CSE 3200 Micro-Computer Graphics
Vectors in CG

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Outline

- What is a vector?
- Importance in CG
- Vector Operations
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  - Addition & Subtraction
  - Scaling vectors
  - Unit Vectors
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What is a vector?

- Vectors are geometric objects that have a **length** and a **direction**
- A vector is *like* a point, in that it is described by a set of coordinates in a given dimension. But there are differences:
  - A point has an absolute position within a coordinate system. A vector has no position; the same vector can appear anywhere.
  - A point has no dimension to it. A vector has a length as well as a direction.

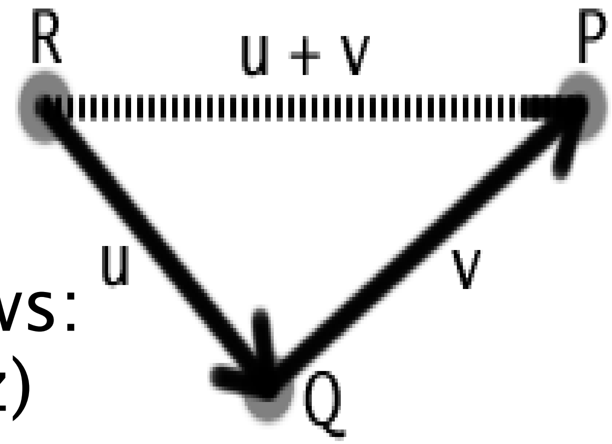
Why are vectors important in CG?

- Analyze shapes
- Determine visibility
- Calculate lighting effects

Vector Operations

- Points and vectors can be used to define one another by adding and subtracting the coordinates. Given that P and Q are points and u and v are vectors, then
  - $P - Q$ is a vector with its tail at Q and its head at P
  - $P + v$ is a new point (P displaced by the quantities in v)
  - $u + v$ is another vector

Vector Operations



- Addition & subtraction as follows:
- a = (ax, ay, az) , b = (bx, by, bz)
  - a + b = (ax+ bx, ay+ by, az+ bz)
  - a – b = (ax – bx, ay – by, az – bz)

- Consider the illustration
  - R = (2, 3, 1), Q = (4, 1, 1), P = (7, 3, 1).
  - Then
    - u = Q – R = (2, –2, 0) and Q = R + u
    - v = P – Q = (3, 2, 0) and P = Q + v
    - u + v = (Q – R) + (P – Q) = P – R = (5, 0, 0)

Vector Operations

- You can change the length of a vector by multiplying it with a scalar value. Given a scalar value *s* and a vector *v = (vx, vy, vz)* then *sv= (svx, svy, svz)*.
  - For example, if s = 0.5 and v = (4, 3, 0) then sv= (2, 1.5, *0*).

- You can *f*ind the length (or *m*agnitude) of a vector using the Pythagorean theorem.
  - Given a vector v = (vx, vy, vz), the magnitude of v is |v| = s*qrt(vx*vx, vy*v*y, vz*vz). For example, *if v = (4, 3, 0) then |v| = 5*.

Unit Vector

- **A unit vector** is a vector of length 1.
- you can find a corresponding unit vector (with the same direction) by dividing each of the coordinate values by the magnitude of the original vector
  - given a vector $v = (vx, vy, vz)$,
    the unit vector is $(vx / |v|, vy / |v|, vz / |v|)$.
  - For example, if $v = (4, 3, 0)$
    the unit vector with the same direction is

$$(4/5, 3/5, 0/5) = (0.8, 0.6, 0).$$

Dot Product

- The dot (or inner) product of 2 vectors produces a scalar value.
- The dot product is used to solve a number of important geometric problems in graphics.
- The dot product for 3-dimensional vectors is solved as follows:
  - If $u = (ux, uy, uz)$ and $v = (vx, vy, vz)$ then
  - $u \cdot v = uxvx + uyvy + uzvz$.
- The dot product has the following properties:
  - Symmetry: $u \cdot v = v \cdot u$
  - Linearity: $(u + w) \cdot v = (u \cdot v) + (w \cdot v)$
  - Homogeneity: $(su) \cdot v = s(u \cdot v)$
  - $|v/ = sqrt(v \cdot v)$
- The dot product can be used to determine the angle between two vectors.

