Average: 89.7
Median: 93

Name: Solutions

CIS603 – Multi-agent Systems

Mid-term Exam

18 October 2012

This exam is closed book, closed notes, closed electronic devices, and open brain. The exam consists of 11 questions or groups of questions. The point value for each question or group of questions is provided. You will have 1.4 hours to take the exam, at which point you will need to turn it in. Thus, if a particular question is taking a particularly long time, skip it and return back to it later. Show your work where applicable, as partial credit is possible. Only legible answers and work can be given credit.

1. (32 points – 4 points each) Indicate whether each statement is *false* (i.e., it contains some untruth) or *true* (otherwise). **If you answer that the statement is *false*, explain why it is false.**

   a. (**True** or **False**) Traditional game theory accounts for situations in which players play ~~either~~ rationally ~~or irrationally~~.

   b. (**True** or **False**) Every normal-form game has at least one Nash equilibrium.

   c. (**True** or **False**) If an algorithm is collectively stable, it can ~~invade~~ all other algorithms.
   avoid being invaded by

   d. (**True** or **False**) The Nash bargaining solution is a pareto optimal solution that ~~minimizes the difference between the players' payoffs~~. maximizes the product of the agents' advantages

   e. (**True** or **False**) In zero-sum games, Nash equilibria only occur when each player plays its maximin strategy.

   f. (**True** or **False**) Every finite extensive-form game can be converted to a normal-form game.

   g. (**True** or **False**) A single agent playing tit-for-tat ~~can~~ cannot invade a population of agents that always defect in the prisoners' dilemma when the discount factor is sufficiently high.
   But a group of TFT agents can if they interact enough

   h. (**True** or **False**) Tit-for-tat is collectively stable in the prisoners' dilemma game when the discount factor (i.e., the probably of playing again) is sufficiently high.

2. (5 points) What is an agent's best strategy (with respect to maximizing its average *per move* payoff) in a repeated prisoners' dilemma with a discount factor near 1 when playing against an associate with an unknown strategy? Prove your answer.

There is no best strategy. It depends on the strategy of one's ~~the~~ associate.

Proof: Best against AD is AD

Best against Never forgive is AC

3. (3 points) What is an agent's best strategy (with respect to maximizing its own expected payoff) in a one-shot prisoners' dilemma played against an associate with an unknown strategy? Briefly explain your answer.

Always defect (AD)

It is strategically dominant

4. (4 points) How will *rational* (self interested) agents play a repeated prisoners' dilemma when it is known that the game will last exactly 10 moves? Explain your answer.

Always defect (AD)

Backwards reasoning: Best strategy in last move for both agents is to always defect. Thus, there is no incentive to cooperate in the 2nd to last move. Same reasoning can be applied to 3rd to last move, etc.

5. (6 points) In discussing Axelrod's prisoners' dilemma tournaments, we identified several characteristics of successful algorithms for repeated games. Name three of these characteristics.

1. Nice
2. Retaliatory (reciprocate cooperation and defection)
3. Forgiveness
4. Clarity

6. (5 points) In an repeated prisoners' dilemma with a discount factor near 1, suppose that the strategy of the first agent is to play tit-for-tat, while the strategy of the second agent is to play the "never forgive" strategy. Is this a Nash equilibrium? Why or why not.

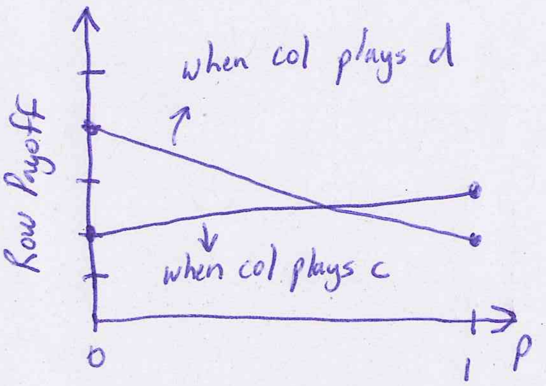
	d	**c**
d	1, 1	5, 0
c	0, 5	3, 3

Yes, because each player's strategy is a best response to the other player's strategy.

