

1. Write a program that verifies if the digits of a number form a permutation, and if they do compute the rank of the permutation using the following formula:

$$\text{Rank}(p) = (p_1 - 1) \cdot (|p| - 1)! + \text{Rank}(p_2, \dots, p_{|p|})$$

2. Write a program that computes the Lucas numbers may thus be defined as follows:

$$L_n := \begin{cases} 2 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ L_{n-1} + L_{n-2} & \text{if } n > 1. \end{cases}$$

3. Write a program that computes the average process queue length, or the average CPU utilization, using a form of exponential moving average

$$S_n = \left[1 - \exp\left(-\frac{t_n - t_{n-1}}{W \cdot 60}\right) \right] Y_n + \exp\left(-\frac{t_n - t_{n-1}}{W \cdot 60}\right) S_{n-1}$$

where $\exp()$ is the exponential function, time for readings t_n is expressed in seconds, and W is the period of time in minutes over which the reading is said to be averaged (the mean lifetime of each reading in the average).

4. Write a program that computes the Bernoulli polynomials based on the following formula:

$$B_m(x) = \sum_{n=0}^m \frac{1}{n+1} \sum_{k=0}^n (-1)^k \binom{n}{k} (x+k)^m.$$