Dot Product

- $u \cdot v > 0$ implies that the angle is **acute**
  - $(-90° < (\theta - \phi) < 90°)$;
- $u \cdot v < 0$ implies the angle is **obtuse**
  - $(90° < (\theta - \phi) < 270°)$;
- $u \cdot v = 0$ implies the angle is **right**
  - $((\theta - \phi) = 90°$ or $(\theta - \phi) = -90°)$,
  - i.e. the vectors are **perpendicular**.

Cross Product

- The cross (or vector) product of 2 vectors produces another vector which is perpendicular (orthogonal) to both of the vectors used to find it
- The cross product is defined in terms of the standard unit vectors ***i***, ***j***, and ***k***, where
  - ***i*** = (1, 0, 0)   ***j*** = (0, 1, 0)   ***k*** = (0, 0, 1)

- The cross product for 3-dimensional vectors
  - If *u = (ux, uy, uz)* and *v = (vx, vy, vz)* then

u x *v = ((uyvz – uzvy)****i*** *+ (uzvx – uxvz)****j*** *+ (uxvy – uyvx)****k****)*.

OR (notice the absence of the I, j and k)

u X v = (*uyvz – uzvy) , (uzvx – uxvz) , (uxvy – uyvx)*

Cross Product

- we can write the cross product as a determinant:

$$u \times v = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

$u \times v = ((u_y v_z - u_z v_y)\boldsymbol{i}\ ,\ (u_z v_x - u_x v_z)\boldsymbol{j}\ ,\ (u_x v_y - u_y v_x)\boldsymbol{k})$.

- The cross product has the following properties:
  - Antisymmetry: $u \times v = -v \times u$
  - Linearity: $u \times (v + w) = (u \times v) + (u \times w)$
  - Homogeneity: $(su) \times v = s(u \times v)$
  - $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}\ ;\ \boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i}\ ;\ \boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}$
  - The result of $u \times v$ is a vector that is perpendicular (orthogonal) to both u and v.

Putting [all of this] together

- **CG operations frequently requires the need to calculate the unit normal vector to a polygon for effects (lighting etc.) and other operations.**
- For these calculations, we assume that the polygon is convex, with its points listed in counter-clockwise order

1. Use the first 3 points of the polygon (*P0, P1, P2*) to define 2 vectors:

   $u = P1 - P0$ and $v = P2 - P0$
2. Find the normal vector using the cross product:

   $n = u \times v$
3. Convert *n* to a unit normal:
   - Find the magnitude of *n*: $|n| = sqrt(n \cdot n)$
   - Divide each of the coordinates of *n* by $|n|$.

Questions?

Review Questions

1. For each of the following, calculate the **coordinates**. Indicate whether the result is a point or a vector.
   - $v + u$, where $v = (-1, 0, 5)$ and $u = (2, 1, 1)$
   - $P + v$, where $P = (1, 2, 3)$ and $v = (-1, -2, -3)$
   - $P - Q$, where $P = (5, 5, 5)$ and $Q = (1, 2, 3)$

Review Questions

2. For each of the following, calculate sv

- $s = 3$, $v = (1, 1, 1)$
- $s = 0.25$, $v = (-4, 8, 2)$

Review Questions

3. For each of the following vectors v and u, calculate the **dot product**. What does the result tell you about the angle between the vectors?
   - v = (1, 0, 0) and u = (0, 1, 0)
   - v = (1, 1, −1) and u = (2, 1, 0)
   - v = (−2, 0, 0) and u = (1, 1, 1)

Review Questions

4. Calculate the **unit normal vector** for the polygon defined by points P0 = (1, 1, 1), P1 = (5, 1, 4) and P2 = (2, 1, 1)