

CHAPTER 8

DEFORMATION AND STRENGTHENING MECHANISMS

PROBLEM SOLUTIONS

Basic Concepts of Dislocations

Characteristics of Dislocations

8.1 *To provide some perspective on the dimensions of atomic defects, consider a metal specimen that has a dislocation density of 10^4 mm^{-2} . Suppose that all the dislocations in 1000 mm^3 (1 cm^3) were somehow removed and linked end to end. How far (in miles) would this chain extend? Now suppose that the density is increased to 10^{10} mm^{-2} by cold working. What would be the chain length of dislocations in 1000 mm^3 of material?*

Solution

The dislocation density is just the total dislocation length per unit volume of material (in this case per cubic millimeters). Thus, the total length in 1000 mm^3 of material having a density of 10^4 mm^{-2} is just

$$(10^4 \text{ mm}^{-2})(1000 \text{ mm}^3) = 10^7 \text{ mm} = 10^4 \text{ m} = 6.2 \text{ mi}$$

Similarly, for a dislocation density of 10^{10} mm^{-2} , the total length is

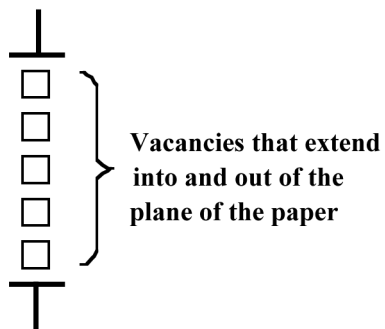
$$(10^{10} \text{ mm}^{-2})(1000 \text{ mm}^3) = 10^{13} \text{ mm} = 10^{10} \text{ m} = 6.2 \times 10^6 \text{ mi}$$

8.2 Consider two edge dislocations of opposite sign and having slip planes that are separated by several atomic distances as indicated in the following diagram. Briefly describe the defect that results when these two dislocations become aligned with each other.



Solution

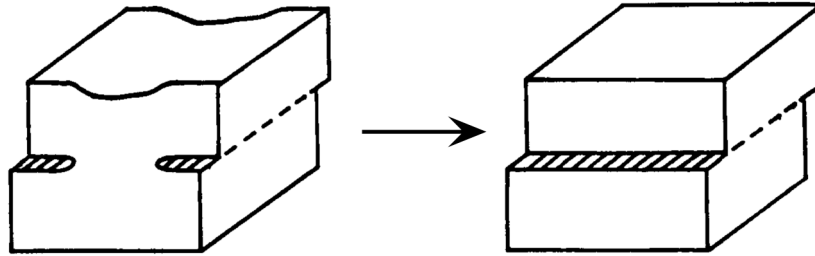
When the two edge dislocations become aligned, a planar region of vacancies will exist between the dislocations as:



8.3 *Is it possible for two screw dislocations of opposite sign to annihilate each other? Explain your answer.*

Solution

It is possible for two screw dislocations of opposite sign to annihilate one another if their dislocation lines are parallel. This is demonstrated in the figure below.



8.4 *For each of edge, screw, and mixed dislocations, cite the relationship between the direction of the applied shear stress and the direction of dislocation line motion.*

Solution

For the various dislocation types, the relationships between the direction of the applied shear stress and the direction of dislocation line motion are as follows:

edge dislocation—parallel

screw dislocation—perpendicular

mixed dislocation—neither parallel nor perpendicular

Slip Systems

8.5 (a) *Define a slip system.*

(b) *Do all metals have the same slip system? Why or why not?*

Solution

(a) A slip system is a crystallographic plane, and, within that plane, a direction along which dislocation motion (or slip) occurs.

(b) All metals do not have the same slip system. The reason for this is that for most metals, the slip system will consist of the most densely packed crystallographic plane, and within that plane the most closely packed direction. This plane and direction will vary from crystal structure to crystal structure.

8.6 (a) Compare planar densities (Section 3.15 and Problem 3.77) for the (100), (110), and (111) planes for FCC.

(b) Compare planar densities (Problem 3.78) for the (100), (110), and (111) planes for BCC.

Solution

(a) For the FCC crystal structure, the planar density for the (110) plane is given in Equation 3.14 as

$$PD_{110}(\text{FCC}) = \frac{1}{4R^2\sqrt{2}} = \frac{0.177}{R^2}$$

Furthermore, the planar densities of the (100) and (111) planes are calculated in Homework Problem 3.77, which are as follows:

$$PD_{100}(\text{FCC}) = \frac{1}{4R^2} = \frac{0.25}{R^2}$$

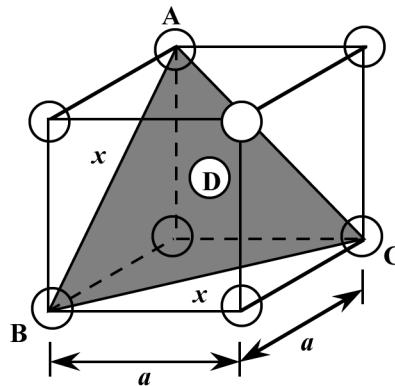
$$PD_{111}(\text{FCC}) = \frac{1}{2R^2\sqrt{3}} = \frac{0.29}{R^2}$$

(b) For the BCC crystal structure, the planar densities of the (100) and (110) planes were determined in Homework Problem 3.78, which are as follows:

$$PD_{100}(\text{BCC}) = \frac{3}{16R^2} = \frac{0.19}{R^2}$$

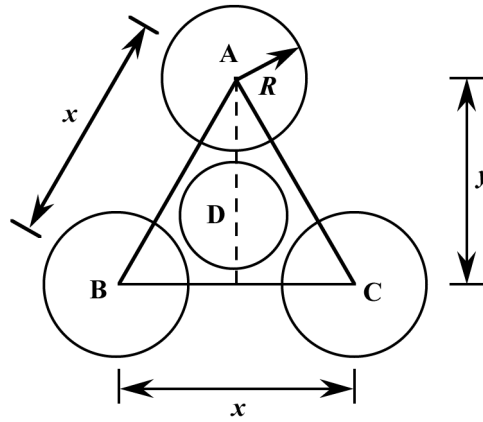
$$PD_{110}(\text{BCC}) = \frac{3}{8R^2\sqrt{2}} = \frac{0.27}{R^2}$$

Below is a BCC unit cell, within which is shown a (111) plane.



(a)

The centers of the three corner atoms, denoted by A, B, and C lie on this plane. Furthermore, the (111) plane does not pass through the center of atom D, which is located at the unit cell center. The atomic packing of this plane is presented in the following figure; the corresponding atom positions from the Figure (a) are also noted.



(b)

Inasmuch as this plane does not pass through the center of atom D, it is not included in the atom count. One sixth of each of the three atoms labeled A, B, and C is associated with this plane, which gives an equivalence of one-half atom.

In Figure (b) the triangle with A, B, and C at its corners is an equilateral triangle. And from Figure (b), the area of this triangle is $\frac{xy}{2}$. The triangle edge length, x , is equal to the length of a face diagonal, as indicated in Figure (a). And its length is related to the unit cell edge length, a , as

$$x^2 = a^2 + a^2 = 2a^2$$

or

$$x = a\sqrt{2}$$

For BCC, $a = \frac{4R}{\sqrt{3}}$ (Equation 3.4), and, therefore,

$$x = \frac{4R\sqrt{2}}{\sqrt{3}}$$

Also, from Figure (b), with respect to the length y we may write

$$y^2 + \left(\frac{x}{2}\right)^2 = x^2$$

which leads to $y = \frac{x\sqrt{3}}{2}$. And substitution for the above expression for x yields

$$y = \frac{x\sqrt{3}}{2} = \left(\frac{4R\sqrt{2}}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{4R\sqrt{2}}{2}$$

Thus, the area of this triangle is equal to

$$\text{AREA} = \frac{1}{2}xy = \left(\frac{1}{2}\right)\left(\frac{4R\sqrt{2}}{\sqrt{3}}\right)\left(\frac{4R\sqrt{2}}{2}\right) = \frac{8R^2}{\sqrt{3}}$$

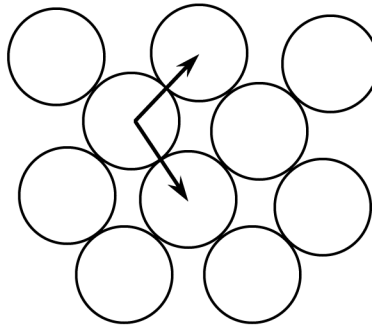
And finally, the planar density for this (111) plane is

$$\text{PD}_{111}(\text{BCC}) = \frac{0.5 \text{ atom}}{\frac{8R^2}{\sqrt{3}}} = \frac{\sqrt{3}}{16R^2} = \frac{0.11}{R^2}$$

8.7 One slip system for the BCC crystal structure is $\{110\}\langle 111 \rangle$. In a manner similar to Figure 8.6b, sketch a $\{110\}$ -type plane for the BCC structure, representing atom positions with circles. Now, using arrows, indicate two different $\langle 111 \rangle$ slip directions within this plane.

Solution

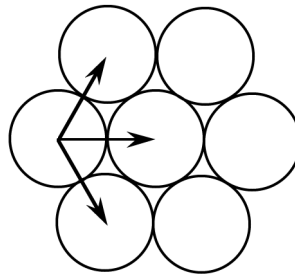
Below is shown the atomic packing for a BCC $\{110\}$ -type plane. The arrows indicate two different $\langle 111 \rangle$ -type directions.



8.8 One slip system for the HCP crystal structure is $\{0001\}\langle 11\bar{2}0 \rangle$. In a manner similar to Figure 8.6b, sketch a $\{0001\}$ -type plane for the HCP structure and, using arrows, indicate three different $\langle 11\bar{2}0 \rangle$ slip directions within this plane. You might find Figure 3.24 helpful.

Solution

Below is shown the atomic packing for an HCP $\{0001\}$ -type plane. The arrows indicate three different $\langle 11\bar{2}0 \rangle$ -type directions.



8.9 Equations 8.1a and 8.1b, expressions for Burgers vectors for FCC and BCC crystal structures, are of the form

$$\mathbf{b} = \frac{a}{2} \langle uvw \rangle$$

where a is the unit cell edge length. The magnitudes of these Burgers vectors may be determined from the following equation:

$$|\mathbf{b}| = \frac{a}{2} (u^2 + v^2 + w^2)^{1/2} \quad (8.13)$$

Determine the values of $|\mathbf{b}|$ for aluminum and chromium. You may want to consult Table 3.1.

Solution

For Al, which has an FCC crystal structure, $R = 0.1431$ nm (Table 3.1) and $a = 2R\sqrt{2} = 0.4047$ nm (Equation 3.1); also, from Equation 8.1a, the Burgers vector for FCC metals is

$$\mathbf{b} = \frac{a}{2} \langle 110 \rangle$$

Therefore, the values for u , v , and w in Equation 8.13 are 1, 1, and 0, respectively. Hence, the magnitude of the Burgers vector for Al is

$$\begin{aligned} |\mathbf{b}| &= \frac{a}{2} \sqrt{u^2 + v^2 + w^2} \\ &= \frac{0.4047 \text{ nm}}{2} \sqrt{(1)^2 + (1)^2 + (0)^2} = 0.2862 \text{ nm} \end{aligned}$$

For Cr, which has a BCC crystal structure, $R = 0.1249$ nm (Table 3.1) and $a = \frac{4R}{\sqrt{3}} = 0.2884$ nm (Equation 3.4); also, from Equation 8.1b, the Burgers vector for BCC metals is

$$\mathbf{b} = \frac{a}{2} \langle 111 \rangle$$

Therefore, the values for u , v , and w in Equation 8.13 are 1, 1, and 1, respectively. Hence, the magnitude of the Burgers vector for Cr is

$$|\mathbf{b}| = \frac{0.2884 \text{ nm}}{2} \sqrt{(1)^2 + (1)^2 + (1)^2} = 0.2498 \text{ nm}$$

8.10 (a) In the manner of Equations 8.1a to 8.1c, specify the Burgers vector for the simple cubic crystal structure. Its unit cell is shown in Figure 3.43. Also, simple cubic is the crystal structure for the edge dislocation of Figure 5.8 and for its motion as presented in Figure 8.1. You may also want to consult the answer to Concept Check 8.1.

