

Image Processing

Image Algebra And Spatial Filters

Image Algebra

There are two primary categories of algebraic operation applied to images: **Arithmetic** and **Logic**.

- ✓ Addition, Subtraction, division and multiplication comprise the arithmetic operation.
- ✓ While AND, OR and NOT make up the logic operation.
- ✓ These operations are performed on two images.
- ✓ Except for the NOT logic operation, This required only one image, and is done on a pixel –by- pixel basis.

$$s(x,y) = f(x,y) + g(x,y)$$

$$d(x,y) = f(x,y) - g(x,y)$$

$$p(x,y) = f(x,y) \times g(x,y)$$

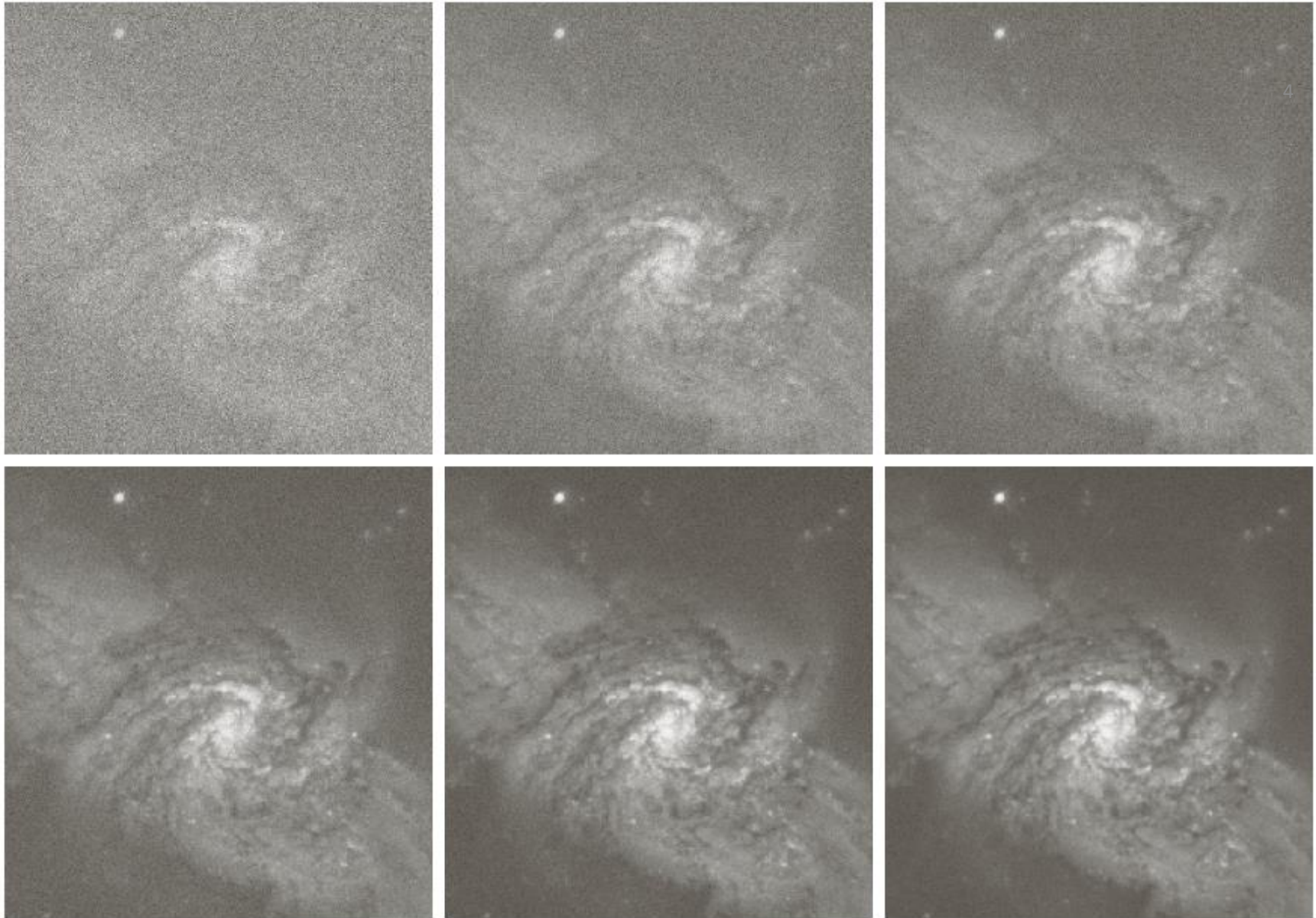
$$v(x,y) = f(x,y) \div g(x,y)$$

- **Addition**

- ✓ is used to combine the information in two images.
- ✓ Applications include development of image restoration algorithm for molding additive noise, and special effects, such as image morphing in motion pictures.

