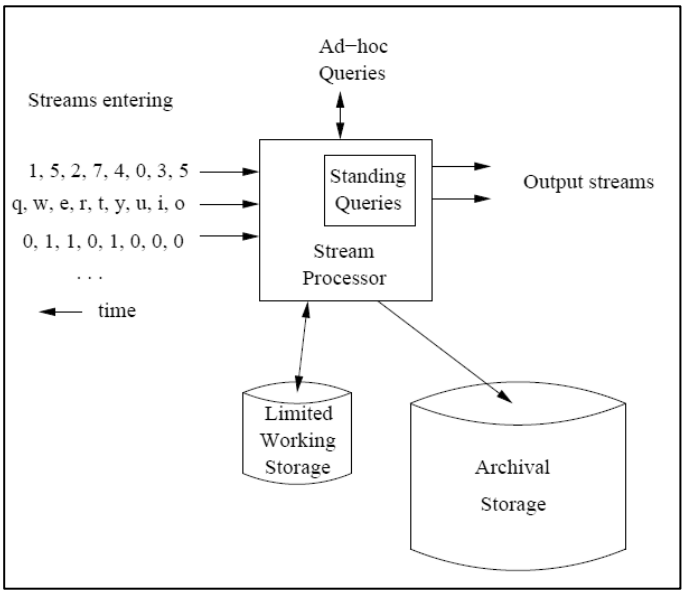
# Big Data Analytics (BDA)

## Mining Big Data Streams

**Stream Data Model:**




**A Data-Stream Management System:** In analogy to a database-management system, we can view a stream processor as a kind of data-management system, the high-level organization of which is suggested in fig. Any number of streams can enter the system. Each stream can provide elements at its own schedule; they need not have the same data rates or data types, and the time between elements of one stream need not be uniform. The fact that the rate of arrival of stream elements is not under the control of the system distinguishes stream processing from the processing of data that goes on within a database-management system. The latter system controls the rate at which data is read from the disk, and therefore never has to worry about data getting lost as it attempts to execute queries.
Streams may be archived in a large archival store, but we assume it is not possible to answer queries from the archival store. It could be examined only under special circumstances using time-consuming retrieval processes. There is also a working store, into which summaries or parts of streams may be placed, and which can be used for answering queries. The working store might be disk, or it might be main memory, depending on how fast we need to process queries. But either way, it is of sufficiently limited capacity that it cannot store all the data from all the streams.

**Examples of Stream Sources:**

**1. Sensor Data:** Imagine a temperature sensor bobbing about in the ocean, sending back to a base station a reading of the surface temperature each hour. The data produced by this sensor is a stream of real numbers. It is not a very interesting stream, since the data rate is so low. It would not stress modern technology, and the entire stream could be kept in main memory, essentially forever. Now, give the sensor a GPS unit, and let it report surface height instead of temperature. The surface height varies quite rapidly compared with temperature, so we might have the sensor send back a reading every tenth of a second. If it sends a 4-byte real number each time, then it produces 3.5 megabytes per day. It will still take some time to fill up main memory, let alone a single disk.

**2. Image Data:** Satellites often send down to earth streams consisting of many terabytes of images per day. Surveillance cameras produce images with lower resolution than satellites, but there can be many of them, each producing a stream of images at intervals like one second. London is said to have six million such cameras, each producing a stream.

**3. Internet and Web Traffic:** A switching node in the middle of the Internet receives streams of IP packets from many inputs and routes them to its outputs. Normally, the job of the switch is to transmit data and not to retain it or query it. But there is a tendency to put more capability

into the switch, e.g., the ability to detect denial-of-service attacks or the ability to reroute packets based on information about congestion in the network.
Web sites receive streams of various types. For example, Google receives several hundred million search queries per day. Yahoo! accepts billions of "clicks" per day on its various sites. Many interesting things can be learned from these streams. For example, an increase in queries like "sore throat" enables us to track the spread of viruses. A sudden increase in the click rate for a link could indicate some news connected to that page, or it could mean that the link is broken and needs to be repaired.

**Stream Queries:**
There are two ways that queries get asked about streams.
**1. Standing queries:** These queries are, in a sense, permanently executing, and produce outputs at appropriate times.
e.g. The stream produced by the ocean-surface-temperature sensor might have a standing query to output an alert whenever the temperature exceeds 25 degrees centigrade. This query is easily answered, since it depends only on the most recent stream element.

