Image processing

DCT and Wavelet Transformation

Discrete Cosine Transform (DCT)

- Just as the fourier series is the starting point in transforming and analyzing periodic functions, the basic step for vectors is the discrete fourier transform (DFT).

- It maps the “time domain” to the “frequency domain.” A vector with N components is written as a combination of N special basis vectors vk. Those are constructed from powers of the complex number $w = e^{2\pi i/N}$:

$$v_k = (1, w^k, w^{2k},\ldots,W^{(n-1)k})$$

- the vectors vk are the columns of the fourier matrix f = fn . those columns are orthogonal.

- A discrete cosine transform (DCT) expresses a finite sequence of data points in terms of a sum of cosine functions oscillating at different frequencies. Dcts are important to numerous applications in science and engineering, from lossy compression of audio (e.G. MP3) and images (e.G. Jpeg)

3. Discrete Cosine transform (DCT)

The discrete cosine transform (DCT) is thae basis for many image compression algorithms they use only cosine function only. Assuming an N*N image , the DCT equation is given by

$$D(u,v)=\alpha(u)\alpha(v)\sum_{r=0}^{N-1}\sum_{c=0}^{N-1} I(r,c)\cos\left[\frac{(2r+1)u\pi}{2N}\right]\cos\left[\frac{(2c+1)v\pi}{2N}\right]$$

$$\alpha(u),\alpha(v)=\begin{cases}\sqrt{\frac{1}{N}} & \text{if } u=0\\ \sqrt{\frac{2}{N}} & \text{if } u>0\end{cases}$$

In matrix form for DCT the equation become as

$$\boxed{\mathbf{D(u,v)}=\begin{cases}\sqrt{\frac{1}{N}} & \textbf{if } u=0\\ \sqrt{\frac{2}{N}}\cos\left(\frac{(2v+1)u\pi}{2N}\right) & \textbf{if } u>0\end{cases}}$$

where u= 0,1,2,...N-1 along rows and v= 0,1,2,...N-1 along columns

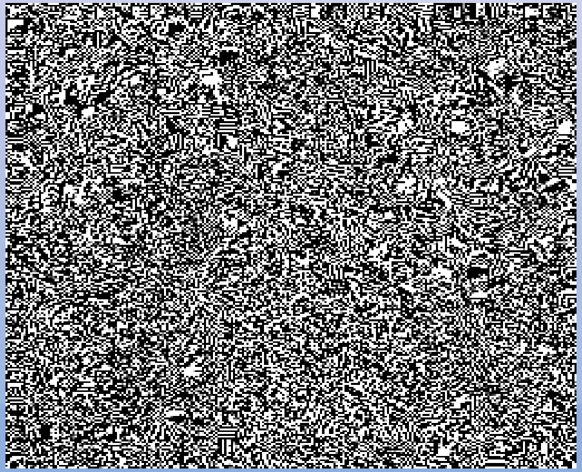
Properties of DCT

- One clear advantage of the DCT over the DFT is that there is no need to
  manipulate complex numbers.
- the DCT is designed to work on pixel values ranging from -0.5 to 0.5 for binary image and from -128 to 127 for gray or color image so the original image block is first "leveled off" by subtract 0.5 or 128 from each entry pixel values.

the general transform equation for DCT is

$$\boxed{\mathbf{A} = \mathbf{D} \;\; \mathbf{I} \;\; \mathbf{D}^{\mathbf{T}}}$$

Original image



DCT

Derive the DCT coefficient matrix of 4*4 block image

$$
\begin{array}{c|cccc}
 & v0 & v1 & v2 & v3 \\
\hline
u0 & \multicolumn{4}{c}{\multirow{4}{*}{$\begin{pmatrix} 00 & 01 & 02 & 03 \\ 10 & 11 & 12 & 13 \\ 20 & 21 & 22 & 23 \\ 30 & 31 & 32 & 33 \end{pmatrix}$}} \\
u1 & & & & \\
u2 & & & & \\
u3 & & & &
\end{array}
$$