- **Subtraction**

- ✓ subtraction of two images is often used to detect motion.
- ✓ consider the case where nothing has changed in a sense; the image resulting from subtraction of two sequential image is filled with zero-a black image.If something has moved in the scene, subtraction produces a nonzero result at the location of movement.
- ✓ Applications include Object tracking , Medical imaging, Law enforcement and Military applications



a	b	c
d	e	f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Ex: To add images I1 and I2 to create I3.

5

3	4	7		6	6	6		3+6	4+6	7+6		9	10	13
3	4	5		4	2	6		3+4	4+2	5+6		7	6	11
2	4	6		3	5	5		2+3	4+5	6+5		5	9	11

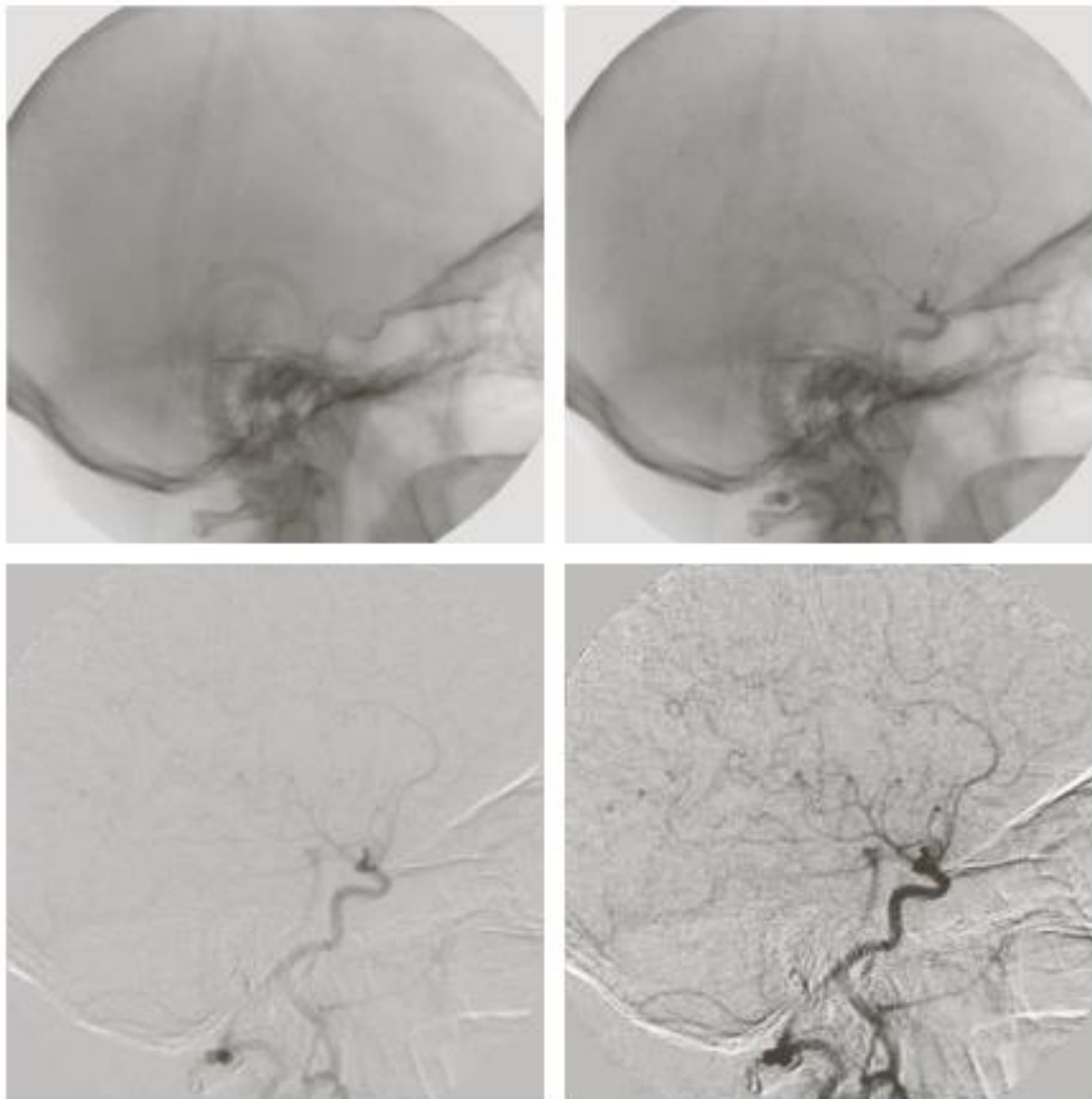