(b) On the basis of Equation 8.13, formulate an expression for the magnitude of the Burgers vector, $|\mathbf{b}|$, for the simple cubic crystal structure.

Solution

(a) This part of the problem asks that we specify the Burgers vector for the simple cubic crystal structure (and suggests that we consult the answer to Concept Check 8.1). This Concept Check asks that we select the slip system for simple cubic from four possibilities. The correct answer is $\{100\}\langle 010\rangle$. Thus, the Burgers vector will lie in a $\langle 010\rangle$ -type direction. Also, the unit slip distance is a (i.e., the unit cell edge length, Figures 5.8 and 8.1). Therefore, the Burgers vector for simple cubic is

$$\mathbf{b} = a\langle 010\rangle$$

Or, equivalently

$$\mathbf{b} = a\langle 100\rangle$$

(b) The magnitude of the Burgers vector, $|\mathbf{b}|$, for simple cubic is

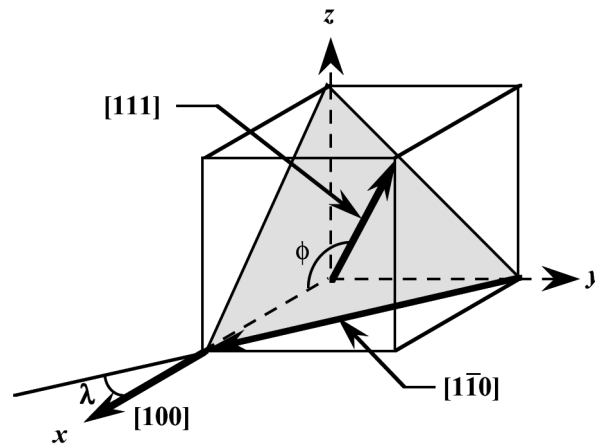
$$|\mathbf{b}| = a(1^2 + 0^2 + 0^2)^{1/2} = a$$

Slip in Single Crystals

8.11 Sometimes $\cos \phi \cos \lambda$ in Equation 8.2 is termed the Schmid factor. Determine the magnitude of the Schmid factor for an FCC single crystal oriented with its $[100]$ direction parallel to the loading axis.

Solution

We are asked to compute the Schmid factor for an FCC crystal oriented with its $[100]$ direction parallel to the loading axis. With this scheme, slip may occur on the (111) plane and in the $[1\bar{1}0]$ direction as noted in the figure below.



The angle between the $[100]$ and $[1\bar{1}0]$ directions, λ , may be determined using Equation 8.6

$$\lambda = \cos^{-1} \left[\frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}} \right]$$

where (for $[100]$) $u_1 = 1$, $v_1 = 0$, $w_1 = 0$, and (for $[1\bar{1}0]$) $u_2 = 1$, $v_2 = -1$, $w_2 = 0$. Therefore, λ is equal to

$$\begin{aligned} \lambda &= \cos^{-1} \left[\frac{(1)(1) + (0)(-1) + (0)(0)}{\sqrt{[(1)^2 + (0)^2 + (0)^2][(1)^2 + (-1)^2 + (0)^2]}} \right] \\ &= \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = 45^\circ \end{aligned}$$

Now, the angle ϕ is equal to the angle between the normal to the (111) plane (which is the [111] direction), and the [100] direction. Again from Equation 8.6, and for $u_1 = 1$, $v_1 = 1$, $w_1 = 1$, and $u_2 = 1$, $v_2 = 0$, and $w_2 = 0$, we have

$$\begin{aligned}\phi &= \cos^{-1} \left[\frac{(1)(1) + (1)(0) + (1)(0)}{\sqrt{[(1)^2 + (1)^2 + (1)^2] [(1)^2 + (0)^2 + (0)^2]}} \right] \\ &= \cos^{-1} \left(\frac{1}{\sqrt{3}} \right) = 54.7^\circ\end{aligned}$$

Therefore, the Schmid factor is equal to

$$\cos \lambda \cos \phi = \cos (45^\circ) \cos (54.7^\circ) = \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{3}} \right) = 0.408$$

8.12 Consider a metal single crystal oriented such that the normal to the slip plane and the slip direction are at angles of 43.1° and 47.9° , respectively, with the tensile axis. If the critical resolved shear stress is 20.7 MPa (3000 psi), will an applied stress of 45 MPa (6500 psi) cause the single crystal to yield? If not, what stress will be necessary?

Solution

This problem calls for us to determine whether or not a metal single crystal having a specific orientation and of given critical resolved shear stress will yield. We are given that $\phi = 43.1^\circ$, $\lambda = 47.9^\circ$, and that the values of the critical resolved shear stress and applied tensile stress are 20.7 MPa (3000 psi) and 45 MPa (6500 psi), respectively. From Equation 8.2

$$\tau_R = \sigma \cos \phi \cos \lambda = (45 \text{ MPa})(\cos 43.1^\circ)(\cos 47.9^\circ) = 22.0 \text{ MPa} \quad (3181 \text{ psi})$$

Since the resolved shear stress (22 MPa) is greater than the critical resolved shear stress (20.7 MPa), the single crystal will yield.

8.13 A single crystal of aluminum is oriented for a tensile test such that its slip plane normally makes an angle of 28.1° with the tensile axis. Three possible slip directions make angles of 62.4° , 72.0° , and 81.1° with the same tensile axis.

(a) Which of these three slip directions is most favored?

(b) If plastic deformation begins at a tensile stress of 1.95 MPa (280 psi), determine the critical resolved shear stress for aluminum.

Solution

We are asked to compute the critical resolved shear stress for Al. As stipulated in the problem, $\phi = 28.1^\circ$, while possible values for λ are 62.4° , 72.0° , and 81.1° .

(a) Slip will occur along that direction for which $(\cos \phi \cos \lambda)$ is a maximum, or, in this case, for the largest $\cos \lambda$. Cosines for the possible λ values are given below.

$$\cos(62.4^\circ) = 0.46$$

$$\cos(72.0^\circ) = 0.31$$

$$\cos(81.1^\circ) = 0.15$$

Thus, the slip direction is at an angle of 62.4° with the tensile axis.

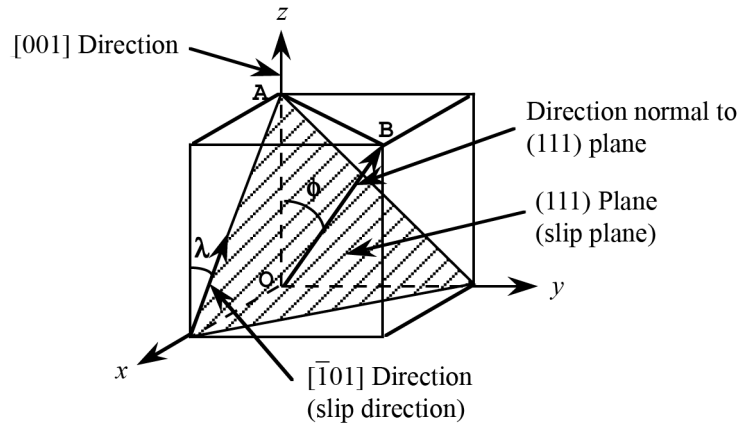
(b) From Equation 8.4, the critical resolved shear stress is just

$$\begin{aligned}\tau_{\text{crss}} &= \sigma_y (\cos \phi \cos \lambda)_{\text{max}} \\ &= (1.95 \text{ MPa}) [\cos (28.1^\circ) \cos (62.4^\circ)] = 0.80 \text{ MPa} \quad (114 \text{ psi})\end{aligned}$$

8.14 Consider a single crystal of silver oriented such that a tensile stress is applied along a $[001]$ direction. If slip occurs on a (111) plane and in a $[\bar{1}01]$ direction and is initiated at an applied tensile stress of 1.1 MPa (160 psi), compute the critical resolved shear stress.

Solution

This problem asks that we compute the critical resolved shear stress for silver. In order to do this, we must employ Equation 8.4, but first it is necessary to solve for the angles λ and ϕ which are shown in the sketch below.



The angle λ is the angle between the tensile axis—i.e., along the $[001]$ direction—and the slip direction—i.e., $[\bar{1}01]$. The angle λ may be determined using Equation 8.6 as

$$\lambda = \cos^{-1} \left[\frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}} \right]$$

where (for $[001]$) $u_1 = 0$, $v_1 = 0$, $w_1 = 1$, and (for $[\bar{1}01]$) $u_2 = -1$, $v_2 = 0$, $w_2 = 1$. Therefore, λ is equal to

$$\begin{aligned} \lambda &= \cos^{-1} \left[\frac{(0)(-1) + (0)(0) + (1)(1)}{\sqrt{[(0)^2 + (0)^2 + (1)^2][(-1)^2 + (0)^2 + (1)^2]}} \right] \\ &= \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = 45^\circ \end{aligned}$$

Furthermore, ϕ is the angle between the tensile axis—the [001] direction—and the normal to the slip plane—i.e., the (111) plane; for this case this normal is along a [111] direction. Therefore, again using Equation 8.6

$$\begin{aligned}\phi &= \cos^{-1} \left[\frac{(0)(1) + (0)(1) + (1)(1)}{\sqrt{[(0)^2 + (0)^2 + (1)^2][(1)^2 + (1)^2 + (1)^2]}} \right] \\ &= \cos^{-1} \left(\frac{1}{\sqrt{3}} \right) = 54.7^\circ\end{aligned}$$

And, finally, using Equation 8.4, the critical resolved shear stress is equal to

$$\begin{aligned}\tau_{\text{crss}} &= \sigma_y (\cos \phi \cos \lambda) \\ &= (1.1 \text{ MPa}) [\cos(54.7^\circ) \cos(45^\circ)] = (1.1 \text{ MPa}) \left(\frac{1}{\sqrt{3}} \right) \left(\frac{1}{\sqrt{2}} \right) = 0.45 \text{ MPa} \quad (65.1 \text{ psi})\end{aligned}$$

8.15 A single crystal of a metal that has the FCC crystal structure is oriented such that a tensile stress is applied parallel to the $[110]$ direction. If the critical resolved shear stress for this material is 1.75 MPa, calculate the magnitude(s) of applied stress(es) necessary to cause slip to occur on the (111) plane in each of the $[1\bar{1}0]$, $[10\bar{1}]$, and $[01\bar{1}]$ directions.

Solution

In order to solve this problem it is necessary to employ Equation 8.4, but first we need to solve for the λ and ϕ angles for the three slip systems.

