CIS 603

Solutions to homework #3

① First game:

	c	d
a	2,2	0,0
b	0,0	1,1

Pure strategy NE: (a,c) and (b,d)

Computing the mixed strategy NE: → See pages 62-63 in your book for more information

We compute the point at which the row player is indifferent between its actions. Let q be the probability that the ~~row~~ column player plays c

$$\left.\begin{aligned} U(a) &= 2q + 0(1-q) = 2q \\ U(b) &= 0q + 1(1-q) = 1-q \end{aligned}\right\}$$

U(a) is the expected payoff for playing a

Set $U(a) = U(b)$ and solve for q

$2q = 1-q = q = \frac{1}{3}$ → the row play is indifferent between actions a + b when $q = \frac{1}{3}$

We do the same thing for the column player, and get the same answer in this case since the game is symmetric

Now check if either player has incentive to deviate when the other player plays the indifferent strategy ⇒ compare the pure strategies against the indifferent strategy

process is the same for both players

$$U\left(\left(\tfrac{1}{3},\tfrac{2}{3}\right),\left(\tfrac{1}{3},\tfrac{2}{3}\right)\right) = \left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right)2 + \left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)0 + \left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)0 + \left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)1 = \tfrac{2}{3}$$

$$U\left((1,0),\left(\tfrac{1}{3},\tfrac{2}{3}\right)\right) = 1\left(\tfrac{1}{3}\right)(2) + 1\left(\tfrac{2}{3}\right)0 + 0\left(\tfrac{1}{3}\right)0 + 0\left(\tfrac{2}{3}\right)0 = \tfrac{2}{3}$$

$$U\left((0,1),\left(\tfrac{1}{3},\tfrac{2}{3}\right)\right) = 0\left(\tfrac{1}{3}\right)2 + 0\left(\tfrac{2}{3}\right)0 + 1\left(\tfrac{1}{3}\right)0 + 1\left(\tfrac{2}{3}\right)1 = \tfrac{2}{3}$$

All utilities are the same so the agents don't have incentive to deviate from their stategy that makes the other player indifferent

Mixed NE: $\left(\tfrac{1}{3},\tfrac{2}{3}\right),\left(\tfrac{1}{3},\tfrac{2}{3}\right)$

Second game:

	a	b
a	4,4	0,3
b	3,0	2,2

Pure-strategy NE: (a,a) and (b,b)

Compute the mixed strategy NE:

Find point of indifference for the row and column players (the same for both players since the game is symmetric)

Let q be the probability that col plays a

$$U(a) = 4q + 0(1-q) = 4q$$

$$U(b) = 3q + 2(1-q) = 3q + 2 - 2q = q + 2$$

Set $U(a) = U(b)$ and solve for q

$$4q = q + 2 \Rightarrow q = 2/3$$ → the row player is indifferent between actions a and b when the column player plays a with probability 2/3

Do the same for the column player

You can verify as in the previous game that: $(2/3, 1/3)(2/3, 1/3)$ is a NE → no player can benefit by unilaterally changing its strategy.

(2) Compare NE w/ maximin strategies

(A) First game

$(1/3, 2/3)$

The maximin strategy pairs form the mixed strategy NE in this game

(B) Second game

always play b: (0,1)

The maximin strategy pairs form the pure strategy NE of (b,b)

In general, the maximin strategy pairs ~~are~~ is not guaranteed to be a NE, though it is in this game.

The maximin strategy pairs do form the point of the unique NE in constant-sum games, however.

(3) The pareto optimal solutions are circled in each game

	c	d
a	2,2	0,0
b	0,0	1,1

	a	b
a	4,4	0,3
b	3,0	2,2

not pareto optimal

(4) This answer is subjective, but you should have shown that you actually thought about it.