a. First Original image

b. Second Original

c. Addition of two images

Figure (2.3): Image Addition.



a	b
c	d

FIGURE 2.28

Digital subtraction angiography.

(a) Mask image.

(b) A live image.

(c) Difference between (a) and (b).

(d) Enhanced difference image.

(Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)



a Original scene



b Same scene later



c. Subtraction of scene a from scene b

Figure (2.4): Image Subtraction.



a. Cameraman image



b. X-ray image of hand



c. Multiplication of two images

Figure (2.5): Image Multiplication.



a. Original image

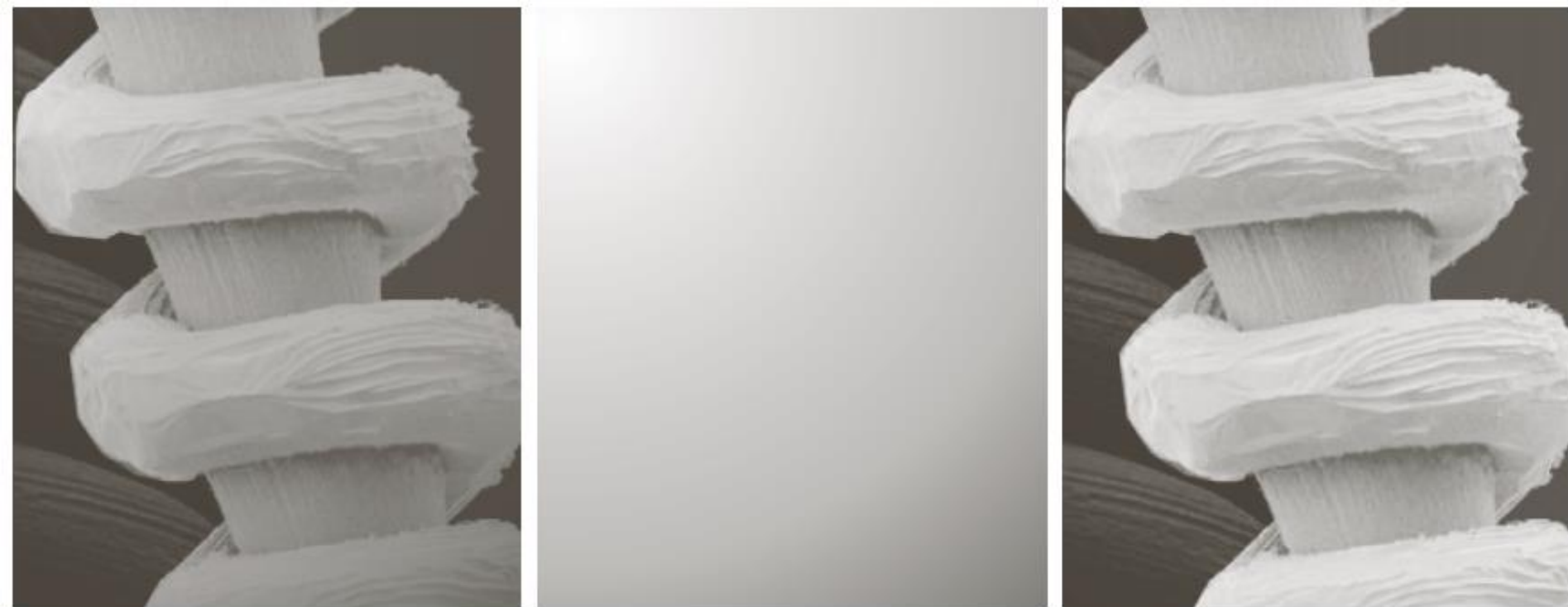


b. Image divided by value < 1



c. Image divided by value > 1

Figure (2.6): Image Division.