For each of these three slip systems, the ϕ will be the same—i.e., the angle between the direction of the applied stress, $[110]$ and the normal to the (111) plane, that is, the $[111]$ direction. The angle ϕ may be determined using Equation 8.6 as

$$\phi = \cos^{-1} \left[\frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}} \right]$$

where (for $[110]$) $u_1 = 1$, $v_1 = 1$, $w_1 = 0$, and (for $[111]$) $u_2 = 1$, $v_2 = 1$, $w_2 = 1$. Therefore, ϕ is equal to

$$\begin{aligned} \phi &= \cos^{-1} \left[\frac{(1)(1) + (1)(1) + (0)(1)}{\sqrt{[(1)^2 + (1)^2 + (0)^2][(1)^2 + (1)^2 + (1)^2]}} \right] \\ &= \cos^{-1} \left(\frac{2}{\sqrt{6}} \right) = 35.3^\circ \end{aligned}$$

Let us now determine λ for the $[1\bar{1}0]$ slip direction. Again, using Equation 8.6 where $u_1 = 1$, $v_1 = 1$, $w_1 = 0$ (for $[110]$), and $u_2 = 1$, $v_2 = -1$, $w_2 = 0$ (for $[1\bar{1}0]$). Therefore, λ is determined as

$$\begin{aligned} \lambda_{[110]-[1\bar{1}0]} &= \cos^{-1} \left[\frac{(1)(1) + (1)(-1) + (0)(0)}{\sqrt{[(1)^2 + (1)^2 + (0)^2][(1)^2 + (-1)^2 + (0)^2]}} \right] \\ &= \cos^{-1} 0 = 90^\circ \end{aligned}$$

Now, we solve for the yield strength for this (111) – $[1\bar{1}0]$ slip system using Equation 8.4 as

$$\begin{aligned}\sigma_y &= \frac{\tau_{\text{crss}}}{(\cos\phi \cos\lambda)} \\ &= \frac{1.75 \text{ MPa}}{\cos(35.3^\circ) \cos(90^\circ)} = \frac{1.75 \text{ MPa}}{(0.816)(0)} = \infty\end{aligned}$$

which means that slip will not occur on this $(111)\text{--}[1\bar{1}0]$ slip system.

Now, we must determine the value of λ for the $(111)\text{--}[10\bar{1}]$ slip system—that is, the angle between the $[110]$ and $[10\bar{1}]$ directions. Again using Equation 8.6

$$\begin{aligned}\lambda_{[110]\text{--}[10\bar{1}]} &= \cos^{-1} \left[\frac{(1)(1) + (1)(0) + (0)(-1)}{\sqrt{[(1)^2 + (1)^2 + (0)^2] [(1)^2 + (0)^2 + (-1)^2]}} \right] \\ &= \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ\end{aligned}$$

Now, we solve for the yield strength for this $(111)\text{--}[10\bar{1}]$ slip system using Equation 8.4 as

$$\begin{aligned}\sigma_y &= \frac{\tau_{\text{crss}}}{(\cos\phi \cos\lambda)} \\ &= \frac{1.75 \text{ MPa}}{\cos(35.3^\circ) \cos(60^\circ)} = \frac{1.75 \text{ MPa}}{(0.816)(0.500)} = 4.29 \text{ MPa}\end{aligned}$$

And finally, for the $(111)\text{--}[01\bar{1}]$ slip system, λ is computed using Equation 8.6 as follows:

$$\begin{aligned}\lambda_{[110]\text{--}[01\bar{1}]} &= \cos^{-1} \left[\frac{(1)(0) + (1)(1) + (0)(-1)}{\sqrt{[(1)^2 + (1)^2 + (0)^2] [(0)^2 + (1)^2 + (-1)^2]}} \right] \\ &= \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ\end{aligned}$$

Thus, since the values of ϕ and λ for this $(110)\text{--}[01\bar{1}]$ slip system are the same as for $(111)\text{--}[10\bar{1}]$, so also will σ_y be the same—viz 4.29 MPa.

8.16 (a) A single crystal of a metal that has the BCC crystal structure is oriented such that a tensile stress is applied in the $[010]$ direction. If the magnitude of this stress is 2.75 MPa, compute the resolved shear stress in the $[\bar{1}11]$ direction on each of the (110) and (101) planes.

(b) On the basis of these resolved shear stress values, which slip system(s) is (are) most favorably oriented?

Solution

(a) This part of the problem asks, for a BCC metal, that we compute the resolved shear stress in the $[\bar{1}11]$ direction on each of the (110) and (101) planes. In order to solve this problem it is necessary to employ Equation 8.2, which means that we first need to solve for angles λ and ϕ for the three slip systems.

For each of these three slip systems, the λ will be the same—i.e., the angle between the direction of the applied stress, $[010]$ and the slip direction, $[\bar{1}11]$. This angle λ may be determined using Equation 8.6

$$\lambda = \cos^{-1} \left[\frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}} \right]$$

where (for $[010]$) $u_1 = 0$, $v_1 = 1$, $w_1 = 0$, and (for $[\bar{1}11]$) $u_2 = -1$, $v_2 = 1$, $w_2 = 1$. Therefore, λ is determined as

$$\begin{aligned} \lambda &= \cos^{-1} \left[\frac{(0)(-1) + (1)(1) + (0)(1)}{\sqrt{[(0)^2 + (1)^2 + (0)^2][(-1)^2 + (1)^2 + (1)^2]}} \right] \\ &= \cos^{-1} \left(\frac{1}{\sqrt{3}} \right) = 54.7^\circ \end{aligned}$$

Let us now determine ϕ for the angle between the direction of the applied tensile stress—i.e., the $[010]$ direction—and the normal to the (110) slip plane—i.e., the $[110]$ direction. Again, using Equation 8.6 where $u_1 = 0$, $v_1 = 1$, $w_1 = 0$ (for $[010]$), and $u_2 = 1$, $v_2 = 1$, $w_2 = 0$ (for $[110]$), ϕ is equal to

$$\phi_{[010]-[110]} = \cos^{-1} \left[\frac{(0)(1) + (1)(1) + (0)(0)}{\sqrt{[(0)^2 + (1)^2 + (0)^2][(1)^2 + (1)^2 + (0)^2]}} \right]$$

$$= \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = 45^\circ$$

Now, using Equation 8.2

$$\tau_R = \sigma \cos \phi \cos \lambda$$

we solve for the resolved shear stress for this slip system as

$$\tau_{R(110)-[\bar{1}11]} = (2.75 \text{ MPa}) [\cos(54.7^\circ) \cos(45^\circ)] = (2.75 \text{ MPa})(0.578)(0.707) = 1.12 \text{ MPa}$$

Now, we must determine the value of ϕ for the $(101)-[\bar{1}11]$ slip system—that is, the angle between the direction of the applied stress, $[010]$, and the normal to the (101) plane—i.e., the $[101]$ direction. Again using Equation 8.6

$$\begin{aligned} \lambda_{[010]-[101]} &= \cos^{-1} \left[\frac{(0)(1) + (1)(0) + (0)(1)}{\sqrt{[(0)^2 + (1)^2 + (0)^2]} \sqrt{[(1)^2 + (0)^2 + (1)^2]}} \right] \\ &= \cos^{-1}(0) = 90^\circ \end{aligned}$$

Thus, the resolved shear stress for this $(101)-[\bar{1}11]$ slip system is

$$\tau_{R(101)-[\bar{1}11]} = (2.75 \text{ MPa}) [\cos(54.7^\circ) \cos(90^\circ)] = (2.75 \text{ MPa})(0.578)(0) = 0 \text{ MPa}$$

(b) The most favored slip system(s) is (are) the one(s) that has (have) the largest τ_R value. Therefore, the $(110)-[\bar{1}11]$ is the most favored since its τ_R (1.12 MPa) is greater than the τ_R value for $(101)-[\bar{1}11]$ (viz., 0 MPa).

8.17 Consider a single crystal of some hypothetical metal that has the FCC crystal structure and is oriented such that a tensile stress is applied along a $[\bar{1}02]$ direction. If slip occurs on a (111) plane and in a $[\bar{1}01]$ direction, compute the stress at which the crystal yields if its critical resolved shear stress is 3.42 MPa.

Solution

This problem asks for us to determine the tensile stress at which a FCC metal yields when the stress is applied along a $[\bar{1}02]$ direction such that slip occurs on a (111) plane and in a $[\bar{1}01]$ direction; the critical resolved shear stress for this metal is 3.42 MPa. To solve this problem we use Equation 8.4; however it is first necessary to determine the values of ϕ and λ . These determinations are possible using Equation 8.6. Now, λ is the angle between $[\bar{1}02]$ and $[\bar{1}01]$ directions. Therefore, relative to Equation 8.6 let us take $u_1 = -1$, $v_1 = 0$, and $w_1 = 2$, as well as $u_2 = -1$, $v_2 = 0$, and $w_2 = 1$. This leads to

$$\begin{aligned}\lambda &= \cos^{-1} \left[\frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}} \right] \\ &= \cos^{-1} \left\{ \frac{(-1)(-1) + (0)(0) + (2)(1)}{\sqrt{[(-1)^2 + (0)^2 + (2)^2][(-1)^2 + (0)^2 + (1)^2]}} \right\} \\ &= \cos^{-1} \left(\frac{3}{\sqrt{10}} \right) = 18.4^\circ\end{aligned}$$

Now for the determination of ϕ , the normal to the (111) slip plane is the $[111]$ direction. Again using Equation 8.6, where we now take $u_1 = -1$, $v_1 = 0$, $w_1 = 2$ (for $[\bar{1}02]$), and $u_2 = 1$, $v_2 = 1$, $w_2 = 1$ (for $[111]$). Thus,

$$\begin{aligned}\phi &= \cos^{-1} \left\{ \frac{(-1)(1) + (0)(1) + (2)(1)}{\sqrt{[(-1)^2 + (0)^2 + (2)^2][(1)^2 + (1)^2 + (1)^2]}} \right\} \\ &= \cos^{-1} \left(\frac{3}{\sqrt{15}} \right) = 39.2^\circ\end{aligned}$$

It is now possible to compute the yield stress (using Equation 8.4) as

$$\sigma_y = \frac{\tau_{\text{crss}}}{\cos \phi \cos \lambda} = \frac{3.42 \text{ MPa}}{\left(\frac{3}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{15}}\right)} = 4.65 \text{ MPa}$$

8.18 *The critical resolved shear stress for iron is 27 MPa (4000 psi). Determine the maximum possible yield strength for a single crystal of Fe pulled in tension.*

Solution

In order to determine the maximum possible yield strength for a single crystal of Fe pulled in tension, we simply employ Equation 8.5 as

$$\sigma_y = 2\tau_{\text{crss}} = (2)(27 \text{ MPa}) = 54 \text{ MPa} \quad (8000 \text{ psi})$$

Deformation by Twinning

8.19 *List four major differences between deformation by twinning and deformation by slip relative to mechanism, conditions of occurrence, and final result.*

Solution

Four major differences between deformation by twinning and deformation by slip are as follows: (1) with slip deformation there is no crystallographic reorientation, whereas with twinning there is a reorientation; (2) for slip, the atomic displacements occur in atomic spacing multiples, whereas for twinning, these displacements may be other than by atomic spacing multiples; (3) slip occurs in metals having many slip systems, whereas twinning occurs in metals having relatively few slip systems; and (4) normally slip results in relatively large deformations, whereas only small deformations result for twinning.

Strengthening by Grain Size Reduction

8.20 *Briefly explain why small-angle grain boundaries are not as effective in interfering with the slip process as are high-angle grain boundaries.*

Solution

Small-angle grain boundaries are not as effective in interfering with the slip process as are high-angle grain boundaries because there is not as much crystallographic misalignment in the grain boundary region for small-angle, and therefore not as much change in slip direction.

8.21 *Briefly explain why HCP metals are typically more brittle than FCC and BCC metals.*

Solution

Hexagonal close packed metals are typically more brittle than FCC and BCC metals because there are fewer slip systems in HCP.

8.22 *Describe in your own words the three strengthening mechanisms discussed in this chapter (i.e., grain size reduction, solid-solution strengthening, and strain hardening). Explain how dislocations are involved in each of the strengthening techniques.*

These three strengthening mechanisms are described in Sections 8.9, 8.10, and 8.11.

8.23 (a) From the plot of yield strength versus (grain diameter)^{-1/2} for a 70 Cu–30 Zn cartridge brass in Figure 8.15, determine values for the constants σ_0 and k_y in Equation 8.7.