**2. ad-hoc query:** a question asked once about the current state of a stream or streams. If we want the facility to ask a wide variety of ad-hoc queries, a common approach is to store a sliding window of each stream in the working store. A sliding window can be the most recent n elements of a stream, for some n, or it can be all the elements that arrived within the last t time units e.g., one day. If we regard each stream element as a tuple, we can treat the window as a relation and query it with any SQL query. Of course, the stream-management system must keep the window fresh, deleting the oldest elements as new ones come in.

**Issues in Stream Processing:**
**1**. First, streams often deliver elements very rapidly. We must process elements in real time, or we lose the opportunity to process them at all, without accessing the archival storage. Thus, it often is important that the stream-processing algorithm is executed in main memory, without access to secondary storage or with only rare accesses to secondary storage.
**2**. Moreover, even when streams are "slow," there may be many such streams. Even if each stream by itself can be processed using a small amount of main memory, the requirements of all the streams together can easily exceed the amount of available main memory.

**Sampling Data in a Stream (Sampling Techniques):**
A search engine receives a stream of queries, and it would like to study the behavior of typical users. We assume the stream consists of tuples (user, query, time). Suppose that we want to answer queries such as "What fraction of the typical user's queries were repeated over the past month?" Assume also that we wish to store only 1/10th of the stream elements.
**1. Obtaining a Representative Sample**
Like many queries about the statistics of typical users, cannot be answered by taking a sample of each user's search queries. Thus, we must strive to pick 1/10th of the users, and take all their searches for the sample, while taking none of the searches from other users. If we can store a list of all users, and whether or not they are in the sample, then we could do the following. Each time a search query arrives in the stream, we look up the user to see whether or not they are in the sample. If so, we add this search query to the sample, and if not, then not. However, if we have no record of ever having seen this user before, then we generate a random integer between 0 and 9. If the number is 0, we add this user to our list with value "in," and if the number is other than 0, we add the user with the value "out."

That method works as long as we can afford to keep the list of all users and they're in/out decision in main memory, because there isn't time to go to disk for every search that arrives. By using a hash function, one can avoid keeping the list of users. That is, we hash each user name to one of ten buckets, 0 through 9. If the user hashes to bucket 0, then accept this search query for the sample, and if not, then not.

**2. The General Sampling Problem:**

The running example is typical of the following general problem. Our stream consists of tuples with n components. A subset of the components are the key components, on which the selection of the sample will be based. In our running example, there are three components – user, query, and time – of which only user is in the key. However, we could also take a sample of queries by making query be the key, or even take a sample of user-query pairs by making both those components form the key.

To take a sample of size a/b, we hash the key value for each tuple to b buckets, and accept the tuple for the sample if the hash value is less than a. If the key consists of more than one component, the hash function needs to combine the values for those components to make a single hash-value. The result will be a sample consisting of all tuples with certain key values. The selected key values will be approximately a/b of all the key values appearing in the stream.

**3. Varying the Sample Size:**

Often, the sample will grow as more of the stream enters the system. In our running example, we retain all the search queries of the selected 1/10th of the users, forever. As time goes on, more searches for the same users will be accumulated, and new users that are selected for the sample will appear in the stream.

If we have a budget for how many tuples from the stream can be stored as the sample, then the fraction of key values must vary, lowering as time goes on. In order to assure that at all times, the sample consists of all tuples from a subset of the key values, we choose a hash function h from key values to a very large number of values 0, 1, . . . ,B−1. We maintain a threshold t, which initially can be the largest bucket number, B − 1. At all times, the sample consists of those tuples whose key K satisfies $h(K) \leq t$. New tuples from the stream are added to the sample if and only if they satisfy the same condition.

If the number of stored tuples of the sample exceeds the allotted space, we lower t to t−1 and remove from the sample all those tuples whose key K hashes to t. For efficiency, we can lower t by more than 1, and remove the tuples with several of the highest hash values, whenever we need to throw some key values out of the sample. Further efficiency is obtained by maintaining an index on the hash value, so we can find all those tuples whose keys hash to a particular value quickly.