a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

* *The Logic Operation*

The Logic operation AND, OR and NOT form a complete set .meaning that any other logic operation (XOR, NOR, NAND) can be performed by combining of these basic elements. Note they operate in a bit-wise fashion on pixel data.

Ex We are performed a logic AND on two images. Two corresponding pixel values are 111_{10} in one image and 88_{10} in the second image .

The corresponding bit string $111_{10}=01101111_2$ and $88_{10}=01011000_2$.

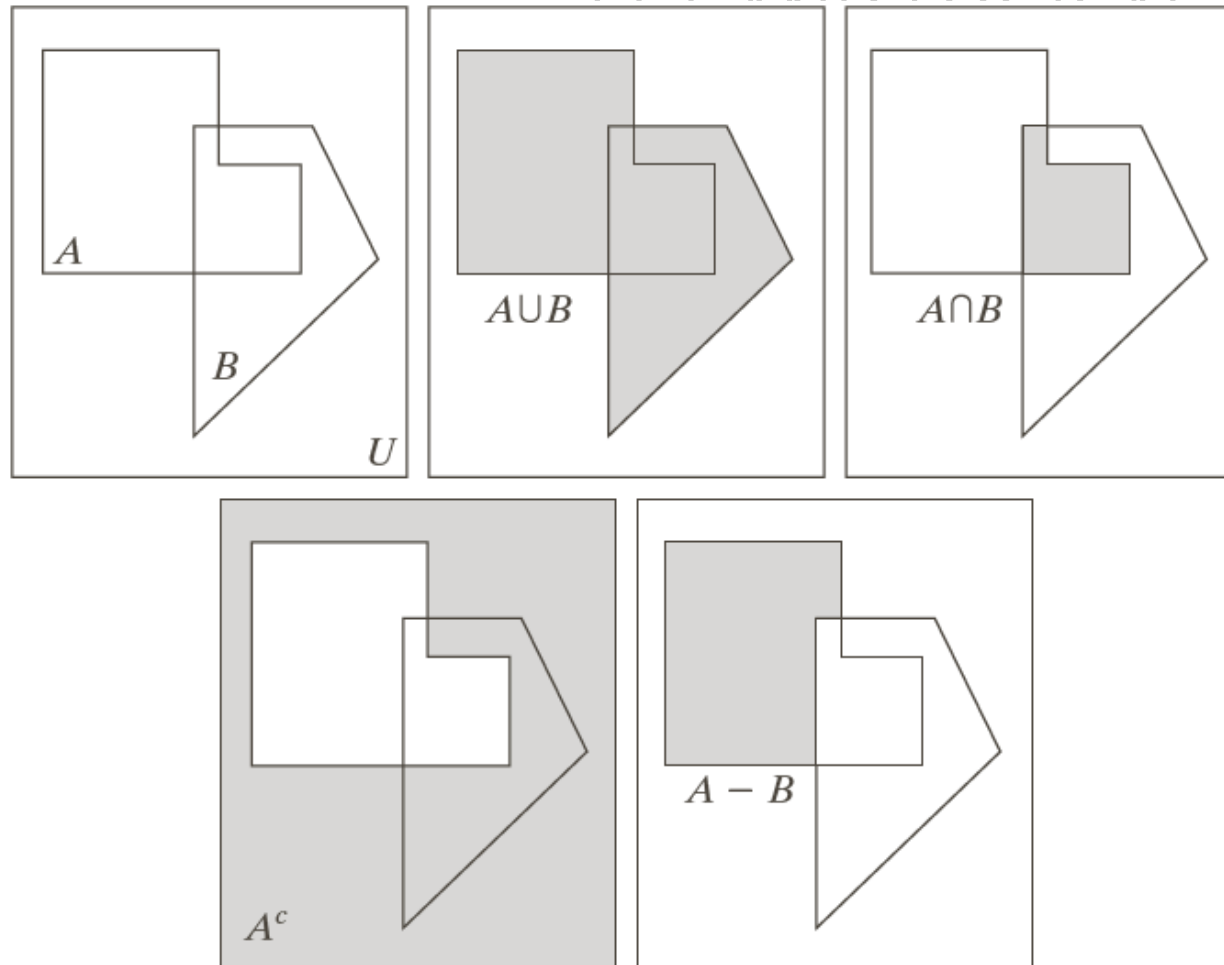
01101111₂

AND 01011000₂

01001000₂

- The logic operations AND, OR is used to **combine** the information in ¹¹two images. This may do for special effects.
- But more useful application for image analysis is to perform a mask operation.
- Use AND and OR as a simple method to extract Region of interest (ROI) from an image.(this process is called masking).
- The **Not** operator creates a negative of the original image by inverting each bit within each pixel values.
- **Example:** A white square ANDed with an image will allow only the portion of the image coincident with the square to appear in the output image with the background turned black; and a black square ORd with an image will allow only the part of the image corresponding to the black square to appear in the output image but will turn the rest of the image white.

This process is called **image masking**



a	b	c
d	e	

FIGURE 2.31

(a) Two sets of coordinates, A and B , in 2-D space. (b) The union of A and B . (c) The intersection of A and B . (d) The complement of A . (e) The difference between A and B . In (b)–(e) the shaded areas represent the member of the set operation indicated.

