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Part I: Crypto

Chapter 3: Symmetric Key Crypto

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Symmetric Key Crypto

The chief forms of beauty are order and symmetry...
— Aristotle

“You boil it in sawdust: you salt it in glue:
You condense it with locusts and tape:
Still keeping one principal object in view —
To preserve its symmetrical shape.”
— Lewis Carroll, *The Hunting of the Snark*

Symmetric Key Crypto

- ❑ Stream cipher — based on one-time pad
 - Except that key is relatively short
 - ▮ Key is stretched into a long **keystream**
 - ▮ Keystream is used just like a one-time pad
- ❑ Block cipher — based on codebook concept
 - ▮ Block cipher key determines a codebook
 - ▮ Each key yields a different codebook
 - ▮ Employs both “confusion” and “diffusion”

Stream Ciphers



Stream Ciphers

- ❑ Once upon a time, not so very long ago, stream ciphers were the king of crypto
- ❑ Today, not as popular as block ciphers
- ❑ We'll discuss two stream ciphers...
- ❑ A5/1
 - ▮ Based on shift registers
 - ▮ Used in GSM mobile phone system
- ❑ RC4
 - ▮ Based on a changing lookup table
 - ▮ Used many places

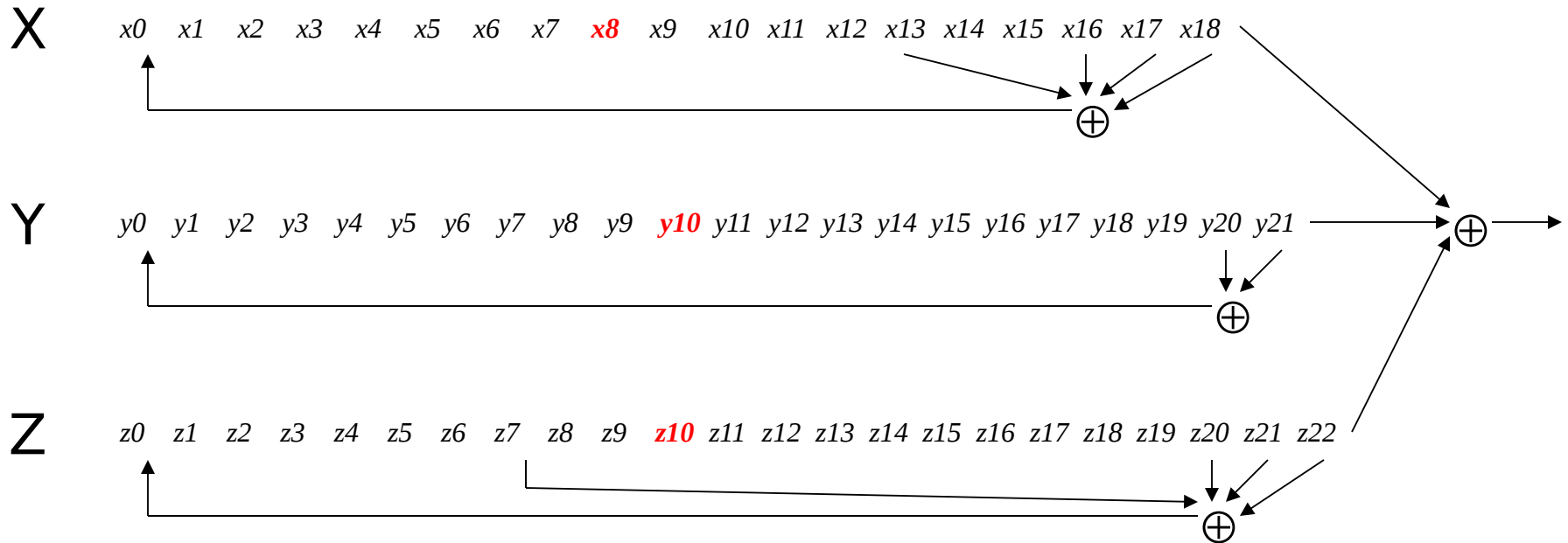
A5/1: Shift Registers

- A5/1 uses 3 *shift registers*
 - X: 19 bits ($x_0, x_1, x_2, \dots, x_{18}$)
 - ▮ Y: 22 bits ($y_0, y_1, y_2, \dots, y_{21}$)
 - ▮ Z: 23 bits ($z_0, z_1, z_2, \dots, z_{22}$)