7. (8 points) What are the row and column player's maximin strategies and values in the following matrix game? Show your work.

		q c	$1-q$ d
p	**a**	3, -1	2, 2
$1-p$	**b**	2, 4	4, 3

Row Player



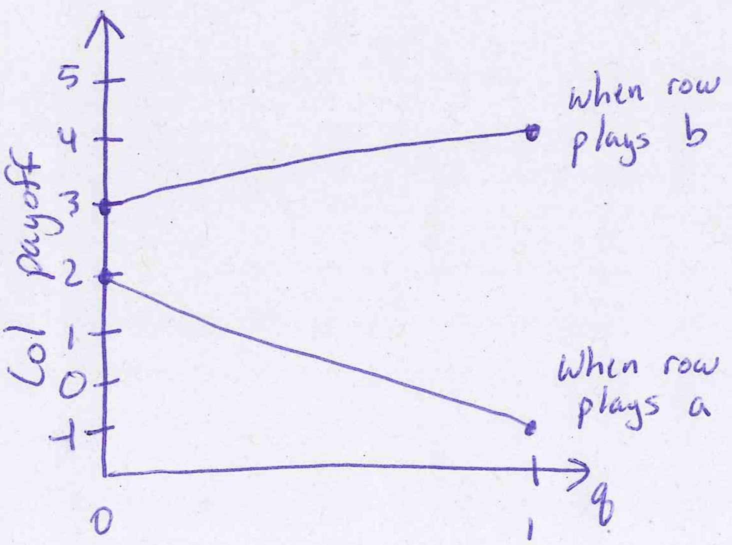
Findpoint of intersection

$$p(3)+(1-p)2 = p(2)+(1-p)4$$
$$3p+2-2p = 2p+4-4p$$
$$2+p = 4-2p$$
$$3p = 2$$
$$p = 2/3$$

maximin strategy:
play a w/ prob $2/3$
play b w/ prob $1/3$

maximin value:
$\left(\frac{2}{3}\right)3+\left(1-\frac{2}{3}\right)2 = 8/3$

Column Player



maximin strategy:
always play d ($q=0$)

maximin value: 2

No point of intersection for $q \in [0,1]$

8 (6 points) What are the pure strategy Nash equilibria of the following two matrix games?

	c	d
a	4, 4	0, 3
b	3, 0	2, 2

	c	d
a	2, 4	4, 3
b	3, 0	2, 2

None

9 (5 points) What is/are the pareto optimal solution(s) of the following matrix game?

	a	b
c	3, 3	2, 2
d	5, 0	4, 2

10 (7 points) Is the joint strategy in which the row player plays action **a** with probability 0.5 and the column player plays action **c** with probability 0.5 a Nash equilibrium? Show your work.

	c	d
a	2, 4	4, 3
b	4, 0	2, 1

We must show that each player is playing a best response:

Row player's utility given that col plays $\left(\frac{1}{2}, \frac{1}{2}\right)$

$$U((0.5, 0.5)) = \frac{1}{4}(2) + \frac{1}{4}(4) + \frac{1}{4}(4) + \frac{1}{4}(2) = \frac{1}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} = 3$$

Check against pure strategies

$$\left.\begin{aligned} U((1,0)) &= \tfrac{1}{2}(2) + \tfrac{1}{2}(4) = 1 + 2 = 3 \\ U((0,1)) &= \tfrac{1}{2}(4) + \tfrac{1}{2}(2) = 2 + 1 = 3 \end{aligned}\right\} \Rightarrow$$

Row player is playing a best response

Col player's utility given that plays $\left(\frac{1}{2}, \frac{1}{2}\right)$

$$U((0.5, 0.5)) = \frac{1}{4}(4) + \frac{1}{4}(3) + \frac{1}{4}(0) + \frac{1}{4}(1) = 1 + \frac{3}{4} + \frac{1}{4} = 2$$

