Heuristic Optimization Algorithm for a Set of Air Conditioners on a Smart Grid

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Abstract

This paper introduces an energy scheduling and optimization algorithm for a set of air conditioners in residential and/or commercial buildings. The main objective of this study is to introduce an heuristic algorithm that is able to reduce electricity bills, keeping the temperature within comfort levels in the building, increase the utilization of domestic renewable power, and reduce the computation time. Cumulative Round Linear Programming (CRLP) is a heuristic algorithm that uses relaxation in order to convert Mixed Integer Linear Programming (MILP) problem, which is NP-hard problem, into LP problem to reduce the computation time. It uses load shifting to increase the utilization of domestic renewable power by scheduling the load on times where there is enough renewable power even this period is peak hours in terms of cost. The results show that CRLP can be used to schedule the load of large number of AC units in very short time and the difference in cost between CRLP and the optimal solution found with a MILP is not considerable.

Categories and Subject Descriptors

G [Mathematics of Computing Optimization], G.1.6 Optimization: Constrained optimization, C4 [PERFORMANCE OF SYSTEMS]: Modeling technique;

General Terms

Algorithms and Performance.

Keywords

Renewable energy, Distributed renewable resources, Linear programming (LP), Mixed Integer Linear Programming (MILP), Scheduling algorithm, Demand-Side Management (DSM), Demand Response (DR), Smart Grid, Home Automation.

1. INTRODUCTION

The remarkable increase in the world population has caused a serious problem in the power sector due to the fact that residential buildings typically consume about 48% of the available power, this growth in population has risen the demand on power in general, which causes an increase in fossil fuel and electricity price. If growth continuous at this pace, the new power plants (traditional or/and renewable) is not going to cover the demand in the future [1]. In addition, study [2] states that space heating devours a high percentage, about 68%, of consumed power in residential building in Europe, whereas water heating and other electrical appliances are consuming 14% and 13% respectively. In addition, the power sector can be improved in many aspects. For instance, improving the generation side using more renewable resources, building or developing power plants to enhance their productivity and to become more environmentally friendly. This includes, enhancing the consumption side using Demand Side Management (DSM) and Demand Response (DR) to encourage consumers to change their power consumption. Reducing the gas emission by 20%, increasing efficiency by 20% and 20% power from renewable resources are the main three goals of the energy sector in EU by 2020 [3].

The concept of Smart Grid is a relatively new. The Smart Grid is an enhanced electrical grid where information and communications technology can be used between the grid entities to improve the power system and increase the profit of consumers, distributors and generation companies. The key features of the Smart Grid are reliability, flexibility, efficiency, sustainability, peak curtailment, demand response, market-enabling, platform for advanced services, manageability of the available resources. In addition, Smart Grid needs new technology in the residential building. For instance, integrated communications, sensing and measurements, smart meters, advanced control, advanced components, smart power generation, etc [4, 5].

Studies [6, 7] use Mixed Integer Linear Programing (MILP) to optimally schedule the load in residential buildings. These studies have proposed a control algorithm that finds an optimal schedule for a set of household appliances, including just one AC unit. However, the computation time to find the optimal solution is very long (around 415 seconds), this is because they use a huge number of integer variables in their model [6]. Also, this number does not scale up well with the number of appliances. In addition, all renewable power is stored in battery first and appliances take the renewable power form the battery, which additionally exercises power efficiency because of power battery losses around 15-20 % of the energy in charging and discharging modes [7].

The Bee project [8–10] has proposed some studies regarding home automation and energy demand management. In addition, paper [8] has investigated the performance of an MILP optimization algorithm in order to decrease the electricity bills, by comparing cooperative and non-cooperative methods. In addition, an energy management optimization model has been studied for single and multi-users cases using MILP in order to optimize the household's appliances load [9]. Furthermore, online and off-line scheduling algorithms using MILP have been investigated in [10] in order to decrease the cost of consumed power. Although this project is good contribution to the area, it has not tackled the computation time problem.

In studies [11, 12], the authors have used MILP in their optimization algorithm in order to optimize the power consumption in the residential building. In paper [11], two techniques are implemented, which are anticipation and reactive management techniques. However, the computation time has not been taken in their consideration. In [12], ancillary services and optimal energy management has been proposed, there is no essential difference between this study and the previous one in terms of computation time and scheduling mechanism. The computation time has not been studied and they still use the same mechanism.

Heating, ventilation and air conditioning systems have been studied before [13–22]. These studies use varaus algorithms to optimize the load of air conditioning systems. In addition, these researches use batteries or storage systems in their model to accommodate their surplus of domestic renewable power. Although these studies add good knowledge to the field, they have not tackled the computation time problem, especially when there are more than one AC unit, which adds complexity to the problem. In addition, all of these studies use MILP, which is NP-hard, to solve the problem.

There are three main issues with the approaches proposed in the literature. Firstly, the computation time, since MILP is an NP-hard problem, which means that in some cases where the number of AC units is large, MILP is very slow. The second issue is related to the use of battery systems in residential building. There are some disadvantages of using batteries in residential building such as safety issues, capital cost, power efficiency, maintenance and replacement cost, and the fact that it takes space in the building. (In commercial buildings the space is costly). Finally, there are fair payment issues in Feed-In Tariff. For example, consumers in Malta get paid more than the retail price in first eight years (26 cent) to encourage people to install domestic renewable generators, whereas in the UK the export tariff is less than the retail price (4.5 pence). In contrast, there are some countries where there is no FIT, such as Libya. Therefore, we would consider this point in our system by increasing the utilization of domestic renewable resources.

The importance of combinatorial optimization comes from the fact that there are many problems that can be solved with it. In contrast, the computation time of large combinatorial optimization problems is very long and it needs powerful machines to solve these problems [23]. For example, the complexity of IP/MILP stems from putting constraints on all/some of the variables in the model. Furthermore, LP relaxation is used to improve the computation time. LP relaxation works by omitting all integers constraints on variables in IL/MILP [24] .

To address the issues above, we introduce a fast heuristic optimization scheduling algorithm based on LP, called Cumulative Round Linear Programing (CRLP), that optimizes the load of air conditioning system (K AC units). The main goals of CRLP are to reduce the cost of the consumed power from the grid by using load shifting, increasing the utilization of domestic renewable power, keep temperature inside the flat comfortable based on user's preferences, and improve the computation time dramatically by using LP relaxation. The algorithm uses predicted generation of renewable power from domestic resources (solar and wind power), provided electricity price and user preferences as input, and it schedules the load of air conditioning system based on these data.

The first and the main contribution of this paper is that it uses a relaxation technique, LP relaxation, to convert an MILP (exact algorithm) into an LP (heuristic algorithm) to improve the computation time remarkably. The second contribution is that because there is no batteries in our model, CRLP schedules the load of air conditioning system with regard to presence of domestic renewable power and electricity price so that the utilization of renewable power is improved. This means that the load can be shifted to peak hours if there is enough renewable power at that time.

The rest of this research paper is organized as follows. Section 2 details the system definition and modelling of system entities, Problem formulation is presented in section 3, the fourth section illustrates the result of CRLP, LP and MILP which is followed by discussions in Section 5. Finally, the paper ends with conclusions.

2. SYSTEM DEFINITION

Our system is well-suited for modelling a building in a smart grid environment. We assume that the building is provided with both solar and wind renewable energy sources, it has a smart meter that receives real-time information about the electricity price from the electricity provider and calculate the exported and imported power to/from the grid and K air conditioning units (AC). Furthermore, these AC units have different specifications and they heat or cool separated rooms/offices in the building. We focus on AC units because as mentioned, they consume the highest percentage of total consumed power in the residential and commercial building.