**Filtering Streams:**

Another common process on streams is selection, or filtering. We want to accept those tuples in the stream that meet a criterion. Accepted tuples are passed to another process as a stream, while other tuples are dropped. If the selection criterion is a property of the tuple that can be calculated (e.g., the first component is less than 10), then the selection is easy to do. The problem becomes harder when the criterion involves lookup for membership in a set. It is especially hard, when that set is too large to store in main memory.

**Bloom Filter:**

A Bloom filter consists of:

1. An array of n bits, initially all 0's.

2. A collection of hash functions h1, h2, . . . , hk. Each hash function maps "key" values to n buckets, corresponding to the n bits of the bit-array.
3. A set S of m key values.
The purpose of the Bloom filter is to allow through all stream elements whose keys are in S, while rejecting most of the stream elements whose keys are not in S.
To initialize the bit array, begin with all bits 0. Take each key value in S and hash it using each of the k hash functions. Set to 1 each bit that is hi(K) for some hash function hi and some key value K in S.
To test a key K that arrives in the stream, check that all of h1(K), h2(K), . . . , hk(K) are 1's in the bit-array. If all are 1's, then let the stream element through. If one or more of these bits are 0, then K could not be in S, so reject the stream element.

**Analysis of Bloom Filtering**

If a key value is in S, then the element will surely pass through the Bloom filter. However, if the key value is not in S, it might still pass. We need to understand how to calculate the probability of a false positive, as a function of n, the bit-array length, m the number of members of S, and k, the number of hash functions.
The model to use is throwing darts at targets. Suppose we have x targets and y darts. Any dart is equally likely to hit any target. After throwing the darts, how many targets can we expect to be hit at least once? The analysis goes as follows:
• The probability that a given dart will not hit a given target is $(x - 1)/x$.
• The probability that none of the y darts will hit a given target is $((x-1)/x)^y$. We can write this expression as $(1 - 1/x)^{x^{\wedge}( y/x )}$
• Using the approximation $(1-\epsilon)^{1/\epsilon} = 1/e$ for small $\epsilon$ (re we conclude that the probability that none of the y darts hit a given target is $e^{-y/x}$.

**Counting Distinct Elements in a Stream:**

A compared to sampling and filtering – it is somewhat tricky to do what we want in a reasonable amount of main memory, so we use a variety of hashing and a randomized algorithm to get approximately what we want with little space needed per stream.

**1. The Count-Distinct Problem**

Suppose stream elements are chosen from some universal set. We would like to know how many different elements have appeared in the stream, counting either from the beginning of the stream or from some known time in the past.
The obvious way to solve the problem is to keep in main memory a list of all the elements seen so far in the stream. Keep them in an efficient search structure such as a hash table or search tree, so one can quickly add new elements and check whether or not the element that just arrived on the stream was already seen. As long as the number of distinct elements is not too great, this structure can fit in main memory and there is little problem obtaining an exact answer to the question how many distinct elements appear in the stream.
However, if the number of distinct elements is too great, or if there are too many streams that need to be processed at once (e.g., Yahoo! wants to count the number of unique users viewing each of its pages in a month), then we cannot store the needed data in main memory. There are several options. We could use more machines, each machine handling only one or several of the streams. We could store most of the data structure in secondary memory and batch stream elements so whenever we brought a disk block to main memory there would be many tests and updates to be performed on the data in that block. Or we could use the strategy where we only

estimate the number of distinct elements but use much less memory than the number of distinct elements.

**2. The Flajolet-Martin Algorithm**

The idea behind the Flajolet-Martin Algorithm is that the more different elements we see in the stream, the more different hash-values we shall see. As we see more different hash-values, it becomes more likely that one of these values will be "unusual." The particular unusual property we shall exploit is that the value ends in many 0's, although many other options exist.

Whenever we apply a hash function h to a stream element a, the bit string h(a) will end in some number of 0's, possibly none. Call this number the tail length for a and h. Let R be the maximum tail length of any a seen so far in the stream. Then we shall use estimate 2R for the number of distinct elements seen in the stream.

This estimate makes intuitive sense. The probability that a given stream element a has h(a) ending in at least r 0's is $2^{-r}$. Suppose there are m distinct elements in the stream. Then the probability that none of them has tail length at least r is $(1 - 2^{-r})^m$. This sort of expression should be familiar by now.