A5/1: Keystream

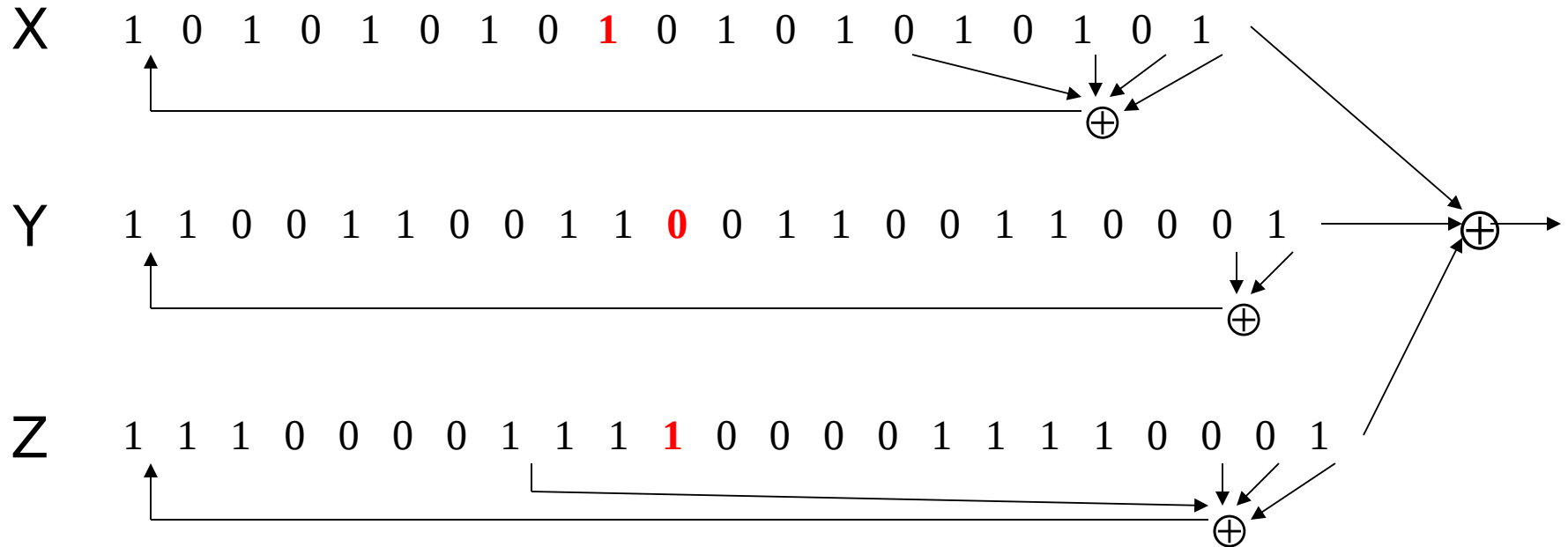
- ❑ At each step: $m = \text{maj}(x_8, y_{10}, z_{10})$
 - ▮ Examples: $\text{maj}(0,1,0) = 0$ and $\text{maj}(1,1,0) = 1$
- ❑ If $x_8 = m$ then *X steps*
 - ▮ $t = x_{13} \oplus x_{16} \oplus x_{17} \oplus x_{18}$
 - ▮ $x_i = x_{i-1}$ for $i = 18, 17, \dots, 1$ and $x_0 = t$
- ❑ If $y_{10} = m$ then *Y steps*
 - ▮ $t = y_{20} \oplus y_{21}$
 - ▮ $y_i = y_{i-1}$ for $i = 21, 20, \dots, 1$ and $y_0 = t$
- ❑ If $z_{10} = m$ then *Z steps*
 - ▮ $t = z_7 \oplus z_{20} \oplus z_{21} \oplus z_{22}$
 - ▮ $z_i = z_{i-1}$ for $i = 22, 21, \dots, 1$ and $z_0 = t$
- ❑ Keystream **bit** is $x_{18} \oplus y_{21} \oplus z_{22}$

A5/1



- ❑ Each variable here is a single bit
- ❑ Key is used as **initial fill** of registers
- ❑ Each register steps (or not) based on $\text{maj}(x_8, y_{10}, z_{10})$
- ❑ Keystream bit is XOR of rightmost bits of registers

A5/1



- ❑ In this example, $m = \text{maj}(x_8, y_{10}, z_{10}) = \text{maj}(\mathbf{1}, \mathbf{0}, \mathbf{1}) = \mathbf{1}$
- ❑ Register X steps, Y does not step, and Z steps
- ❑ Keystream bit is XOR of right bits of registers
- ❑ Here, keystream bit will be $0 \oplus 1 \oplus 0 = 1$

Shift Register Crypto

- ❑ Shift register crypto efficient in hardware
- ❑ Often, slow if implement in software
- ❑ In the past, very popular
- ❑ Today, more is done in software due to fast processors
- ❑ Shift register crypto still used some
 - ❑ Resource-constrained devices

RC4

- ❑ A self-modifying lookup table
- ❑ Table always contains a permutation of the byte values 0,1,...,255
- ❑ Initialize the permutation using key
- ❑ At each step, RC4 does the following
 - ▢ Swaps elements in current lookup table
 - ▢ Selects a keystream byte from table
- ❑ Each step of RC4 produces a **byte**
 - ▢ Efficient in software
- ❑ Each step of A5/1 produces only a bit
 - ▢ Efficient in hardware

RC4 Initialization

- `S[]` is permutation of `0,1,...,255`
- `key[]` contains `N` bytes of key

```
for i = 0 to 255
    S[i] = i
    K[i] = key[i (mod N)]
next i
j = 0
for i = 0 to 255
    j = (j + S[i] + K[i]) mod 256
    swap(S[i], S[j])
next i
i = j = 0
```

RC4 Keystream

- For each keystream byte, swap elements in table and select byte

```
i = (i + 1) mod 256
```

```
j = (j + S[i]) mod 256
```

```
swap(S[i], S[j])
```

```
t = (S[i] + S[j]) mod 256
```

```
keystreamByte = S[t]
```

- Use keystream bytes like a one-time pad
- **Note:** first 256 bytes should be discarded
 - Otherwise, related key attack exists

Stream Ciphers

- ❑ Stream ciphers were popular in the past
 - Efficient in hardware
 - ▮ Speed was needed to keep up with voice, etc.
 - ▮ Today, processors are fast, so software-based crypto is usually more than fast enough
- ❑ Future of stream ciphers?
 - ▮ Shamir declared “the death of stream ciphers”
 - ▮ May be greatly exaggerated...

