

# Distributed Computer Systems Engineering

CIS 508: Lecture 12  
Byzantine Problem

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# Recall: What are Failures

| Type of failure   | Description  |
|---|--|
| Crash failure   | A server halts, but is working correctly until it halts  |
| Omission failure<br>Receive omission<br>Send omission         | A server fails to respond to incoming requests<br>A server fails to receive incoming messages<br>A server fails to send messages |
| Timing failure  | A server's response lies outside the specified time interval   |
| Response failure<br>Value failure<br>State transition failure | The server's response is incorrect<br>The value of the response is wrong<br>The server deviates from the correct flow of control |
| <b>Arbitrary failure</b>                                      | <b>A server may produce arbitrary responses at arbitrary times</b>   |

- Fail-stop failure: Failed parties may notify (or detected by) others in advance
- Fail-silent failure: Not certain if a silent response is a failure or not
- *Arbitrary failure*: Byzantine failure, can be malicious and coordinated

# Need for Theories

- Designing distributed computer systems is challenging
- *Big questions that consider arbitrary failures*
  - *What can be achieved (theoretically or practically)?*
  - *What cannot be achieved (thus no hope to go beyond that)?*
- Need for precise descriptions and problem formulations
  - Enabling formal theoretical studies and proofs
  - Obtaining mathematical insight to implement practical solution
- A widely studied problem: **Byzantine agreement**
  - A simplified setting of fault tolerance consensus problem
  - Various applications in distributed computing:
    - Mission critical infrastructure control, aviation, military,

# A little back story ...

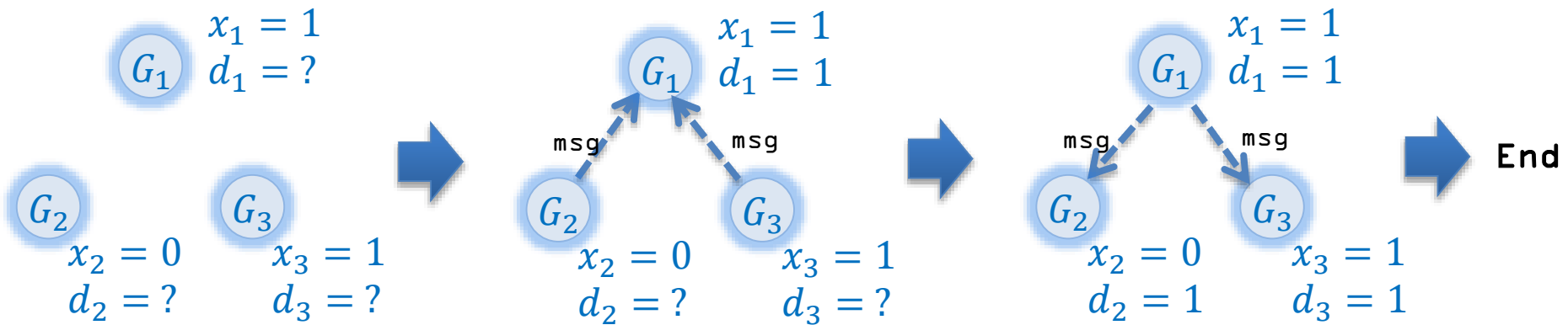


- In 1453 AD, city of Constantinople (controlled by Byzantine Empire) was under siege by troops of Ottoman Empire
- Ottoman battalions camped around the city preparing for attack
- The battalions could communicate with each other by (unreliable) messenger service
- Multiple battalions must attack together in order to win the battle
- Generals of Ottoman battalions must agree on a common time to launch the attack
- However, some (unidentified) generals were corrupted. Aimed at disrupting the attack, they would give false information to confuse the loyal generals
- How can the Ottoman generals proceed to plan their attack in a distributed manner, in the presence of corrupted generals and unreliable messengers?

