COMPUTING $d^{d+o(d)}$ –APPROXIMATE EQUILIBRIA IN WEIGHTED CONGESTION GAMES WITH DEGREE-d POLYNOMIAL COSTS

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PRELIMINARIES

- A POLYNOMIAL WEIGHTED CONGESTION GAME OF DEGREE d CONSISTS OF:
  - A SET OF PLAYERS, EACH WITH A POSITIVE WEIGHT
  - A SET OF RESOURCES E, EACH RESOURCE'S COST FUNCTION IS A DEGREE-d INCREASING POLYNOMIAL IN THE SUM OF WEIGHTS OF PLAYERS USING IT
  - THE STRATEGY SPACE FOR EACH PLAYER IS SOME COLLECTION OF SUBSETS OF E
  - EACH PLAYER PAYS THE SUM OF COSTS OF CHOSEN RESOURCES
- PURE NASH EQUILIBRIA ARE COMPUTATIONALLY HARD FOR CONGESTION GAMES WITHOUT ASSUMING POLYNOMIAL COSTS:
  - COMPUTING EQUILIBRIA IS PLS-COMPLETE [FABRIKANT ET AL.]
  - EVEN COMPUTING ρ-APPROXIMATE EQUILIBRIA IS PLS-COMPLETE [SKOPALIK, VOECKING]
  - DETERMINING EXISTENCE OF EQUILIBRIA IS STRONGLY NP-COMPLETE [DUNKEL, SCHULZ]
- BUT, FOR POLYNOMIAL WEIGHTED CONGESTION GAMES OF DEGREE d, CARAGIANNIS ET AL. GAVE AN EFFICIENT ALGORITHM FOR COMPUTING $d^{2d+o(d)}$-APPROXIMATE EQUILIBRIA

RESULTS

- WE IMPROVE THE ALGORITHM BY CARAGIANNIS ET AL. IN PARTICULAR:
  - APPROXIMATION GUARANTEE IMPROVES FROM $d^{2d+o(d)}$ TO $d^{d+o(d)}$
  - THE ORIGINAL ALGORITHM FINDS $d^{d+o(d)}$-APPROXIMATE EQUILIBRIA OF A PROXY GAME CALLED Ψ-GAME (WHOSE ρ-APPROXIMATE EQUILIBRIA MAP TO $d!$ρ-APPROXIMATE EQUILIBRIA OF INPUT CONGESTION GAME). BY CONTRAST, OUR ALGORITHM APPLIES DIRECTLY TO THE INPUT GAME.
    - **COROLLARY**: FROM ANY STATE OF A POLYNOMIAL WEIGHTED CONGESTION GAME OF DEGREE d, ONE CAN GET TO A $d^{d+o(d)}$-APPROXIMATE EQUILIBRIUM VIA A POLY-TIME COMPUTABLE SEQUENCE OF INDIVIDUAL BEST RESPONSES
- NEW INGREDIENTS IN THE ALGORITHM'S ANALYSIS:
  - WE USE A $(d+1)$-APPROXIMATE POTENTIAL FUNCTION OF THE INPUT GAME [CHRISTODOULOU ET AL.]
  - THE IMPROVED ALGORITHM'S APPROXIMATION GUARANTEE IS ASYMPTOTIC TO THE PRICE OF ANARCHY OF $(d+1)$-APPROXIMATE EQUILIBRIA OF INPUT GAME.
  - THIS PRICE OF ANARCHY IS $d^{d+o(d)}$, AS A COROLLARY OF THESE APPROXIMATE-PRICE-OF-ANARCHY RESULTS WE GIVE FOR THE CLASS OF POLYNOMIAL WEIGHTED CONGESTION GAMES OF DEGREE d:
    - THE PRICE OF ANARCHY OF ρ-APPROXIMATE EQUILIBRIA IS $\Phi_{d,\rho}^{d+1}$, WHERE $\Phi_{d,\rho}$ IS THE UNIQUE POSITIVE ROOT OF THE EQUATION $\rho(x+1)^d = x^{d+1}$
    - THE PRICE OF ANARCHY OF ρ-APPROXIMATE EQUILIBRIA IS $\leq \left(\frac{d}{\mathcal{W}(d/\rho)}\right)^{d+1}$, WHERE $\mathcal{W}$ IS THE LAMBERT-W FUNCTION