Block Ciphers



(Iterated) Block Cipher

- ❑ Plaintext and ciphertext consist of fixed-sized blocks
- ❑ Ciphertext obtained from plaintext by iterating a **round function**
- ❑ Input to round function consists of **key** and **output** of previous round
- ❑ Usually implemented in software

Feistel Cipher: Encryption

- ❑ **Feistel cipher** is a type of block cipher, not a specific block cipher
- ❑ Split plaintext block into left and right halves: $P = (L_0, R_0)$
- ❑ For each round $i = 1, 2, \dots, n$, compute
$$L_i = R_{i-1}$$
$$R_i = L_{i-1} \oplus F(R_{i-1}, K_i)$$
where F is **round function** and K_i is **subkey**
- ❑ Ciphertext: $C = (L_n, R_n)$

Feistel Cipher: Decryption

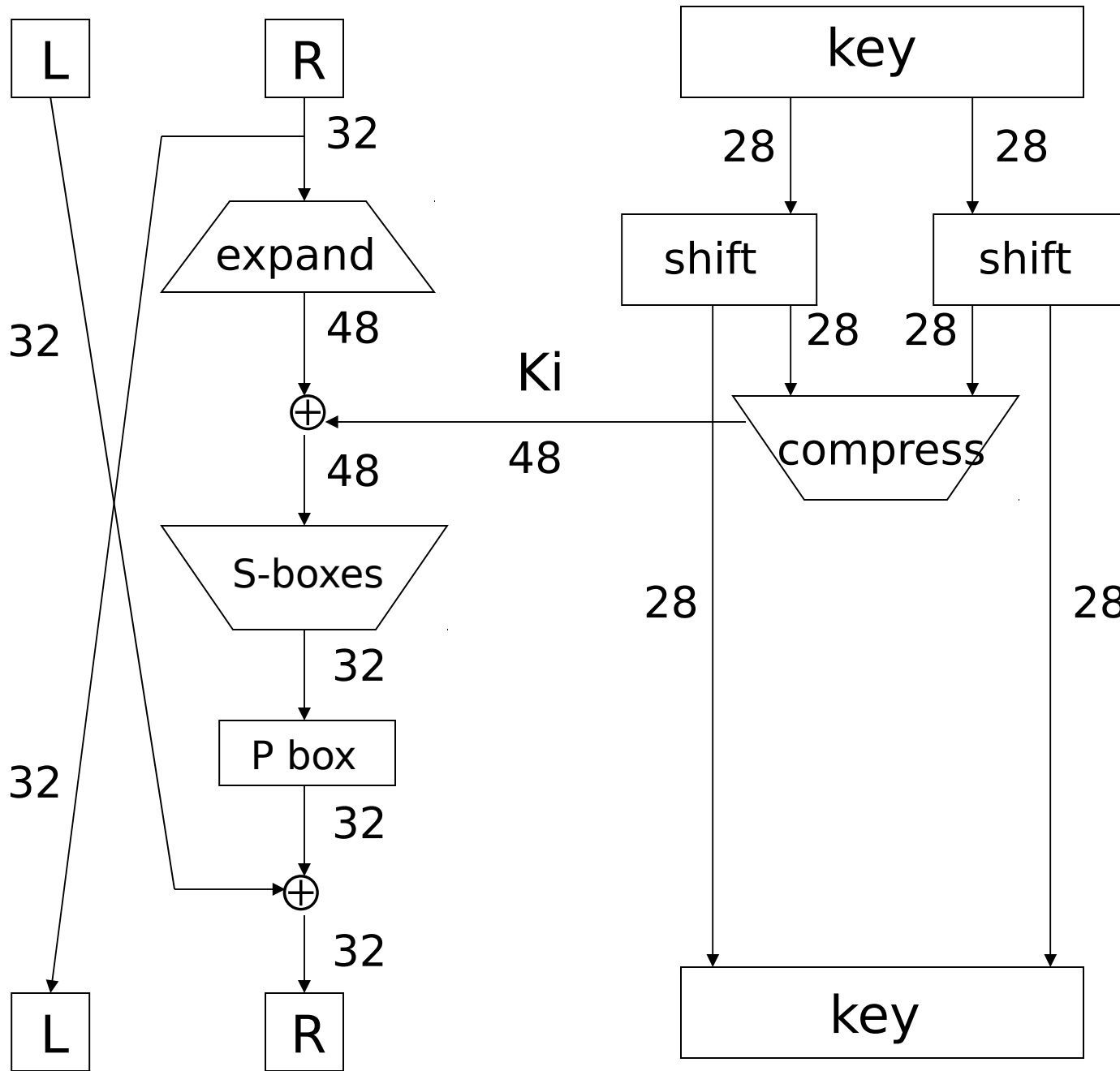
- ❑ Start with ciphertext $C = (L_n, R_n)$
- ❑ For each round $i = n, n-1, \dots, 1$, compute
$$R_{i-1} = L_i$$
$$L_{i-1} = R_i \oplus F(R_{i-1}, K_i)$$
where F is round function and K_i is subkey
- ❑ Plaintext: $P = (L_0, R_0)$
- ❑ Formula “works” for any function F
 - But only secure for certain functions F