2.1 System modelling

2.1.1 *Time horizon and parameter division*

The time horizon, T, is split into a sequence of N time intervals of length τ (all quantities involved are assumed to be constant in each of these intervals, see Figure 1). In what follows we will use the following variables. The allocated grid power to K AC units at time slot t is $L_g(t)$, the allocated renewable power to K AC units at time slot t is $L_r(t)$, allocated power to the i^{th} AC unit at time slot t is $P_i(t)$, the allocated renewable power to be exported to the grid at time slot t is $E(t)$, the internal room temperature of the i^{th} AC unit at time slot t is $T^i_{in}(t)$. In contrast, the given data are the following: the external temperature at time slot t is $T_{out}(t)$, the grid electricity price at time slot t is $\lambda(t)$, the predicted solar power at time slot t is $P_s(t)$, the predicted wind power at time t is $P_w(t)$, the export rate in the feed-in tariff is ζ. In addition, all the variables and given data are sampled at the same rate τ. Then, the algorithm will allocate the optimal power to the each AC unit over time T in order to increase the utilization of domestic renewable power and decrease the consumed grid power as well as maintaining the internal temperature within the comfort level.

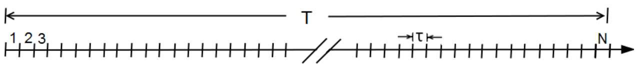


Figure 1: Division of time horizon

2.1.2 *PV array modelling*

It is very difficult to estimate the output of PV array down to less than an hour interval because most of the meteorological stations give hourly average data. Therefore, all the experimental results are based on the assumption that the predicted domestic renewable power is constant during each hour. Also, we assume that this prediction is correct. The output of any PV array can be estimated by using hourly average predicted solar radiation. Many formulas are being used to estimate the output of a PV array but in this case study, equation (1) is used to estimate the output of a PV array.

$$P_s(t) = \eta_e \cdot \eta_d \cdot \eta_c \cdot \eta_w \cdot A_s \cdot I_T(t) \qquad \forall t : t \in [1, \dots, N] \tag{1}$$

Where $P_s(t)$ is the solar power at time t and η_e is the efficiency of a solar cell, η_d is the degradation factor of the PV array, η_c is the efficiency of the power conditioning devices, η_w is the wiring efficiency of the PV array system, A_s is the PV array surface area, and I_T is the total hourly radiation in W/m^2 [**?**].

2.1.3 *Wind turbine modelling*

Although it is very difficult to predict wind speed down to less than an hour, the output of wind turbine can be estimated by using hourly average predicted wind speed provided by meteorological stations. Regarding the equation for estimating wind power, there are more than one formula for wind turbine modeling but the most common one is presented in equation (2).

$$P_w(t) = \begin{cases} 0 & \nu_{C_{out}} \leqslant \nu(t) < \nu_{C_{in}} \\ \frac{1}{2} \cdot c_w \cdot \rho \cdot A_w . \nu(t)^3 & \nu_{C_{in}} \leqslant \nu(t) < \nu_r \\ P_{rate} & \nu_r \leqslant \nu(t) < \nu_{C_{out}} \end{cases}$$

$$\forall t : t \in [1, \dots, N] \tag{2}$$

where $P_w(t)$ is the output power of the wind turbine, c_w is the power coefficient of the wind turbine, $A_w = \pi \cdot r^2$ is the swept area of wind turbine, ρ is the air density and $v(t)$ is a the wind speed and P_{rate} is the maximum output power. In order to use this formula, some approximation and assumption must be made. For instance, ρ depends on weather variant parameters and it is very difficult to measure them or get them from meteorological stations. Therefore, we need to approximate it. Also, we use the average wind speed to predict $P_w(t)$ and we assume that it is constant over the time slot of one hour [25].

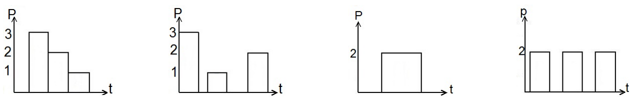
2.1.4 *Air Conditioner (AC) modelling*

Air conditioning system is really difficult to model because it depends on inside and outside temperatures, state of the windows and doors, and isolation of the building. Also, to schedule and control any electric appliance, we must know its way of working and its design issues. For instance, a washing machine can not be switched off once started until it finishes its job. There are many classifications of household appliances that has been illustrated in studies [**?**, 19, 26, 27]. For example, there are deferrable and non-deferrable appliances, controllable and uncontrollable appliances, and interruptible and uninterruptible appliances. AC unit is a controllable appliance and it could be classified under interruptible and uninterruptible or deferrable and non-deferrable. For instance, let us assume that the AC works as heater. If the temperature inside the building is equal or less than the lower acceptable limit of the inside temperature then the AC unit becomes a non-deferrable appliance and it should be switched ON until the temperature reaches the comfort level that has been chosen by the resident or occupier, whereas if the temperature is within the given comfort level, then the AC becomes deferrable or interruptible appliance.

In this study, the problem is to optimize the power profile of K AC units with different or the same power demand. Furthermore, these AC units could be identical or different in terms of power efficiency. For instance, the working levels could be as follows: Off level that consumes 0 KWH, minimum level that consumes 1 KW, medium level that consumes 2 KW, and maximum level that consumes 3 KW. In addition, the power profile of the AC is not constant or does not have a specific shape. Figure 2 illustrates the possible shapes of a power profile of AC unit with four working levels (0, 1, 2 and 3) that can be allocated to AC. The value of the allocated power to AC, $P(t)$, is specified by the optimization algorithm based on the grid electricity price, temperature constraints and availability of renewable power, (more details later in the next section). Therefore, the power profile is not known in advance. Equation (3) illustrates the consumed power of an AC unit.

$$P_i(t) = \begin{cases} P_o^i & OFF \\ P_1^i & \ell_1 \\ \vdots & \vdots \\ P_{max}^i & \ell_{max} \end{cases} \quad \forall t : t \in [1, \ldots, N] \qquad (3)$$

Where $P_i(t)$ is the power demand of the i^{th} AC unit at time slot t, P_1^i and P_{max}^i are the power demands of the i^{th} AC unit when it works in its minimum (ℓ_1) and maximum levels (ℓ_m). In addition, the values of $P_1^i$and P_{max}^i are assumed to be integers, but they could be real.



