Frequancy Transformation

Lecture 7

Image Transforms

- An image transform can be applied to an image to convert it from one domain to another. Viewing an image in domains such as frequency or Hough space enables the identification of features that may not be as easily detected in the spatial domain. Common image transforms include:
- Hough Transform, used to find lines in an image
- Radon Transform, used to reconstruct images from fan-beam and parallel-beam projection data
- Discrete Cosine Transform, used in image and video compression
- Discrete Fourier Transform, used in filtering and frequency analysis
- Wavelet Transform, used to perform discrete wavelet analysis, denoise, and fuse images

Transformation is a function. A function that maps one set to another set after performing some operations.

- we use transform equation to make the transformation from spatial to frequency domain this equation use the inner product for getting the output image . The general form of transformation equation is

$$\mathbf{A} = \mathbf{H} \ \mathbf{I} \ \mathbf{H}^{\mathbf{T}}$$

A : output image matrix

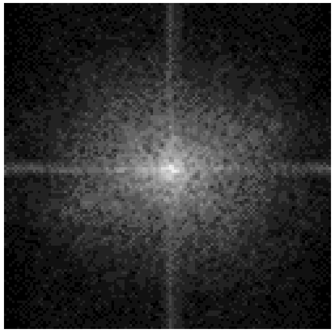
I : input image matrix

H : transform matrix

H^T: inverse of transform matrix

- The size of matrices are n*n were n is either 4 or 8 so the input image , output image and transform matrices are 4*4 block or 8*8 block

The Fourier Transform



- The Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components.
- The output of the transformation represents the image in the Fourier or frequency domain, while the input image is the spatial domain equivalent.
- In the Fourier domain image, each point represents a particular frequency contained in the spatial domain image.
- The Fourier transform is a representation of an image as a sum of complex exponentials of varying magnitudes, frequencies, and phases. The Fourier transform plays a critical role in a broad range of image processing applications, including enhancement, analysis, restoration, and compression..تحويل فوريه هو تمثيل صورة كمجموعة من الأسي المعقدة من مختلف الأحجام، والترددات، والمراحل

- The DFT is the sampled Fourier Transform and therefore does not contain all frequencies forming an image,
- but only a set of samples which is large enough to fully describe the spatial domain image.
- The number of frequencies corresponds to the number of pixels in the spatial domain image, i.e. the image in the spatial and Fourier domain are of the same size.

$$F(u,v) = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} I(x,y)\, e^{-j2\pi\frac{(ux+vy)}{N}} \qquad \textbf{...1}$$

J is the imaginary part of the complex number equal to $\sqrt{-1}$ while

A complex number is a number consisting of a real part and an imaginary part

$$e^{\pm jx} = cosx \pm j\, sinx$$

So we can write the discrete Fourier transform equation as

$$F(u,v) = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} I(x,y) \left[\cos\left(\frac{2\pi}{N}(ux+vy)\right) - j \sin\left(\frac{2\pi}{N}(ux+vy)\right) \right] \quad \ldots 2$$

From equation (1) for DFT we can construct matrix F (u , v) as follow equation

Formula 1 $$\boxed{F(u,v) = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{uv}{N}}}$$

Where u takes values 0,1,2, … N-1 along each column and v takes the same values along each raw

We need the following rules in construct the DFT matrix

Formula 2 $$\boxed{e^{-j\frac{2\pi}{N}\times X} = e^{-j\frac{2\pi}{N}\times[mod_N(X)]}}$$

Formula 3 $$\boxed{e^{\pm jx} = cosx \pm j\, sinx}$$

The DFT of 4*4 image can be obtain with **u= 0,1,2,3** along columns , and **v= 0,1,2,3** along rows

$$\begin{pmatrix} 00 & 10 & 20 & 30 \\ 01 & 11 & 21 & 31 \\ 02 & 12 & 22 & 32 \\ 03 & 13 & 23 & 33 \end{pmatrix}$$

By apply formula 1 we obtain

$$F = \frac{1}{\sqrt{4}} \begin{pmatrix} e^{-j\frac{2\pi}{4}\times 0*0} & e^{-j\frac{2\pi}{4}\times 1*0} & e^{-j\frac{2\pi}{4}\times 2*0} & e^{-j\frac{2\pi}{4}\times 3*0} \\ e^{-j\frac{2\pi}{4}\times 0*1} & e^{-j\frac{2\pi}{4}\times 1*1} & e^{-j\frac{2\pi}{4}\times 2*1} & e^{-j\frac{2\pi}{4}\times 3*1} \\ e^{-j\frac{2\pi}{4}\times 0*2} & e^{-j\frac{2\pi}{4}\times 1*2} & e^{-j\frac{2\pi}{4}\times 2*2} & e^{-j\frac{2\pi}{4}\times 3*2} \\ e^{-j\frac{2\pi}{4}\times 0*3} & e^{-j\frac{2\pi}{4}\times 1*3} & e^{-j\frac{2\pi}{4}\times 2*3} & e^{-j\frac{2\pi}{4}\times 3*3} \end{pmatrix}$$