We can rewrite it as $((1 - 2^{-r})^{2r})^{m2^{\wedge}-r}$. Assuming r is reasonably large, the inner expression is of the form $(1 - \epsilon)^{1/\epsilon}$, which is approximately 1/e. Thus, the probability of not finding a stream element with as many as r 0's at the

end of its hash value is $e^{-m2^{\wedge}-r}$. We can conclude:

1. If m is much larger than 2r, then the probability that we shall find a tail of length at least r approaches 1.
2. If m is much less than 2r, then the probability of finding a tail length at least r approaches 0.

We conclude from these two points that the proposed estimate of m, which is 2R (recall R is the largest tail length for any stream element) is unlikely to be either much too high or much too low.

**3. Combining Estimates**

There are two methods to combine estimates. We can combine the two methods. First, group the hash functions into small groups, and take their average. Then, take the median of the averages. It is true that an occasional outsized $2^R$ will bias some of the groups and make them too large. However, taking the median of group averages will reduce the influence of this effect almost to nothing. Moreover, if the groups themselves are large enough, then the averages can be essentially any number, which enables us to approach the true value m as long as we use enough hash functions. In order to guarantee that any possible average can be obtained, groups should be of size at least a small multiple of $\log_2^m$.

**4. Space Requirements**

Observe that as we read the stream it is not necessary to store the elements seen. The only thing we need to keep in main memory is one integer per hash function; this integer records the largest tail length seen so far for that hash function and any stream element. If we are processing only one stream, we could use millions of hash functions, which is far more than we need to get a close estimate. Only if we are trying to process many streams at the same time would main memory constrain the number of hashes functions, we could associate with any one stream. In practice, the time it takes to compute hash values for each stream element would be the more significant limitation on the number of hash functions we use.

**Counting Ones in a Window**

Suppose we have a window of length N on a binary stream. We want at all times to be able to answer queries of the form "how many 1's are there in the last k bits?" for any $k \leq N$.

**1. The Cost of Exact Counts**

Suppose we want to be able to count exactly the number of 1's in the last k bits for any $k \leq N$. Then we claim it is necessary to store all N bits of the window, as any representation that used fewer than N bits could not work. In proof, suppose we have a representation that uses fewer than N bits to represent the N bits in the window. Since there are $2^N$ sequences of N bits, but fewer than $2^N$ representations, there must be two different bit strings w and x that have the same representation. Since $w \neq x$, they must differ in at least one bit. Let the last $k-1$ bit of w and x agree, but let them differ on the kth bit from the right end.

e.g. If w = 0101 and x = 1010, then k = 1, since scanning from the right, they first disagree at position 1. If w = 1001 and x = 0101, then k = 3, because they first disagree at the third position from the right.

Suppose the data representing the contents of the window is whatever sequence of bits represents both w and x. Ask the query "how many 1's are in the last k bits?" The query-answering algorithm will produce the same answer, whether the window contains w or x, because the algorithm can only see their representation. But the correct answers are surely different for these two bit-strings. Thus, we have proved that we must use at least N bits to answer queries about the last k bits for any k.

**2. The Datar-Gionis-Indyk-Motwani (DGIM) Algorithm**

Each bit of the stream has a timestamp, the position in which it arrives. The first bit has timestamp 1, the second has timestamp 2, and so on. Since we only need to distinguish positions within the window of length N, we shall represent timestamps modulo N, so they can be represented by $\log_2 N$ bits. If we also store the total number of bits ever seen in the stream (i.e., the most recent timestamp) modulo N, then we can determine from a timestamp modulo N where in the current window the bit with that timestamp is.