(a) Uninterruptable Multilevel power profile

(b) Interruptible multilevel power profile

(c) Uninterruptible uni-level power profile

(d) Interruptible uni-level power profile

Figure 2: Possible AC power profiles

2.2 Air Conditioner (AC) constraints

2.2.1 *Temperature and time constraints*

The user must determine the periods of time that the controller or the algorithm has to keep the room temperature within the desired comfort level. For example, if the residents leave their house in the morning for work and school and come back home in the evening, then the controller has to keep the temperature within comfort levels in the evening period, from 6:00 PM to 6:00 AM.

$$T_{min}^i \leqslant T_{in}^i(t) \leqslant T_{max}^i \ \forall t : t \in \left[\beta_1^i, ..., \delta_1^i\right] \cup \left[\beta_2^i, \ldots, \delta_2^i\right]$$

$$, \ldots, \cup \left[\beta_m^i, \ldots, \delta_m^i\right] \qquad (4)$$

Where β_1^i, δ_1^i, β_M^i and δ_M^i are the start and end times of the 1^{st} and m^{th} period in which the i^{th} AC unit must keep the room temperature $T_{in}^i(t)$ within the comfort level [T_{min}^i, T_{max}^i], and m is the number of periods in which the AC unit must keep the temperature within the comfort level. To set a schedule for K AC units, the relationship between the consumed power $P_i(t)$ by the i^{th} AC and room temperature $T_{in}^i(t)$ must be known. Moreover, the temperature inside the room, actually, depends on many other aspects such as state of the windows and doors, the size of the room, the number of residents occupying the space, the external temperature, and the isolation material that has been used in the building. In addition, there are many models that convey this relationship [13–22]. In this paper, we prefer the model that has been used in study [15, 17].

$$T_{in}^i(t) = \epsilon_i \cdot T_{in}^i(t-1) + (1 - \epsilon_i) \cdot [T_{out}(t) - \eta_i \cdot P_i(t)]$$

$$\forall t : t \in [1, \ldots, N] \qquad (5)$$

Where $T_{out}(t)$ is the external temperature at time slot t, ϵ_i is the system inertia of the i^{th} AC unit , $\eta_i = {}^{\eta_{ac}}/_{A_i}$ is efficiency of system, η_{ac} is the efficiency of AC unit, A_i is the thermal conductivity (KW/°C), and $P_i(t)$ is the allocated power to the i^{th} AC unit at time slot t. In addition, in case of cooling, $\eta_i \times P_i(t)$ is negative, whereas in case of heating, $\eta_i \times P(t)$ is positive.

2.2.2 *Available domestic renewable power constraint*

The summation of allocated domestic renewable power $L_r(t)$ and allocated surplus renewable power to grid $E(t)$ must be equal to the predicted domestic renewable power $P_r(t)$ at any time slot t . Furthermore, $L_r(t)$ and $E(t)$ are continuous variables. $P_r(t)$ is given, see equation (6) .

$$P_r(t) = P_s(t) + P_w(t) \qquad \forall t : t \in [1, \ldots, N] \qquad (6)$$

$$L_r(t) + E(t) = P_r(t) \qquad \forall t : t \in [1, \ldots, N] \quad (7)$$

$$0 \leq L_r(t),\, E(t) \leq P_r(t) \qquad \forall t : t \in [1, \ldots, N] \quad (8)$$

2.3 Grid and domestic renewable power consumption constraint

The allocated power $P_i(t)$ to the i^{th} AC unit could be from the grid $L_g(t)$ and/or from domestic renewable resources $L_r(t)$. In addition, $L_g(t)$ and $L_r(t)$ are continuous variables, whereas $P_i(t)$ is an integer variable represented in equation (10).

$$L_g(t) + L_r(t) = \sum_{i=1}^{K} P_i(t) \quad \forall t : t \in [1, \ldots, N] \quad (9)$$

$$P_i(t) = \{P_0^i,\, P_1^i, \ldots, P_{max}^i\} \quad \forall t : t \in [1, \ldots, N] \quad (10)$$

3. PROBLEM FORMULATION AND OPTIMIZATION

The main objective is to optimize the power consumption of K AC units in residential building or offices building. In addition, the objective function is represented by the following equation.

$$C_T = \min \sum_{t=1}^{N} \lambda(t) \cdot L_g(t) + \xi \cdot L_r(t) - \zeta \cdot E(t) \quad (11)$$

where C_T is the total cost of the allocated power to the AC units over the time horizon $T = \tau \times N$, $\lambda(t)$ is the electricity price given by the electricity company at time slot t, ξ is the cost of domestic renewable power and ζ is the expert rate in the Feed In Tariff. In addition, ξ and ζ are constant over T. As we can see from the objective function, equation (11), and aforementioned system constraints, this system is a linear system with some integer variables, $\mathbf{P_i}$, and some continuous variables, $\mathbf{L_g}$, $\mathbf{L_r}$, $\mathbf{E}$ and C_T, this problem formulation is known as a Mixed Integer Linear Programming (MILP). Furthermore, MILP is a NP-hard problem. Therefore, the computation time to solve it could be very long, and it could goes to infinity if the number of AC units, K, or the number of integer variables in the model is large. Therefore, relaxation technique, LP relaxation, will be exploited to convert this MILP problem to an LP problem by using the CRLP algorithm. $\mathbf{\bar{P}_i}$, $\mathbf{\bar{L}_g}$, $\mathbf{\bar{L}_r}$, $\mathbf{\bar{E}}$ and $\bar{C}_T$ will represent the MILP variables, $\mathbf{\hat{P}_i}$, $\mathbf{\hat{L}_g}$, $\mathbf{\hat{L}_r}$, $\mathbf{\hat{E}}$ and $\hat{C}_T$ will represent LP variables, whereas $\mathbf{\widetilde{P}_i}$, $\mathbf{\widetilde{L}_g}$, $\mathbf{\widetilde{L}_r}$, $\mathbf{\widetilde{E}}$ and $\widetilde{C}_T$ will represent CRLP variables.

The relaxation process of the MILP is replacing the constraint that each variable must be integer, in equation (10), by a weaker constraint, in equation (12). As we explained, only the $\mathbf{\bar{P}}$ array contains integer variables. By applying relaxation, constraint (10) will be replaced with constraint (12).

$$P_i(t) \in [0,\, P_{max}^i]\, \forall t : t \in [1, \ldots, N] \quad (12)$$

Although, the relaxation of constraint 10 improves the computation time dramatically, the allocated power to the i^{th} AC units by LP, $\mathbf{\hat{P}_i}$, is not practical because the allocated power to the i^{th} AC must be an integer values between 0 and P_{max}^i KW per time unit so that it can be used as control signal to the AC. In other words, the allocated power to AC unit must match one of the AC's working levels (0, P_1^i, ..., P_{max}^i).

To cope with this technical issue, we have proposed an algorithm, CRLP, to convert the optimal continuous allocated power to the i^{th} AC unit, $\mathbf{\hat{P}_i}$, to integer values, $\mathbf{\widetilde{P}_i}$. Also it calculates $\mathbf{\widetilde{L}_r}$, $\mathbf{\widetilde{L}_g}$, $\mathbf{\widetilde{E}}$ and $\widetilde{C}_T$ by using $\mathbf{\hat{P}_i}$, in Algorithm 1 and Algorithm 2 . After solving the problem using the LP solver, the optimal allocated power to the AC units will contain the continuous values that minimize the cost under the aforementioned constraints but this power profile could not be applied on the AC units because the allocated values to AC do not match AC's working levels (0, 1,..., P_{max}). Therefore, we need to convert the optimal allocated power to AC units, $\mathbf{\hat{P}}$, to integer values, $\mathbf{\widetilde{P}}$. This is done in Algorithm 1. Then Algorithm 2 will calculate the other variables. Algorithms 1 and 2 illustrate how Cumulative Rounding Linear Programming (CRLP) works.

The pseudo code of algorithm 1 shows how does CRLP convert the continuous values to integers. See the following example for detailed explanation.

Algorithm 1 CRLP

```
for i = 1 to K do
  S ← 0; c ← 0;
  for t = 1 to N do
    if P̂_i(t) is an integer then
      P̃_i(t) ← P̂_i(t);
    else
      S ← P̂_i(t) + c;
      for j = 0 to P_max do
        if j − θ ≤ S < J + θ then
          P̃_i(t) ← j;
          c ← S − j;
        end if
      end for
    end if
  end for
  Call Algorithm 2
end for
```

Example

To illustrate the mechanism of converting continuous/real values to integers because of constraints (3), let us consider this numerical example. Assume that we have one AC unit, and we have 8 time slots and the optimal allocated power to the AC unit calculated by LP is $\hat{P}(t)$. In addition, assume that $\theta = 0.5$. First of all, the CRLP algorithm will check if the value is already integer or not, if it is integer then it goes to the next value. Otherwise, CRLP sums the continuous values with the next continuous value until the sum becomes greater than or equal 0.5. Then, the sum will be rounded to the nearest integer according to Algorithm 1.