From apply formula 2 we obtain

$$\mathbf{F} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{\pi}{2}} & e^{-j\pi} & e^{-j\frac{3\pi}{2}} \\ 1 & e^{-j\pi} & 1 & e^{-j\pi} \\ 1 & e^{-j\frac{3\pi}{2}} & e^{-j\pi} & e^{-j\frac{\pi}{2}} \end{pmatrix}$$

by apply formula 3 we obtain

$$e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$

$$e^{-j\pi} = \cos\pi - j\sin\pi = -1$$

$$e^{-j\frac{3\pi}{2}} = \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} = j$$

angle	cos	Sin
90(pi/2)	0	1
180(pi)	-1	0
270(3pi/2)	0	-1

So the discrete fourier transform (DFT)

$$\mathbf{F} = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}$$

From the DFT matrix we can see the transformation matrices are symmetric this mean

$$\mathbf{F} = \mathbf{F}^{T}$$

So the general transform equation for DFT is

$$\boxed{\mathbf{A} = \mathbf{F} \quad \mathbf{I} \quad \mathbf{F}}$$

EX1: Calculate the DFT transform for the image I(4,4)

$$I = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{A} = \frac{1}{4} \underbrace{\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}}_{F} \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{I} \underbrace{\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}}_{F}$$

$$\mathbf{A} = \frac{1}{4} \underbrace{\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}}_{F} \underbrace{\begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}}_{IF} \quad \mathbf{A} = \frac{1}{4} \underbrace{\begin{pmatrix} 4 & -4 & 4 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{F\,IF} = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Compute the real and imaginary parts of discrete Fourier transform of the image

$$I = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}$$

$$A = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 2 & -1-j & 0 & j-1 \\ 2 & -1-j & 0 & j-1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 4 & -2-2j & 0 & 2j-2 \\ -2j-2 & 2j & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 2j-2 & 2 & 0 & -2j \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -\frac{1+j}{2} & 0 & \frac{j-1}{2} \\ -\frac{1+j}{2} & \frac{j}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ \frac{j-1}{2} & \frac{1}{2} & 0 & -\frac{j}{2} \end{pmatrix} \quad Re(A) = \begin{pmatrix} 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad Im(A) = \begin{pmatrix} 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

Derive the DFT transform matrix of 8 * 8 image

by apply formula 1 we obtain

$$F=\frac{1}{\sqrt{8}}\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & e^{-j\frac{2\pi}{8}\times 1} & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 3} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 5} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 7}\\
1 & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 8} & e^{-j\frac{2\pi}{8}\times 10} & e^{-j\frac{2\pi}{8}\times 12} & e^{-j\frac{2\pi}{8}\times 14}\\
1 & e^{-j\frac{2\pi}{8}\times 3} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 9} & e^{-j\frac{2\pi}{8}\times 12} & e^{-j\frac{2\pi}{8}\times 15} & e^{-j\frac{2\pi}{8}\times 18} & e^{-j\frac{2\pi}{8}\times 21}\\
1 & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 8} & e^{-j\frac{2\pi}{8}\times 12} & e^{-j\frac{2\pi}{8}\times 16} & e^{-j\frac{2\pi}{8}\times 20} & e^{-j\frac{2\pi}{8}\times 24} & e^{-j\frac{2\pi}{8}\times 28}\\
1 & e^{-j\frac{2\pi}{8}\times 5} & e^{-j\frac{2\pi}{8}\times 10} & e^{-j\frac{2\pi}{8}\times 15} & e^{-j\frac{2\pi}{8}\times 20} & e^{-j\frac{2\pi}{8}\times 25} & e^{-j\frac{2\pi}{8}\times 30} & e^{-j\frac{2\pi}{8}\times 35}\\
1 & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 12} & e^{-j\frac{2\pi}{8}\times 18} & e^{-j\frac{2\pi}{8}\times 24} & e^{-j\frac{2\pi}{8}\times 30} & e^{-j\frac{2\pi}{8}\times 36} & e^{-j\frac{2\pi}{8}\times 42}\\
1 & e^{-j\frac{2\pi}{8}\times 7} & e^{-j\frac{2\pi}{8}\times 14} & e^{-j\frac{2\pi}{8}\times 21} & e^{-j\frac{2\pi}{8}\times 28} & e^{-j\frac{2\pi}{8}\times 35} & e^{-j\frac{2\pi}{8}\times 42} & e^{-j\frac{2\pi}{8}\times 49}
\end{pmatrix}$$