# Simple Model

- A set of processes (*generals*):  $G = \{G_1, G_2, \dots, G_n\}$ 
  - A subset in  $G$  are correct processes (*loyal generals*):  $L \subseteq G$
  - A subset in  $G$  are faulty processes (*traitorous generals*):  $T \subseteq G$
  - Let  $|G| = n$  and  $|T| = t$
- Each process  $G_i \in L$ 
  - has an initial *binary* value  $x_i \in \{0,1\}$
  - must produce an *binary* output  $d_i \in \{0,1\}$  at the end of consensus
- We assume that the communication between processes is
  - *synchronous*: the processes have perfectly synchronized clocks and a message is guaranteed to be delivered in one time unit
  - *reliable*: messages can neither be forged nor corrupted nor lost
  - *authenticated*: the identity of the sender is known to the receiver
  - *point-to-point*: the underlying topology is that of a complete graph

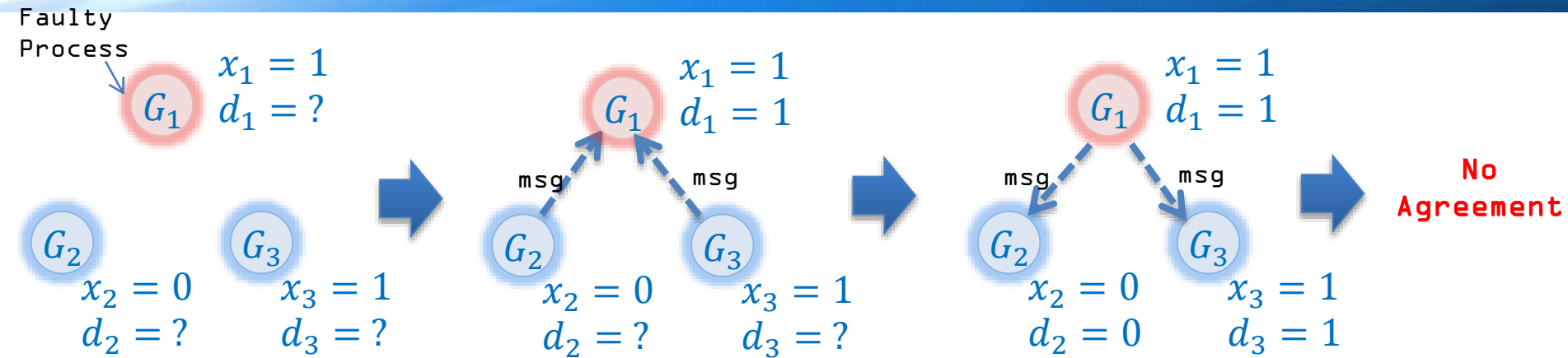
# Desirable Protocol



- Processes have a protocol of message exchanges among them
- A desirable protocol shall satisfy:
  - Non-triviality* : If all processes have the same input value  $x$ , then the only possible output of the normal processes is  $x$ . More formally,
    - If for all  $G_i \in G$  such that  $x_i = x$ , then  $d_i = x$  for all  $G_i \in L$
  - Agreement* : The loyal generals should agree on the decision. That is,
    - For all  $G_i, G_j \in L$ , we have  $d_i = d_j$
  - Finite running time* : The protocol must terminate
- Note that we **do not** assume voting by majority vote!



# Faulty Process



- Remember that any processes can be *faulty*
- Wrong information can be propagated by faulty processes
  - Faulty processes may ignore or send inconsistent messages to others
  - Faulty processes may collude together to disrupt the correct processes
- Identities of faulty processes are unknown
- But we know that there are **at most**  $t$  faulty processes
  - And there can be no faulty process
- A protocol is  **$t$ -resilient** if it tolerates up to  $t$  faulty processes