Table 1: CRLP mechanism example

t	1	2	3	4	5	6	7	8
$\hat{P}(t)$	0.3	0	1.8	1	2.5	0.2	0.5	1.1
S	0.3	-	2.1	-	2.6	-0.2	0.3	1.4
c	0.3	-	0.1	-	-0.4	-0.2	0.3	0.4
$\widetilde{P}(t)$	0	0	2	1	3	0	0	1

Algorithm 2 Calculate CRLP's variables

for $t = 1$ **to** N **do**
  if $\sum_{i=1}^{K} \widetilde{P}_i(t) < P_r(t)$ **then**
    $\widetilde{L}_g(t) \leftarrow 0;$
    $\widetilde{L}_r(t) \leftarrow \sum_{i=1}^{K} \widetilde{P}_i(t);$
    $\widetilde{E}(t) \leftarrow P_r(t) - \sum_{i=1}^{K} \widetilde{P}_i(t);$
  else if $\sum_{i=1}^{K} \widetilde{P}_i(t) = P_r(t)$ **then**
    $\widetilde{L}_g(t) \leftarrow 0;$
    $\widetilde{L}_r(t) \leftarrow P_r(t);$
    $\widetilde{E}(t) \leftarrow 0;$
  else
    $\widetilde{L}_g(t) \leftarrow \sum_{i=1}^{K} \widetilde{P}_i(t) - P_r(t);$
    $\widetilde{L}_r(t) \leftarrow P_r(t);$
    $\widetilde{E}(t) \leftarrow 0;$
  end if
  $\widetilde{C}_T \leftarrow \widetilde{C}_T + \lambda(t) \cdot \widetilde{L}_g(t) + \xi \cdot \widetilde{L}_r(t) - \zeta \cdot \widetilde{E}$
end for

4. RESULTS

All the experiments in this paper have been done on a PC with an Intel(R) core(TM) i7-2600 CPU @ 3.4 GHZ, RAM is 16 GB, 64-bit Operating System (windows 7). In addition, Gurobi has been used to solve LP and MILP problems, whereas the Java was the main tools to build our model.

Two case studies will be demonstrated in order to explain the advantages and the disadvantages of LP, MILP and CRLP algorithm. Also, these case studies compare between MILP, LP and CRLP in terms of cost and computation time.

4.1 Case study 1

The main goal of the first case study is to compare between LP, MILP and CRLP in terms of cost. Let us assume that we have a studio flat with just one AC unit. This flat is provided with domestic renewable resources: PV array (3.5 KWH) and wind turbine (1.5 KW). In addition, this building connected to a smart grid via a smart meter, and the building is provided with a controller that optimizes the load of the AC unit over time horizon T taking into account the electricity price and presence of domestic renewable power and occupier's temperature preferences. Moreover, this AC unit has four working levels, the first level is Off that consumes 0 KWH, the second level is Minimum level, that consumes 1 KWH, the third level is Medium level and it consumes 2 KWH and the last level is Maximum level and it consumes 3 KWH. The main task of the controller is to optimize the allocated power to the AC and to the grid (exported power) in order to decrease the cost, increase the utilization of domestic renewable power and keep the temperature inside the comfort levels (T_{min}= 18.0 and T_{max}= 22.0 °C) in two periods of the day, from β_1= 6:00 to δ_1= 12:00 and from β_2= 18:00 to δ_2 = 23:00.

Furthermore, the input data are: $\varepsilon = 0.965$, $\eta = 20$, $A = 0.9$ KW/°C, $\xi = 0.0$ Pence/KWH, $\zeta = 5.0$ Pence/KWH, $\tau = 10$ minutes, $N = 144$ time slot . Also, electricity price, predicted renewable power and predicted external temperature are shown in Figure 3. In this case study, all the variable quantities are assumed to be constant in each time slot.

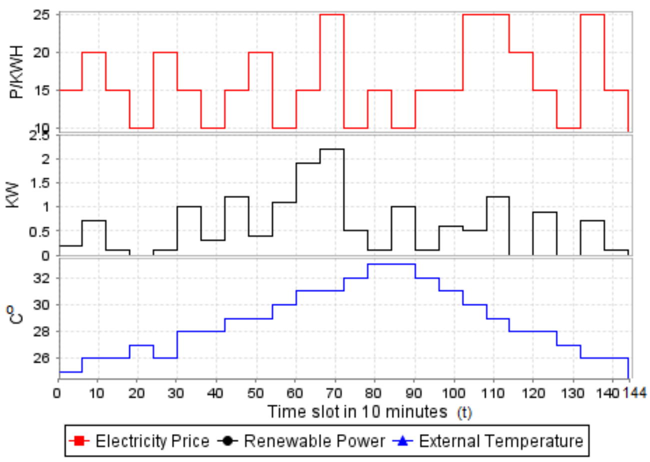


Figure 3: Input data

Firstly, a comparison study between MILP, LP and CRLP algorithms in terms of cost will be illustrated with details. Figure 4 shows the optimal solution of the problem using MILP. Furthermore, the first chart, in Figure 4, explains the allocated power to AC unit, $\bar{\mathbf{P}}$, whereas the second chart, in the same figure, illustrates the allocated amount of renewable power that will be exported to grid (blue line), $\bar{\mathbf{E}}$, the allocated power to AC form grid (red line), $\bar{\mathbf{L}}_{\mathbf{g}}$, and the allocated power from domestic renewable resources to the AC (green line), $\bar{\mathbf{L}}_{\mathbf{r}}$. All the variables in charts 1 and 2 are measured in KW. As we can see from the second chart, the controller does not export renewable power when grid power is allocated to AC unit. Vice versa, there is no allocated grid power to AC unit when there is allocated renewable power to be exported to grid. Finally, the last chart in the Figure depicts the internal temperature in the flat. The cost of running the AC using MILP is shown in Table 2.

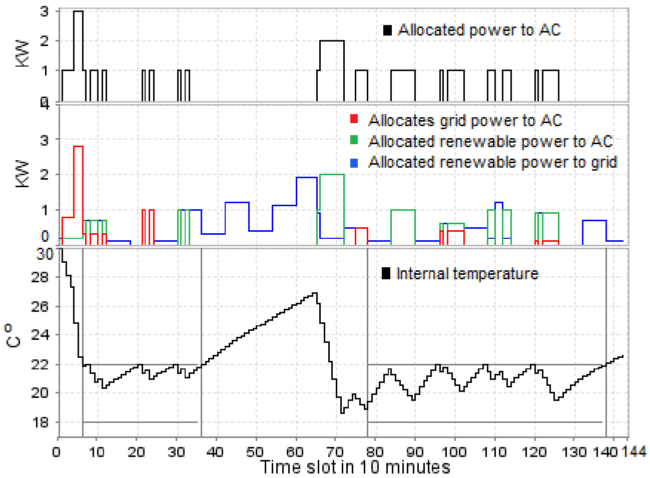


Figure 4: MILP results

Figure 5 portrays the results of LP and CRLP in detail. The first chart in Figure 5 shows the optimal allocated power to the AC unit by the LP algorithm, $\hat{\mathbf{P}}$, and the allocated power by the CRLP algorithm, $\widetilde{\mathbf{P}}$. Although there is a slight difference between $\hat{\mathbf{P}}$ and $\widetilde{\mathbf{P}}$, the optimal running time of the AC that is provided by CRLP is within the optimal time that has been chosen by the LP. In contrast, the cost of the power is not the same, this is because there are differences between ($\hat{\mathbf{L}}_{\mathbf{g}}$, $\hat{\mathbf{L}}_{\mathbf{r}}$ and $\hat{\mathbf{E}}$) and ($\widetilde{\mathbf{L}}_{\mathbf{g}}$, $\widetilde{\mathbf{L}}_{\mathbf{r}}$ and $\widetilde{\mathbf{E}}$) which are shown in chart 2, 3 and 4, respectively, in Figure 5. As a result, $\hat{C}_T$ is different from $\tilde{C}_T$. See Table 2 and 3.