(b) Now predict the yield strength of this alloy when the average grain diameter is 1.0×10^{-3} mm.

Solution

(a) Perhaps the easiest way to solve for σ_0 and k_y in Equation 8.7 is to pick two values each of σ_y and $d^{-1/2}$ from Figure 8.15, and then solve two simultaneous equations, which may be created. For example

$d^{-1/2}$ (mm) ^{-1/2}	σ_y (MPa)
4	75
12	175

The two equations are thus

$$75 = \sigma_0 + 4k_y$$

$$175 = \sigma_0 + 12k_y$$

Solution of these equations yield the values of

$$k_y = 12.5 \text{ MPa}(\text{mm})^{1/2} \left[1810 \text{ psi}(\text{mm})^{1/2} \right]$$

$$\sigma_0 = 25 \text{ MPa} \text{ (3630 psi)}$$

(b) When $d = 1.0 \times 10^{-3}$ mm, $d^{-1/2} = 31.6 \text{ mm}^{-1/2}$, and using Equation 8.7,

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

$$= (25 \text{ MPa}) + [12.5 \text{ MPa}(\text{mm})^{1/2}] (31.6 \text{ mm}^{-1/2}) = 420 \text{ MPa} \text{ (61,000 psi)}$$

8.24 The lower yield point for an iron that has an average grain diameter of 5×10^{-2} mm is 135 MPa (19,500 psi). At a grain diameter of 8×10^{-3} mm, the yield point increases to 260 MPa (37,500 psi). At what grain diameter will the lower yield point be 205 MPa (30,000 psi)?

Solution

The best way to solve this problem is to first establish two simultaneous expressions of Equation 8.7, solve for σ_0 and k_y , and finally determine the value of d when $\sigma_y = 205$ MPa. The data pertaining to this problem may be tabulated as follows:

σ_y	d (mm)	$d^{-1/2}$ (mm) ^{-1/2}
135 MPa	5×10^{-2}	4.47
260 MPa	8×10^{-3}	11.18

The two equations thus become

$$135 \text{ MPa} = \sigma_0 + (4.47) k_y$$

$$260 \text{ MPa} = \sigma_0 + (11.18) k_y$$

Which yield the values, $\sigma_0 = 51.7$ MPa and $k_y = 18.63 \text{ MPa}(\text{mm})^{1/2}$. At a yield strength of 205 MPa

$$205 \text{ MPa} = 51.7 \text{ MPa} + \left[18.63 \text{ MPa}(\text{mm})^{1/2} \right] d^{-1/2}$$

or $d^{-1/2} = 8.23 \text{ (mm)}^{-1/2}$, which gives $d = 1.48 \times 10^{-2}$ mm.

8.25 *If it is assumed that the plot in Figure 8.15 is for non-cold-worked brass, determine the grain size of the alloy in Figure 8.19; assume its composition is the same as the alloy in Figure 8.15.*

Solution

This problem asks that we determine the grain size of the brass for which is the subject of Figure 8.19. From Figure 8.19a, the yield strength of brass at 0%CW is approximately 175 MPa (26,000 psi). This yield strength from Figure 8.15 corresponds to a $d^{-1/2}$ value of approximately $12.0 \text{ (mm)}^{-1/2}$. Thus,

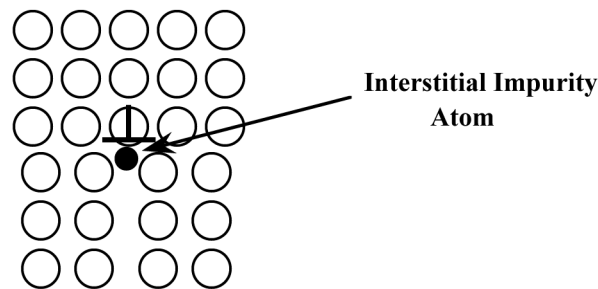
$$d = (d^{-1/2})^{-2} = [12.0 \text{ (mm)}^{-1/2}]^{-2} = 6.94 \times 10^{-3} \text{ mm}$$

Solid-Solution Strengthening

8.26 In the manner of Figures 8.17b and 8.18b, indicate the location in the vicinity of an edge dislocation at which an interstitial impurity atom would be expected to be situated. Now briefly explain in terms of lattice strains why it would be situated at this position.

Solution

Below is shown an edge dislocation and where an interstitial impurity atom would be located. Compressive lattice strains are introduced by the impurity atom. There will be a net reduction in lattice strain energy when these lattice strains partially cancel tensile strains associated with the edge dislocation; such tensile strains exist just below the bottom of the extra half-plane of atoms (Figure 8.4).



Strain Hardening

8.27 (a) Show, for a tensile test, that

$$\%CW = \left(\frac{\epsilon}{\epsilon + 1} \right) \times 100$$

if there is no change in specimen volume during the deformation process (i.e., $A_0 l_0 = A_d l_d$).

(b) Using the result of part (a), compute the percent cold work experienced by naval brass (the stress-strain behavior of which is shown in Figure 7.12) when a stress of 400 MPa (58,000 psi) is applied.

Solution

(a) From Equation 8.8

$$\%CW = \left(\frac{A_0 - A_d}{A_0} \right) \times 100 = \left(1 - \frac{A_d}{A_0} \right) \times 100$$

Which means that

$$\%CW = \left(1 - \frac{l_0}{l_d} \right) \times 100$$

since the conservation of volume stipulation given in the problem statement—i.e., $A_0 l_0 = A_d l_d$ —may be rearranged to read

$$\frac{A_d}{A_0} = \frac{l_0}{l_d}$$

Now, from the definition of engineering strain (Equation 7.2)

$$\epsilon = \frac{l_d - l_0}{l_0} = \frac{l_d}{l_0} - 1$$

Or, rearranging this expression gives

$$\frac{l_0}{l_d} = \frac{1}{\epsilon + 1}$$

Substitution for l_0 / l_d into the %CW expression above gives

$$\%CW = \left(1 - \frac{l_0}{l_d}\right) \times 100 = \left(1 - \frac{1}{\epsilon + 1}\right) \times 100 = \left(\frac{\epsilon}{\epsilon + 1}\right) \times 100$$

(b) From the engineering stress-strain plot of Figure 7.12, a stress of 400 MPa (58,000 psi) corresponds to a strain of 0.13. Therefore, using the above expression yields

$$\%CW = \left(\frac{\epsilon}{\epsilon + 1}\right) \times 100 = \left(\frac{0.13}{0.13 + 1.00}\right) \times 100 = 11.5\%CW$$

8.28 Two previously undeformed cylindrical specimens of an alloy are to be strain hardened by reducing their cross-sectional areas (while maintaining their circular cross sections). For one specimen, the initial and deformed radii are 16 and 11 mm, respectively. The second specimen, with an initial radius of 12 mm, must have the same deformed hardness as the first specimen; compute the second specimen's radius after deformation.

Solution

In order for these two cylindrical specimens to have the same deformed hardness, they must be deformed to the same percent cold work. For the first specimen, using Equation 8.8, the percent cold work is equal to

$$\begin{aligned}\%CW &= \frac{A_0 - A_d}{A_0} \times 100 = \frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \times 100 \\ &= \frac{\pi (16 \text{ mm})^2 - \pi (11 \text{ mm})^2}{\pi (16 \text{ mm})^2} \times 100 = 52.7\%CW\end{aligned}$$

For the second specimen, the deformed radius is computed using the above equation and solving for r_d as

$$r_d = r_0 \sqrt{1 - \frac{\%CW}{100}}$$

For a percent cold work of 52.7% and an r_0 value of 12 mm, r_d is equal to

$$r_d = (12 \text{ mm}) \sqrt{1 - \frac{52.7\%CW}{100}} = 8.25 \text{ mm}$$

8.29 Two previously undeformed specimens of the same metal are to be plastically deformed by reducing their cross-sectional areas. One has a circular cross section, and the other is rectangular; during deformation the circular cross section is to remain circular, and the rectangular is to remain rectangular. Their original and deformed dimensions are as follows:

	Circular (diameter, mm)	Rectangular (mm)
Original dimensions	15.2	125 × 175
Deformed dimensions	11.4	75 × 200

Which of these specimens will be the hardest after plastic deformation, and why?

Solution

The hardest specimen will be the one that has experienced the greatest degree of cold work. Therefore, all we need do is to compute the %CW for each specimen using Equation 8.8. For the circular one

$$\begin{aligned}\%CW &= \left[\frac{A_0 - A_d}{A_0} \right] \times 100 \\ &= \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \right] \times 100 \\ &= \left[\frac{\pi \left(\frac{15.2 \text{ mm}}{2} \right)^2 - \pi \left(\frac{11.4 \text{ mm}}{2} \right)^2}{\pi \left(\frac{15.2 \text{ mm}}{2} \right)^2} \right] \times 100 = 43.8\%CW\end{aligned}$$

For the rectangular one

$$\%CW = \left[\frac{(125 \text{ mm})(175 \text{ mm}) - (75 \text{ mm})(200 \text{ mm})}{(125 \text{ mm})(175 \text{ mm})} \right] \times 100 = 31.4\%CW$$

Therefore, the deformed circular specimen will be harder because its percent cold work is greater than the rectangular one.

8.30 A cylindrical specimen of cold-worked copper has a ductility (%EL) of 25%. If its cold-worked radius is 10 mm (0.40 in.), what was its radius before deformation?

Solution

This problem calls for us to calculate the precold-worked radius of a cylindrical specimen of copper that has a cold-worked ductility of 25%EL. From Figure 8.19c, copper that has a ductility of 25%EL will have experienced a deformation of about 11%CW. For a cylindrical specimen, Equation 8.8 becomes

$$\%CW = \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \right] \times 100$$

Solving this expression for r_0

$$r_0 = \frac{r_d}{\sqrt{1 - \frac{\%CW}{100}}}$$

and for r_d and %CW values of 10 mm and 11%CW, respectively, r_0 is equal to

$$r_0 = \frac{10 \text{ mm}}{\sqrt{1 - \frac{11.0}{100}}} = 10.6 \text{ mm} \quad (0.424 \text{ in.})$$

8.31 (a) *What is the approximate ductility (%EL) of a brass that has a yield strength of 275 MPa (40,000 psi)?*

(b) *What is the approximate Brinell hardness of a 1040 steel having a yield strength of 690 MPa (100,000 psi)?*

Solution

(a) In order to solve this problem, it is necessary to consult Figures 8.19a and 8.19c. From Figure 8.19a, a yield strength of 275 MPa for brass corresponds to 10%CW. A brass that has been cold-worked 10% will have a ductility of about 43%EL (Figure 8.19c).

(b) This portion of the problem asks for the Brinell hardness of a 1040 steel having a yield strength of 690 MPa (100,000 psi). From Figure 8.19a, a yield strength of 690 MPa for a 1040 steel corresponds to about 10%CW. A 1040 steel that has been cold worked 10% will have a tensile strength of about 780 MPa (Figure 8.19b). Finally, using Equation 7.25a

$$HB = \frac{TS(\text{MPa})}{3.45} = \frac{780 \text{ MPa}}{3.45} = 226$$

8.32 Experimentally, it has been observed for single crystals of a number of metals that the critical resolved shear stress τ_{crss} is a function of the dislocation density ρ_D as

$$\tau_{crss} = \tau_0 + A\sqrt{\rho_D}$$

where τ_0 and A are constants. For copper, the critical resolved shear stress is 2.10 MPa (305 psi) at a dislocation density of 10^5 mm^{-2} . If it is known that the value of A for copper is $6.35 \times 10^{-3} \text{ MPa-mm}$ (0.92 psi-mm), compute the τ_{crss} at a dislocation density of 10^7 mm^{-2} .