SET AND LOGICAL OPERATIONS

a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

SET AND LOGICAL OPERATIONS

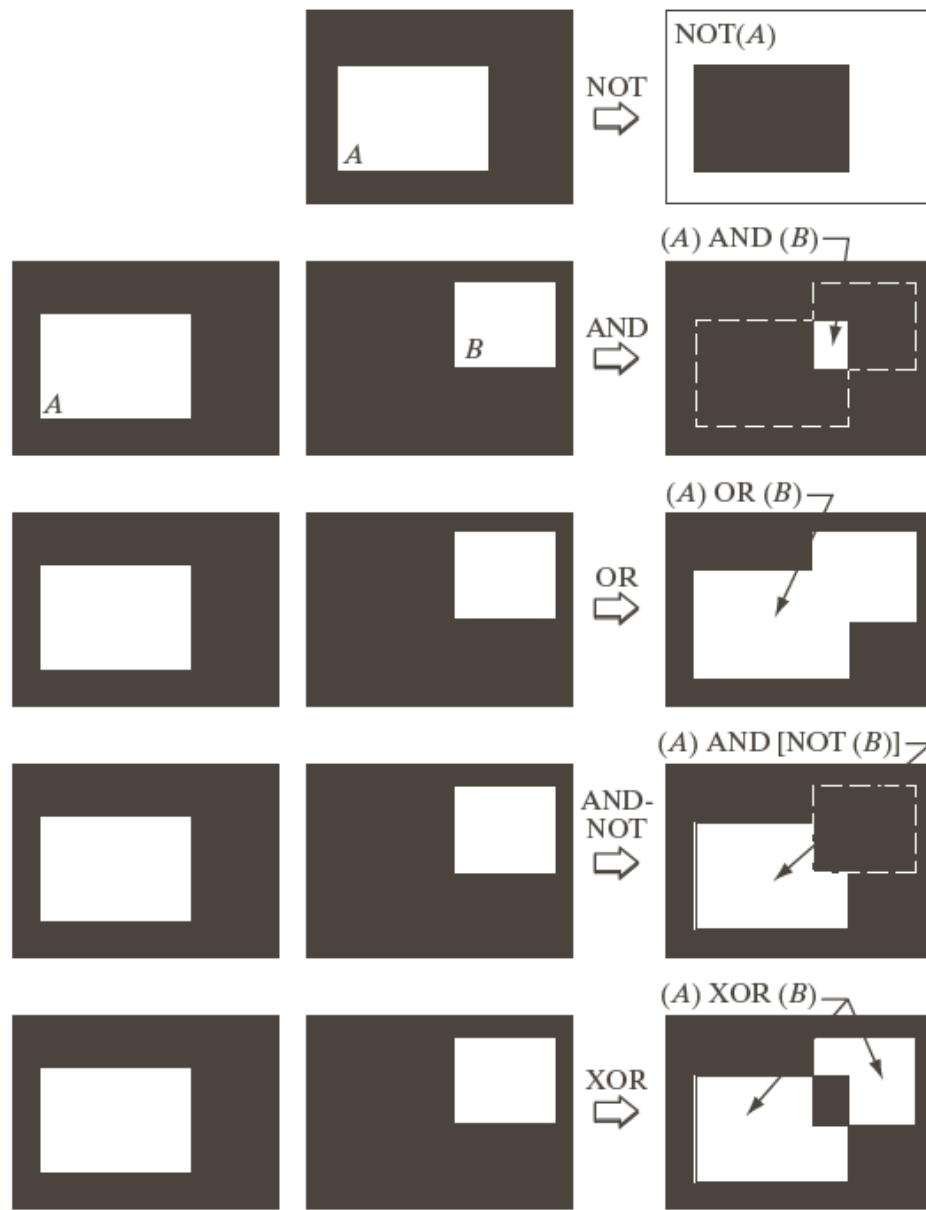


FIGURE 2.33

Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.



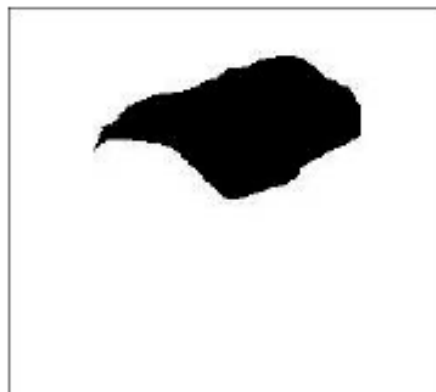
a. Original image



b. Image mask (AND)



c. ANDing a and b



d. Image mask (OR)



e. ORing a and d

Figure (2.7): Image masking.



a. Original image



b. Image after NOT operation.

Figure (2.8): Complement Image.

- Mean and median filters are used primarily to conceal or remove noise,
- a mean filter adds “softer” look to an image.
- The enhancement filter high lights edges and details within the image.

Spatial filters are implemented with convolution masks. Because convolution mask operation provides a result that is weighted sum of the values of a pixel and its neighbors, it is called a linear filter.

Effects the convolution mask can **be predicated based** on the general pattern.

For example:

- **If** the coefficients of the mask sum to one, the average brightness of the image will be retained.
- **If** the coefficients of the mask sum to zero, the average brightness will be lost and will return a dark image.
- **If** the coefficients of the mask are alternatively positive and negative, the mask is a filter that returns edge information only.
- **If** the coefficients of the mask are all positive, it is a filter that will blur the image.

The Mean filters

are essentially averaging filter.

They operate on local groups of pixel called **neighborhoods** and replace the center pixel with an average of the pixels in this neighborhood.