# Impossibility Theory

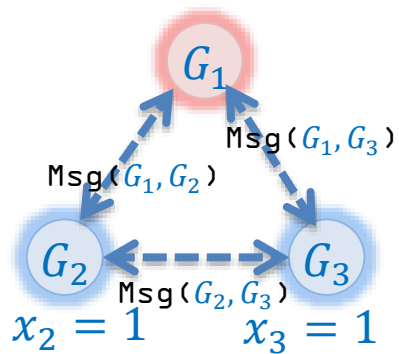
There is **no**  $t$ -resilient protocol for Byzantine agreement, when  $t \geq n/3$

- Implications:
  - Impossible to design a resilient protocol that can tolerate more than one third of faulty processes
  - Ability to tolerate more faults requiring weaken some of assumptions
- Basic idea of proof
  - Suppose that there exists such a  $t$ -resilient protocol
  - Find an instance of execution under the assumption of *non-triviality, agreement, finite running time* that can generate a contradiction



# Impossibility Theory

There is **no**  $t$ -resilient protocol for Byzantine agreement, when  $t \geq n/3$



- We first assume that there are 3 processes ( $n = 3$ )
- No correct process knows the identity of the faulty process
- Suppose that there exists a 1-resilient protocol satisfying
  - *non-triviality, agreement, finite running time*
- We consider three cases:

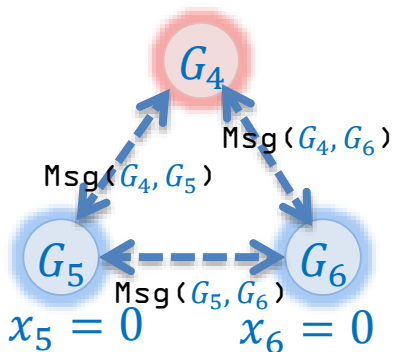
1)  $G_1 \in T; G_2, G_3 \in L; x_2 = x_3 = 1$ :

Whatever  $G_1$  tells  $G_2$  and  $G_3$ , at the end  $d_2 = d_3 = 1$   
 Because the protocol satisfies non-triviality

2)  $G_4 \in T; G_5, G_6 \in L; x_5 = x_6 = 0$ :

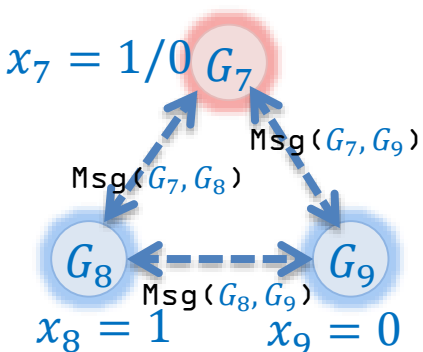
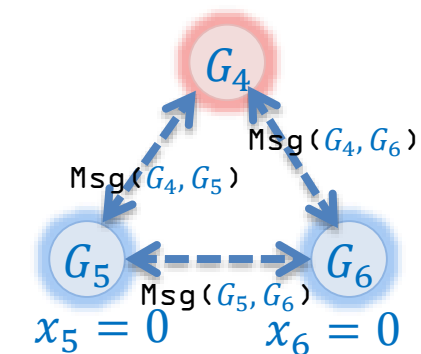
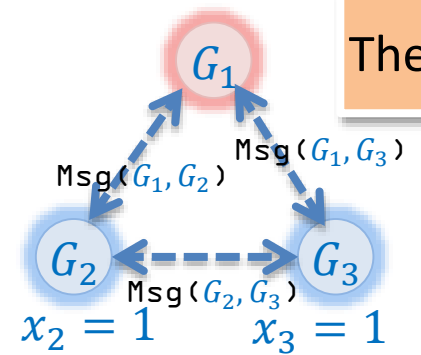
Whatever  $G_4$  tells  $G_5$  and  $G_6$ , at the end  $d_5 = d_6 = 0$   
 Because the protocol satisfies non-triviality

3) ...