$$
D = \begin{pmatrix}
1/2 & 1/2 & 1/2 & 1/2 \\
\sqrt{\frac{2}{4}}\cos(\frac{\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{3\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{5\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{7\pi}{8}) \\
\sqrt{\frac{2}{4}}\cos(\frac{2\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{6\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{10\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{14\pi}{8}) \\
\sqrt{\frac{2}{4}}\cos(\frac{3\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{9\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{15\pi}{8}) & \sqrt{\frac{2}{4}}\cos(\frac{21\pi}{8})
\end{pmatrix}
$$

$$
\mathbf{D} = \begin{pmatrix}
0.5 & 0.5 & 0.5 & 0.5 \\
0.65 & 0.27 & -0.27 & -0.65 \\
0.5 & -0.5 & -0.5 & 0.5 \\
0.27 & -0.65 & 0.65 & -0.27
\end{pmatrix}
$$

EX1: Calculate the DCT transform for the image I(4,4)

$$I = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \text{leveled off} \quad I = \begin{pmatrix} -0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \end{pmatrix}$$

$$A = \overset{D}{\begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{pmatrix}} * \overset{I}{\begin{pmatrix} -0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \end{pmatrix}} * \overset{D^T}{\begin{pmatrix} 0.5 & 0.65 & 0.5 & 0.27 \\ 0.5 & 0.27 & -0.5 & -0.65 \\ 0.5 & -0.27 & -0.5 & 0.65 \\ 0.5 & -0.65 & 0.5 & -0.27 \end{pmatrix}}$$

$$A = \overset{D}{\begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{pmatrix}} * \overset{I\,D^T}{\begin{pmatrix} 0 & -0.38 & 0 & -0.92 \\ 0 & 0.38 & 0 & 0.92 \\ 0 & -0.38 & 0 & -0.92 \\ 0 & 0.38 & 0 & 0.92 \end{pmatrix}} \quad A = \overset{D\,I\,D^T}{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -0.29 & 0 & -0.71 \\ 0 & 0 & 0 & 0 \\ 0 & -0.71 & 0 & -1.71 \end{pmatrix}}$$

Derive the DCT coefficient matrix of 8 * 8 block image

$$
\begin{pmatrix}
00 & 01 & 02 & 03 & 04 & 05 & 06 & 07 \\
10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 \\
20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 \\
30 & 31 & 32 & 33 & 34 & 35 & 36 & 37 \\
40 & 41 & 42 & 43 & 44 & 45 & 46 & 47 \\
50 & 51 & 52 & 53 & 54 & 55 & 56 & 57 \\
60 & 61 & 62 & 63 & 64 & 65 & 66 & 67 \\
70 & 71 & 72 & 73 & 74 & 75 & 76 & 77
\end{pmatrix}
$$

$$
D = \begin{pmatrix}
\sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} \\
\sqrt{\frac{2}{8}}\cos(\frac{\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{3\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{5\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{7\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{9\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{12\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{15\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{18\pi}{16}) \\
\sqrt{\frac{2}{8}}\cos(\frac{2\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{6\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{10\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{14\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{18\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{22\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{26\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{30\pi}{16}) \\
\sqrt{\frac{2}{8}}\cos(\frac{3\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{9\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{15\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{21\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{27\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{33\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{39\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{45\pi}{16}) \\
\sqrt{\frac{2}{8}}\cos(\frac{4\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{12\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{20\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{28\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{36\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{44\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{52\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{60\pi}{16}) \\
\sqrt{\frac{2}{8}}\cos(\frac{5\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{15\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{25\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{35\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{45\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{55\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{65\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{75\pi}{16}) \\
\sqrt{\frac{2}{8}}\cos(\frac{6\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{18\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{30\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{42\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{54\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{66\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{78\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{90\pi}{16}) \\
\sqrt{\frac{2}{8}}\cos(\frac{7\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{21\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{35\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{49\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{63\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{77\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{91\pi}{16}) & \sqrt{\frac{2}{8}}\cos(\frac{105\pi}{16})
\end{pmatrix}
$$