Solution

We are asked in this problem to compute the critical resolved shear stress at a dislocation density of 10^7 mm^{-2} . It is first necessary to compute the value of the constant τ_0 (in the equation provided in the problem statement) from the one set of data. Rearranging this expression such that τ_0 is the dependent variable, and incorporating appropriate values given in the problem statement

$$\begin{aligned}\tau_0 &= \tau_{crss} - A\sqrt{\rho_D} \\ &= 2.10 \text{ MPa} - (6.35 \times 10^{-3} \text{ MPa-mm})\left(\sqrt{10^5 \text{ mm}^{-2}}\right) = 0.092 \text{ MPa (13.3 psi)}\end{aligned}$$

Now, the critical resolved shear stress may be determined at a dislocation density of 10^7 mm^{-2} as

$$\begin{aligned}\tau_{crss} &= \tau_0 + A\sqrt{\rho_D} \\ &= (0.092 \text{ MPa}) + (6.35 \times 10^{-3} \text{ MPa-mm})\sqrt{10^7 \text{ mm}^{-2}} = 20.2 \text{ MPa (2920 psi)}\end{aligned}$$

Recovery
Recrystallization
Grain Growth

8.33 *Briefly cite the differences between recovery and recrystallization processes.*

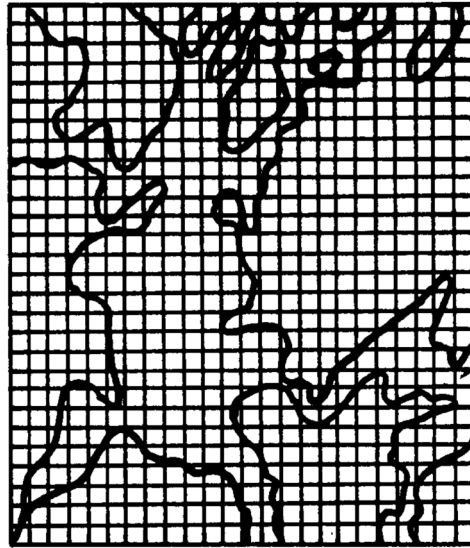
Solution

For recovery, there is some relief of internal strain energy by dislocation motion; however, there are virtually no changes in either the grain structure or mechanical characteristics. During recrystallization, on the other hand, a new set of strain-free grains forms, and the material becomes softer and more ductile.

8.34 Estimate the fraction of recrystallization from the photomicrograph in Figure 8.21c.

Solution

Below is shown a square grid onto which is superimposed the recrystallized regions from the micrograph. The fraction of recrystallization may be computed by dividing the number of squares that cover recrystallized areas by the total number of squares. Approximately 400 squares lie within the recrystallized areas, and since there are 672 total squares, the fraction recrystallization is approximately $400/672$ or about 0.60.



8.35 *Explain the differences in grain structure for a metal that has been cold worked and one that has been cold worked and then recrystallized.*

Solution

During cold-working, the grain structure of the metal has been distorted to accommodate the deformation. Recrystallization produces grains that are equiaxed and smaller than the parent grains.

8.36 (a) *What is the driving force for recrystallization?*

(b) *For grain growth?*

Solution

(a) The driving force for recrystallization is the difference in internal energy between the strained and unstrained material.

(b) The driving force for grain growth is the reduction in grain boundary energy as the total grain boundary area decreases.

8.37 (a) From Figure 8.25, compute the length of time required for the average grain diameter to increase from 0.01 to 0.1 mm at 500°C for this brass material.

(b) Repeat the calculation at 600°C.

Solution

(a) At 500°C, the time necessary for the average grain diameter to grow to increase from 0.01 to 0.1 mm is approximately 3500 min.

(b) At 600°C the time required for this same grain size increase is approximately 150 min.

8.38 The average grain diameter for a brass material was measured as a function of time at 650°C, which is shown in the following table at two different times:

<i>Time (min)</i>	<i>Grain Diameter (mm)</i>
30	3.9×10^{-2}
90	6.6×10^{-2}

(a) What was the original grain diameter?

(b) What grain diameter would you predict after 150 min at 650°C?

Solution

(a) Using the data given and Equation 8.9 (taking $n = 2$), we may set up two simultaneous equations with d_0 and K as unknowns; thus

$$(3.9 \times 10^{-2} \text{ mm})^2 - d_0^2 = (30 \text{ min})K$$

$$(6.6 \times 10^{-2} \text{ mm})^2 - d_0^2 = (90 \text{ min})K$$

Solution of these expressions yields a value for d_0 , the original grain diameter, of

$$d_0 = 0.01 \text{ mm},$$

and a value for K of $4.73 \times 10^{-5} \text{ mm}^2/\text{min}$

(b) At 150 min, the diameter d is computed using a rearranged form of Equation 8.9 [and incorporating values of d_0 and K in part (a)] as

$$d = \sqrt{d_0^2 + Kt}$$

$$= \sqrt{(0.01 \text{ mm})^2 + (4.73 \times 10^{-5} \text{ mm}^2/\text{min})(150 \text{ min})} = 0.085 \text{ mm}$$

8.39 *An undeformed specimen of some alloy has an average grain diameter of 0.040 mm. You are asked to reduce its average grain diameter to 0.010 mm. Is this possible? If so, explain the procedures you would use and name the processes involved. If it is not possible, explain why.*

Solution

Yes, it is possible to reduce the average grain diameter of an undeformed alloy specimen from 0.040 mm to 0.010 mm. In order to do this, plastically deform the material at room temperature (i.e., cold work it), and then anneal at an elevated temperature in order to allow recrystallization and some grain growth to occur until the average grain diameter is 0.010 mm.

8.40 Grain growth is strongly dependent on temperature (i.e., rate of grain growth increases with increasing temperature), yet temperature is not explicitly included in Equation 8.9.

- (a) Into which of the parameters in this expression would you expect temperature to be included?
- (b) On the basis of your intuition, cite an explicit expression for this temperature dependence.

Solution

- (a) The temperature dependence of grain growth is incorporated into the constant K in Equation 8.9.
- (b) The explicit expression for this temperature dependence is of the form

$$K = K_0 \exp\left(-\frac{Q}{RT}\right)$$

in which K_0 is a temperature-independent constant, the parameter Q is an activation energy, and R and T are the gas constant and absolute temperature, respectively.

8.41 *A non-cold-worked brass specimen of average grain size 0.008 mm has a yield strength of 160 MPa (23,500 psi). Estimate the yield strength of this alloy after it has been heated to 600°C for 1000 s, if it is known that the value of k_y is 12.0 MPa-mm^{1/2} (1740 psi-mm^{1/2}).*

Solution

In order to solve this problem, it is first necessary to calculate the constant σ_0 in Equation 8.7. Rearrangement of this expression and incorporating values of σ_y , k_y , and d provided in the problem statement allows computation of σ_0 as

$$\begin{aligned}\sigma_0 &= \sigma_y - k_y d^{-1/2} \\ &= 160 \text{ MPa} - (12.0 \text{ MPa} \cdot \text{mm}^{1/2})(0.008 \text{ mm})^{-1/2} = 25.8 \text{ MPa (4046 psi)}\end{aligned}$$

Next, we must determine the average grain size after the heat treatment. From Figure 8.25 at 600°C after 1000 s (16.7 min) the average grain size of a brass material is about 0.020 mm. Therefore, we compute σ_y at this new grain size using Equation 8.7 as follows:

$$\begin{aligned}\sigma_y &= \sigma_0 + k_y d^{-1/2} \\ &= 25.8 \text{ MPa} + (12.0 \text{ MPa} \cdot \text{mm}^{1/2})(0.020 \text{ mm})^{-1/2} = 111 \text{ MPa (16,300 psi)}\end{aligned}$$

Crystalline Ceramics (Deformation Mechanisms for Ceramic Materials)

8.42 *Cite one reason why ceramic materials are, in general, harder yet more brittle than metals.*

Solution

Crystalline ceramics are harder yet more brittle than metals because they (ceramics) have fewer slip systems, and, therefore, dislocation motion is highly restricted.

Deformation of Semicrystalline Polymers

8.43 *In your own words, describe the mechanisms by which semicrystalline polymers (a) elastically deform and (b) plastically deform, and (c) by which elastomers elastically deform.*

(a) and (b) The mechanisms by which semicrystalline polymers elastically and plastically deform are described in Section 8.17.

(c) The explanation of the mechanism by which elastomers elastically deform is provided in Section 8.19.

**Factors That Influence the Mechanical Properties of Semicrystalline
Polymers
Deformation of Elastomers**

8.44 Briefly explain how each of the following influences the tensile modulus of a semicrystalline polymer and why:

- (a) *Molecular weight*
- (b) *Degree of crystallinity*
- (c) *Deformation by drawing*
- (d) *Annealing of an undeformed material*
- (f) *Annealing of a drawn material*

Solution

- (a) The tensile modulus is not directly influenced by a polymer's molecular weight.
- (b) Tensile modulus increases with increasing degree of crystallinity for semicrystalline polymers. This is due to enhanced secondary interchain bonding which results from adjacent aligned chain segments as percent crystallinity increases. This enhanced interchain bonding inhibits relative interchain motion.
- (c) Deformation by drawing also increases the tensile modulus. The reason for this is that drawing produces a highly oriented molecular structure, and a relatively high degree of interchain secondary bonding.
- (d) When an undeformed semicrystalline polymer is annealed below its melting temperature, the tensile modulus increases.
- (e) A drawn semicrystalline polymer that is annealed experiences a decrease in tensile modulus as a result of a reduction in chain-induced crystallinity, and a reduction in interchain bonding forces.

8.45 Briefly explain how each of the following influences the tensile or yield strength of a semicrystalline polymer and why:

- (a) *Molecular weight*
- (b) *Degree of crystallinity*
- (c) *Deformation by drawing*
- (d) *Annealing of an undeformed material*

Solution

(a) The tensile strength of a semicrystalline polymer increases with increasing molecular weight. This effect is explained by increased chain entanglements at higher molecular weights.

(b) Increasing the degree of crystallinity of a semicrystalline polymer leads to an enhancement of the tensile strength. Again, this is due to enhanced interchain bonding and forces; in response to applied stresses, interchain motions are thus inhibited.

(c) Deformation by drawing increases the tensile strength of a semicrystalline polymer. This effect is due to the highly oriented chain structure that is produced by drawing, which gives rise to higher interchain secondary bonding forces.

(d) Annealing an undeformed semicrystalline polymer produces an increase in its tensile strength.

8.46 *Normal butane and isobutane have boiling temperatures of -0.5 and -12.3°C (31.1 and 9.9°F), respectively. Briefly explain this behavior on the basis of their molecular structures, as presented in Section 4.2.*

Solution

Normal butane has a higher melting temperature as a result of its molecular structure (Section 4.2). There is more of an opportunity for van der Waals bonds to form between two molecules in close proximity to one another than for isobutane because of the linear nature of each normal butane molecule.

8.47 The tensile strength and number-average molecular weight for two poly(methyl methacrylate) materials are as follows:

<i>Tensile Strength (MPa)</i>	<i>Number-Average Molecular Weight (g/mol)</i>
107	40,000
170	60,000

Estimate the tensile strength at a number-average molecular weight of 30,000 g/mol.