Data Encryption Standard

- ❑ **DES** developed in 1970's
- ❑ Based on IBM's Lucifer cipher
- ❑ DES was U.S. government standard
- ❑ DES development was controversial
 - ▮ NSA secretly involved
 - ▮ Design process was secret
 - ▮ Key length reduced from 128 to 56 bits
 - ▮ Subtle changes to Lucifer algorithm

DES Numerology

- ❑ DES is a Feistel cipher with...
 - ▮ 64 bit block length
 - ▮ 56 bit key length
 - ▮ 16 rounds
 - ▮ 48 bits of key used each round (subkey)
- ❑ Each round is simple (for a block cipher)
- ❑ Security depends heavily on “S-boxes”
 - ▮ Each S-boxes maps 6 bits to 4 bits



One
Round
of
DES

DES Expansion Permutation

□ Input 32 bits

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

□ Output 48 bits

31	0	1	2	3	4	3	4	5	6	7	8
7	8	9	10	11	12	11	12	13	14	15	16
15	16	17	18	19	20	19	20	21	22	23	24
23	24	25	26	27	28	27	28	29	30	31	0

DES S-box

- ❑ 8 “substitution boxes” or S-boxes
- ❑ Each S-box maps 6 bits to 4 bits
- ❑ S-box number 1

input bits (0,5)

↓

input bits (1,2,3,4)

		0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
<hr/>																	
00		1110	0100	1101	0001	0010	1111	1011	1000	0011	1010	0110	1100	0101	1001	0000	0111
01		0000	1111	0111	0100	1110	0010	1101	0001	1010	0110	1100	1011	1001	0101	0011	1000
10		0100	0001	1110	1000	1101	0110	0010	1011	1111	1100	1001	0111	0011	1010	0101	0000
11		1111	1100	1000	0010	0100	1001	0001	0111	0101	1011	0011	1110	1010	0000	0110	1101

DES P-box

❑ Input 32 bits

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

❑ Output 32 bits

15	6	19	20	28	11	27	16	0	14	22	25	4	17	30	9
1	7	23	13	31	26	2	8	18	12	29	5	21	10	3	24

DES Subkey

- ❑ 56 bit DES key, numbered 0,1,2,...,55
- ❑ Left half key bits, LK

49	42	35	28	21	14	7
0	50	43	36	29	22	15
8	1	51	44	37	30	23
16	9	2	52	45	38	31

- ❑ Right half key bits, RK

55	48	41	34	27	20	13
6	54	47	40	33	26	19
12	5	53	46	39	32	25
18	11	4	24	17	10	3

DES Subkey

- For rounds $i=1, 2, \dots, 16$
 - Let $LK = (LK \text{ circular shift left by } r_i)$
 - Let $RK = (RK \text{ circular shift left by } r_i)$
 - Left half of subkey K_i is of LK bits

13	16	10	23	0	4	2	27	14	5	20	9
22	18	11	3	25	7	15	6	26	19	12	1

- Right half of subkey K_i is RK bits

12	23	2	8	18	26	1	11	22	16	4	19
15	20	10	27	5	24	17	13	21	7	0	3

DES Subkey

- ❑ For rounds 1, 2, 9 and 16 the shift r_i is 1, and in all other rounds r_i is 2
- ❑ Bits 8,17,21,24 of LK omitted each round
- ❑ Bits 6,9,14,25 of RK omitted each round
- ❑ **Compression permutation** yields 48 bit subkey K_i from 56 bits of LK and RK
- ❑ **Key schedule** generates subkey

DES Last Word (Almost)

- ❑ An initial permutation before round 1
- ❑ Halves are swapped after last round
- ❑ A final permutation (inverse of initial perm) applied to (R16,L16)
- ❑ None of this serves security purpose

Security of DES

- ❑ Security depends heavily on S-boxes
 - Everything else in DES is linear
- ❑ Thirty+ years of intense analysis has revealed no “back door”
- ❑ Attacks, essentially exhaustive key search
- ❑ **Inescapable conclusions**
 - ▮ Designers of DES knew what they were doing
 - ▮ Designers of DES were way ahead of their time

