CIS604: Techniques in Artificial Intelligence

Final Exam, Fall, 2012 - Solutions

Duration: 120 minutes

Instructions:

- This is a closed-book exam.
- Answer *all* questions.
- You may use a calculator.
- Start with the easy questions, and don't spend too much time on one question until you covered the others.
- If you are caught cheating, you will be asked to leave and will receive 0 for this exam.

Student Name: –

Student ID: –

Problem	Points Obtained	Points Possible
1		5
2		10
3		6
4		14
5		16
6		6
7		7
Total		64

1 General (5 points)

- If all action costs are equal, uniform cost search is the same as breadth first search. **T**
- If all action costs are equal and the heuristic function h is 0 for all states, depth first search is equivalent to A* search. **F**
- MDP instances with a discount factor $\gamma = 1$ tend to emphasize near-term rewards. **F**
- A CSP with only boolean variables can always be solved in polynomial time. **F**
- Any search problem can be represented as a Markov Decision Problem. **T**

2 Bayesian Networks (10 points)

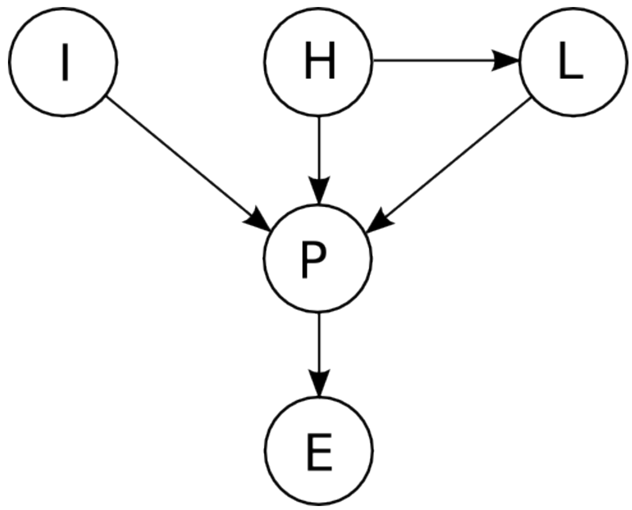
1) (4 points)

Consider Bayesian networks with n boolean variables. Each node has maximum 3 parents. Estimate an upper limit of the size of all conditional probability tables (CPTs) together. Compare that to the full joint distribution table of n boolean variables. Provide values for $n = 20$ and argue why limiting the number of parents in Bayesian Networks is advantageous, but also point out the disadvantage.

Solution A CPT with max 3 parents holds max $2^4 = 16$ entries. We have n such tables, hence the combined size of table entries is $O(n2^4)$. In contrast, the full joint distribution table holds 2^n entries. For 20 nodes, this means that the CPTs of the Bayesian Network is represented by 320 versus 1048576 entries in the full joint distribution. This shows that for larger n, the full joint distribution, which is growing exponentially, becomes infeasable to handle in contrast to the CPTs of the Bayesian Network. The disadvantage of limiting the number of parents is the loss of accuracy, which might happen, if some variables are slightly influencing others but the connections are ignored in order not to exceed the limit for number of parents.

2) (6 points)

Given below is a Bayesian Network with the Boolean variables $I = Intelligent$, $H = Honest$, $P = Popular$, $L = LotsOfCampaignFunds$, $E = Elected$.



The corresponding CPTs are:

H	$P(L \mid H)$
T	.3
F	.9

I	H	L	$P(P \mid I, H, L)$
T	T	T	.9
T	T	F	.4
T	F	T	.8
T	F	F	.2
F	T	T	.8
F	T	F	.3
F	F	T	.8
F	F	F	.1

P	$P(E \mid P)$
T	.6
F	.1

P(I)
0.5

P(H)
0.1

- Calculate the value of $P(i, h, \neg l, p, \neg e)$.
  Remember: The joint distribution can be evaluated using the Bayesian Network topology:

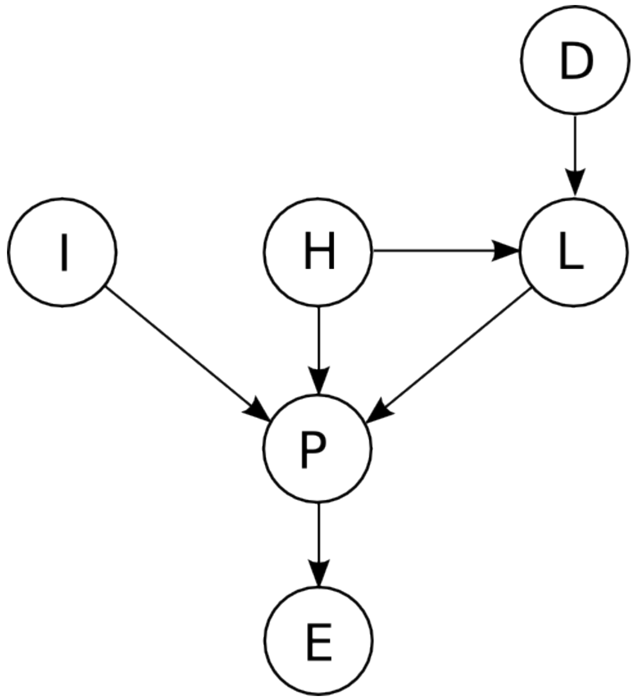
$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} (x_i \mid parents(x_i))$$

- Suppose we want to add the node *Donation*. Argue, which existing variables would be impacted. Draw the resulting network with the appropriate addition of nodes and arcs.

Solution:

$$P(i, h, \neg l, p, \neg e) = P(i)P(h)P(\neg l \mid h)P(p \mid i, h, \neg l)P(\neg e \mid p)$$
$$= .5 \times .1 \times .7 \times .4 \times .4 = 0.0056$$

Adding a node for donations can yield to a number of possible solutions, given a plausible explanation. The most simple is depicted below.



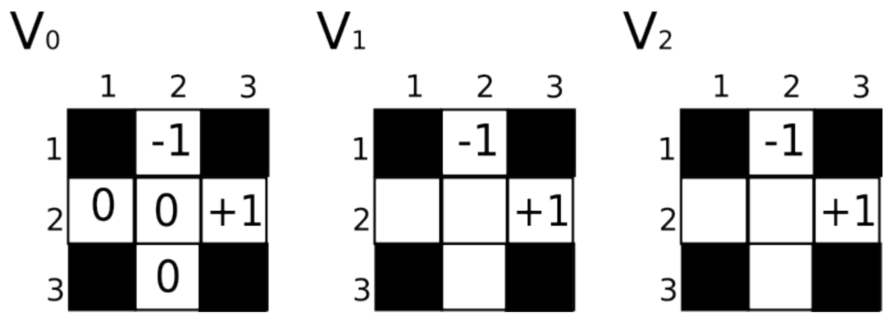
3 Propositional Logic (5 points)

Decide whether the following propositions are satisfiable, valid or contradictory.

1. $\neg A$ **satisfiable**
2. $A \rightarrow A$ **valid**
3. $A \rightarrow B$ **satisfiable**
4. $(A \rightarrow B) \rightarrow (\neg A \rightarrow \neg B)$ **satisfiable**
5. $A \vee B \vee \neg B$ **valid**
6. $(A \rightarrow B) \leftrightarrow ((A \wedge C) \rightarrow B)$ **satisfiable**

4 MDP - Bellmann updates (14 points)

Given below is a Grid MDP with the discount factor $\gamma = 1$ and all rewards for non-terminal states are -0.1. The actions are N, S, W, E, with 80% success rate and 10% probability to drift to either side of the intended direction. Also assume positions $< 2, 1 >$ and $< 3, 2 >$ are terminal states with the indicated payoffs.



Apply the Bellman update in order to calculate utility values for all nonterminal states $V_1(s)$ and $V_2(s)$. After calculating V_2, did the algorithm converge? **Remember:** The Bellman update equation:

$$V_{i+1}(s) = \max_a \sum_{s'} P(s' \mid s, a)\big[R(s) + \gamma V_i(s')\big]$$

where $P(s' \mid s, a)$ is the transition probability associated with action a, and $R(s)$ is the immediate transition reward (at s).