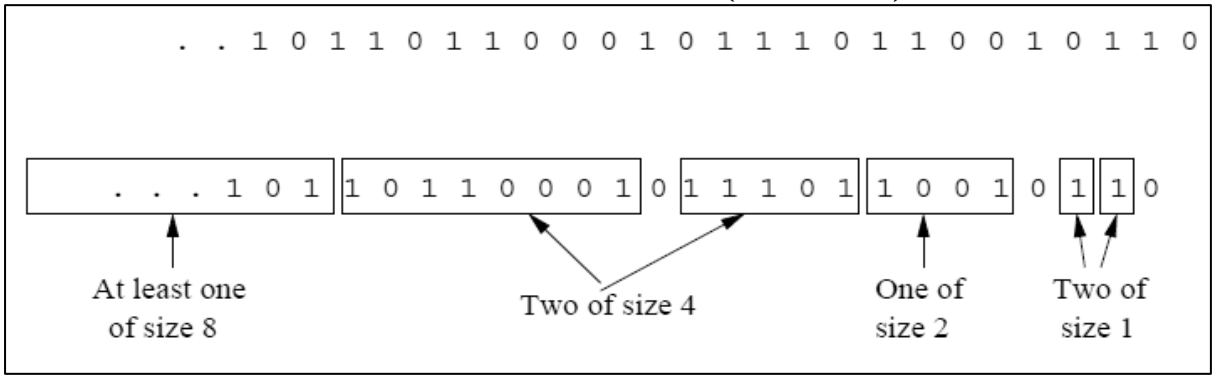
We divide the window into buckets,5 consisting of:

1. The timestamp of its right (most recent) end.
2. The number of 1's in the bucket. This number must be a power of 2, and we refer to the number of 1's as the size of the bucket.

To represent a bucket, we need $\log_2 N$ bits to represent the timestamp (modulo N) of its right end. To represent the number of 1's we only need $\log_2 \log_2 N$ bits. The reason is that we know this number i is a power of 2, say $2^j$, so we can represent i by coding j in binary. Since j is at most $\log_2 N$, it requires $\log_2 \log_2 N$ bits. Thus, O(logN) bits suffice to represent a bucket.

There are six rules that must be followed when representing a stream by buckets.

- The right end of a bucket is always a position with a 1.
- Every position with a 1 is in some bucket.
- No position is in more than one bucket.
- There are one or two buckets of any given size, up to some maximum size.
- All sizes must be a power of 2.
- Buckets cannot decrease in size as we move to the left (back in time).



**3. Query answering in the DGIM algorithm:**

Suppose we are asked how many 1's there are in the last k bits of the window, for some $1 \leq k \leq N$. Find the bucket b with the earliest timestamp that includes at least some of the k most recent bits. Estimate the number of 1's to be the sum of the sizes of all the buckets to the right (more recent) than bucket b, plus half the size of b itself.

Suppose the stream is 1 0 1 1 0 1 1 0 0 0 1 0 1 1 1 0 1 1 0 0 1 0 1 1 0 and k = 10. Then the query asks for the number of 1's in the ten rightmost bits, which happen to be 0110010110. Let the current timestamp (time of the rightmost bit) be t. Then the two buckets with one 1, having timestamps $t - 1$ and $t - 2$ are completely included in the answer. The bucket of size 2, with timestamp $t - 4$, is also completely included. However, the rightmost bucket of size 4, with timestamp $t-8$ is only partly included. We know it is the last bucket to contribute to the answer, because the next bucket to its left has timestamp less than $t - 9$ and thus is completely out of the window. On the other hand, we know the buckets to its right are completely inside the range of the query because of the existence of a bucket to their left with timestamp $t - 9$ or greater.

Our estimate of the number of 1's in the last ten positions is thus 6. This number is the two buckets of size 1, the bucket of size 2, and half the bucket of size 4 that is partially within range. Of course, the correct answer is 5.

Suppose the above estimate of the answer to a query involves a bucket b of size $2^j$ that is partially within the range of the query. Let us consider how far from the correct answer c our estimate could be. There are two cases: the estimate could be larger or smaller than c.

Case 1: The estimate is less than c. In the worst case, all the 1's of b are actually within the range of the query, so the estimate misses half bucket b, or $2^{j-1}$ 1's. But in this case, c is at least $2^j$; in fact it is at least $2^{j+1} - 1$, since there is at least one bucket of each of the sizes $2^{j-1}$, $2^{j-2}$, . . . , 1. We conclude that our estimate is at least 50% of c.

Case 2: The estimate is greater than c. In the worst case, only the rightmost bit of bucket b is within range, and there is only one bucket of each of the sizes smaller than b. Then $c = 1 + 2^{j-1} + 2^{j-2} + \cdots + 1 = 2^j$ and the estimate we give is $2^{j-1} + 2^{j-1} + 2^{j-2} + \cdots + 1 = 2^j + 2^{j-1} - 1$. We see that the estimate is no more than 50% greater than c.