# Impossibility Theory

There is **no**  $t$ -resilient protocol for Byzantine agreement, when  $t \geq n/3$



- We consider three cases:

1)  $G_1 \in T; G_2, G_3 \in L; x_2 = x_3 = 1; d_2 = d_3 = 1$

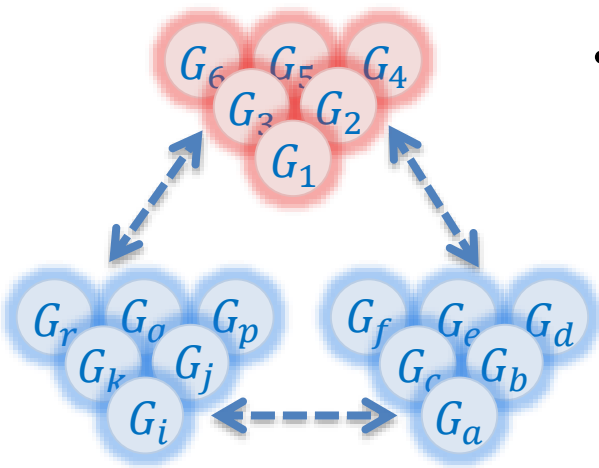
2)  $G_4 \in T; G_5, G_6 \in L; x_5 = x_6 = 0; d_5 = d_6 = 0$

3)  $G_7 \in T; G_8, G_9 \in L; x_8 = 1; x_9 = 0$ :

- Suppose  $G_7$  tells  $G_8$  in the same manner as  $G_3$  tells  $G_2$  in Case 1). That is,  $G_7$  reacts to  $G_8$  as if  $x_7 = 1$
- Meanwhile, suppose  $G_7$  tells  $G_9$  in the same manner as  $G_5$  tells  $G_6$  in Case 2). That is,  $G_7$  reacts to  $G_9$  as if  $x_7 = 0$
- From Case 1), at the end  $d_8 = 1$
- From Case 2), at the end  $d_9 = 0$
- Contradiction! The protocol cannot satisfy agreement  $d_8 \neq d_9$

# Impossibility Theory

There is **no**  $t$ -resilient protocol for Byzantine agreement, when  $t \geq n/3$



- We consider  $n = 3t > 3$ :
- We can divide the processes into three equal groups
  - All faulty processes into one group
  - Other correct processes into two other groups
  - Pick one leader from each group
  - Suppose all processes in a group have same initial value  $x_i$  as the leader
- We seek the agreement among the three leaders
  - Equivalent to Byzantine agreement of 3 processes
- If there exists  $t$ -resilient protocol for  $n = 3t > 3$ , then there exists 1-resilient protocol for  $n = 3$
- Contradiction!

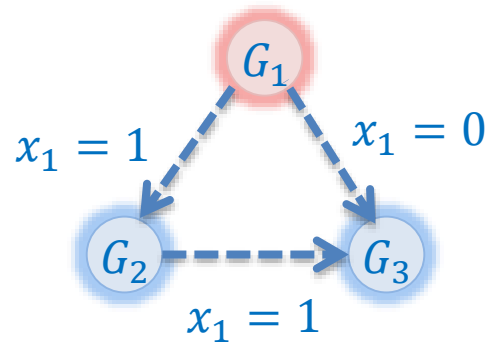
# Any solution?

- Impossibility theory implies  $t < n/3$  is necessary for Byzantine agreement
- Does there really exist a  $t$ -resilient protocol, when  $t < n/3$ ?
- Answer is Yes!
- Assume some deterministic tie-breaking rule (e.g. 1 always wins)

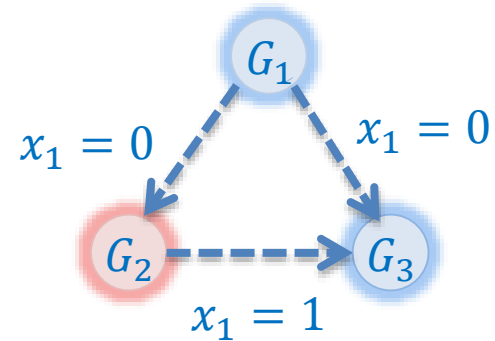
A simple gossiping based solution:

1. Each process will re-announce the initial values that are announced by others
2. Each process will keep track of all the announced and re-announced initial values
3. Majority vote will be used to determine the correct initial values
4. Since the number of faulty processes  $t < n/3$ , correct processes always win, even if there is an even split of votes among correct process

# Intuition



Inconsistency



$t$

$2t + 1$

$t$  ( $x_1 = 0$ )

$t + 1$  ( $x_1 = 1$ )

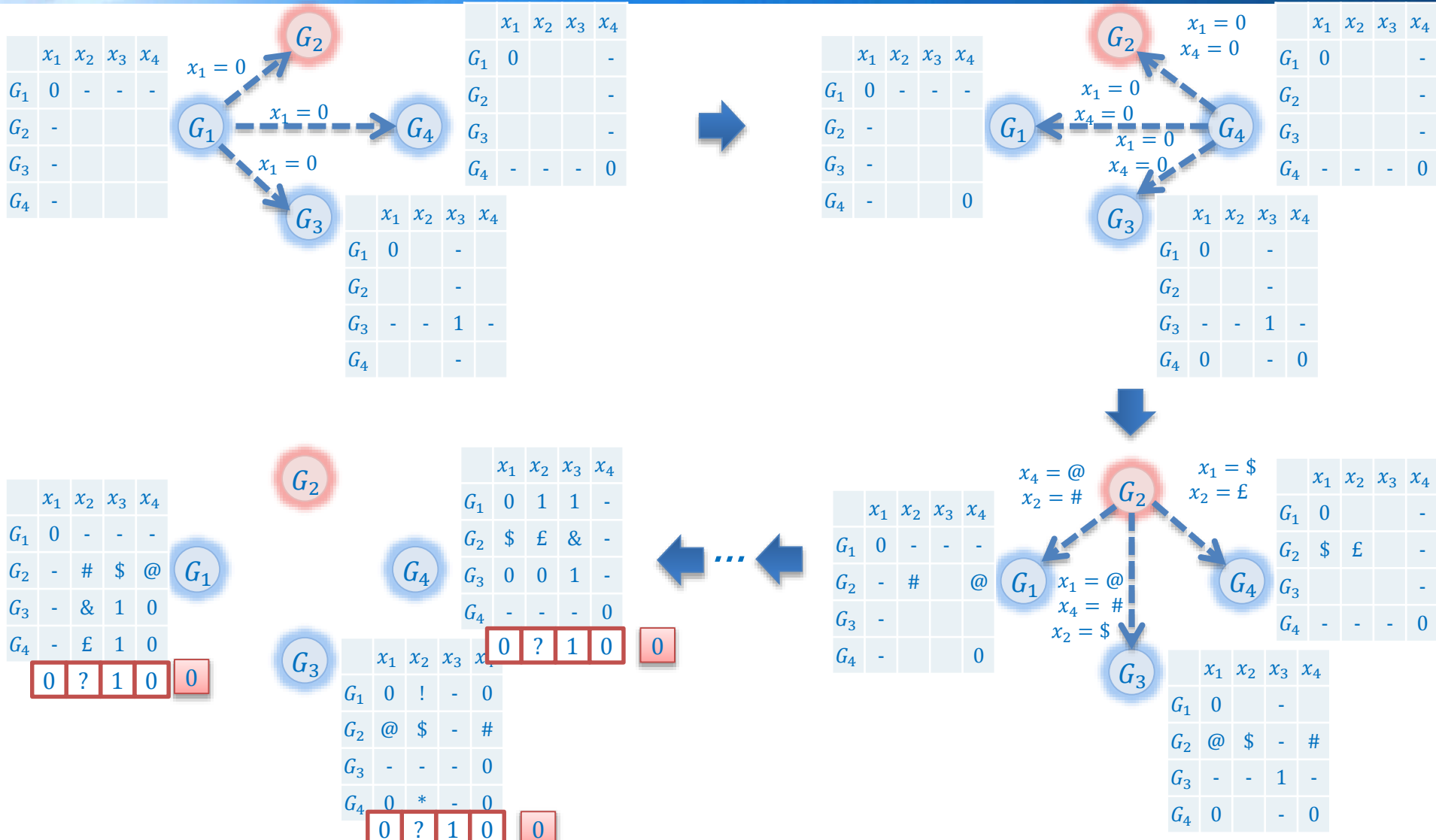
Faulty Processes

Correct Processes

# A Simple Solution

1. Each process  $G_i$  announces its initial value  $x_i$  to all other processes
2. Iteratively, each process  $G_i$  announces  $x_j(k)$  to other process  $G_j$  that is learnt from other process  $G_k$  in previous steps
3. Each process  $G_i$  constructs a table of all the initial values  $x_j(k)$  for other process  $G_j$  learnt from other process  $G_k$
4. Each process  $G_i$  carries out majority vote to determine the initial value  $x_j$  for other process  $j$  from the table using  $\{x_j(k): G_k \in G \setminus \{G_i, G_j\}\}$