Solution

$$V_{i+1}(s) = \max_a \sum_{s'} P(s' \mid s, a)\big[R(s) + \gamma V_i(s')\big]$$

For $s =< 2, 2 >$ we compare the expected utility for the four possible actions:

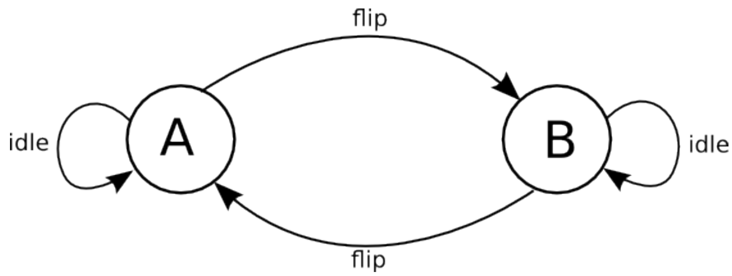
$$\begin{array}{lrr} N: & 0.8 * (-0.1 + (-1)) + 0.1 * (-0.1 + 1) + 0.1 * (-0.1 + 0) = & -0.8 \\ \mathbf{E}: & 0.8 * (-0.1 + 1) + 0.1 * (-0.1 + 0) + 0.1 * (-0.1 + (-1)) = & \mathbf{0.6} \\ W: & 0.8 * (-0.1 + 0) + 0.1 * (-0.1 + 0) + 0.1 * (-0.1 + (-1)) = & -0.2 \\ S: & 0.8 * (-0.1 + 0) + 0.1 * (-0.1 + 0) + 0.1 * (-0.1 + 1) = & 0 \end{array}$$

Therefore the best action is E and hence $V_1(< 2, 2 >) = 0.6$. For states $< 1, 2 >$, $< 2, 3 >$, all actions lead to states of value 0. Hence $V_1(< 1, 2 >) = V_1(< 2, 3 >) = 0.8 * (-0.1) + 0.1 * (-0.1) + 0.1 * (-0.1) = -0.1$

$V_2 < 1, 2 >= -0.8 * (-0.1 + 0.6) + 0.2 * (-0.1)$ $V_2 < 2, 2 >= 0.8 * (-0.1 + 1) + 0.1 * (-0.1 + -1) + 0.1 * (-0.1 - 0.1)$ It is easy to see that e.g. $V_3(< 2, 2 >) \neq V_3(< 2, 2 >)$, so the value iteration process did not converge.

5 Policy Iteration (16 points)

Consider a Markov Decision Process with two states, A and B. The rewards are $R(A) = 3$ and $R(B) = 2$. The discount factor is $\gamma = 0.5$. In each state two actions are possible, *flip* (changing to the other state) and *idle* (staying in the same state).



1)

Apply the Policy iteration algorithm to the deterministic case, where all actions lead to the state as indicated in the figure above: Fill out the table.

	π^0	V^{π^0}	π^1	V^{π^1}	π^2
A	*idle*				
B	*idle*				

Note that Policy evaluation, when solved directly (without Value iteration), leads to a simple linear equation system. Show each step of the calculation in detail.

Remember: Policy evaluation equation:

$$V^{\pi}(s) = \sum_{s'} P(s' \mid s, a)\big[R(s) + \gamma V^{\pi}(s')\big]$$

where $P(s' \mid s, a)$ is the transition probability associated with action a, and $R(s)$ is the immediate transition reward (at s). Hint: this formula can be simplified for the deterministic case.