As described the CRLP changes the optimal allocated power to the AC by the LP, $\hat{\mathbf{P}}$, into $\widetilde{\mathbf{P}}$ in order to make sure that the allocated power to AC at any time slot t is an integer, and match an one of power levels of the AC. As a result, the temperature inside the flat will be affected as well, and it may exceed the upper or lower limit of comfortable temperature that has been set by occupier. In the worst case the temperature set by CRLP, $\widetilde{\mathbf{T}}_{\mathbf{in}}$ would not be very different from the temperature corresponding to the solution of the LP, $\hat{\mathbf{T}}_{\mathbf{in}}$. Let us imagine that in any time slot, the allocated power to the AC by the LP is $P(t) = 0.49$ KW, so it will be rounded to 0 and the carry will be equal to $c = 0.49$ and in the next slot the allocated power to AC is $P(t+1) = 2.1$ KW. Then, the sum will be $S = 2.1 + 0.49 = 2.59$ KW, which will be rounded to 3.0 KW. As a result, the difference in this case is 3-2.1 = 0.9 KW. We can calculate the effect of this change in room temperature, $\widetilde{\mathbf{T}}_{\mathbf{in}}$. Let us assume that at any time slot t, the internal temperature is $T_{in}(t) = 20$ °C and the external temperatures is $T_{out}(t) = 30$ °C. So, the internal temperature, $\hat{T}_{in}(t)$, will be 18.83 °C, if the AC unit consumes 2.1 KW in the next 10 minutes, whereas $\tilde{T}_{in}$ = 18.0 °C, if the AC unit consumed 3 KW in the next 10 minutes. ($\tilde{P}(t) = 3$ KW). So, the difference in temperature between LP and CRLP in the worst case will be around $\mp$ 0.8. Furthermore, the difference between $\hat{T}_{in}(t)$ and $\tilde{T}_{in}(t)$ is also depends on the time interval. Table 4 shows the average absolute error in temperature (AEE) in °C (equation (13) is used to calculate AAE).

$$AAE = \frac{\sum_{t=1}^{N} \mid \hat{T}_{in}(t) - \tilde{T}_{in}(t) \mid}{N} \tag{13}$$

Table 2: MILP, LP, CRLP statistics

	Power in KWH		
MILP	$\sum_{t=0}^{t=N} \bar{L}_g(t)$=2.53	$\sum_{t=0}^{t=N} \bar{L}_r(t)$=6.30	$\sum_{t=0}^{t=N} \bar{E}(t)$=8.60
LP	$\sum_{t=0}^{t=N} \hat{L}_g(t)$=1.43	$\sum_{t=0}^{t=N} \hat{L}_r(t)$=6.94	$\sum_{t=0}^{t=N} \hat{E}(t)$=7.96
CRLP	$\sum_{t=0}^{t=N} \tilde{L}_g(t)$=2.83	$\sum_{t=0}^{t=N} \tilde{L}_r(t)$=5.50	$\sum_{t=0}^{t=N} \tilde{E}(t)$=9.40

Table 3: Total Cost

	$\bar{C}_T$	$\hat{C}_T$	$\tilde{C}_T$
Total cost	-6.90 Pences	-18.80 Pences	-0.50 Pences

Table 2 illustrates the total allocated grid power to the AC unit in KWH, the total allocated domestic renewable power to the AC unit in KWH and the total allocated renewable power to be exported to the grid in KWH. Table 3 shows the total cost of MILP, LP and CRLP in Pence. As we can see from the Table 3, the best cost is achieved by the LP, whereas the worst achieved by CRLP which is expected because CRLP is not exact. Furthermore, CRLP does not give optimal solution but its solution is close to the optimal solution found by LP. Despite the fact that LP gives the best solution, it is not practical and can not be applied directly to the AC unit. Despite the fact that CRLP gives the worst cost ($\tilde{C}$) with the current setting and input data, it is not always $\hat{C}_T \leq \bar{C}_T \leq \tilde{C}_T$. In different setting and different input data, CRLP could give

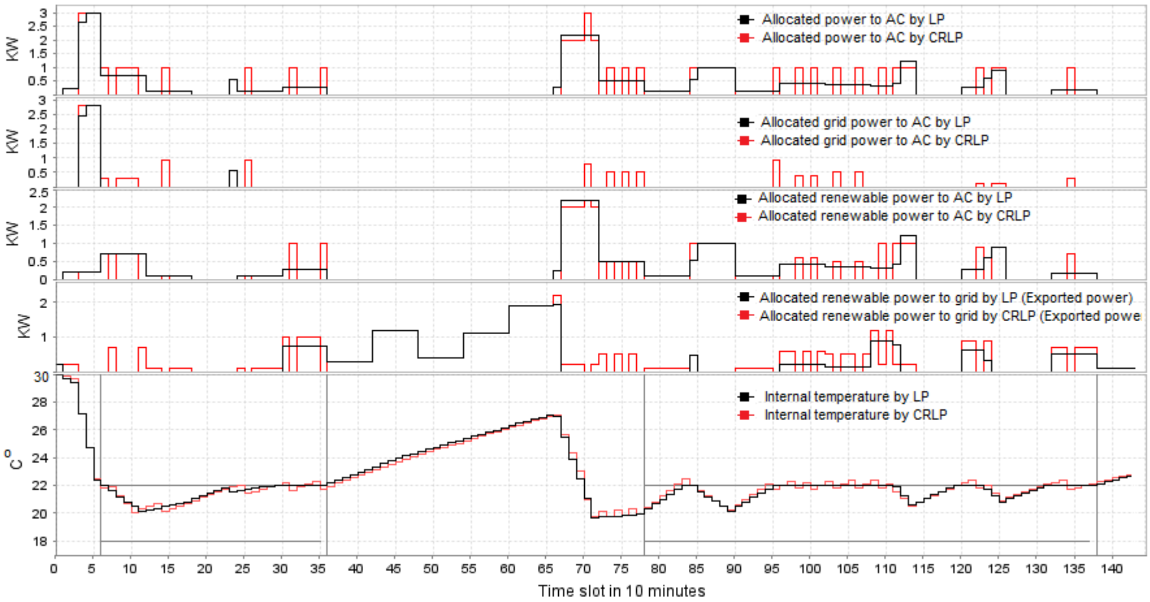


Figure 5: LP vs CRLP

better solution than MILP ($\hat{C_T} \leq \tilde{C_T} \leq \bar{C_T}$). So far, let us say MILP is the best with this sitting and input data disregarding the computation time which will be investigated in case study two.