Block Cipher Notation

- ❑ P = plaintext block
- ❑ C = ciphertext block
- ❑ Encrypt P with key K to get ciphertext C
 - ❑ $C = E(P, K)$
- ❑ Decrypt C with key K to get plaintext P
 - ❑ $P = D(C, K)$
- ❑ Note: $P = D(E(P, K), K)$ and $C = E(D(C, K), K)$
 - ❑ But $P \neq D(E(P, K_1), K_2)$ and $C \neq E(D(C, K_1), K_2)$ when $K_1 \neq K_2$

Triple DES

- ❑ Today, 56 bit DES key is too small
 - Exhaustive key search is feasible
- ❑ But DES is everywhere, so what to do?
- ❑ **Triple DES** or **3DES** (112 bit key)
 - ▮ $C = E(D(E(P, K_1), K_2), K_1)$
 - ▮ $P = D(E(D(C, K_1), K_2), K_1)$
- ❑ Why Encrypt-Decrypt-Encrypt with 2 keys?
 - ▮ Backward compatible: $E(D(E(P, K), K), K) = E(P, K)$
 - ▮ And 112 bits is enough

3DES

- ❑ Why not $C = E(E(P, K), K)$?
 - Trick question --- it's still just 56 bit key
- ❑ Why not $C = E(E(P, K1), K2)$?
- ❑ A (semi-practical) **known plaintext** attack
 - ▮ Pre-compute table of $E(P, K1)$ for every possible key $K1$ (resulting table has 256 entries)
 - ▮ Then for each possible $K2$ compute $D(C, K2)$ until a match in table is found
 - ▮ When match is found, have $E(P, K1) = D(C, K2)$
 - ▮ Result gives us keys: $C = E(E(P, K1), K2)$

Advanced Encryption Standard

- ❑ Replacement for DES
- ❑ AES competition (late 90's)
 - NSA openly involved
 - ▮ Transparent process
 - ▮ Many strong algorithms proposed
 - ▮ Rijndael Algorithm ultimately selected
(pronounced like “Rain Doll” or “Rhine Doll”)
- ❑ Iterated block cipher (like DES)
- ❑ Not a Feistel cipher (unlike DES)

AES Overview

- ❑ **Block size:** 128 bits (others in Rijndael)
- ❑ **Key length:** 128, 192 or 256 bits (independent of block size)
- ❑ 10 to 14 rounds (depends on key length)
- ❑ Each round uses 4 functions (3 “layers”)
 - ByteSub (nonlinear layer)
 - ▮ ShiftRow (linear mixing layer)
 - ▮ MixColumn (nonlinear layer)
 - ▮ AddRoundKey (key addition layer)

AES ByteSub

- Treat 128 bit block as 4x4 byte array

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \longrightarrow \text{ByteSub} \longrightarrow \begin{bmatrix} b_{00} & b_{01} & b_{02} & b_{03} \\ b_{10} & b_{11} & b_{12} & b_{13} \\ b_{20} & b_{21} & b_{22} & b_{23} \\ b_{30} & b_{31} & b_{32} & b_{33} \end{bmatrix}.$$

- ByteSub is AES's "S-box"
- Can be viewed as nonlinear (but invertible) composition of two math operations

AES “S-box”

Last 4 bits of input

First 4
bits of
input

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

AES ShiftRow

- Cyclic shift rows

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \longrightarrow \text{ShiftRow} \longrightarrow \begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{11} & a_{12} & a_{13} & a_{10} \\ a_{22} & a_{23} & a_{20} & a_{21} \\ a_{33} & a_{30} & a_{31} & a_{32} \end{bmatrix}$$

AES MixColumn

- Invertible, linear operation applied to each column

$$\begin{bmatrix} a_{0i} \\ a_{1i} \\ a_{2i} \\ a_{3i} \end{bmatrix} \xrightarrow{\text{MixColumn}} \begin{bmatrix} b_{0i} \\ b_{1i} \\ b_{2i} \\ b_{3i} \end{bmatrix} \quad \text{for } i = 0, 1, 2, 3$$

- Implemented as a (big) lookup table

AES AddRoundKey

- ❑ XOR subkey with block

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \oplus \begin{bmatrix} k_{00} & k_{01} & k_{02} & k_{03} \\ k_{10} & k_{11} & k_{12} & k_{13} \\ k_{20} & k_{21} & k_{22} & k_{23} \\ k_{30} & k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} b_{00} & b_{01} & b_{02} & b_{03} \\ b_{10} & b_{11} & b_{12} & b_{13} \\ b_{20} & b_{21} & b_{22} & b_{23} \\ b_{30} & b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Block

Subkey

- ❑ RoundKey (subkey) determined by **key schedule** algorithm

AES Decryption

- ❑ To decrypt, process must be invertible
- ❑ Inverse of MixAddRoundKey is easy, since “ \oplus ” is its own inverse
- ❑ MixColumn is invertible (inverse is also implemented as a lookup table)
- ❑ Inverse of ShiftRow is easy (cyclic shift the other direction)
- ❑ ByteSub is invertible (inverse is also implemented as a lookup table)