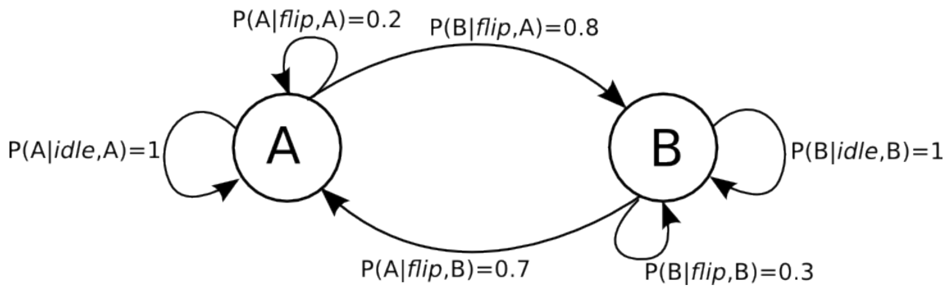
Remember: Policy update is given as:

$$\pi_{k+1}(s) = \arg\max_{a} \sum_{s'} P(s' \mid s, a)\big[R(s) + \gamma V^{\pi}(s')\big]$$

2)

Apply the Policy iteration algorithm to the non-deterministic case, with action outcomes as indicated in the figure below. Fill out the table below. Show each step of the calculation in detail.

	π^0	V^{π^0}	π^1	V^{π^1}	π^2
A	*idle*				
B	*idle*				



6 Description Logic (6 points)

Consider the following description logic knowledge base:

$$
\begin{aligned}
KB \quad = \quad & \{a : C, \quad b : D, \quad c : C, \quad d : E, \quad b : C, \\
& E \sqsubseteq C, \\
& C \sqsubseteq \forall R.D, \\
& \langle a, c \rangle : R, \\
& \langle d, c \rangle : R\}
\end{aligned}
$$

Suppose we are using a multiple-model semantics with $\Delta = \{w, x, y, z\}$. State whether each of the interpretation below is a model of KB. Explain your answers.

1. $C^I = \{w, x, z\}$; $D^I = \{x, y\}$; $E^I = \{y\}$; $a^I = x$; $b^I = y$; $c^I = z$; $d^I = w$; $R^I = \{(x, z), (w, z)\}$.

2. $C^I = \{w, x, z\}$; $D^I = \{x, y\}$; $E^I = \{w\}$; $a^I = w$; $b^I = x$; $c^I = y$; $d^I = z$; $R^I = \{(w, y), (z, y)\}$

Solution Several answers might be possible. Sample answers: The first interpretation is not a model of KB. $d^I = w$ but wE^I. The second interpretation is not a model of KB. $c^I = y$ but yC^I.

7 Conditional probability (7 points)

Let D be a binary variable that denotes "disease". Let $T \in \{+, -\}$ be a variable denoting whether a medical test comes positive or negative. Suppose $P(D) = 0.05$, $P(+|D) = 0.98$, $P(+|\neg D) = 0.1$. Answer the following question:

1. What is $P(\neg D)$?

2. What is $P(-|D)$?

3. What is $P(+, D)$?
4. What is $P(-, D)$?
5. What is $P(+, \neg D)$?
6. What is $P(-, \neg D)$?
7. What is $P(D|+)$?

Solution

1. $P(\neg D) = 1 - P(D) = 1 - 0.05 = 0.95$
2. $P(-|D) = 1 - P(+|D) = 1 - 0.98 = 0.02$
3. $P(+, D) = P(+|D)P(D) = 0.98 * 0.05 = 0.049$
4. $P(-, D) = P(-|D)P(D) = 0.02 * 0.05 = 0.001$
5. $P(+, \neg D) = P(+|\neg D)P(\neg D) = 0.1 * 0.95 = 0.095$
6. $P(-, \neg D) = P(-|\neg D)P(\neg D) = (1-P(+|\neg D))P(\neg D) = 0.9*0.95 = 0.855$
7. $P(D|+) = \frac{P(D,+)}{P(+)} = \frac{P(D,+)}{P(+,D)+P(+,\neg D)} = \frac{0.049}{(0.049+0.095)} = 0.34027.$
   Note that we used the law of total probability in the denominator.
   Or using Bayes Law: $P(D|+) = \frac{P(+|D)P(D)}{P(+)} = \frac{0.98 \times 0.05}{(0.049+0.095)} = 0.34027$