Check against pure strategies:

$$\left.\begin{aligned} U((1,0)) &= \tfrac{1}{2}(4) + \tfrac{1}{2}(0) = 2 \\ U((0,1)) &= \tfrac{1}{2}(3) + \tfrac{1}{2}(1) = 2 \end{aligned}\right\} \Rightarrow$$

Col player is playing a best response

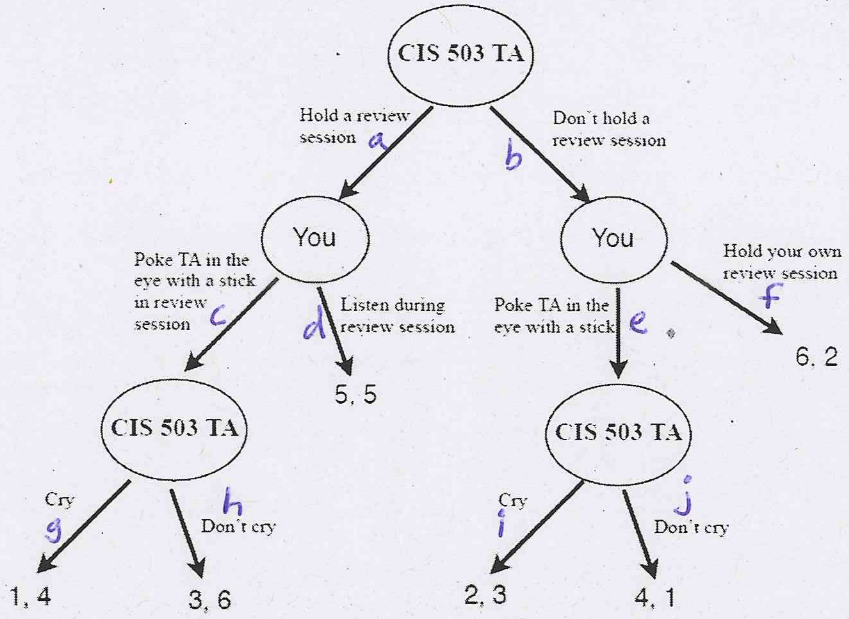
$\therefore$ Both are playing a best response, so it is a NE

TA's pure strategies

(a,g,i)
(a,h,i)
(a,g,j)
(a,h,j)
(b,g,i)
(b,h,i)
(b,h,j)
(b,g,j)

You's pure strats

(c,e)
(c,f)
(d,e)
(d,f)



11. (12 points) Convert the above extensive-form game to normal form. Payoffs to the TA are listed first. Determine the pure-strategy Nash equilibria of the game and indicate which of these Nash equilibria are sub-game perfect and which are not.

	c,e	c,f	d,e	d,f
a,g,i	1,4	1,4	5,5	5,5
a,h,i	3,6	3,6	5,5	5,5
a,g,j	1,4	1,4	5,5	5,5
a,h,j	3,6	3,6	5,5	5,5
b,g,i	2,3	6,2	2,3	6,2
b,h,i	2,3	6,2	2,3	6,2
b,g,j	4,1	6,2	4,1	6,2
b,h,j	4,1	6,2 ★	4,1	6,2

Circle indicates NE

★ indicates sub-game perfection

12. (7 points) For the following repeated matrix game with a high discount factor (near 1), describe a pair of trigger strategies that form a Nash equilibrium of the repeated game in which each agent receives a payoff of 4. Briefly explain why this pair of strategies constitutes a Nash equilibrium.

	c	**d**
a	5, 2	3, 1
b	4, 4	2, 10

[Multiple answers are possible]

Row player: ① As long as col plays 'c', play 'b'

② If col ever plays 'd', play 'a' foreverafter

Col player: ① As long as row plays 'b', play 'c'

② If row ever plays 'a', play 'd' forever after