A Few Other Block Ciphers

- ❑ Briefly...
 - IDEA
 - ▮ Blowfish
 - ▮ RC6
- ❑ More detailed...
 - ▮ TEA

IDEA

- ❑ Invented by James Massey
 - ▮ One of the giants of modern crypto
- ❑ IDEA has 64-bit block, 128-bit key
- ❑ IDEA uses **mixed-mode arithmetic**
- ❑ Combine different math operations
 - ▮ IDEA the first to use this approach
 - ▮ Frequently used today

Blowfish

- ❑ Blowfish encrypts 64-bit blocks
- ❑ Key is variable length, up to 448 bits
- ❑ Invented by Bruce Schneier
- ❑ Almost a Feistel cipher

$$R_i = L_{i-1} \oplus K_i$$

$$L_i = R_{i-1} \oplus F(L_{i-1} \oplus K_i)$$

- ❑ The round function F uses 4 S-boxes
 - ▮ Each S-box maps 8 bits to 32 bits
- ❑ **Key-dependent S-boxes**
 - ▮ S-boxes determined by the key

RC6

- ❑ Invented by Ron Rivest
- ❑ Variables
 - ▮ Block size
 - ▮ Key size
 - ▮ Number of rounds
- ❑ An AES finalist
- ❑ Uses **data dependent rotations**
 - ▮ Unusual for algorithm to depend on plaintext

Time for TEA

- ❑ Tiny Encryption Algorithm (TEA)
- ❑ 64 bit block, 128 bit key
- ❑ Assumes 32-bit arithmetic
- ❑ Number of rounds is variable (32 is considered secure)
- ❑ Uses “weak” round function, so large number of rounds required

TEA Encryption

Assuming 32 rounds:

$(K[0], K[1], K[2], K[3]) = 128 \text{ bit key}$

$(L, R) = \text{plaintext (64-bit block)}$

$\text{delta} = 0x9e3779b9$

$\text{sum} = 0$

for $i = 1$ to 32

$\text{sum} += \text{delta}$

$L += ((R \ll 4) + K[0]) \wedge (R + \text{sum}) \wedge ((R \gg 5) + K[1])$

$R += ((L \ll 4) + K[2]) \wedge (L + \text{sum}) \wedge ((L \gg 5) + K[3])$

next i

$\text{ciphertext} = (L, R)$

TEA Decryption

Assuming 32 rounds:

$(K[0], K[1], K[2], K[3]) = 128 \text{ bit key}$

$(L, R) = \text{ciphertext (64-bit block)}$

$\text{delta} = 0x9e3779b9$

$\text{sum} = \text{delta} \ll 5$

for $i = 1$ to 32

$R \leftarrow ((L \ll 4) + K[2]) \wedge (L + \text{sum}) \wedge ((L \gg 5) + K[3])$

$L \leftarrow ((R \ll 4) + K[0]) \wedge (R + \text{sum}) \wedge ((R \gg 5) + K[1])$

$\text{sum} \leftarrow \text{sum} + \text{delta}$

next i

$\text{plaintext} = (L, R)$

TEA Comments

- ❑ **Almost** a Feistel cipher
 - ▮ Uses + and - instead of \oplus (XOR)
- ❑ Simple, easy to implement, fast, low memory requirement, etc.
- ❑ Possibly a “related key” attack
- ❑ eXtended TEA (XTEA) eliminates related key attack (slightly more complex)
- ❑ Simplified TEA (STEAs) — insecure version used as an example for cryptanalysis

Block Cipher Modes

Multiple Blocks

- ❑ How to encrypt multiple blocks?
- ❑ Do we need a new key for each block?
 - As bad as (or worse than) a one-time pad!
- ❑ Encrypt each block independently?
- ❑ Make encryption depend on previous block?
 - ▮ That is, can we “chain” the blocks together?
- ❑ How to handle partial blocks?
 - ▮ We won't discuss this issue

Modes of Operation

- ❑ Many modes — we discuss 3 most popular
- ❑ Electronic Codebook (**ECB**) mode
 - ▮ Encrypt each block independently
 - ▮ Most obvious, but has a serious weakness
- ❑ Cipher Block Chaining (**CBC**) mode
 - ▮ Chain the blocks together
 - ▮ More secure than ECB, virtually no extra work
- ❑ Counter Mode (**CTR**) mode
 - ▮ Block ciphers acts like a stream cipher
 - ▮ Popular for random access

ECB Mode

- ❑ Notation: $C = E(P, K)$
- ❑ Given plaintext $P_0, P_1, \dots, P_m, \dots$
- ❑ Most obvious way to use a block cipher:

Encrypt

$$C_0 = E(P_0, K)$$

$$C_1 = E(P_1, K)$$

$$C_2 = E(P_2, K) \dots$$

Decrypt

$$P_0 = D(C_0, K)$$

$$P_1 = D(C_1, K)$$

$$P_2 = D(C_2, K) \dots$$

- ❑ For fixed key K , this is “electronic” version of a codebook cipher (without additive)
 - With a different codebook for each key

ECB Cut and Paste

- Suppose plaintext is

Alice digs Bob. Trudy digs Tom.