Table 4: Average absolute error in Temperature

τ	20 min	15 min	10 min	5 min	1 min
AAE	0.46 °C	0.36 °C,	0.32 °C	0.24 °C	0.15 °C

4.2 Case study 2

The main purpose of this case study is to demonstrate the performance of CRLP and MILP algorithm in terms of computation time by using K AC units. Firstly, let us assume that we have a building with 3 different AC units, $K = 3$, that are used to cool three separated rooms in the same flat or in different flats in two periods of a specific day. In addition, AC unit 1 has three working levels (0, 1 and 2 KW) and the parameters of this AC unit are $\eta = 18$, $\epsilon = 0.955$, $A = 0.9$ KW/°C, $\tau = 10$ minutes, $N = 144$ time slots, the first period is from $\beta_1 = $ 01:00 to $\delta_1 = $ 06:00, and the second period is from $\beta_2 = $ 13:00 to $\delta_2 = $ 23:00, and the AC unit must maintain the inside temperature within, $T_{min} = 19.0$ °C and $T_{max} = 23.0$ °C. The second AC unit has four working levels (0, 1, 2 and 3 KW), and its parameters have the following values $\eta = 20$, $\epsilon = 0.965$, $A = 0.95$ KW/°C, $\tau = 10$ minutes, $N = 144$ time slots, AC unit 2 must maintain the temperature within $T_{min} = 18.0$ °C and $T_{max} = 22.0$ °C in two periods of the day. The first period starts at $\beta_1 = $ 03:00 and finishes at $\delta_1 = $ 09:00, and the second period starts at $\beta_2 = $ 15:00 and ends at $\delta_2 = $ 20:00. Finally, the third AC unit has five working levels (0, 1, 2, 3 and 4 KW) and its parameters have the following values: $\eta = 25$, $\epsilon = 0.975$, $A = 0.98$ KW/°C, $\tau = 10$ minutes, $N = 144$ time slot. This AC must keep the temperature within the comfort level in flat/room 3, i.e. Within $T_{min} = 17.0$ °C and $T_{max}=$ 21.0 °C for two periods of the day. The fires period starts at $\beta_1 = $ 07:00 and ends at $\delta_1 = $ 13:00, the second period starts at $\beta_2 = $ 16:00 and ends at $\delta_2 = $ 22:00. In addition, these three AC units share the same domestic renewable power resources. The electricity price, the predicted renewable power and the external temperature that are used in this case study are the same as in case study 1, Figure 3.

Figure 6 illustrates in details the allocated renewable and the grid power to AC units. The first chart shows the allocated power to first AC unit by LP and CRLP, $\hat{\mathbf{P_1}}$ and $\tilde{\mathbf{P}}_\mathbf{1}$. The second chart shows the allocated power to the second AC unit by LP and CRLP algorithms, $\hat{\mathbf{P_2}}$ and $\tilde{\mathbf{P}}_\mathbf{2}$, whereas the third chart shows the allocated power to the third AC unit by LP and CRLP, $\hat{\mathbf{P_3}}$ and $\tilde{\mathbf{P}}_\mathbf{3}$. The last chart shows the total allocated power to AC units in the building. The computation time of CRLP is 0.154 Second, whereas MILP could not provide a solution in 24 hours. Figure 7, portrays the internal temperature of the three flats or rooms in the building. The first chart shows $\hat{\mathbf{T}}^\mathbf{1}_\mathbf{in}$and $\tilde{\mathbf{T}}^\mathbf{1}_\mathbf{in}$, the second chart depicts $\hat{\mathbf{T}}^\mathbf{2}_\mathbf{in}$and $\tilde{\mathbf{T}}^\mathbf{2}_\mathbf{in}$, and $\hat{\mathbf{T}}^\mathbf{3}_\mathbf{in}$and $\tilde{\mathbf{T}}^\mathbf{3}_\mathbf{in}$ are illustrated in the third chart. In the next part, we will show the effect of K and τ on MILP, LP and CRLP.

As we explained above, MILP could not solve the problem for 3 AC units when time slot τ=10 minutes.

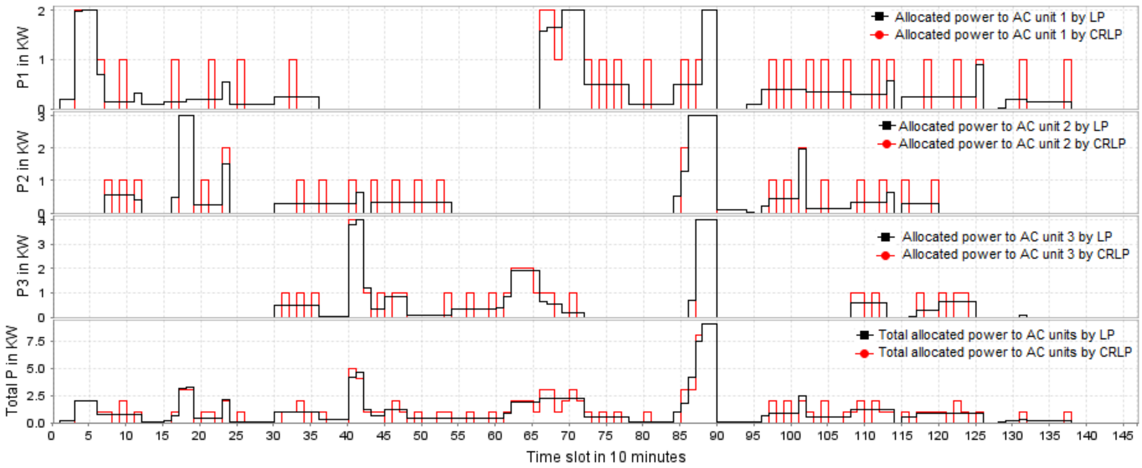


Figure 6: Power load of 3 AC units

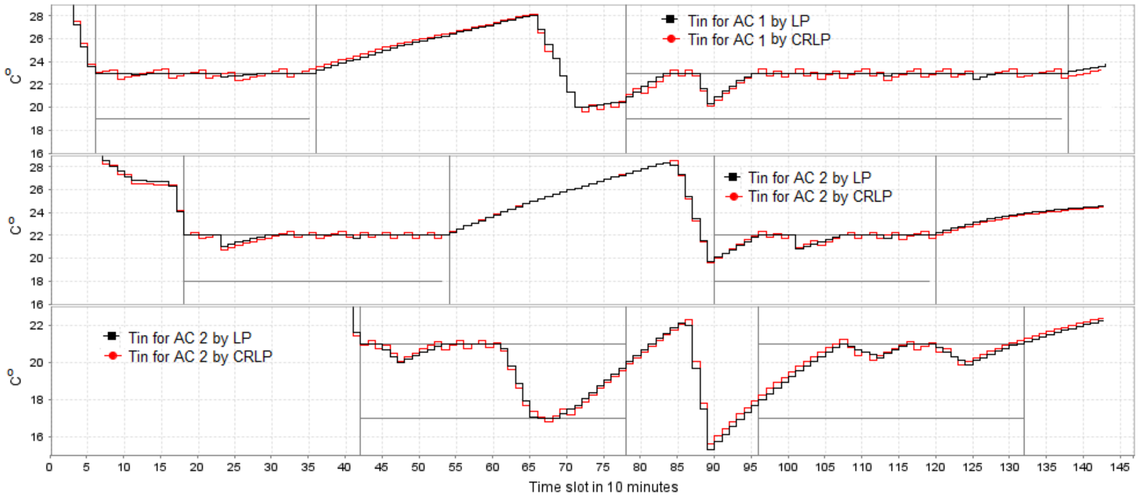


Figure 7: Internal temperature for ACs

Therefore, we will examine the effect of K and τ on computation time of MILP, LP and CRLP. For this problem all the result in Tables 5, 6 and 7 are calculated with 95% confidence interval. Firstly, Table 5 shows the computation time of MILP in seconds for different values of K and different values of τ to illustrate the effect of these parameters on computation time. As we can see from the table, the computation time increases dramatically when K increases for the same value of time slot , τ. On the other hand, the computation time rises remarkably when τ decreases, which means increases in the number of time slots in the problem, N. As a result, the number of continuous and integer variables in the problem rises. From this table, looks like there is a positive exponential correlation between K and the computation time and negative exponential correlation between τ and the computation time of the MILP problem.