This replacement is done with a convolution mask such as the following 3X3 mask.

Note that

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

the coefficient of this mask sum to one.

the sum of all are = $(1/9+1/9+1/9+1/9+1/9+1/9+1/9+1/9+1/9) = 1$

- the image brightness will be retained , and the coefficients are all positive .
- This type of mean filter smooths out local variations within an image.
- it essentially a low pass filter,
- a low filter can be used to attenuate image noise that is composed primarily of high frequencies components.

10	4	2	7
8	22	11	6
1	5	7	9
4	6	8	0

×

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=

10	4	2	7
8	7.7	11	6
1	5	7	9
4	6	8	0

10	4	2	7
8	7.7	11	6
1	5	7	9
4	6	8	0

×

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=

10	4	2	7
8	7.7	14.2	6
1	5	7	9
4	6	8	0

10	4	2	7
8	7.7	14.2	6
1	5	7	9
4	6	8	0

×

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=

10	4	2	7
8	7.7	14.2	6
1	6.7	7	9
4	6	8	0

10	4	2	7
8	7.7	14.2	6
1	6.7	7	9
4	6	8	0

×

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=

10	4	2	7
8	7.7	14.2	6
1	6.7	6.9	9
4	6	8	0

MEDIAN FILTER

- ✓ The median filter is a non linear filter (order filter).
- ✓ These filters are based on a specific type of image statistics called order statistics.
- ✓ Typically, these filters operate on a small sub image, “Window”, and replace the center pixel value (similar to the convolution process).
- ✓ **Order statistics** is a technique that arranges the entire pixel in sequential order, given an $N \times N$ window (W) the pixel values can be ordered from smallest to the largest.

