CIS507: Design & Analysis of Algorithms
Quiz 6, Spring 2012
(VERSION WITH ANSWERS)

Duration: 20 minutes
Total weight: 5%

Student Name: –

Student ID: –

Problem	Points Obtained	Points Possible
1		1
2		4
Total		5

Cheat list:
P: problems for which a polynomial time algorithm exists.
NP: problems which are polynomial time verifiable.
$CoNP$: problems which are complements of problems in NP.
$NP - Hard$: problems to which every problem in NP can be reduced in polynomial time.
$CoNP - Hard$: problems which are complements of $NP - Hard$ problems.
$NP - Complete = NP \cap NP - Hard$
$CoNP - Complete$: problems which are complements of $NP - Complete$ problems. Equivalently: problems in $CoNP$ to which every problem in $CoNP$ can be reduced in polynomial time.

1 True/False (1 point)

1. **(0.5 point)** Suppose there exists a $coNP - complete$ problem L that is also in NP. Then $coNP \subseteq NP$.

2. **(0.5 point)** Suppose there exists a $coNP - complete$ problem L that is also in NP. Then $NP \subseteq coNP$.

ANSWER:

1. True. Consider arbitrary problem $L' \in coNP$. Since L is $coNP - Complete$, then $L' \leq_p L$. And since L is in NP, then L' is also in NP.

2. True. Consider arbitrary problem $L'' \in NP$. Therefore problem "is $x \notin L''$?" is in $coNP$. Since L is $coNP - Complete$, then "is $x \notin L''$?" $\leq_p L$. Therefore, "is $x \in L''$?" $\leq_p$ "is $x \notin L$?". But we assumed L is in NP, so $x \notin L$?" is in $coNP$. Therefore, "is $x \in L''$?" is also $coNP$.

2 Multiple Choice (4 points)

For each of the following, circle all (zero or more) correct answer(s). You will be graded 0.25 points on each choice.

1. **(1.5 point)** 3-SAT is:
   (a) P (b) NP (c) CoNP
   (d) NP-Hard (e) coNP-Hard (f) NP-Complete

2. **(1.5 point)** Consider the problem of determining, for a given boolean formula, whether every assignment to the variables satisfies it. The problem is:
   (a) EXP (b) NP (c) CoNP
   (d) NP-Hard (e) coNP-Hard (f) CoNP-Complete

3. **(1 point)** Showing a polynomial time reduction from 3-SAT to problem X proves that X is:
   (a) P (b) NP (c) NP-Complete (d) NP-Hard

4. **(1 point)** If X is NP-Complete, this implies that X is:
   (a) NP (b) EXP (c) P (d) NP-Hard

ANSWER:

1. (b), (d), (f): 3-SAT is NP-complete. So it is also NP, NP-Hard.

2. (a), (c), (e), (f): Tautology is the complimentary problem to Satisfiability, which is NP-complete, so Tautology is CoNP-complete, which also implies it is CoNP, CoNP-hard. EXP incompasses all of coNP.

3. (d) This only shows NP-hardness, since X is not NP.

4. (a), (b), (d): Definition of NP-complete implies NP and $NP - Hard$. And $NP \subseteq EXP$.