Solution

This problem gives us the tensile strengths and associated number-average molecular weights for two poly(methyl methacrylate) materials and then asks that we estimate the tensile strength for $\bar{M}_n = 30,000$ g/mol. Equation 8.11 cites the dependence of the tensile strength on \bar{M}_n . Thus, using the data provided in the problem statement, we may set up two simultaneous equations from which it is possible to solve for the two constants TS_∞ and A . These equations are as follows:

$$107 \text{ MPa} = TS_\infty - \frac{A}{40,000 \text{ g/mol}}$$

$$170 \text{ MPa} = TS_\infty - \frac{A}{60,000 \text{ g/mol}}$$

Thus, the values of the two constants are: $TS_\infty = 296$ MPa and $A = 7.56 \times 10^6$ MPa-g/mol. Substituting these values into Equation 8.11 for $\bar{M}_n = 30,000$ g/mol leads to

$$\begin{aligned} TS &= TS_\infty - \frac{A}{30,000 \text{ g/mol}} \\ &= 296 \text{ MPa} - \frac{7.56 \times 10^6 \text{ MPa} \cdot \text{g/mol}}{30,000 \text{ g/mol}} \\ &= 44 \text{ MPa} \end{aligned}$$

8.48 The tensile strength and number-average molecular weight for two polyethylene materials are as follows:

<i>Tensile Strength (MPa)</i>	<i>Number-Average Molecular Weight (g/mol)</i>
85	12,700
150	28,500

Estimate the number-average molecular weight that is required to give a tensile strength of 195 MPa.

Solution

This problem gives us the tensile strengths and associated number-average molecular weights for two polyethylene materials and then asks that we estimate the \bar{M}_n that is required for a tensile strength of 195 MPa. Equation 8.11 cites the dependence of the tensile strength on \bar{M}_n . Thus, using the data provided in the problem statement, we may set up two simultaneous equations from which it is possible to solve for the two constants TS_∞ and A . These equations are as follows:

$$85 \text{ MPa} = TS_\infty - \frac{A}{12,700 \text{ g/mol}}$$

$$150 \text{ MPa} = TS_\infty - \frac{A}{28,500 \text{ g/mol}}$$

Thus, the values of the two constants are: $TS_\infty = 202 \text{ MPa}$ and $A = 1.489 \times 10^6 \text{ MPa-g/mol}$. Solving for \bar{M}_n in Equation 8.11 and substituting $TS = 195 \text{ MPa}$ as well as the above values for TS_∞ and A leads to

$$\begin{aligned} \bar{M}_n &= \frac{A}{TS_\infty - TS} \\ &= \frac{1.489 \times 10^6 \text{ MPa-g/mol}}{202 \text{ MPa} - 195 \text{ MPa}} = 213,000 \text{ g/mol} \end{aligned}$$

8.49 For each of the following pairs of polymers, do the following: (1) state whether it is possible to decide whether one polymer has a higher tensile modulus than the other; (2) if this is possible, note which has the higher tensile modulus and cite the reason(s) for your choice; and (3) if it is not possible to decide, state why.

(a) Random acrylonitrile-butadiene copolymer with 10% of possible sites crosslinked; alternating acrylonitrile-butadiene copolymer with 5% of possible sites crosslinked

(b) Branched and syndiotactic polypropylene with a degree of polymerization of 5000; linear and isotactic polypropylene with a degree of polymerization of 3000

(c) Branched polyethylene with a number-average molecular weight of 250,000 g/mol; linear and isotactic poly(vinyl chloride) with a number-average molecular weight of 200,000 g/mol

Solution

(a) No, it is not possible. The random acrylonitrile-butadiene copolymer will tend to a lower degree of crystallinity than the alternating acrylonitrile-butadiene copolymer inasmuch as random copolymers don't normally crystallize. On this basis only, the alternating material would have a higher modulus inasmuch as tensile modulus increases with degree of crystallinity. On the other hand, the random copolymer has a higher degree of crosslinking (10% versus 5% for the alternating copolymer), and on this basis only would have the higher tensile modulus—an increase in crosslinking leads to an increase in E . Thus, this determination is not possible; with regard to degree of crystallinity the alternating material has the higher E , whereas the random copolymer would have a higher E value on the basis of degree of crosslinking.

(b) Yes, it is possible. The linear and isotactic polypropylene will have a greater tensile modulus. Linear polymers are more likely to crystallize than branched ones. The likelihood of crystallization for both syndiotactic and isotactic polypropylene is about the same, and, therefore, degree of crystallization is not a factor. Furthermore, tensile modulus is relatively insensitive to degree of polymerization (i.e., molecular weight)—the fact that branched PP has the higher molecular weight is not important.

(c) No, it is not possible. Linear polymers have higher degrees of crystallization (and higher tensile moduli) than branched polymers—on this basis, the PVC material should have the higher value of E . On the other hand, PVC has a more complex repeat unit structure than does polyethylene, which means that, on this basis, the PE would have a higher degree of crystallinity and also a greater tensile modulus. Also, tensile modulus is relatively independent of number-average molecular weight. Therefore, this determination is not possible since it is not possible to determine which of the two materials has the greater degree of crystallinity.

8.50 For each of the following pairs of polymers, do the following: (1) state whether it is possible to decide whether one polymer has a higher tensile strength than the other; (2) if this is possible, note which has the higher tensile strength and cite the reason(s) for your choice; and (3) if it is not possible to decide, state why.

(a) Syndiotactic polystyrene having a number-average molecular weight of 600,000 g/mol; atactic polystyrene having a number-average molecular weight of 500,000 g/mol

(b) Random acrylonitrile-butadiene copolymer with 10% of possible sites crosslinked; block acrylonitrile-butadiene copolymer with 5% of possible sites crosslinked

(c) Network polyester; lightly branched polypropylene

Solution

(a) Yes it is possible. The syndiotactic polystyrene has the higher tensile strength. Syndiotactic polymers are more likely to crystallize than atactic ones; the greater the crystallinity, the higher the tensile strength. Furthermore, the syndiotactic also has a higher molecular weight; increasing molecular weight also enhances the strength.

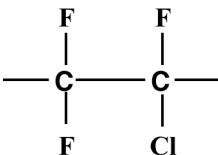
(b) No it is not possible. The random acrylonitrile-butadiene copolymer has more crosslinking; increased crosslinking leads to an increase in strength. However, the block copolymeric material will most likely have a higher degree of crystallinity; and increasing crystallinity improves the strength.

(c) Yes it is possible. The network polyester will display a greater tensile strength. Relative chain motion is much more restricted than for the lightly branched polypropylene since there are many more of the strong covalent bonds for the network structure.

8.51 *Would you expect the tensile strength of polychlorotrifluoroethylene to be greater than, the same as, or less than that of a polytetrafluoroethylene specimen having the same molecular weight and degree of crystallinity? Why?*

Solution

The strength of a polychlorotrifluoroethylene having the repeat unit structure



will be greater than for a polytetrafluoroethylene having the same molecular weight and degree of crystallinity. The replacement of one fluorine atom within the PTFE repeat unit with a chlorine atom leads to a higher interchain attraction, and, thus, a stronger polymer. Furthermore, poly(vinyl chloride) is stronger than polyethylene (Table 7.2) for the same reason.

8.52 For each of the following pairs of polymers, plot and label schematic stress–strain curves on the same graph [i.e., make separate plots for parts (a) to (c)].

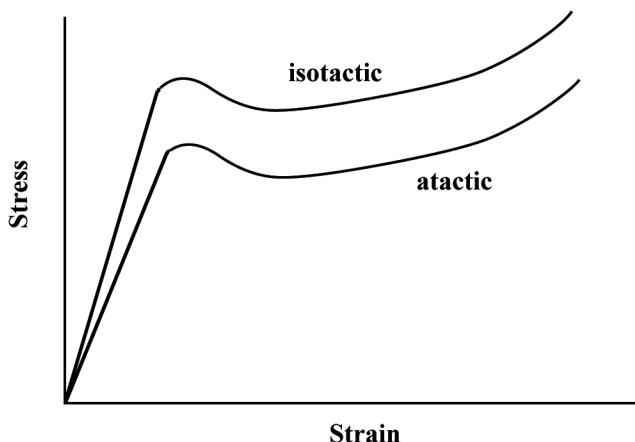
(a) Isotactic and linear polypropylene having a weight-average molecular weight of 120,000 g/mol; atactic and linear polypropylene having a weight-average molecular weight of 100,000 g/mol

(b) Branched poly(vinyl chloride) having a degree of polymerization of 2000; heavily crosslinked poly(vinyl chloride) having a degree of polymerization of 2000

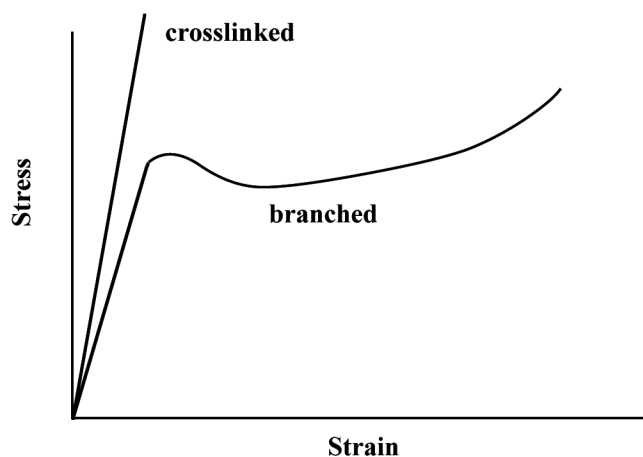
(c) Poly(styrene-butadiene) random copolymer having a number-average molecular weight of 100,000 g/mol and 10% of the available sites crosslinked and tested at 20°C; poly(styrene-butadiene) random copolymer having a number-average molecular weight of 120,000 g/mol and 15% of the available sites crosslinked and tested at -85°C. Hint: poly(styrene-butadiene) copolymers may exhibit elastomeric behavior.

Solution

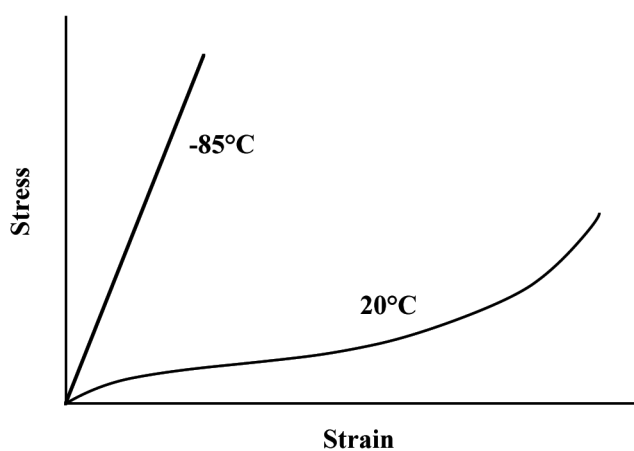
(a) Shown below are the stress-strain curves for the two polypropylene materials. These materials will display the stress-strain behavior of a normal plastic, curve *B* in Figure 7.22. However, the isotactic/linear will have a higher degree of crystallinity (since isotactic are more likely to crystallize than atactic/linear), and therefore, will have a higher tensile modulus and strength. Furthermore, the isotactic/linear also has a higher molecular weight which also leads to an increase in strength.



(b) Shown below are the stress-strain curves for the two polyvinyl chloride materials. The branched PVC will probably display the stress-strain behavior of a plastic, curve *B* in Figure 7.22. However, the heavily crosslinked PVC will undoubtedly have a higher tensile modulus, and also a higher strength, and will most likely fail in a brittle manner—as curve *A*, Figure 7.22; these are the typical characteristics of a heavily crosslinked polymer.



(c) Shown below are the stress-strain curves for the two poly(styrene-butadiene) random copolymers. The copolymer tested at 20°C will display elastomeric behavior (curve *C* of Figure 7.22) inasmuch as it is a random copolymer that is lightly crosslinked; furthermore, the temperature of testing is above its glass transition temperature. On the other hand, since -85°C is below the glass transition temperature of the poly(styrene-butadiene) copolymer, the stress-strain behavior under these conditions is as curve *A* of Figure 7.22.



8.53 *List the two molecular characteristics that are essential for elastomers.*

Solution

Two molecular characteristics essential for elastomers are: (1) they must be amorphous, having chains that are extensively coiled and kinked in the unstressed state; and (2) there must be some crosslinking.

