geq 0 \, , \, y \geq 0$

$y \leq -2x + 32$

$y \leq -x + 18$

$y \leq -1/3\,x + 12$

$x \geq 0 \, , \, y \geq 0$



FUNDAMENTAL THEOREM OF LP

- The max / min value of the objective function
  - Is achieved
  - At one of the vertices of the FS

Ex: FMP

Vertex (x,y)	Profit = 80x+70y
A(14,4)	1400
B(9,9)	1350
C(0,12)	840
D(0,0)	0
E(16,0)	1280

A METHOD FOR SOLVING LP

- Mathematical Modeling
  - Organize the data
  - Identify the unknown quantities
  - Define corresponding variables
  - Translate the restrictions into system of linear inequalities
  - Form the objective function
- Graph the feasible set
- Determine the vertices of the feasible set
- Determine the optimal point.

  Evaluate the objective function at each vertex.

A METHOD FOR SOLVING LP

Ex: FMP

- Mathematical Modeling: Max $Z = 80x + 70y$

  St : $6x + 3y \leq 96$

  $1x + 1y \leq 18$

  $2x + 6y \leq 72$

  $x \geq 0 , y \geq 0$

- Graph the feasible set
- Determine the vertices of the feasible set

  A(14,4), B(9,9), C(0,12), D(0,0), E(16,0)

- Determine the optimal point.

Vertex (x,y)	Profit = 80x+70y
A(14,4)	1400
B(9,9)	1350
C(0,12)	840
D(0,0)	0
E(16,0)	1280

→ x = 14, y = 4, z = 1400

LP FORMULATION

- Investment planning problem
- Transportation shipping problem

Investment planning problem

Ex:

Invest $ 30 million in:

- Treasury notes
- Bonds
- Stocks

Constraints:

1. At least $3 million / each type
2. At least half the money invested in bonds and treasury notes
3. The amount invested in Bonds not exceed twice the amount invested in Treasury notes]

And, Annual yields:

* Treasury Notes:	7%
* Bonds	8%
* Stocks	9%

How should the money be allocated to produce the largest return?

Investment planning problem

- Let

  x = the amount to be invested in Treasury Notes

  y = the amount to be invested in Bonds

Therefore, the amount to be invested in Stocks is: 30 – (x + y)

- We have:

	Treasury notes	Bonds	Stocks
Yield	.07	.08	.09
Variables	x	y	30 - (x + y)

Investment planning problem

Constraints

1. At least 3 (million dollars) must be invested in each category, then

$$x \geq 3$$

$$y \geq 3$$

$$30 - (x + y) \geq 3$$

2. At least $ 15 million must be invested in Treasury notes and Bonds

$$x + y \geq 15$$

3. The amount invested in Bonds must not exceed twice the amount invested in Treasury notes:

$$y \leq 2x$$

Investment planning problem

The objective:

Max Z = .07x + .08y + .09[30 – (x + y)] = .07 x +.08y + 2.7 - .09x - .09y

= 2.7 - .02x - .01y

Therefore, the model of problem can be summarized as follows:

Max Z = 2.7 - .02x - .01y

Subject to:

$$x \geq 3$$

$$y \geq 3$$

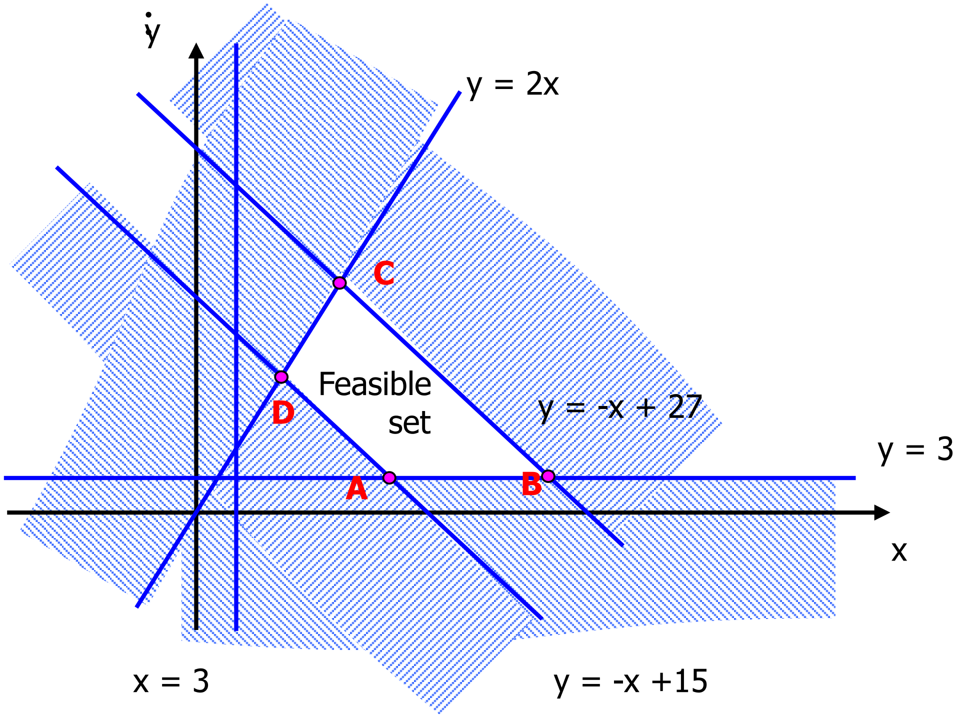
$$30 - (x + y) \geq 3$$

$$x + y \geq 15$$

$$y \leq 2x$$

Investment planning problem

Solution



A = (12, 3) → Z = $2.43 millions

B = (24, 3) → Z = $2.19 millions

C = (9, 18) → Z = $2.34 millions

D = (5, 10) → Z = $2.50 millions

Thus,

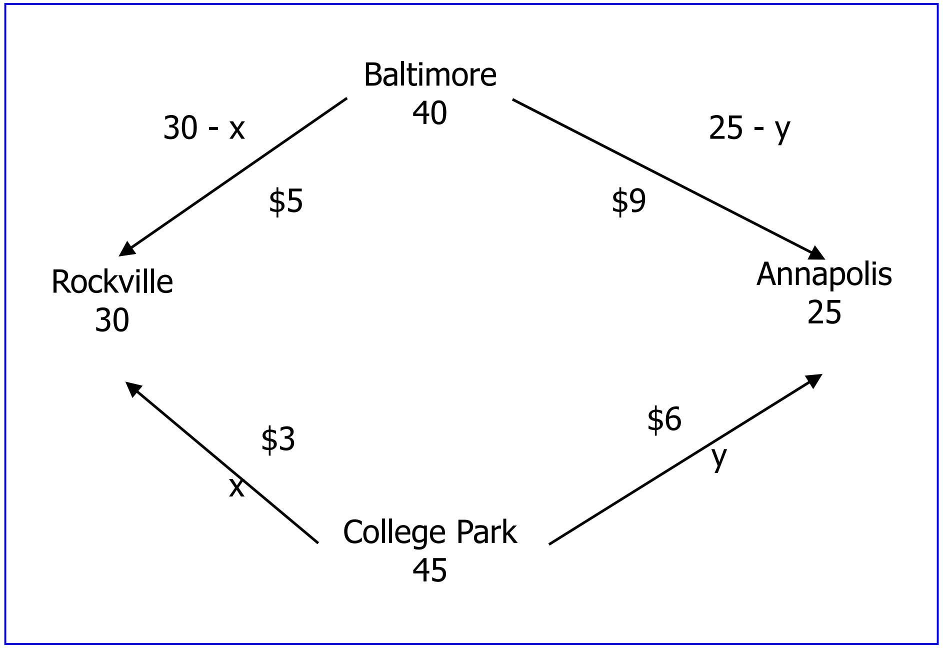
x = $5, y = 10

Or we invest :

- $5 millions in Treasury notes
- $10 millions in Bonds
- $15 millions in Stocks

Transportation shipping problem

Ex:



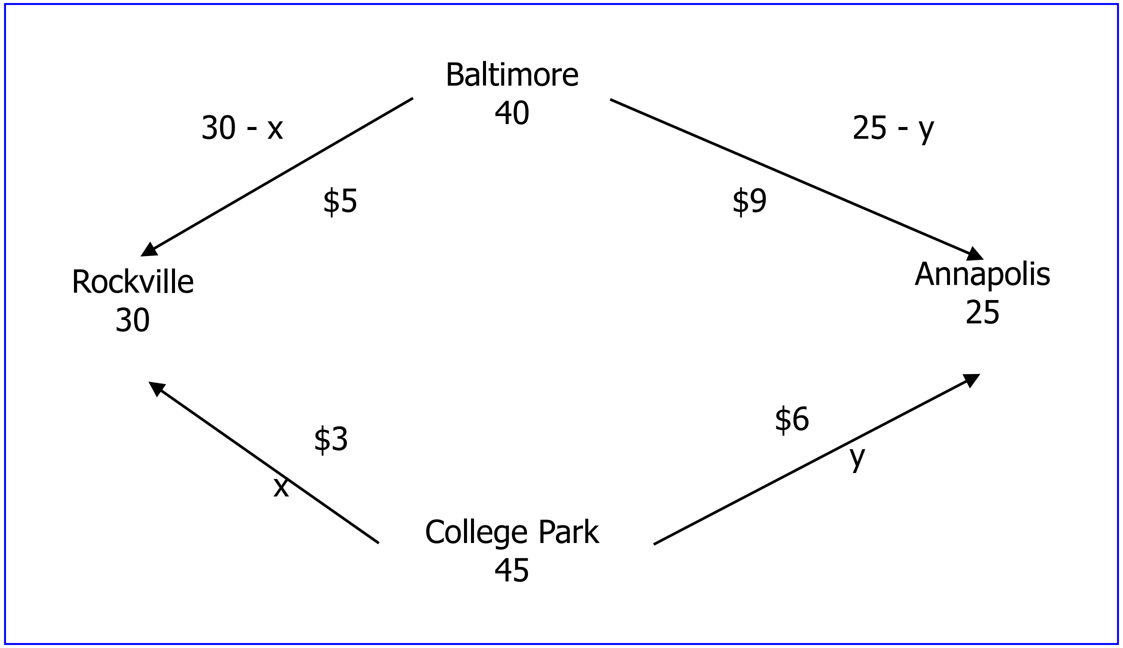
Maryland TV dealer:

- Stores in Annapolis & Rockville
  - Store orders
- Warehouses in College park & Baltimore.
  - Warehouse stocks
- Cost of shipping

→ The most economy way to supply the requested TV sets to the 2 stores?

Transportation shipping problem

Ex:



Variables:

x: No. of TV sets to be shipped from College park to Rockvill

y: No. of TV sets to be shipped from College park to Annapolis

Thus,

30 – x: No. of TV sets to be shipped from Baltimore to Rockvill

25 – y: No. of TV sets to be shipped from Baltimore to Annapolis

Transportation shipping problem

Constraints:

- Nonegative conditions:

$$x \geq 0$$

$$y \geq 0$$

$$30 - x \geq 0$$

$$25 - y \geq 0$$

- Warehouse cannot ship more TV sets than it has in stock

$$55 - x - y \leq 40$$

$$-x - y \leq -15$$

$$x + y \geq 0$$

Transportation shipping problem

Objective function:

Min Z = 3x + 6y + 5(30 – x) + 9(25 – y) = 3x + 6y + 150 – 5x + 225 – 9y

= 375 – 2x – 3y

Therefore, the model can be summarized as follows,

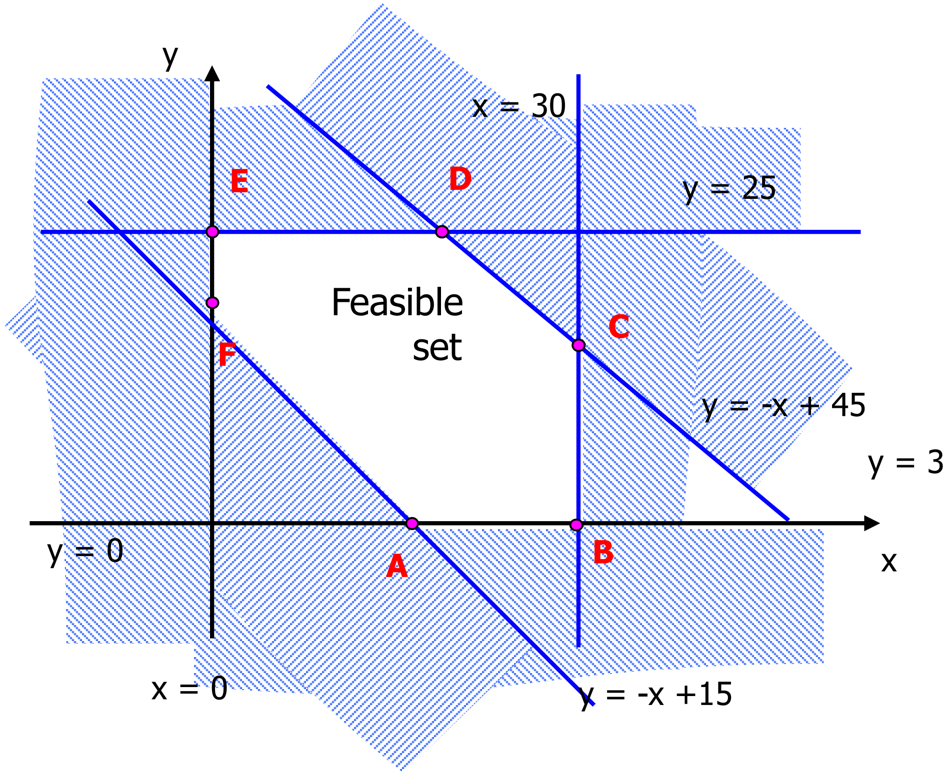
Min Z = 375 – 2x – 3y

Subject to:

$$\begin{cases} x \geq 0, & y \geq 0 \\ x \leq 30, & y \leq 25 \\ x + y \geq 15 & \\ x + y \leq 45 & \end{cases}$$

Transportation shipping problem

Solution:



Vertex:

A = (15, 0) → Z = 345

B = (30, 0) → Z = 315

C = (30, 15) → Z = 270

D = (20, 25) → Z = 260

E = (0, 25) → Z = 300

F = (0, 15) → Z = 330

SHADOW PRICES

- The shadow price for a constraint
  - The max. price
  - Willing to pay for additional constraint unit

  Ex: the Furniture manufacturing problem

  Finishing constraint : $x + y \leq 18$

  The max. price - Willing to pay for additional finishing hour
- How to find SP for finishing
  - How much was the profit increased due to the additional hour for finishing.
- The SP associated w. a constraint
  - The change in the value of ?
  - Per unit change of the constraint , RHS resource.

SHADOW PRICES

Ex: Furniture Manufacturing Problem

Max Z = 80x + 70y

St:
$$\begin{cases} 6x+3y \le 96 \\ x+y \le 18 \\ 2x+6y \le 72 \\ x \ge 0, y \ge 0 \end{cases}$$

→ x = 14, y = 4, Z = 1400

What is the shadow price for the finishing constraint?

$$x+y \le 18 \rightarrow x+y \le 19$$

The new problem,

Max Z = 80x + 70y

Subject to:
$$\begin{cases} 6x+3y \le 96 \\ x+y \le 19 \\ 2x+6y \le 72 \\ x \ge 0, y \ge 0 \end{cases}$$

→ x = 13, y = 6, Z = 1460

The new maximum profit is 1,460.

→ SP for the finishing constraint : $60

FINITE MATHEMATICS
& ITS APPLICATIONS

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