Table 6 gives the computation time of the LP algorithm with diverse values of K and τ, to demonstrate their effect on LP's computation time. The computation time is measured in milliseconds in this table. As we can see, there is positive strong correlation between computation time and K. Also, there is a negative correlation between τ and the computation time of LP. Although there is a correlation between computation time, τ and K, its effect is not remarkable compared with its effect in MILP.

Table 5: MILP Computation time in seconds

K \ τ	30 min	20 min	10 min	5 min	1 min
1	0.11	0.25	8.104	311.4	5118.8
5	6.25	95.3	∞	∞	∞
10	169.3	56164.8	∞	∞	∞
15	228.9	∞	∞	∞	∞
20	1009.4	∞	∞	∞	∞
25	∞	∞	∞	∞	∞

The computation time of CRLP will not change significantly from the computation time of LP, because CRLP is based on LP. The only difference is that the rounding algorithm adds some time to LP's computation time. Table 7 supports this claim. Therefore, from case studies 1 and 2, we can say that LP is always the best optimization algorithm but unfortunately its solution is not practical. So, the choice must be made between MILP and CRLP.

Table 6: LP computation time in milliseconds

K \ τ	30 min	20 min	10 min	5 min	1 min
1	0.066	0.072	0.085	0.083	0.266
5	0.071	0.094	0.105	0.133	0.532
10	0.088	0.098	0.136	0.201	0.755
15	0.093	0.117	0.157	0.249	1.093
20	0.101	0.133	0.186	0.260	1.401
25	0.114	0.134	0.204	0.377	1.855
50	0.145	0.184	0.276	0.614	3.753
100	0.205	0.250	0.516	1.092	8.349

Case study 1 reveals MILP as the best with this input data and settings, and if the number of AC unit in the system is 1, $K = 1$, and time slot is between 30 minutes and 10 minutes, whereas case study 2 confirms that CRLP is the best in terms of computation time when there are more than one AC unit, $K > 1$, and the time slot, τ is small .

Table 7: CRLP computation time in milliseconds

K \ τ	30 min	20 min	10 min	5 min	1 min
1	0.068	0.073	0.091	0.087	0.276
5	0.076	0.095	0.109	0.144	0.574
10	0.090	0.101	0.141	0.212	0.781
15	0.094	0.120	0.165	0.262	1.141
20	0.104	0.137	0.194	0.278	1.461
25	0.118	0.146	0.215	0.389	1.917
50	0.153	0.193	0.294	0.700	3.865
100	0.217	0.269	0.544	1.145	8.501

5. DISCUSSIONS

This paper illustrates how an appropriate CRLP algorithm can be used to solve MILP problems. The results support the hypotheses in problem formulation section, which is that converting an MILP into an LP would improve the computation time dramatically. Actually, when K is large number and τ is small number, MILP could not solve the problem, See Table 5 .

First of all, we will discuss the effect of other parameters on the computation time of MILP and CRLP. For instance, the computation time of MILP is affected by other parameters in the system such as the value of A, ϵ, η, δ, β, T_{min}, T_{max}, whereas CRLP is not affected much by changing the value of these parameters. For instance, the average computation time of MILP is 686.22 seconds when η = 18, and it is 420.18 seconds when η = 20 for the same input data. In contrast, the computation time of CRLP is 0.083 second and 0.081 second, respectively, which means there is almost no effect of η on CRLP's computation time. In addition, the effect of ϵ on the MILP problem is considerable as well. The computation time of MILP is 41.206 seconds when ϵ = 0.945, and it is 0.561 second when ϵ = 0.965, which is really a big difference in terms of computation time. This phenomena is normal in MILP. Also, all the other parameters have big effect on MILP but we can not provide examples here because of the limited pages of the paper.

Secondly, the computation time of MILP is affected as well by the input parameters, T_{out}, P_r, λ. The optimization algorithm makes its decision based on electricity price and domestic renewable power, (equation (11)). If we eliminate the renewable power the computation time would improve dramatically in MILP. In contrast, the computation time of CRLP is not affected by removing or adding domestic renewable power data to the problem or the model. For example, the computation of one AC is 420.18 seconds, and 143.05 when $P_r(t) = 0$ for all t. In contrast, the computation time of CRLP is 0.083 seconds with renewable power and 0.084 seconds without renewable power.

The curve of the relationship between the computation time of MILP and K has increasing exponential shape with unexpected fluctuations. Let us assume that we want to calculate the computation time for K=1 , 2 and 3 by MILP and CRLP. It is not necessarily true that the MILP computation time of the problem with 2 AC units is less than the computation time of the problem with 3 AC units, and greater that the computation time of the problem with 1 AC unit. In CRLP however, the computation time with 1 AC unit < 2 AC units < 3 AC units. For example, the computation time of MILP is 420.18 second for one AC unit, 123.5 seconds for 2 AC units, and 613.9 seconds for 3 AC units, whereas the computation time of CRLP is 0.083, 0.089 and 0.093 seconds, respectively.

Now we will discuss the issues related to CRLP algorithm. The main observation is that the internal temperature that is controlled by the i^{th} AC using CRLP,$\tilde{\mathbf{T}}^{\mathbf{i}}_{\mathbf{in}}$, could exit the comfort temperature levels (T_{min} and T_{max}). The main objective of CRLT is reducing the cost, increasing the utilization of domestic renewable power as much as possible, and maintain the internal temperature within the comfort level. Hence, CRLT allows the temperature to exceed T_{min} and T_{max} in order to decrease the cost. Otherwise, the cost would be higher because the running time of AC will be changed if CRLP keeps constraint (4). Therefore, it is cheaper to relax this constraint as far as the difference in the temperature is not noticeable, which means constraint (4) will be relaxed to $T_{min} + \Delta \leq T^i_{in}(t) \leq T_{max} + \Delta$, $\forall t : t \in [1, \ldots, N]$, where $\Delta < 1\ C^o$, see Figures (5) and (7). The given results have considered this issue, have shown the worst case of Δ, and illustrated the average absolute error, (Table 4).

Finally, there are two main issues related to the model. The first one is that the relationship between consumed power by AC and the room temperature is really complex because it depends on many parameters, not just the consumed power by the AC unit. Moreover, the temperature inside the residential building depends on the size of the flat or rooms, the building or wall isolation, the number of residents in the building, the external temperature, the solar radiance, the state of the windows and doors (closed or opened), etc [22]. All of these factors are very difficult to be predicted. Therefore, we used just power efficiency, external temperatures, room size and building isolation, whereas we neglect the number of resident and we assumed that all windows and doors are closed. So, all these parameters needs to be taken in the consideration and the model must coped with them in the future. The second and last issue is the uncertainty in predicted renewable power. These data depends mainly on weather forecasting, and the accuracy of this data depends on country or the area using this model. For instance, weather forecasting in Mediterranean countries is more reliable than in North Europe, especially in the summer. The error on weather forecasting is outside of the scope of this paper, and more investigations is needed to tackle this issue.