- Assuming 64-bit blocks and 8-bit ASCII:

P0 = "Alice di", P1 = "gs Bob. ",

P2 = "Trudy di", P3 = "gs Tom. "

- Ciphertext: C0,C1,C2,C3

- Trudy cuts and pastes: C0,C3,C2,C1

- Decrypts as

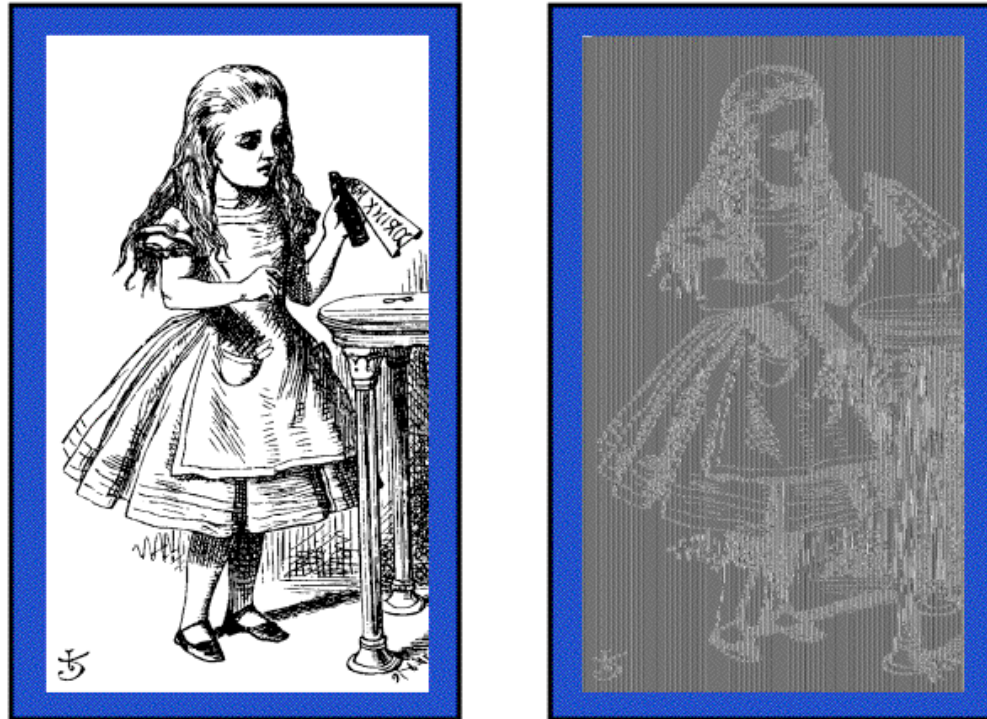
Alice digs Tom. Trudy digs Bob.

ECB Weakness

- ❑ Suppose $P_i = P_j$
- ❑ Then $C_i = C_j$ and Trudy knows $P_i = P_j$
- ❑ This gives Trudy some information, even if she does not know P_i or P_j
- ❑ Trudy might know P_i
- ❑ Is this a serious issue?

Alice Hates ECB Mode

- Alice's uncompressed image, and ECB encrypted (TEA)



- Why does this happen?
- Same plaintext yields same ciphertext!

CBC Mode

- ❑ Blocks are “chained” together
- ❑ A random initialization vector, or IV, is required to initialize CBC mode
- ❑ IV is random, but not secret

Encryption Decryption

$$\begin{aligned}C_0 &= E(IV \oplus P_0, K), & P_0 &= IV \oplus D(C_0, K), \\C_1 &= E(C_0 \oplus P_1, K), & P_1 &= C_0 \oplus D(C_1, K), \\C_2 &= E(C_1 \oplus P_2, K), \dots & P_2 &= C_1 \oplus D(C_2, K), \dots\end{aligned}$$

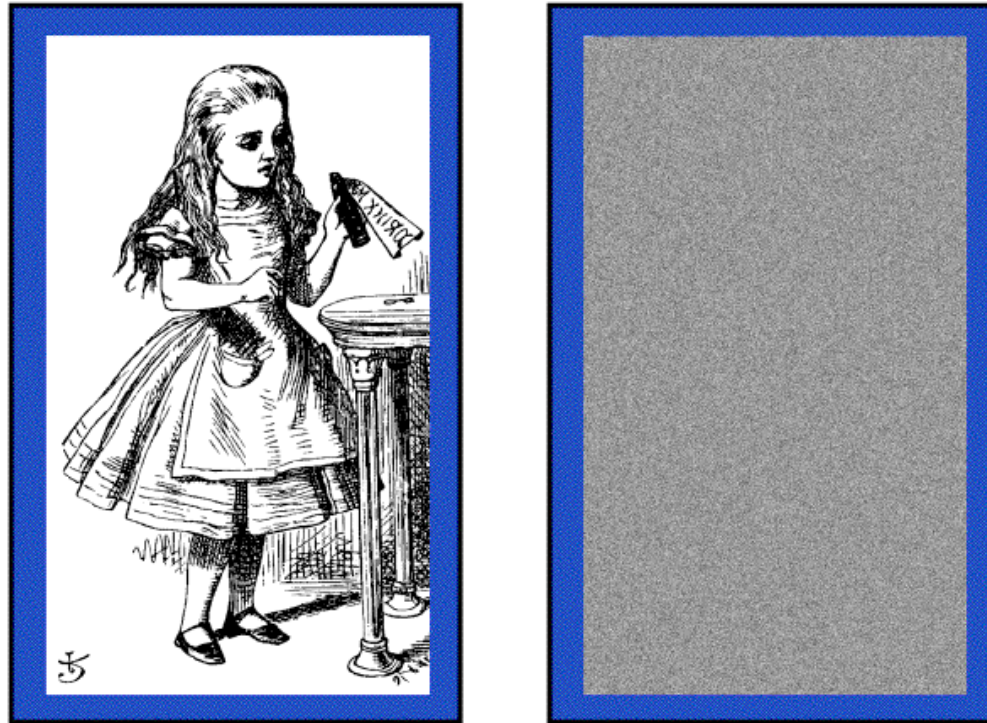
- ❑ Analogous to classic codebook *with additive*

