UCC501 Homework 3 Solutions

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Programming should be fun. So it is with education.

1 Why Hydropower Works?

1.1 Return Period

- Yes.
- Philosophically we would say that **nothing is impossible**. A more mathematical explanation goes as follows.
- According to Wikipedia, the return period is defined as such,

  `A return period, also known as a recurrence interval (sometimes repeat interval) is an estimate of the likelihood of an event, such as an earthquake, flood or a river discharge flow to occur. It is a statistical measurement typically based on historic data denoting the average recurrence interval over an extended period of time, and is usually used for risk analysis (e.g. to decide whether a project should be allowed to go forward in a zone of a certain risk, or to design structures to withstand an event with a certain return period). The following analysis assumes that the probability of the event occurring does not vary over time and is independent of past events.`

To simplify our analysis, we follow the assumption that the probability of the event occurring does not vary over time and is independent of past events.

- Given that the return period of a flood is 50 years, we have the probability of it as such:

$$p = \frac{1}{50} = 0.02 \tag{1}$$

Since in a year whether there is a flood or not is **mutex**, we have

$$q = 1 - p = 0.98 \tag{2}$$

Following the definition of **Combination**[1], we calculate the probability of a 50-year flood occurs in two consecutive years as follows:

$$P \;=\; C_1^{49} * 0.02^2 * 0.98^{48} \tag{3}$$

where C_1^{49} indicates 49 out of 50 slots the two **consecutive** events (in this case, floods) reside. Evaluating equation (3) we have $P = 0.0074$. Apparently it's a positive number, so it is possible.

1.2 Within 20 Years

- Intuitively speaking, the answer should be 20%. Let's prove that.

- The definition of recurrence interval is

$$RI \;=\; \frac{n+1}{m} \tag{4}$$

where n is number of years on record and m is the number of recorded occurrences of the event being considered.

- Then the probability calculation of this question is equivalent to "How likely for a ball to fall into one of the first 20 boxes, given that it is equally likely to fall into any one of 100 boxes". Mathematically speaking,

$$P \;=\; 1 - (0.01)^{20} \tag{5}$$

Evaluating equation (5) we can get $P = 0.18$.

[1] `https://en.wikipedia.org/wiki/Combination`

1.3 Does it rain often?

- We extract the rainfall data manually to one column, then use MATLAB to do the remaining data processing (or information retrieval, machine learning whatsoever). If the data sheet is too big, we will use MATLAB directly to extract information.

- Following the code implemented in `rain.m` we can get **Annual Maximum Series** as follows:

```
Columns 1 through 16

130.6000  101.3000  118.3000  109.0000  110.2000  166.8000
179.6000  125.8000  195.6000  152.3000  120.1000  147.8000
141.0000  156.5000  132.6000  153.9000

Columns 17 through 21

156.4000  161.9000  158.5000  168.2000  183.7000
```

  with the plotted figure shown as below,

- The Gringorten formula is described as such,

$$T = \frac{n + 0.12}{i - 0.44} \tag{6}$$

  From $P = \frac{1}{T}$ we get

$$P = \frac{i - 0.44}{n + 0.12} \tag{7}$$

- Put all these equations into MATLAB codes, we can get the probability of recurrence as follows:

```
Columns 1 through 16

  0.0265    0.0739    0.1212    0.1686    0.2159    0.2633    0.3106
  0.3580    0.4053    0.4527    0.5000    0.5473    0.5947    0.6420
  0.6894    0.7367
```

