1. John Conway's Doomsday Algorithm

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Calendar Facts: Leap Years

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- Years that are divisible by 100, but not 400, such as 1700, 1800, 1900, and 2100, are not leap years.
- Any year that is divisible by 4, but not 100, such as 2012 is also a leap year. Years not divisible by 4, such as 2011 or 2014, are not leap years.

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- Any year that is divisible by 4, but not 100, such as 2012 is also a leap year. Years not divisible by 4, such as 2011 or 2014, are not leap years.

Leap years usually occur every 4 years, 2012, 2016, 2020, . . ., up to 2096. But the first leap year after 2096 will occur in 2104, 8 years later.

The Doomsday Algorithm — John H. Conway

The last of Feb., or of Jan. will do
(Except that in Leap Years it's Jan. 32).
Then for even months use the month's own day,
And for odd ones add 4, or take it away.*

Now to work out your doomsday the orthodox way
Three things you should add to the century day
Dozens, remainder, and fours in the latter,
(If you alter by sevens of course it won't matter)

In Julian times, lackaday, lackaday
Zero was Sunday, centuries fell back a day
But Gregorian 4 hundreds are always Tues.
And now centuries extra take us back twos.

**According to length or simply remember,*
you only subtract for September, or November.

Identifying the Doomsday

In any given year, the doomsday is defined to be the day of the week on which the last day of February falls.

FEBRUARY 2014						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

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Thus, for the year 2014, the doomsday is Friday.

Casting Off Sevens

The rule of *casting off sevens* states that adding or subtracting any multiple of seven from a particular date does not change the day of the week.

FEBRUARY 2014						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

Since February 28, 2014 fell on Friday, so did

$$\begin{aligned}
\text{February } 21, &\qquad 28 - 7 = 21,\\
\text{February } 14, &\qquad 28 - 14 = 14,\\
\text{February } 7, &\qquad 28 - 21 = 7,\\
\text{February } 0, &\qquad 28 - 28 = 0.
\end{aligned}$$

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Date Arithmetic

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$$\begin{aligned} \text{February } 13, 2014 &= \text{Friday} - (28 - 13) \\ &= \text{Friday} - 15 \\ &= \text{Friday} - \ \ 1 \\ &= \text{Thursday} \end{aligned}$$

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Likewise,

$$\begin{aligned}\text{February } 23, 2014 &= \text{Friday} - (28 - 23) \\ &= \text{Friday} - 5 \\ &= \text{Friday} + 2 \\ &= \text{Sunday}\end{aligned}$$

(You should practice this with a calendar nearby to check your accuracy.)

Finding Doomsdays in the Other Months

Four additional rules suffice:

1. Rule of the Even Numbered Months (4, 6, 8, 10, 12).
2. Rule of the Odd Months After April (5, 7, 9, 11).
3. Rule of March (3)
4. Rule of January (1)

Finding Doomsdays in the Other Months

Four additional rules suffice:

1. Rule of the Even Numbered Months (4, 6, 8, 10, 12).
   Match the index of the even numbered month with its date:
   April 4 (4/4) August 8 (8/8) December 12 (12/12)
   June 6 (6/6) October 10 (10/10)
   are all doomsdays. E.g., April 4, 2014 was a Friday
2. Rule of the Odd Months After April (5, 7, 9, 11).
3. Rule of March (3)
4. Rule of January (1)

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1. Rule of the Even Numbered Months (4, 6, 8, 10, 12).
2. Rule of the Odd Months After April (5, 7, 9, 11).
   Remember the mnemonic, "She works 9 to 5 at the 7-11," as
   May 9 (5/9) September 5 (9/5)
   July 11 (7/11) November 7 (11/7)
   are all doomsdays. E.g., September 5, 2014 will be a Friday.
3. Rule of March (3)
4. Rule of January (1)

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1. Rule of the Even Numbered Months (4, 6, 8, 10, 12).
2. Rule of the Odd Months After April (5, 7, 9, 11).
3. Rule of March (3)
   The last day of February falls 1 day before March 1st.
   Thus, March 0 is the doomsday ($1 - 1 = 0$, right?). By the rule of seven, therefore, every date of March that is a multiple of 7 falls on the doomsday. E.g, March 21, 2014 was a Friday.
4. Rule of January (1)

Finding Doomsdays in the Other Months

Four additional rules suffice:

1. Rule of the Even Numbered Months (4, 6, 8, 10, 12).
2. Rule of the Odd Months After April (5, 7, 9, 11).
3. Rule of March (3)
4. Rule of January (1)
   This is the only rule that depends on the year being either a leap year or a non-leap year. Recall that leap years usually occur once every 4 years, and each leap year is usually followed by 3 non-leap years. Thus,

   For leap years, January 4 is a doomsday.
   For non-leap years, January 3 is a doomsday.

   For example, since 2014 is a common year, January 3, 2014 was a Friday.