8.54 Which of the following would you expect to be elastomers and which thermosetting polymers at room temperature? Justify each choice.

- (a) Epoxy having a network structure
- (b) Lightly crosslinked poly(styrene-butadiene) random copolymer that has a glass-transition temperature of -50°C
- (c) Lightly branched and semicrystalline polytetrafluoroethylene that has a glass-transition temperature of -100°C
- (d) Heavily crosslinked poly(ethylene-propylene) random copolymer that has a glass-transition temperature of 0°C
- (e) Thermoplastic elastomer that has a glass-transition temperature of 75°C

Solution

This question asks us to choose from a list of materials those that would be expected to be elastomers and those that would be thermosetting polymers.

(a) Epoxy having a network structure will be a thermoset polymer since it has a network structure. It would not be an elastomer since it does not have a crosslinked chain structure.

(b) A lightly crosslinked poly(styrene-butadiene) random copolymer that has a glass-transition temperature of -50°C will be an elastomer since it (1) is a random copolymer, (2) is lightly crosslinked, and (3) is being used at a temperature above its glass transition. All three of these criteria are requisites for an elastomer.

(c) Lightly branched and semicrystalline PTFE would be neither an elastomer nor a thermoset. It is not crosslinked nor does it have a network structure.

(d) A heavily crosslinked poly(ethylene-propylene) random copolymer would be a thermoset inasmuch as it is heavily crosslinked.

(e) A thermoplastic elastomer that has a glass-transition temperature of 75°C is neither an elastomer nor a thermoset. Since it is a thermoplastic it is not a thermoset. Furthermore, room temperature is below its glass-transition temperature, and, therefore, it will not display elastomeric behavior.

8.55 Ten kilograms of polybutadiene is vulcanized with 4.8 kg of sulfur. What fraction of the possible crosslink sites is bonded to sulfur crosslinks, assuming that, on the average, 4.5 sulfur atoms participate in each crosslink?

Solution

This problem asks that we compute the fraction of possible crosslink sites in 10 kg of polybutadiene when 4.8 kg of S is added, assuming that, on the average, 4.5 sulfur atoms participate in each crosslink bond. Given the butadiene repeat unit in Table 4.5, we may calculate its molecular weight as follows:

$$\begin{aligned} A(\text{butadiene}) &= 4(A_C) + 6(A_H) \\ &= (4)(12.01 \text{ g/mol}) + 6(1.008 \text{ g/mol}) = 54.09 \text{ g/mol} \end{aligned}$$

Which means that in 10 kg of butadiene there are $\frac{10,000 \text{ g}}{54.09 \text{ g/mol}} = 184.9 \text{ mol} = n_{\text{buta}}$.

For the vulcanization polybutadiene, there are two possible crosslink sites per repeat unit—one for each of the two carbon atoms that are doubly bonded. Furthermore, each of these crosslinks forms a bridge between two repeat units. Therefore, we can say that there is the equivalent of one crosslink per repeat unit. Let us now calculate the number of moles of sulfur (n_{sulfur}) that react with the butadiene, by taking the mole ratio of sulfur to butadiene, and then dividing this ratio by 4.5 atoms per crosslink; this yields the fraction of possible sites that are crosslinked. Thus

$$n_{\text{sulfur}} = \frac{4800 \text{ g}}{32.06 \text{ g/mol}} = 149.7 \text{ mol}$$

And

$$\text{fraction sites crosslinked} = \frac{\frac{n_{\text{sulfur}}}{4.5}}{\frac{n_{\text{buta}}}{4.5}} = \frac{149.7 \text{ mol}}{184.9 \text{ mol}} = 0.180$$

8.56 Compute the weight percent sulfur that must be added to completely crosslink an alternating chloroprene-acrylonitrile copolymer, assuming that five sulfur atoms participate in each crosslink.

Solution

For an alternating chloroprene-acrylonitrile copolymer, we are asked to compute the weight percent sulfur necessary for complete crosslinking, assuming that, on the average, five sulfur atoms participate in each crosslink. The chloroprene and acrylonitrile repeat units are shown in Table 4.5, from which it may be noted that there are two possible crosslink sites on each chloroprene repeat unit (one site at each of the two carbon atoms that are doubly bonded), and no possible sites for acrylonitrile; also, since it is an alternating copolymer, the ratio of chloroprene to acrylonitrile repeat units is 1:1. Thus, for each pair of combined chloroprene-acrylonitrile repeat units that crosslink, ten sulfur atoms are required, or, for complete crosslinking, the sulfur-to-(chloroprene-acrylonitrile) ratio is 5:1.

Now, let us consider as our basis, one mole of the combined chloroprene-acrylonitrile repeat unit. In order for complete crosslinking, five moles of sulfur are required. Thus, for us to convert this composition to weight percent, it is necessary to convert moles to mass. The acrylonitrile repeat unit consists of three carbon atoms, three hydrogen atoms, and one nitrogen atom; the chloroprene repeat unit is composed of four carbons, five hydrogens, and one chlorine. This gives a molecular weight for the combined repeat unit of

$$\begin{aligned} m(\text{chloroprene-acrylonitrile}) &= 3(A_C) + 3(A_H) + A_N + 4(A_C) + 5(A_H) + A_{Cl} \\ &= 7(12.01 \text{ g/mol}) + 8(1.008 \text{ g/mol}) + 14.007 \text{ g/mol} + 35.45 \text{ g/mol} = 141.59 \text{ g/mol} \end{aligned}$$

Or, in one mole of this combined repeat unit, there are 141.59 g. Furthermore, for complete crosslinking 5.0 mol of sulfur is required, which amounts to $(5.0 \text{ mol})(32.06 \text{ g/mol}) = 160.3 \text{ g}$. Thus, the concentration of S in weight percent C_S is just

$$C_S = \frac{160.3 \text{ g}}{160.3 \text{ g} + 141.59 \text{ g}} \times 100 = 53.1 \text{ wt\%}$$

8.57 The vulcanization of polyisoprene is accomplished with sulfur atoms according to Equation 8.12. If 57 wt% sulfur is combined with polyisoprene, how many crosslinks will be associated with each isoprene repeat unit if it is assumed that, on the average, six sulfur atoms participate in each crosslink?

Solution

This problem asks for us to determine how many crosslinks form per isoprene repeat unit when 57 wt% sulfur is added. If we arbitrarily consider 100 g of the vulcanized material, 57 g will be sulfur and 43 g will be polyisoprene. Next, let us find how many moles of sulfur and isoprene correspond to these masses. The atomic weight of sulfur is 32.06 g/mol, and thus,

$$\# \text{ moles S} = \frac{57 \text{ g}}{32.06 \text{ g/mol}} = 1.78 \text{ mol}$$

Now, in each isoprene repeat unit there are five carbon atoms and eight hydrogen atoms. Thus, the molecular weight of a mole of isoprene units is

$$(5)(12.01 \text{ g/mol}) + (8)(1.008 \text{ g/mol}) = 68.11 \text{ g/mol}$$

Or, in 43 g of polyisoprene, the number of moles is equal to

$$\# \text{ moles isoprene} = \frac{43 \text{ g}}{68.11 \text{ g/mol}} = 0.631 \text{ mol}$$

Therefore, the ratio of moles of S to the number of moles of polyisoprene is

$$\frac{1.78 \text{ mol}}{0.631 \text{ mol}} : 1 = 2.82 : 1$$

When all possible sites are crosslinked, the ratio of the number of moles of sulfur to the number of moles of isoprene is 6:1; this is because there are two crosslink sites per repeat unit and each crosslink is shared between repeat units on adjacent chains, and there are 6 sulfur atoms per crosslink. Finally, to determine the fraction of sites that are crosslinked, we just divide the actual crosslinked sulfur/isoprene ratio by the completely crosslinked ratio. Or,

$$\text{fraction of repeat unit sites crosslinked} = \frac{2.82/1}{6/1} = 0.470$$

8.58 For the vulcanization of polyisoprene, compute the weight percent of sulfur that must be added to ensure that 8% of possible sites will be crosslinked; assume that, on the average, three sulfur atoms are associated with each crosslink.

Solution

We are asked what weight percent of sulfur must be added to polyisoprene in order to ensure that 8% of possible sites are crosslinked, assuming that, on the average, three sulfur atoms are associated with each crosslink. Table 4.5 shows the repeat unit for cis-isoprene. For each of these units there are two possible crosslink sites; one site is associated with each of the two carbon atoms that are involved in the chain double bond. Since 8% of the possible sites are crosslinked, for each 100 isoprene repeat units 8 of them are crosslinked; actually there are two crosslink sites per repeat unit, but each crosslink is shared by two chains. Furthermore, on the average we assume that each crosslink is composed of 3 sulfur atoms; thus, there must be 3×8 or 24 sulfur atoms added for every 100 isoprene repeat units. In terms of moles, it is necessary to add 24 moles of sulfur to 100 moles of isoprene. The atomic weight of sulfur is 32.06 g/mol, while the molecular weight of isoprene is

$$\begin{aligned} A(\text{isoprene}) &= 5(A_{\text{C}}) + 8(A_{\text{H}}) \\ &= (5)(12.01 \text{ g/mol}) + (8)(1.008 \text{ g/mol}) = 68.11 \text{ g/mol} \end{aligned}$$

The mass of sulfur added (m_{S}) is

$$m_{\text{S}} = (24 \text{ mol})(32.06 \text{ g/mol}) = 769.4 \text{ g}$$

While for isoprene

$$m_{\text{ip}} = (100 \text{ mol})(68.11 \text{ g/mol}) = 6811 \text{ g}$$

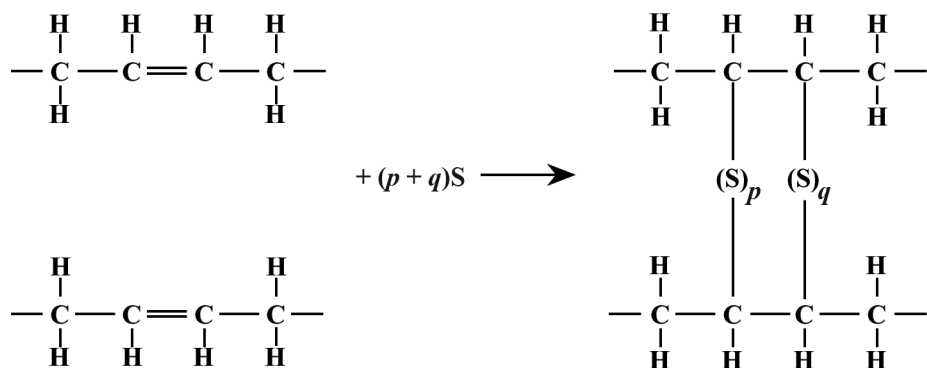
Or, the concentration of sulfur in weight percent (Equation 5.6) is just

$$C_{\text{S}} = \frac{m_{\text{S}}}{m_{\text{S}} + m_{\text{ip}}} \times 100 = \frac{769.4 \text{ g}}{769.4 \text{ g} + 6811 \text{ g}} \times 100 = 10.1 \text{ wt\%}$$

8.59 Demonstrate, in a manner similar to Equation 8.12, how vulcanization may occur in a butadiene rubber.

Solution

The reaction by which a butadiene rubber may become vulcanized is as follows:



DESIGN PROBLEMS

Strain Hardening

Recrystallization

8.D1 *Determine whether it is possible to cold work steel so as to give a minimum Brinell hardness of 225 and at the same time have a ductility of at least 12%EL. Justify your answer.*

Solution

The tensile strength corresponding to a Brinell hardness of 225 may be determined using Equation 7.25a as

$$TS(\text{MPa}) = 3.45 \times HB = (3.45)(225) = 776 \text{ MPa}$$

Furthermore, from Figure 8.19b, in order to achieve a tensile strength of 776 MPa, deformation of at least 9%CW is necessary. Finally, if we cold work the steel to 9%CW, then the ductility is 17%EL from Figure 8.19c (which is greater than 12%EL). Therefore, it *is possible* to meet both of these criteria by plastically deforming the steel.