So the (DCT) matrix of 8*8 image

$$D = \begin{pmatrix} 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\ -0.4904 & 0.4157 & 0.2778 & 0.0975 & -0.0975 & -0.2778 & -0.4157 & -0.4904 \\ 0.4619 & 0.1913 & -0.1913 & -0.4619 & -0.4619 & -0.1913 & 0.1913 & 0.4619 \\ 0.4157 & -0.0975 & -0.4904 & -0.2778 & 0.2778 & 0.4904 & 0.0975 & -0.4157 \\ 0.3536 & -0.3536 & -0.3536 & 0.3536 & 0.3536 & -0.3536 & -0.3536 & 0.3536 \\ 0.2778 & -0.4904 & 0.0975 & 0.4157 & -0.4157 & -0.0975 & 0.4904 & -0.2778 \\ 0.1913 & -0.4619 & 0.4619 & -0.1913 & -0.1913 & 0.4619 & -0.4619 & 0.1913 \\ 0.975 & -0.2778 & 0.4157 & -0.4904 & 0.4904 & -0.4157 & 0.2778 & -0.0975 \end{pmatrix}$$

inverse discrete cosine transform (IDCT)

the general inverse transform equation for DCT is

$$\boxed{I = D^T A\ D}$$

Calculate the IDCT transform for the image I(4,4)

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -0.29 & 0 & -0.71 \\ 0 & 0 & 0 & 0 \\ 0 & -0.71 & 0 & -1.71 \end{pmatrix}$$

$$\begin{array}{cccc} & D^T & A & D \end{array}$$

$$I = \begin{pmatrix} 0.5 & 0.65 & 0.5 & 0.27 \\ 0.5 & 0.27 & -0.5 & -0.65 \\ 0.5 & -0.27 & -0.5 & 0.65 \\ 0.5 & -0.65 & 0.5 & -0.27 \end{pmatrix} * \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -0.29 & 0 & -0.71 \\ 0 & 0 & 0 & 0 \\ 0 & -0.71 & 0 & -1.71 \end{pmatrix} * \begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{pmatrix}$$

$$I = \begin{pmatrix} 0.5 & 0.65 & 0.5 & 0.27 \\ 0.5 & 0.27 & -0.5 & -0.65 \\ 0.5 & -0.27 & -0.5 & 0.65 \\ 0.5 & -0.65 & 0.5 & -0.27 \end{pmatrix} * \begin{pmatrix} 0 & 0 & 0 & 0 \\ -0.38 & 0.38 & -0.38 & 0.38 \\ 0 & 0 & 0 & 0 \\ -0.92 & 0.92 & -0.92 & 0.92 \end{pmatrix}$$

$$I = \begin{pmatrix} -0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \end{pmatrix} + 0.5 \longrightarrow I = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

WAVELET

Filtering

After the image has been transformed into the frequency domain , we may want to modify the resulting spectrum

1. high-pass filter:- use to remove the low- frequency information which will tend to sharpen the image

2. low-pass filter:- use to remove the high- frequency information which will tend to blurring or smoothing the image

3. Band-pass filter:- use to extract the(low , high) frequency information in specific parts of spectrum

4. Band-reject:- use to eliminate frequency information from specific part of the spectrum

4. Discrete wavelet transform (DWT)

Definition:-The wavelet transform is really a family transform contains not just frequency information but also spatial information Numerous filters can be used to implement the wavelet transform and the two commonly used are the Haar and Daubechies wavelet transform .

Wavelet families

1. Haar wavelet
2. Daubechies families (D2,D3,D4,D5,D6,…..,D20)
3. biorthogonal wavelet
4. Coiflets wavelet
5. Symlets wavelet
6. Morlet wavelet

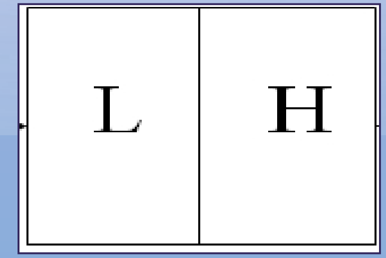
Implement DWT:- The discrete wavelet transform of signal or image is calculated by passing it through a series of filters called filter bank which contain levels of low-pass filter (L) and high- pass (H) simultaneously . They can be used to implement a wavelet transform by first convolving them with rows and then with columns.