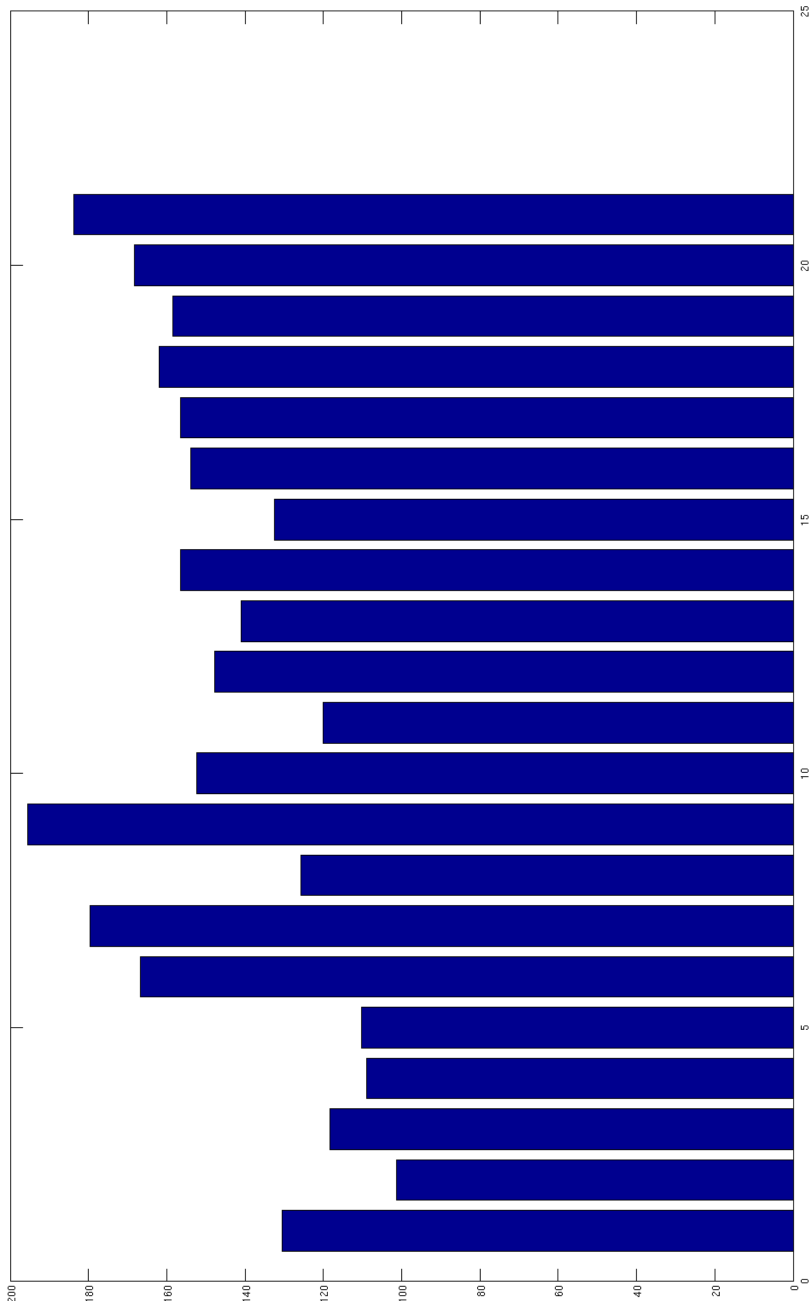

Figure 1: Annual Maximum Series

```
  Columns 17 through 21

    0.7841    0.8314    0.8788    0.9261    0.9735
```

- The return periods are just the reverse of probability of recurrence, as per below.

```
Columns 1 through 16

   37.7143   13.5385    8.2500    5.9326    4.6316    3.7986    3.2195
    2.7937    2.4673    2.2092    2.0000    1.8270    1.6815    1.5575
    1.4505    1.3573

  Columns 17 through 21

    1.2754    1.2027    1.1379    1.0798    1.0272
```

- Therefore from figure 2 we can infer $145.7mm$ is the rainfall of 50% exceedance.

2 Gone with the Wind

2.1 The good, The bad

- The pros of utilizing wind energy may be plotted as follows:
  - Before the death of sun, this type of energy is **renewable**.
  - In the "Subsidy Free" scenario, the levelized cost of wind energy is cheap among other renewable energy types[2].
  - New types of wind farms like the offshore one levitate the issue of high land cost and noise.
- I would like to list the cons here.