	5	5	6	5
	3	4	2	3
	3	4	7	7
	1	2	6	6

- We first sort the value in order of size (3,3,4,4,5,5,5,6,7) ;
- then we select the middle value , un this case it is 5.
- This 5 is then placed in center location.

A median filter can use a neighborhood of any size, but 3X3, 5X5 and 7X7 are typical.

5	5	6	5
3	5	5	3
3	4	7	7
1	2	6	6

5	5	6
3	<u>4</u>	5
3	4	7

5 5 6 3 4 5 3 4 7

The Sorting are :-

3 3 4 4 5 5 6 6 7

5	5	6
3	<u>5</u>	5
3	4	7

5	6	5
5	<u>2</u>	3
3	7	6

5 6 5 5 2 3 3 7 4

The Sorting are :-

2 3 3 4 5 5 5 6 7

5	6	5
5	<u>5</u>	3
3	7	6

5	5	6	5
3	<u>5</u>	<u>5</u>	3
3	4	7	7
1	2	6	6

3	5	5
3	<u>4</u>	7
1	2	6

3 5 5 3 4 7 1 2 6

The Sorting are :-

1 2 3 3 4 5 5 6 7

3	5	5
3	4	7
1	2	6

5	5	3
4	<u>7</u>	7
2	6	6

5 5 3 4 7 7 2 6 6

The Sorting are :-

2 3 4 5 5 6 6 7 7

5	5	3
4	<u>5</u>	7
2	6	6



5	5	6	5
3	<u>5</u>	<u>5</u>	3
3	<u>4</u>	<u>5</u>	7
1	2	6	6

Enhancement filter.

The enhancement filter concenter here including laplacian type and difference type. **This type is tend** to bring out , or enhance, details in the image.

The laplacian filter will enhance will details in all directions equally. the

two 3x3 convolution masks for the laplacian type filters are :-

0	-1	0
-1	5	-1
0	-1	0

0	-2	0
-2	5	-2
0	-2	0

The **Difference** filters will enhance details in the directions specific to the mask selected. here are four difference filter convolution masks. corresponding to the line in the vertical, horizontal, diagonal1 and diagonal2