The outputs giving the detail coefficients (d) from the high-pass filter and course approximation coefficients (a) from the low-pas filter .

decomposition algorithm

1. Convolve the low-pass filter with the rows of image and the result is (L) band with size (N/2,N).

2. Convolve the high-pass filter with the rows of image and the result is (H) band with size (N/2,N).

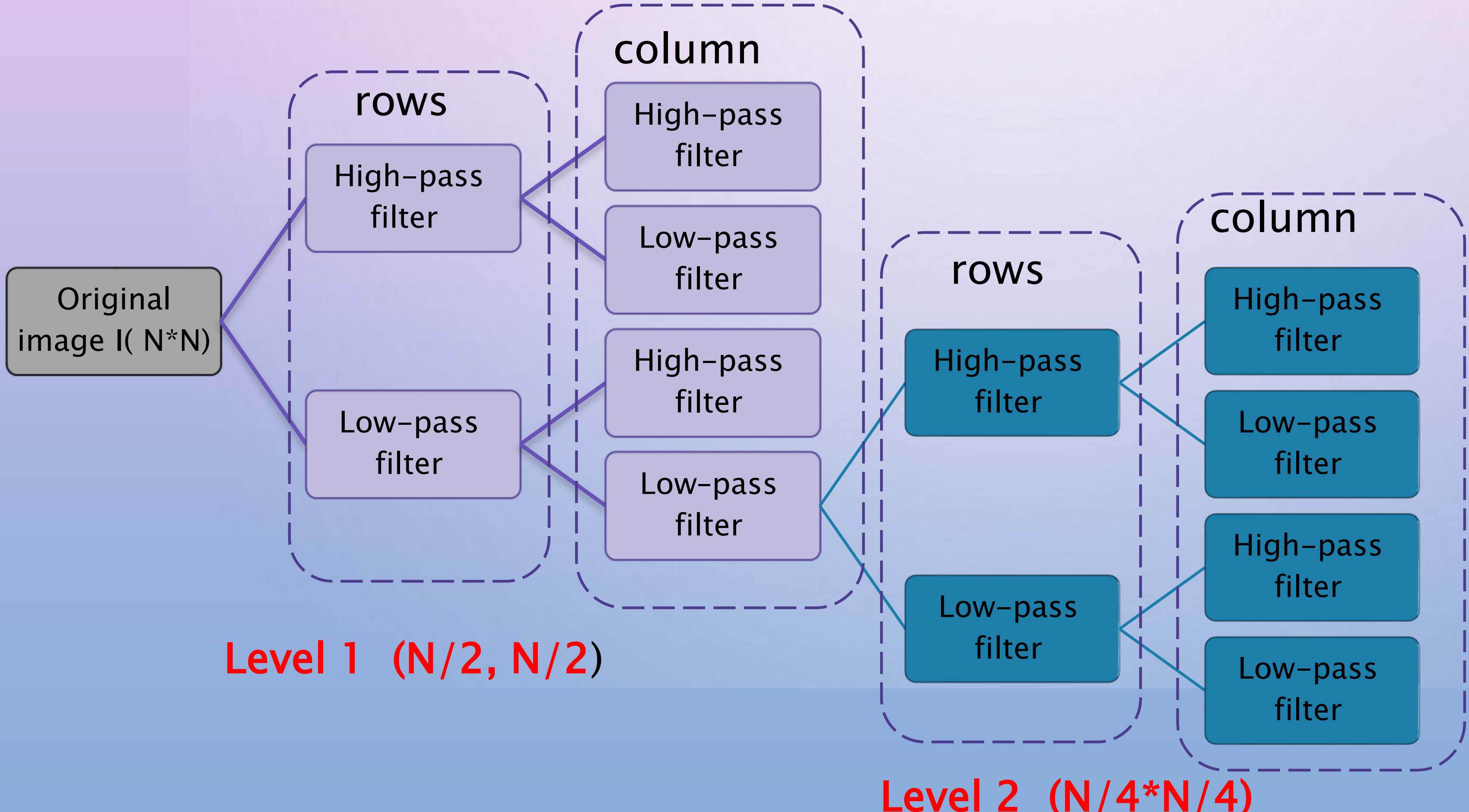


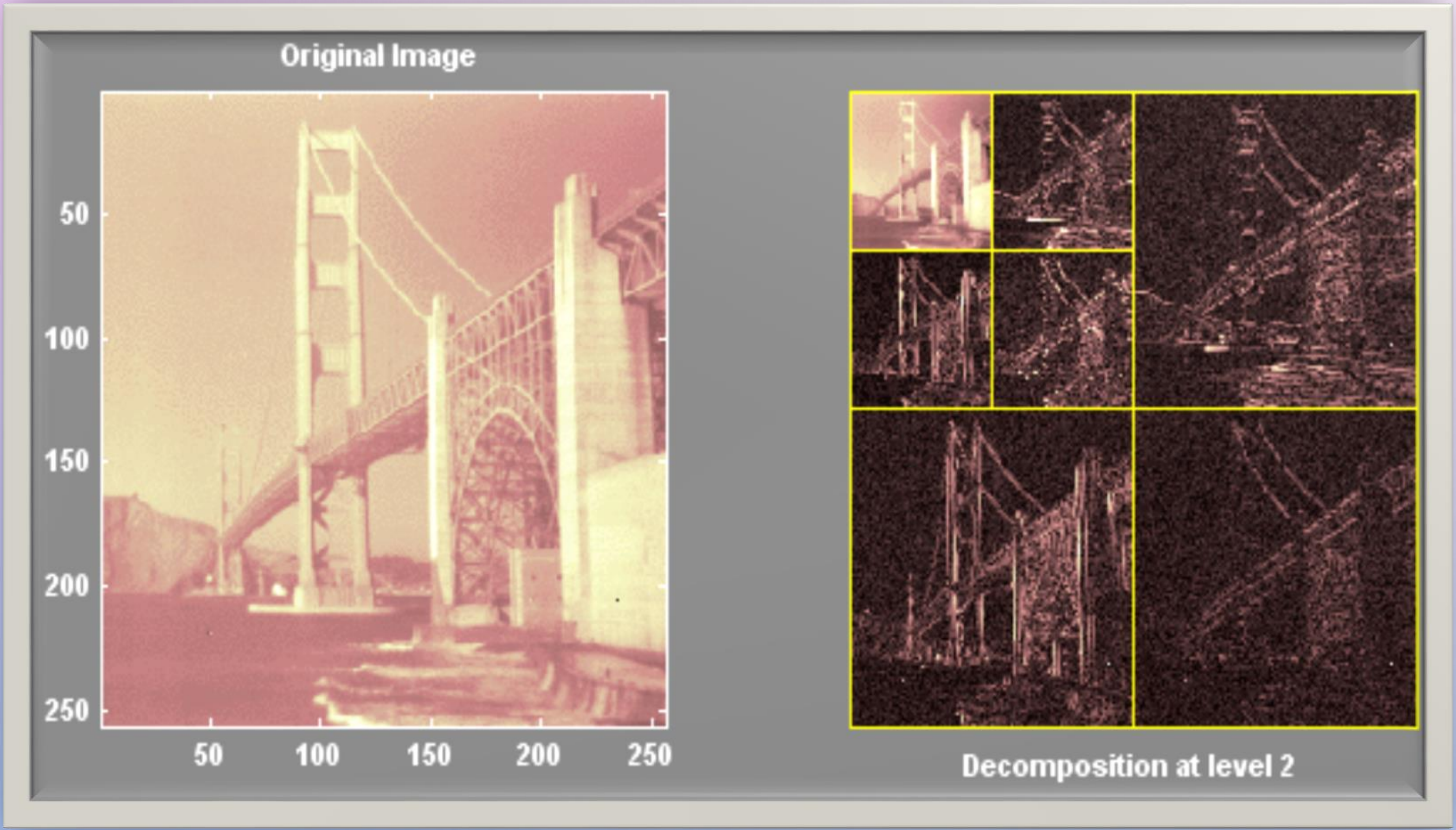
3. Convolve the columns of (L) band with low-pass filter and the result is (LL) band with size (N/2,N/2).
4. Convolve the columns of (L) band with high-pass filter and the result is (LH) band with size (N/2,N/2).
5. Convolve the columns of (H) band with low-pass filter and the result is (HL) band with size (N/2,N/2).
6. Convolve the columns of (H) band with high-pass filter and the result is (HH) band with size (N/2,N/2).