5. A final majority vote is carried out on the all determined initial values of all processes  $\{x_j: G_j \in G\}$  to determine output  $d_i$

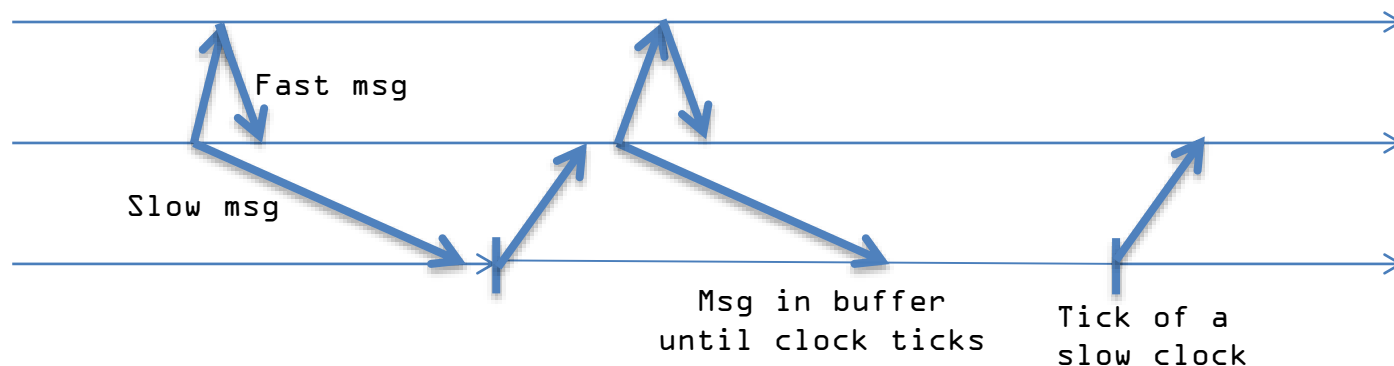
# A simple solution





# Asynchronous Communication

- In asynchronous message-passing systems
  - Messages can be delayed in delivery for arbitrarily long time
  - Messages can be re-ordered in arbitrary manner
    - First message can come last
  - Processes can be crashed, or restart with no previous memory
  - Each process runs according to its own clock
    - Clocks not synchronized



# Asynchronous Communication

- Intuition of asynchrony:
  - Processes cannot tell whether another process is crashed or just messages delayed
  - If they wait, then they might wait forever
  - If they decide and wait no more, they might find that the other process can come to a different decision if they decide to wait (or their clocks run slower)
  - Handling asynchrony is impossible

There is **no** 1-resilient protocol for Byzantine agreement in the presence of asynchrony, for any  $n$



# References

- Reference book
  - Introduction to Distributed Algorithms  
*Gerard Tel, Cambridge University Press*