[2] `http://www.e3s-center.org/pubs/227/1-13Majumdar.pdf`, page 16

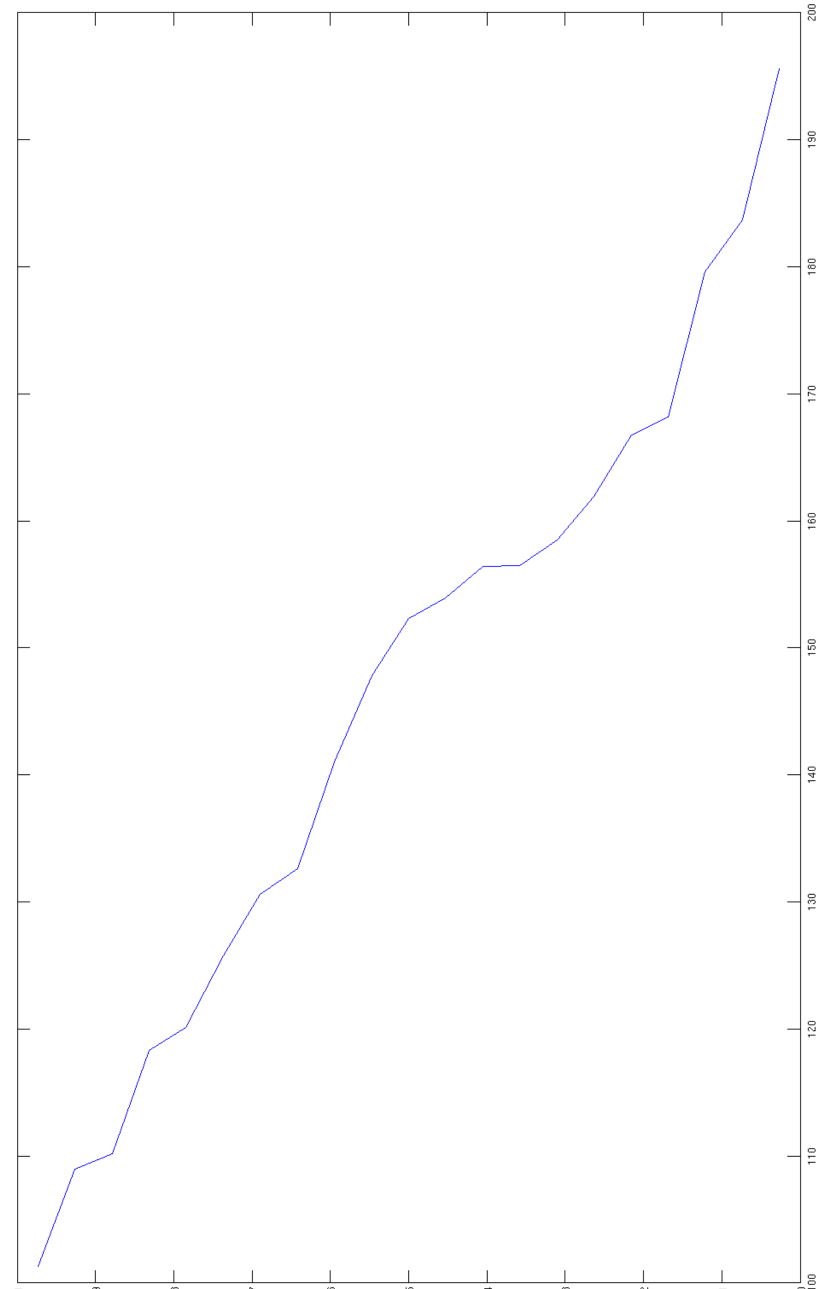

Figure 2: Probability of Recurrence

- When built on land, the noise of mainstream wind farm is annoying. Therefore, residents could be against the whole building plan.
- This type of energy generation is heavily climate, or rather geologically dependent.
- The maintenance of wind farms are difficult when compared with traditional fossil and hydro power plant.

- **I think utilizing wind energy is generally viable**. Though I am a big fan of SciFi and nuclear energy, I would say wind energy is viable. As the research and development of wind energy goes, there are turbines in use that can make the "most" of wind, namely the low speed wind. This is important because the majority part of the world do not "enjoy" high wind speed, when the low-speed wind are available to use, due to the almost no cost nature of wind, it can generate highly competitive electricity, in terms of price.

2.2 How Powerful Could Wind Be?

- From the conditions (parameters, configurations, etc) we have, we can calculate the **blade tip speed** using the equation below.

$$u = \frac{v_{rotation} * \pi * D}{60} \tag{8}$$

where D is the diameter of the turbine. We have $L = 60m$ for blade length. Under the assumption that $D = 2 * L$ we can evaluate equation (8), getting $u = 24\pi ms^{-1}$.