CBC Mode

- ❑ Identical plaintext blocks yield different ciphertext blocks — this is good!
- ❑ If C1 is garbled to, say, G then
$$P1 \neq C0 \oplus D(G, K), P2 \neq G \oplus D(C2, K)$$
- ❑ But $P3 = C2 \oplus D(C3, K), P4 = C3 \oplus D(C4, K), \dots$
- ❑ Automatically recovers from errors!
- ❑ Cut and paste is still possible, but more complex (and will cause garbles)

Alice Likes CBC Mode

- Alice's uncompressed image, Alice CBC encrypted (TEA)



- Why does this happen?
- Same plaintext yields different ciphertext!

Counter Mode (CTR)

- CTR is popular for random access
- Use block cipher like a stream cipher

Encryption Decryption

$$C_0 = P_0 \oplus E(IV, K), \quad P_0 = C_0 \oplus E(IV, K),$$

$$C_1 = P_1 \oplus E(IV+1, K), \quad P_1 = C_1 \oplus E(IV+1, K),$$

$$C_2 = P_2 \oplus E(IV+2, K), \dots P_2 = C_2 \oplus E(IV+2, K), \dots$$

- CBC can also be used for random access
 - With a significant limitation...

Integrity

Data Integrity

- ❑ **Integrity** — detect unauthorized writing (i.e., modification of data)
- ❑ Example: Inter-bank fund transfers
 - Confidentiality may be nice, integrity is critical
- ❑ Encryption provides **confidentiality** (prevents unauthorized disclosure)
- ❑ Encryption alone does **not** provide integrity
 - One-time pad, ECB cut-and-paste, etc.

MAC

- ❑ Message Authentication Code (MAC)
 - Used for data **integrity**
 - ▮ Integrity **not** the same as confidentiality
- ❑ MAC is computed as **CBC residue**
 - ▮ That is, compute CBC encryption, saving only final ciphertext block, the MAC

MAC Computation

- MAC computation (assuming N blocks)

$$C_0 = E(IV \oplus P_0, K),$$

$$C_1 = E(C_0 \oplus P_1, K),$$

$$C_2 = E(C_1 \oplus P_2, K), \dots$$

$$C_{N-1} = E(C_{N-2} \oplus P_{N-1}, K) = \text{MAC}$$

- MAC sent with IV and plaintext
- Receiver does same computation and verifies that result agrees with MAC
- Note: receiver must know the key K

Does a MAC work?

- Suppose Alice has 4 plaintext blocks
- Alice computes
$$\mathbf{C0} = E(\text{IV} \oplus P0, K), \mathbf{C1} = E(\mathbf{C0} \oplus P1, K),$$
$$\mathbf{C2} = E(\mathbf{C1} \oplus P2, K), \mathbf{C3} = E(\mathbf{C2} \oplus P3, K) = \mathbf{MAC}$$
- Alice sends IV, P0, P1, P2, P3 and **MAC** to Bob
- Suppose Trudy changes P1 to X
- Bob computes
$$\mathbf{C0} = E(\text{IV} \oplus P0, K), \mathbf{C1} = E(\mathbf{C0} \oplus X, K),$$
$$\mathbf{C2} = E(\mathbf{C1} \oplus P2, K), \mathbf{C3} = E(\mathbf{C2} \oplus P3, K) = \mathbf{MAC} \neq \mathbf{MAC}$$
- That is, error propagates into **MAC**
- Trudy can't make **MAC** == **MAC** without K

Confidentiality and Integrity

- ❑ Encrypt with one key, MAC with another key
- ❑ Why not use the same key?
 - Send last encrypted block (MAC) twice?
 - ▮ This cannot add any security!
- ❑ Using different keys to encrypt and compute MAC works, even if keys are related
 - ▮ But, twice as much work as encryption alone
 - ▮ Can do a little better — about 1.5 “encryptions”
- ❑ Confidentiality and integrity with same work as one encryption is a research topic

Uses for Symmetric Crypto

- ❑ Confidentiality
 - Transmitting data over insecure channel
 - ▮ Secure storage on insecure media
- ❑ Integrity (MAC)
- ❑ Authentication protocols (later...)
- ❑ Anything you can do with a hash function (upcoming chapter...)