LL	HL
LH	HH

This six steps of wavelet decomposition are repeated to further increase the detailed and approximation coefficients decomposed with high and low pass filters <u>.</u>
<u>in each level we start from (LL) band .</u>

the DWT decomposition of input image I(N,N) show as follow for two levels of filter bank




Original Image
50
100
150
200
250
50
100
150
200
250
Decomposition at level 2

Haar wavelet transform

The Haar equation to calculate the approximation coefficients and detailed coefficients given as follow if si represent the input vector

Low -pass filter for Haar wavelet

L0 = 0.5

L1= 0.5

high-pass filter for Haar wavelet

H0 = 0.5

H1= - 0.5

$$\boxed{a_i = \frac{s_i + s_{i+1}}{2}} \quad \boxed{d_i = \frac{s_i - s_{i+1}}{2}}$$

approximation coefficients detailed coefficients

Calculate the Haar wavelet for the following image

117	101	104	138	161	152	170	132
120	111	125	143	154	154	151	136
113	144	140	162	168	179	184	151
108	151	156	181	159	145	152	134
110	151	154	135	114	95	100	121
135	169	134	108	107	110	112	147
149	150	125	132	129	156	163	159
135	107	132	149	141	150	156	135

L

109	121	156.5	151
115.5	134	154	143.5
128.5	151	173.5	167.5
129.5	168.5	152	143
130.5	144.5	104.5	110.5
152	121	108.5	129.5
149.5	129.5	142.5	161
121	140.5	145.5	145

H

8	−17	4.5	19
4.5	−9	0	7.5
−15.5	−11	−5.5	16.5
−21.5	−12.5	7	9
−20.5	9.5	9.5	−10.5
−17	13	−1.5	−17.5
−0.5	−2.5	−13.5	2
14	−8.5	−4.5	10.5

Then convolution the columns of result with low and high – pass filters

L

109	121	156.5	151
115.5	134	154	143.5
128.5	151	173.5	167.5
129.5	168.5	152	143
130.5	144.5	104.5	110.5
152	121	108.5	129.5
149.5	129.5	142.5	161
121	140.5	145.5	145

H

8	-17	4.5	19
4.5	-9	0	7.5
-15.5	-11	-5.5	16.5
-21.5	-12.5	7	9
-20.5	9.5	9.5	-10.5
-17	13	-1.5	-17.5
-0.5	-2.5	-13.5	2
14	-8.5	-4.5	10.5

LL1

112.25	127.5	155.25	147.25
129	159.75	162.75	155.25
141.25	132.75	106.5	120
135.25	135	144	153.25

HL1

6.25	-13	2.25	13.25
-18.5	-11.75	0.75	12.75
-18.75	11.25	4	-14
6.75	-5.5	-9	6.25

-3.25	-6.5	1.25	3.75
-0.5	-8.75	10.75	12.25
-10.75	11.75	-2	-9.5
14.25	-5.5	-1.5	7.75

LH1

1.75	-4	2.25	5.75
3	0.75	-6.25	3.75
-1.75	-1.75	5.5	3.5
-7,75	3	-4.5	-4.25

HH1

in the next level we start with (LL1) band

LL1

112.25	127.5	155.25	147.25
129	159.75	162.75	155.25
141.25	132.75	106.5	120
135.25	135	144	153.25