- After the first step, we have $u = 24\pi$ and $v = 15$, we can evaluate equation (9) to get tip speed ratio.

$$\lambda = \frac{u}{v} \tag{9}$$

$\lambda = 5.0265$, where v stands for wind speed.

- From the figure given in this question we can infer (well, roughly since the exact function of that curve is not presented. Somehow if we do want we can do **curve fitting**) $C_p = 0.275$.

- From the general (namely, **ideal**) wind power calculation formula (10) we know how to calculate the power of a wind turbine.

$$P = \frac{1}{2}\rho A v^3 C_p \tag{10}$$

Therefore we pre-calculate A as $A = \pi L^2$. Evaluating the whole formula we can get (as can be verified from MATLAB codes) $P = 6.2136 MW$. Given air pressure and air temperature we use the ρ value from corresponding wiki page[3].

- For power density we only need to reform equation (10) as follows.

$$PD = P/A = \frac{1}{2}\rho v^3 C_p \tag{11}$$

Put what we have into this function (in some manner, this equation can be regarded as a function. In this case, it's regarding A, the area a turbine blade swept.) $PD = 5.4940 * 10^{-4} MW \cdot m^{-2}$.

- For this question we just evaluate our MATLAB codes. Total power should be $1.6329 * 10^7 MJ$.

3 How Did the Red Sun Arise?

3.1 All Solar Leads to Energy

- The major difference lies in the way to capture the solar power and transform it into electricity—which is at least one of the building blocks of modern industry.

- Solar Photovoltaic does the transformation directly by way of semiconductors. The key part is that when temperature rises, the electron jumps out of its default orbit therefore forms the current.

- Concentrating Solar Power (CSP) does the transformation to electricity in a passive way. The intermediate part is mechanical, like turbines or other engines.

[3] `https://en.wikipedia.org/wiki/Density_of_air`

3.2 Is Solar Energy Really the Ultimate Solution?

- Let the solar array size be x, then following the assumption given in this question, we have

$$x * 4.5 * 365 * 0.7 = 11000 * 0.6 \tag{12}$$

Solve this equation we get $x = 5.7404$.

- Since the present technology (as given in question, those solar panels in lab are **way too** expensive) requires us to put 5 solar panels to generate $1kW$ power, we need $5 * x$ solar panels. Evaluating the function we get 28.7019, in practice it should be 29 or 30 solar panels to meet the need.

4 Chernobyl

4.1 Smaller than Smaller

- To be short and in Dr. Youssef's rhetoric, fussion is "marriage" and fission is "divorce".

- To be specific, Fission is a reaction when the nucleus of an atom, having captured a neutron, splits into two or more nuclei, and in so doing, releases a significant amount of energy as well as more neutrons. Fusion is a process where nuclei collide and join together to form a heavier atom, usually deuterium and tritium. When this happens a considerable amount of energy gets released at extremely high temperatures[4].

4.2 Decay, Mortal and Immortal

- $^{245}_{96}Cm \rightarrow ^{243}_{94}Pu + ^{4}_{2}He$ The so called alpha stuff is just $^{4}_{2}He$.

- $^{139}_{51}Sb$

- For this problem let the initial quantity be N_0 and half-life quantity be $N_{\frac{1}{2}}$, half-life of half-life quantity be $N_{\frac{1}{4}}$. Therefore we should be able

[4] http://www.iea.org/topics/nuclearfissionandfusion/

to get equations as follows, where λ is a constant, t_1 and t_2 are the corresponding time to decay.

$$\begin{aligned} N_{\frac{1}{2}} &= e^{-\lambda t_1} \\ N_{\frac{1}{4}} &= e^{-\lambda t_2} \end{aligned} \tag{13}$$

- According to the question, we normalize the parameters, where $N_{\frac{1}{2}} = \frac{1}{2}$ and $N_{\frac{1}{4}} = \frac{1}{4}$ and $t_1 = 28.8$, solve equation (13) we get $t_2 = 166.198$ years.