   Alternatively, following the poem, January 31 is the doomsday in a non-leap year; but in a leap year use January 32.

“Practice makes perfect.”

- On what day of the week was July 4, 2014?

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  1. Friday is the doomsday for 2014.

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- On what day of the week was July 4, 2014?
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  2. From the rule for the odd months (after April), July 11, 2014 was also Friday.

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- On what day of the week was July 4, 2014?
  1. Friday is the doomsday for 2014.
  2. From the rule for the odd months (after April), July 11, 2014 was also Friday.
  3. Using date arithmetic,

$$\begin{aligned} \text{July } 4, 2014 &= \text{Friday} - (11 - 4) \\ &= \text{Friday} - 7 \\ &= \text{Friday}. \end{aligned}$$

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  1. Friday is the doomsday for 2014.
  2. From the rule of even months, October 10, 2014 falls on Friday.

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- On what day of the week will be October 31, 2014?

1. Friday is the doomsday for 2014.
2. From the rule of even months, October 10, 2014 falls on Friday.
3. Using date arithmetic,

$$\begin{aligned} \text{October } 31, 2014 &= \text{Friday} + (31 - 10) \\ &= \text{Friday} + 21 \\ &= \text{Friday}. \end{aligned}$$

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- On what day of the week was July 4, 2014?
- On what day of the week will be October 31, 2014?
- On what day of the week will be December 25, 2014?

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- On what day of the week will be October 31, 2014?
- On what day of the week will be December 25, 2014?

1. Friday is the doomsday for 2014.
2. From the rule of even months, December 12, 2014 falls on Friday.

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- On what day of the week will be October 31, 2014?
- On what day of the week will be December 25, 2014?

1. Friday is the doomsday for 2014.
2. From the rule of even months, December 12, 2014 falls on Friday.
3. Using date arithmetic,

$$\begin{aligned} \text{December } 25, 2014 &= \text{Friday} + (25 - 12) \\ &= \text{Friday} + 13 \\ &= \text{Friday} - 1 \\ &= \text{Thursday}. \end{aligned}$$

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- On what day of the week will be October 31, 2014?
- On what day of the week will be December 25, 2014?
- What is the date of the next Thanksgiving?

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- On what day of the week was July 4, 2014?
- On what day of the week will be October 31, 2014?
- On what day of the week will be December 25, 2014?
- What is the date of the next Thanksgiving?
  Note that Thanksgiving (in the USA) is always celebrated on the 4th Thursday of November. The 1st Thursday occurs on or before the doomsday November 7th; the 4th Thursday occurs $3 \times 7 = 21$ days after the 1st Thursday.
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  3. Thus the first Thursday is November 6, 2014.

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  2. From the rule of odd months, November 7, 2014 falls on Friday.
  3. Thus the first Thursday is November 6, 2014.
  4. Using date arithmetic,

$$\begin{aligned} \text{4th Thursday in Nov. 2014} &= 6 + 3 \times 7 \\ &= 6 + 21 \\ &= 27. \end{aligned}$$

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- On what day of the week will be October 31, 2014?
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- What is the date of the 3rd Saturday in January 2014?

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- On what day of the week will be December 25, 2014?
- What is the date of the next Thanksgiving?
- What is the date of the 3rd Saturday in January 2014?

1. Friday is the doomsday for 2014.
2. From the rule of January (in a common year), January 3, 2014 fell on Friday. Since this means January 1st fell on $\text{Friday} - (3 - 1) = \text{Friday} - 2 = \text{Wednesday}$, the first Saturday fell after January 3rd.

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3. Using date arithmetic,

$$\begin{aligned} \text{3rd Saturday in January 2014} &= 3 + (\text{Saturday} - \text{Friday}) + 2 \times 7 \\ &= 3 + 1 + 14 \\ &= 18, \quad (\text{January 18, 2014}). \end{aligned}$$

Finding Doomsdays in Successive and Previous Years

- Moving forward:

  Since $365 = 52 \times 7 + 1$, the doomsday advances 1 day as we move into a non-leap year. Thus the doomsday will be Saturday in 2015 (which should make for a great Halloween).

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  Since $366 = 52 \times 7 + 2$, the doomsday advances 2 days as we move into a leap year. Thus the doomsday in 2016 (a leap year) is $\text{Saturday} + 2 = \text{Monday}$.

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- Moving backward:

  The flip side is that as we move backwards *from* a non-leap year, the doomsday retreats by 1 day. Thus the doomsday in 2013 was $\text{Friday} - 1 = \text{Thursday}$; and in 2012 was $\text{Thursday} = 1 = \text{Wednesday}$

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- Moving backward:

  The flip side is that as we move backwards *from* a non-leap year, the doomsday retreats by 1 day. Thus the doomsday in 2013 was $\text{Friday} - 1 = \text{Thursday}$; and in 2012 was $\text{Thursday} = 1 = \text{Wednesday}$

  Likewise, moving backwards *from* a leap year results in a retreat of 2 days. For example, since 2012 was a leap year, the doomsday in 2011 was $\text{Wednesday} - 2 = \text{Monday}$. Since 2011 was a non-leap year, the doomsday in 2010 was $\text{Monday} - 1 = \text{Sunday}$.