8.D2 Determine whether it is possible to cold work brass so as to give a minimum Brinell hardness of 120 and at the same time have a ductility of at least 20%EL. Justify your answer.

Solution

According to Figure 7.31, a Brinell hardness of 120 corresponds to a tensile strength of 440 MPa (63,500 psi). Furthermore, from Figure 8.19b, in order to achieve a tensile strength of 440 MPa, deformation of at least 26%CW is necessary. Finally, if we are to achieve a ductility of at least 20%EL, then a maximum deformation of 23%CW is possible from Figure 8.19c. Therefore, it is *not possible* to meet both of these criteria by plastically deforming brass.

8.D3 A cylindrical specimen of cold-worked steel has a Brinell hardness of 250.

(a) Estimate its ductility in percent elongation.

(b) If the specimen remained cylindrical during deformation and its original radius was 5 mm (0.20 in.), determine its radius after deformation.

Solution

(a) From Figure 7.31, a Brinell hardness of 250 corresponds to a tensile strength of 860 MPa (125,000 psi), which, from Figure 8.19b, requires a deformation of 25%CW. Furthermore, 25%CW yields a ductility of about 11%EL for steel, Figure 8.19c.

(b) We are now asked to determine the radius after deformation if the uncold-worked radius is 5 mm (0.20 in.). From Equation 8.8 and for a cylindrical specimen

$$\%CW = \left(\frac{A_0 - A_d}{A_0} \right) \times 100 = \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \right] \times 100$$

Now, solving for r_d from this expression, we get

$$r_d = r_0 \sqrt{1 - \frac{\%CW}{100}}$$

And, since $r_0 = 5$ mm and %CW = 25, we have

$$r_d = (5 \text{ mm}) \sqrt{1 - \frac{25}{100}} = 4.33 \text{ mm} \quad (0.173 \text{ in.})$$

8.D4 *It is necessary to select a metal alloy for an application that requires a yield strength of at least 345 MPa (50,000 psi) while maintaining a minimum ductility (%EL) of 20%. If the metal may be cold worked, decide which of the following are candidates: copper, brass, and a 1040 steel. Why?*

Solution

For each of these alloys, the minimum cold work necessary to achieve the yield strength may be determined from Figure 8.19a, while the maximum possible cold work for the ductility is found in Figure 8.19c. These data are tabulated below.

	Yield Strength (<u>> 345 MPa</u>)	Ductility (<u>> 20%EL</u>)
Steel	Any %CW	< 5%CW
Brass	> 20%CW	< 23%CW
Copper	> 54%CW	< 15%CW

Thus, both the 1040 steel and brass are possible candidates since for these alloys there is an overlap of percents coldwork to give the required minimum yield strength and ductility values.

8.D5 A cylindrical rod of 1040 steel originally 15.2 mm (0.60 in.) in diameter is to be cold worked by drawing; the circular cross section will be maintained during deformation. A cold-worked tensile strength in excess of 840 MPa (122,000 psi) and a ductility of at least 12%EL are desired. Furthermore, the final diameter must be 10 mm (0.40 in.). Explain how this may be accomplished.

Solution

First let us calculate the percent cold work and attendant tensile strength and ductility if the drawing is carried out without interruption. From Equation 8.8

$$\begin{aligned}\%CW &= \left(\frac{A_0 - A_d}{A_0} \right) \times 100 = \frac{\pi \left(\frac{d_0}{2} \right)^2 - \pi \left(\frac{d_d}{2} \right)^2}{\pi \left(\frac{d_0}{2} \right)^2} \times 100 \\ &= \frac{\pi \left(\frac{15.2 \text{ mm}}{2} \right)^2 - \pi \left(\frac{10 \text{ mm}}{2} \right)^2}{\pi \left(\frac{15.2 \text{ mm}}{2} \right)^2} \times 100 = 56\%CW\end{aligned}$$

At 56%CW, the steel will have a tensile strength on the order of 920 MPa (133,000 psi) (Figure 8.19b), which is adequate; however, the ductility will be less than 10%EL [Figure 8.19c], which is insufficient.

Instead of performing the drawing in a single operation, let us initially draw some fraction of the total deformation, then anneal to recrystallize, and, finally, cold-work the material a second time in order to achieve the final diameter, tensile strength, and ductility.

Reference to Figure 8.19b indicates that 20%CW is necessary to yield a tensile strength of 840 MPa (122,000 psi). Similarly, a maximum of 21%CW is possible for 12%EL (Figure 8.19c). The average of these extremes is 20.5%CW. Again using Equation 8.8, if the final diameter after the first drawing is d'_0 , then

$$20.5\%CW = \frac{\pi \left(\frac{d'_0}{2} \right)^2 - \pi \left(\frac{10 \text{ mm}}{2} \right)^2}{\pi \left(\frac{d'_0}{2} \right)^2} \times 100$$

And solving the above expression for d'_0 , yields

$$d'_0 = \frac{10 \text{ mm}}{\sqrt{1 - \frac{20.5\%CW}{100}}} = 11.2 \text{ mm (0.45 in.)}$$

8.D6 A cylindrical rod of copper originally 16.0 mm (0.625 in.) in diameter is to be cold worked by drawing; the circular cross section will be maintained during deformation. A cold-worked yield strength in excess of 250 MPa (36,250 psi) and a ductility of at least 12%EL are desired. Furthermore, the final diameter must be 11.3 mm (0.445 in.). Explain how this may be accomplished.

Solution

Let us first calculate the percent cold work and attendant yield strength and ductility if the drawing is carried out without interruption. From Equation 8.8

$$\begin{aligned}\%CW &= \left(\frac{A_0 - A_d}{A_0} \right) \times 100 = \frac{\pi \left(\frac{d_0}{2} \right)^2 - \pi \left(\frac{d_d}{2} \right)^2}{\pi \left(\frac{d_0}{2} \right)^2} \times 100 \\ &= \frac{\pi \left(\frac{16.0 \text{ mm}}{2} \right)^2 - \pi \left(\frac{11.3 \text{ mm}}{2} \right)^2}{\pi \left(\frac{16.0 \text{ mm}}{2} \right)^2} \times 100 = 50\%CW\end{aligned}$$

At 50%CW, the copper will have a yield strength on the order of 330 MPa (48,000 psi), Figure 8.19a, which is adequate; however, the ductility will be about 4%EL, Figure 8.19c, which is insufficient.

Instead of performing the drawing in a single operation, let us initially draw some fraction of the total deformation, then anneal to recrystallize, and, finally, cold work the material a second time in order to achieve the final diameter, yield strength, and ductility.

Reference to Figure 8.19a indicates that 21%CW is necessary to give a yield strength of 250 MPa. Similarly, a maximum of 23%CW is possible for 12%EL (Figure 8.19c). The average of these two values is 22%CW, which we will use in the calculations. Thus, to achieve both the specified yield strength and ductility, the copper must be deformed to 22%CW. If the final diameter after the first drawing is d'_0 , then, using Equation 8.8

$$22\%CW = \frac{\pi \left(\frac{d'_0}{2} \right)^2 - \pi \left(\frac{11.3 \text{ mm}}{2} \right)^2}{\pi \left(\frac{d'_0}{2} \right)^2} \times 100$$

And, solving for d'_0 from the above expression yields

$$d'_0 = \frac{11.3 \text{ mm}}{\sqrt{1 - \frac{22\%CW}{100}}} = 12.8 \text{ mm (0.50 in.)}$$

8.D7 A cylindrical 1040 steel rod having a minimum tensile strength of 865 MPa (125,000 psi), a ductility of at least 10%EL, and a final diameter of 6.0 mm (0.25 in.) is desired. Some steel stock of diameter 7.94 mm (0.313 in.) that has been cold worked 20% is available. Describe the procedure you would follow to obtain this material. Assume that 1040 steel experiences cracking at 40%CW.

Solution

This problem calls for us to cold work some 1040 steel stock that has been previously cold worked in order to achieve minimum tensile strength and ductility values of 865 MPa (125,000 psi) and 10%EL, respectively, while the final diameter must be 6.0 mm (0.25 in.). Furthermore, the material may not be deformed beyond 40%CW. Let us start by deciding what percent coldwork is necessary for the minimum tensile strength and ductility values, assuming that a recrystallization heat treatment is possible. From Figure 8.19b, at least 25%CW is required for a tensile strength of 865 MPa. Furthermore, according to Figure 8.19c, 10%EL corresponds a maximum of 30%CW. Let us take the average of these two values (i.e., 27.5%CW), and determine what previous specimen diameter is required to yield a final diameter of 6.0 mm. For cylindrical specimens, Equation 8.8 takes the form

$$\%CW = \frac{\pi \left(\frac{d_0}{2} \right)^2 - \pi \left(\frac{d_d}{2} \right)^2}{\pi \left(\frac{d_0}{2} \right)^2} \times 100$$

Solving for the original diameter d_0 yields

$$d_0 = \frac{d_d}{\sqrt{1 - \frac{\%CW}{100}}} = \frac{6.0 \text{ mm}}{\sqrt{1 - \frac{27.5\%CW}{100}}} = 7.05 \text{ mm} \quad (0.278 \text{ in.})$$

Now, let us determine its undeformed diameter, realizing that a diameter of 7.94 mm corresponds to 20%CW. Again solving for d_0 using the above equation and assuming $d_d = 7.94$ mm yields

$$d_0 = \frac{d_d}{\sqrt{1 - \frac{\%CW}{100}}} = \frac{7.94 \text{ mm}}{\sqrt{1 - \frac{20\%CW}{100}}} = 8.88 \text{ mm} \quad (0.350 \text{ in.})$$

At this point let us see if it is possible to deform the material from 8.88 mm to 7.05 mm without exceeding the 40%CW limit. Again employing Equation 8.8

$$\%CW = \frac{\pi \left(\frac{8.88 \text{ mm}}{2} \right)^2 - \pi \left(\frac{7.05 \text{ mm}}{2} \right)^2}{\pi \left(\frac{8.88 \text{ mm}}{2} \right)^2} \times 100 = 37\%CW$$

which is less than 40%CW—the maximum amount of deformation before cracking.

In summary, the procedure that can be used to produce the desired material would be as follows: cold work the as-received stock to 7.05 mm (0.278 in.), heat treat it to achieve complete recrystallization, and then cold work the material again to 6.0 mm (0.25 in.), which will give the desired tensile strength and ductility.

Fundamentals of Engineering Questions and Problems

8.1FE *Plastically deforming a metal specimen near room temperature will generally lead to which of the following property changes?*

- (A) an increased tensile strength and a decreased ductility*
- (B) a decreased tensile strength and increased in ductility*
- (C) an increased tensile strength and an increased ductility*
- (D) a decreased tensile strength and a decreased ductility*

Solution

The correct answer is A. Plastically deforming (or strain hardening) a metal increases the dislocation density; this produces an increase in tensile strength and a decrease in ductility.

8.2FE *A dislocation formed by adding an extra half-plane of atoms to a crystal is referred to as a(n)*

(A) screw dislocation

(B) vacancy dislocation

(C) interstitial dislocation

(D) edge dislocation

Solution

The correct answer is D. A dislocation formed by adding an extra half-plane of atoms to a crystal is referred to as an edge dislocation.

8.3FE *The atoms surrounding a screw dislocation experience which kinds of strains?*

(A) tensile strains

(B) shear strains

(C) compressive strains

(D) Both B and C

Solution

The correct answer is B. The atoms surrounding a screw dislocation experience only *shear* strains.