Vertical

0	1	0
0	1	0
0	-1	0

Horizontal

0	0	0
1	1	-1
0	0	0

diagonal1

1	0	0
0	1	0
0	0	-1

diagonal2

0	0	1
0	1	0
-1	0	0

Apply the Laplaci filter mask1

2	3	5	4
1	5	6	7
2	3	4	11

2	3	5	4
1	<u>12</u>	<u>2</u>	7
2	3	9	11

2	3	5
1	5	6
2	3	9

×

0	-1	0
-1	5	-1
0	-1	0

=

$$-3-1+25-6-3=12$$

3	5	4
12	6	7
3	9	11

×

0	-1	0
-1	5	-1
0	-1	0

=

$$-12-5+30-7-4=2$$

Apply the Difference filter , main diagonal

2	3	5	4
1	5	6	7
2	3	9	11

2	3	5	4
1	<u>-2</u>	<u>-2</u>	7
2	3	9	11

2	3	5
1	5	6
2	3	9

×

1	0	0
0	1	0
0	0	-1

=

$$2+5-9=-2$$

3	5	4
5	6	7
3	9	11

×

1	0	0
0	1	0
0	0	-1

=

$$3+6-11=-2$$

2	3	5	4
1	5	6	7
2	3	9	11

Apply the Difference filter horizontal mask

30

2	3	5	4
1	0	-1	7
2	3	9	11

2	3	5
1	5	6
2	3	9

×

0	0	0
1	1	-1
0	0	0

=

$$1+5-6=0$$

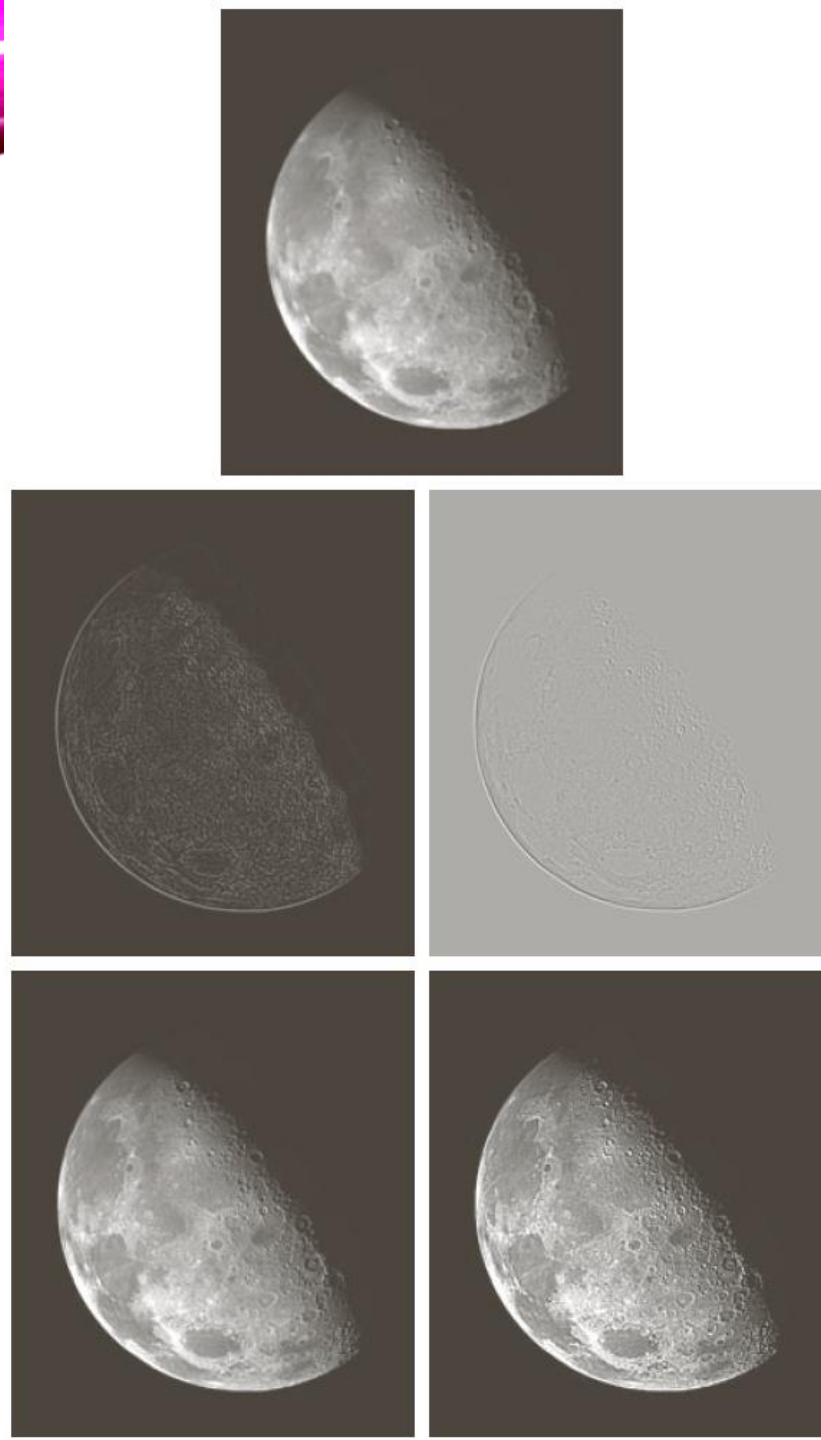
3	5	4
0	6	7
3	9	1

×

0	0	0
1	1	-1
0	0	0

=

$$0+6-7=-1$$



a	
b	c
d	e

FIGURE 3.38

(a) Blurred image of the North Pole of the moon.

(b) Laplacian without scaling.

(c) Laplacian with scaling.

(d) Image sharpened using the mask in Fig. 3.37(a).

(e) Result of using the mask in Fig. 3.37(b).

(Original image courtesy of NASA.)