6. CONCLUSION

This paper has examined the performance of MILP, LP and our CRLP algorithm, in terms of computation time and running cost with a set of AC units in residential and commercial building with dynamic electricity price and availability of domestic renewable resources environment. In addition, according to our results, LP algorithm has achieved the cheapest schedule in terms of cost and fastest computation time but, unfortunately, LP is not practical. Therefore, our CRLP algorithm is a provably efficient alternative to traditional MILP solution when the number of AC units is large (number of variables in the model is large) because MILP, which is NP-hard problem, cannot solve the problem in reasonable time. On the other hand, the results, also, illustrate that the computation time of MILP is effected badly by using predicted renewable power in the model, and it also affected with the value of other parameters which makes that its computation time is not predictable. Although CRLP is heuristic and not exact algorithm, the results proves that it is performing well in models with large number of variables.

7. REFERENCES

[1] Aldo V Da Rosa. *Fundamentals of renewable energy processes*. Academic Press, 2005.

[2] Anastasios I Dounis and Christos Caraiscos. Advanced control systems engineering for energy and comfort management in a building environment a review. *Renewable and Sustainable Energy Reviews*, 13(6):1246–1261, 2009.

[3] Silviu Nistor, Jianzhong Wu, Mahesh Sooriyabandara, and J Ekanayake. Cost optimization of smart appliances. In *International Conference and Exhibition on Innovative Smart Grid Technologies (ISGT Europe), 2011 2nd IEEE PES*, pages 1–5. IEEE, 2011.

[4] Saima Aman, Yogesh Simmhan, and Viktor K Prasanna. Energy management systems: state of the art and emerging trends. *Communications Magazine, IEEE*, 51(1):114–119, 2013.

[5] Roberto Rigolin Ferreira Lopes, Rikke Stoud Platou, Sverre Hendseth, Nunzio Marco Torrisi, Kristoffer Nyborg Gregertsen, and Geir Mathisen. Deploying third party services at smart grids end users using broadband links. In *Innovative Smart Grid Technologies Europe (ISGT EUROPE), 2013 4th IEEE/PES*, pages 1–5. IEEE, 2013.

[6] Tanguy Hubert and Santiago Grijalva. Modeling for residential electricity optimization in dynamic pricing environments. *IEEE TRANSACTIONS ON SMART GRID*, 2012.

[7] Tanguy Hubert and Santiago Grijalva. Realizing smart grid benefits requires energy optimization algorithms at residential level. In *Innovative Smart Grid Technologies (ISGT), 2011 IEEE PES*, pages 1–8. IEEE, 2011.

[8] A Barbato, Antonio Capone, Giuliana Carello, M Delfanti, M Merlo, and A Zaminga. Cooperative and non-cooperative house energy optimization in a smart grid perspective. In *World of Wireless, Mobile and Multimedia Networks (WoWMoM), 2011 IEEE International Symposium on a*, pages 1–6. IEEE, 2011.

[9] A Barbato, A Capone, G Carello, M Delfanti, M Merlo, and A Zaminga. House energy demand optimization in single and multi-user scenarios. In *2011 IEEE International Conference on Smart Grid Communications (SmartGridComm)*, pages 345–350. IEEE, 2011.

[10] A Barbato and G Carpentieri. Model and algorithms for the real time management of residential electricity demand. In *Energy Conference and Exhibition (ENERGYCON), 2012 IEEE International*, pages 701–706. IEEE, 2012.

[11] TT Ha Pham, Frédéric Wurtz, and Seddik Bacha. Optimal operation of a pv based multi-source system and energy management for household application. In *IEEE International Conference on Industrial Technology,ICIT 2009*, pages 1–5. IEEE, 2009.

[12] Cédric Clastres, TT Ha Pham, Frédéric Wurtz, and Seddik Bacha. Ancillary services and optimal household energy management with photovoltaic production. *Energy*, 35(1):55–64, 2010.

[13] Yi Zong, Lucian Mihet-Popa, Daniel Kullmann, Anders Thavlov, Oliver Gehrke, and Henrik W Bindner. Model predictive controller for active demand side management with pv self consumption in an intelligent building. In *2012 3rd IEEE PES International Conference and Exhibition on Innovative Smart Grid Technologies (ISGT Europe)*, pages 1–8. IEEE, 2012.

[14] KM Tsui and SC Chan. Demand response optimization for smart home scheduling under real-time pricing. *IEEE TRANSACTIONS ON SMART GRID*, 2012.

[15] Zhi Chen, Lei Wu, and Yong Fu. Real time price based demand response management for residential appliances via stochastic optimization and robust optimization. *IEEE TRANSACTIONS ON SMART GRID*, 2012.

[16] Duy Long Ha, Florent Frizon De Lamotte, and Quoc Hung Huynh. Real time dynamic multilevel optimization for demand side load management. In *2007 IEEE International Conference on Industrial Engineering and Engineering Management*, pages 945–949. IEEE, 2007.

[17] Marija Ilic, Jason W Black, and Jill L Watz. Potential benefits of implementing load control. In *Power Engineering Society Winter Meeting, 2002. IEEE*, volume 1, pages 177–182. IEEE, 2002.

[18] Mohammad Chehreghani Bozchalui, Syed Ahsan Hashmi, Hussin Hassen, Claudio A Cañizares, and Kankar Bhattacharya. Optimal operation of residential energy hubs in smart grids. *IEEE TRANSACTIONS ON SMART GRID*, 2012.

[19] Na Li, Lijun Chen, and Steven H Low. Optimal demand response based on utility maximization in power networks. In *Power and Energy Society General Meeting, 2011 IEEE*, pages 1–8. IEEE, 2011.

[20] Grégory De Oliveira, Mireille Jacomino, Duy Long Ha, and Stéphane Ploix. Optimal power control for smart homes. In *18th IFAC World Congress, elsevier, Ed*, 2011.

[21] Yi Zong, Daniel Kullmann, Anders Thavlov, Oliver Gehrke, and Henrik W Bindner. Active load management in an intelligent building using model predictive control strategy. In *PowerTech, 2011 IEEE Trondheim*, pages 1–6. IEEE, 2011.

[22] Y-W Chen, Xiuxing Chen, and Nicholas Maxemchuk. The fair allocation of power to air conditioners on a smart grid. *IEEE Transactions on Smart Grid*, 3(4):2188–2195, 2012.

[23] Karla L Hoffman. Combinatorial optimization: Current successes and directions for the future. *Journal of computational and applied mathematics*, 124(1):341–360, 2000.

[24] Wayne L Winston. *Introduction to mathematical programming:applications and algorithms.* International Thomson publishing, second edition edition, 1995.

[25] Robert Gasch and Jochen Twele. *Wind power plants: fundamentals, design, construction and operation.* Springer, jun 2012.

[26] E Matallanas, M Castillo-Cagigal, A Gutiérrez, F Monasterio-Huelin, E Caamaño-Martín, D Masa, and J Jiménez-Leube. Neural network controller for active demand side management with pv energy in the residential sector. *Applied Energy*, 91(1):90–97, 2012.

[27] Marc Beaudin, Hamidreza Zareipour, and Antony Schellenberg. Residential energy management using a moving window algorithm. In *2012 3rd IEEE PES International Conference and Exhibition on Innovative Smart Grid Technologies (ISGT Europe)*, pages 1–8. IEEE, 2012.