Conway's 12 year rule

In 2000 (and as it turns out for every year evenly divisible by 400), the doomsday was Tuesday. Advancing forward by 12 years entails jumping forward into 3 successive leap years: 2004, 2008, 2012, and $12 - 3 = 9$ non-leap years. Thus the total number of days that the doomsday advances is

$$\begin{aligned}
\text{doomsday advance} &= 2 \times \#\text{ of leap years} + 1 \times \#\text{ of non-leap years} \\
&= 2 \times 3 + 1 \times (12 - 3) \\
&= 6 + 9 \\
&= 15 \\
&= 1, \quad \text{(casting off sevens)}.
\end{aligned}$$

Hence the doomsday in 2012 is $\text{Tuesday} + 1 = \text{Wednesday}$.

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&= 2 \times 3 + 1 \times (12 - 3) \\
&= 6 + 9 \\
&= 15 \\
&= 1, \quad \text{(casting off sevens)}.
\end{aligned}$$

Hence the doomsday in 2012 is $\text{Tuesday} + 1 = \text{Wednesday}$.

This turns out always to be the case,

$$12 \text{ years forward} \Longrightarrow 1 \text{ day advance},$$

as long as we don't jump into (or over) a year that is evenly divisible by 100, but not 400, e.g., 2100.

Applying the 12 year rule

Now pick any year in the 21st century, say 2067. Here's how we compute its doomsday:

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1. Subtract 2000 (Tuesday) from 2067, to obtain 67.
2. Count how many times 12 divides evenly into the difference. (Here we obtain $67 \div 12 = 5 \text{ R } 7$.)
3. Count how many times 4 divides evenly into the remainder. (Here we obtain $7 \div 4 = 1 \text{ R } 3$.)

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3. Count how many times 4 divides evenly into the remainder. (Here we obtain $7 \div 4 = 1 \text{ R } 3$.)
4. Add the quotient (5) and remainder (7) in Step 2 to the quotient in Step 3 (1), and cast off sevens. (Here we obtain $5 + 7 + 1 = 6 = -1$.

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1. Subtract 2000 (Tuesday) from 2067, to obtain 67.
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4. Add the quotient (5) and remainder (7) in Step 2 to the quotient in Step 3 (1), and cast off sevens. (Here we obtain $5 + 7 + 1 = 6 = -1$.
5. Add this number to the doomsday in Step 1: $\text{Tuesday} - 1 = \text{Monday}$.

In summary,

$$\begin{aligned}
\text{Doomsday of } 2067 &= \text{Tuesday} + \lfloor 67/12 \rfloor + 67 \bmod 12 + \lfloor (67 \bmod 12)/4 \rfloor \\
&= \text{Tuesday} + 5 + 7 + 1 \\
&= \text{Tuesday} + 6 \\
&= \text{Tuesday} - 1 \\
&= \text{Monday}
\end{aligned}$$

Doomsdays in Other Centuries

Applying the 12 year rule to a 100 year interval results in an advance of

$$\lfloor 100/12 \rfloor + 100 \bmod 12 + \lfloor (100 \bmod 12)/4 \rfloor = 8 + 4 + 1 = 13 = -1 \text{days}.$$

However, this value would only be correct if the largest year divisible by 100 was a leap year. If not, then it is one day too many.
Thus the doomsday for 2100 is $\text{Tuesday} - 2 = \text{Sunday}$.
By similar calculation one can derive the following doomsdays for the centuries from 1500 through 2500.

Gregorian Centuries by Doomsday						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
1700		1600	1500			
2100		2000	1900		1800	
2500		2400	2300		2200	

Doomsdays by Year

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1948	1949	1950	1945	1946	1947	1943
1954	1955	1961	1951	1957	1952	1959
1965	1960	1967	1956	1963	1958	1964
1971	1966	1972	1962	1968	1969	1970
1976	1977	1978	1973	1974	1975	1981
1982	1983	1989	1979	1985	1980	1987
1993	1988	1995	1984	1991	1986	1992
1999	1994	2000	1990	1996	1997	1998
2004	2005	2007	2001	2002	2003	2009
2010	2011	2017	2007	2013	2008	2015
2021	2016	2023	2012	2019	2014	2020
2027	2022	2028	2018	2024	2025	2026
2032	2033	2034	2029	2030	2031	2037
2038	2039	2045	2035	2041	2036	2043
2049	2044	2051	2040	2047	2042	2048
2055	2050	2056	2046	2052	2053	2054