by apply formula 2 we obtain

$$F=\frac{1}{\sqrt{8}}\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & e^{-j\frac{2\pi}{8}\times 1} & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 3} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 5} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 7}\\
1 & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 0} & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 6}\\
1 & e^{-j\frac{2\pi}{8}\times 3} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 1} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 7} & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 5}\\
1 & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 0} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 0} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 0} & e^{-j\frac{2\pi}{8}\times 4}\\
1 & e^{-j\frac{2\pi}{8}\times 5} & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 7} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 1} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 3}\\
1 & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 0} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 2}\\
1 & e^{-j\frac{2\pi}{8}\times 7} & e^{-j\frac{2\pi}{8}\times 6} & e^{-j\frac{2\pi}{8}\times 5} & e^{-j\frac{2\pi}{8}\times 4} & e^{-j\frac{2\pi}{8}\times 3} & e^{-j\frac{2\pi}{8}\times 2} & e^{-j\frac{2\pi}{8}\times 1}
\end{pmatrix}$$

$$F=\frac{1}{\sqrt{8}}\begin{pmatrix}
1 & e^{-j\frac{\pi}{4}} & e^{-j\frac{\pi}{2}} & e^{-j\frac{3\pi}{4}} & e^{-j\pi} & e^{-j\frac{5\pi}{4}} & e^{-j\frac{3\pi}{2}} & e^{-j\frac{7\pi}{4}} \\
1 & e^{-j\frac{\pi}{2}} & e^{-j\pi} & e^{-j\frac{3\pi}{2}} & 1 & e^{-j\frac{\pi}{2}} & e^{-j\pi} & e^{-j\frac{3\pi}{2}} \\
1 & e^{-j\frac{3\pi}{4}} & e^{-j\frac{3\pi}{2}} & e^{-j\frac{\pi}{4}} & e^{-j\pi} & e^{-j\frac{7\pi}{4}} & e^{-j\frac{\pi}{2}} & e^{-j\frac{5\pi}{4}} \\
1 & e^{-j\pi} & 1 & e^{-j\pi} & 1 & e^{-j\pi} & 1 & e^{-j\pi} \\
1 & e^{-j\frac{5\pi}{4}} & e^{-j\frac{\pi}{2}} & e^{-j\frac{7\pi}{4}} & e^{-j\pi} & e^{-j\frac{\pi}{4}} & e^{-j\frac{3\pi}{2}} & e^{-j\frac{3\pi}{4}} \\
1 & e^{-j\frac{3\pi}{2}} & e^{-j\pi} & e^{-j\frac{\pi}{2}} & 1 & e^{-j\frac{3\pi}{2}} & e^{-j\pi} & e^{-j\frac{\pi}{2}} \\
1 & e^{-j\frac{7\pi}{4}} & e^{-j\frac{3\pi}{2}} & e^{-j\frac{5\pi}{4}} & e^{-j\pi} & e^{-j\frac{3\pi}{4}} & e^{-j\frac{\pi}{2}} & e^{-j\frac{\pi}{4}}
\end{pmatrix}$$

by apply formula 3 we obtain

$$e^{-j\frac{\pi}{4}}=\cos(45)-j\sin(45)=\frac{1}{\sqrt{2}}-\frac{j}{\sqrt{2}}$$

$$e^{-j\frac{3\pi}{4}}=\cos(135)-j\sin(135)=-\frac{1}{\sqrt{2}}-\frac{j}{\sqrt{2}}$$