After apply convolution rows of (LL1) band with law-pass ,high pass filters

L2

119.875	151.25
144.375	159
137	113.25
135.125	148.75

H2

-7.625	4
-15.375	3.75
4.24	-6.75
0.125	-4.625

Then convolution the columns of result with low and high -pass filters

LL2

132.125	155.125
136.062	130.937

HL2

-11.5	3.875
2.1875	-5.687

-12.25	-3.875
0.9375	-17.687

LH2

3.875	0.125
2.0625	-1.062

HH2

So the decomposition for two levels are

LL2

132.125	155.125
136.062	130.937

HL2

-11.5	3.875
2.1875	-5.687

LH2

-12.25	-3.875
0.9375	-17.687

HH2

3.875	0.125
2.0625	-1.062

HL1

6.25	-13	2.25	13.25
-18.5	-11.75	0.75	12.75
-18.75	11.25	4	-14
6.75	-5.5	-9	6.25

LH1

-3.25	-6.5	1.25	3.75
-0.5	-8.75	10.75	12.25
-10.75	11.75	-2	-9.5
14.25	-5.5	-1.5	7.75

HH1

1.75	-4	2.25	5.75
3	0.75	-6.25	3.75
-1.75	-1.75	5.5	3.5
-7,75	3	-4.5	-4.25

inverse Haar wavelet transform

To reconstructs the original image we use the following equations

$$\mathbf{S_i = a_i + d_i} \qquad \mathbf{S_{i+1} = a_i - d_i}$$

First begin with (LL2) band and apply the above equations with each corresponded element of (HL2) band rows , and the same work apply between LH2 and HH2 rows

LL2		HL2	
132.125	155.125	-11.5	3.875
136.062	130.937	2.1875	-5.687
-12.25	-3.875	3.875	0.125
0.9375	-17.687	2.0625	-1.062
LH2		HH2	

120.625	143.625	159	151.25
138.25	133.874	125.25	136.624
-8.375	-16.125	-3.75	-4
3	-1.125	-18.749	-16.625

Then apply the above equations with columns of new matrix as showing

120.625	143.625	159	151.25
138.25	133.874	125.25	136.624
-8.375	-16.125	-3.75	-4
3	-1.125	-18.749	-16.625

112.25	127.5	155.25	147.25
129	159.75	162.75	155.25
141.25	132.75	106.5	120
135.25	135	144	153.25

LL1

LL^1 | HL^1

112.25	127.5	155.25	147.25	6.25	−13	2.25	13.25
129	159.75	162.75	155.25	−18.5	−11.75	0.75	12.75
141.25	132.75	106.5	120	−18.75	11.25	4	−14
135.25	135	144	153.25	6.75	−5.5	−9	6.25

−3.25	−6.5	1.25	3.75	1.75	−4	2.25	5.75
−0.5	−8.75	10.75	12.25	3	0.75	−6.25	3.75
−10.75	11.75	−2	−9.5	−1.75	−1.75	5.5	3.5
14.25	−5.5	−1.5	7.75	−7,75	3	−4.5	−4.25

HH^1

LH^1

118.5	106	114.5	140.5	157.5	153	160.5	134

118.5	106	114.5	140.5	157.5	153	160.5	134
110.5	147.5	148	171.5	163.5	162	168	142.5
122.5	160	144	121.5	110.5	102.5	106	134
142	128.5	129.5	140.5	135	153	159.5	147

−1.5	−5	−10.5	−2.5	3.5	−1	9.5	−2
2.5	−3.5	−8	−9.5	4.5	17	16	8.5
−12.5	−9	10	13.5	3.5	−7.7	−6	−13
6.5	22	−2.5	−8.5	−6	3	3.5	12

117
120
113
108

I=

117	101	104	138	161	152	170	132
120	111	125	143	154	154	151	136
113	144	140	162	168	179	184	151
108	151	156	181	159	145	152	134
110	151	154	135	114	95	100	121
135	169	134	108	107	110	112	147
149	150	125	132	129	156	163	159
135	107	132	149	141	150	156	135

Home work

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16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99