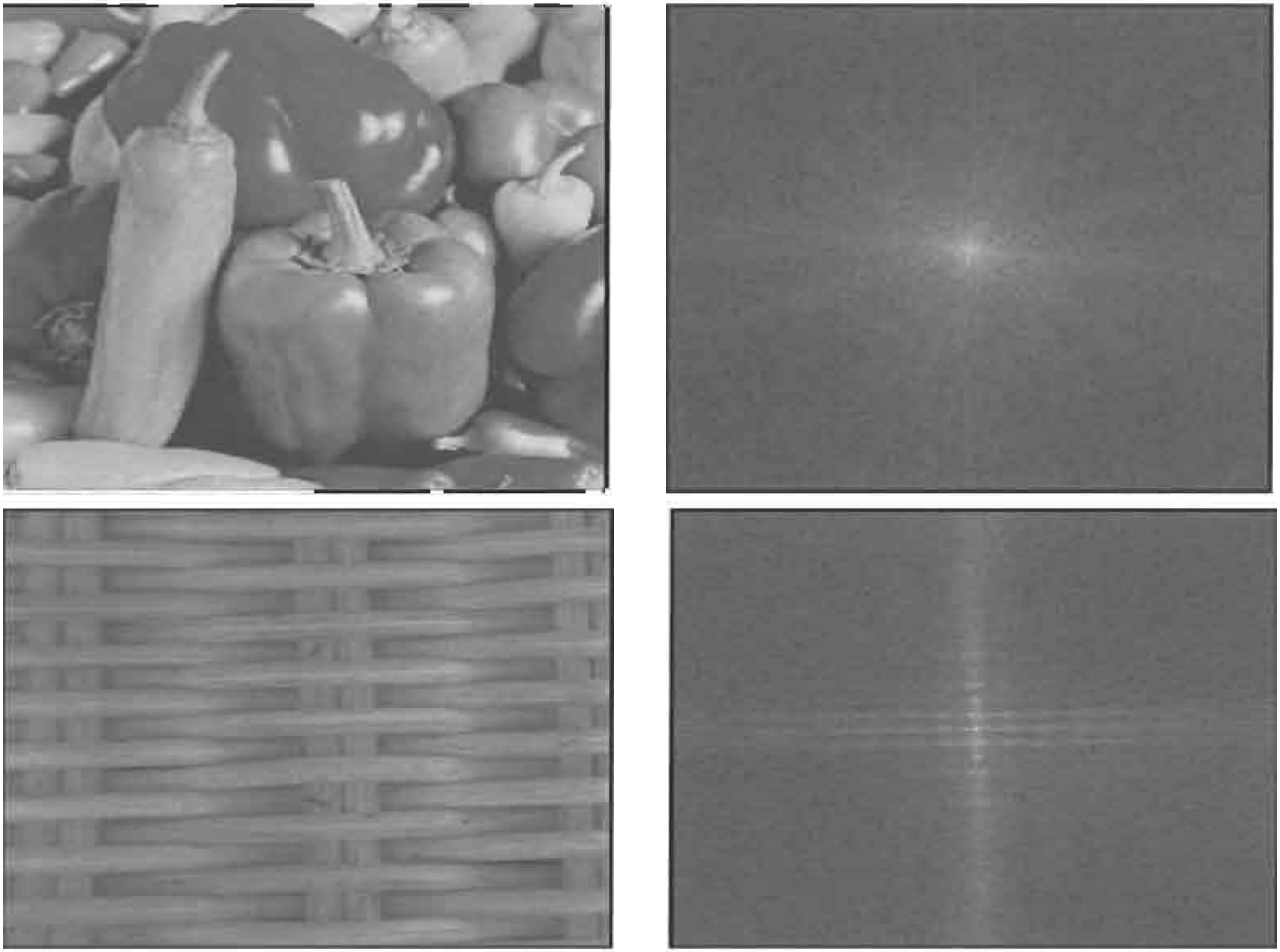
$$e^{-j\frac{5\pi}{4}}=\cos(225)-j\sin(225)=-\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}$$

$$e^{-j\frac{7\pi}{4}}=\cos(315)-j\sin(315)=\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}$$

angle	cos	Sin
45 (pi/4)	$1/\sqrt{2}$	$1/\sqrt{2}$
90(pi/2)	0	1
135(3 pi/4)	$-1/\sqrt{2}$	$1/\sqrt{2}$
180(pi)	-1	0
225(5pi/4)	$-1/\sqrt{2}$	$-1/\sqrt{2}$
270(3pi/2)	0	-1
315(7pi/4)	$1/\sqrt{2}$	$-1/\sqrt{2}$

So the discrete fourier transform (DFT)

$$
F = \frac{1}{\sqrt{8}}
\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & j & \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & j & \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & -j & \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & j & -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & j & -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & -j & \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}
\end{pmatrix}
$$



Inverse discrete Fourier transform (IDFT)

The inverse of discrete Fourier transform use to get the original image from transform image by the equation

$$\boxed{\overline{\mathbf{F}}(\mathbf{u},\mathbf{v}) = \frac{\mathbf{1}}{\sqrt{\mathbf{N}}}\mathbf{e}^{\mathbf{j2\pi\frac{uv}{N}}}}$$

So the IDFT matrix if N=4 is

$$\mathbf{F} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix}$$

And the general inverse transform

$$\boxed{\mathbf{I} = \overline{\mathbf{F}} \quad \mathbf{A} \quad \overline{\mathbf{F}}}$$

Calculate the IDFT of example 1 when N=4

$$\overline{\mathbf{F}} \qquad \mathbf{A} \qquad \overline{\mathbf{F}}$$

$$\mathbf{I} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix}$$

$$\mathbf{I} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix} \begin{pmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{I} = \frac{1}{4}\begin{pmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 \end{pmatrix} \longrightarrow \mathbf{I} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- https://homepages.inf.ed.ac.uk/rbf/HIPR2/wksheets.htm
- https://www.mathworks.com/help/images/fourier-transform.html