An approach à la de Vries to compact Hausdorff spaces and closed relations

M. Abbadini[1,*] and I. Di Liberti[2]

[1] University of Salerno, Italy. Department of Mathematics.
marco.abbadini.uni@gmail.com

[2] Stockholm University, Sweden. Department of Mathematics.
diliberti.math@gmail.com

After Tarski's seminal works, topological accounts have become more and more relevant in the general theory of modal logic. The duality between modal algebras and coalgebras for the Vietoris functor on Stone spaces, which generalizes Stone duality for Boolean algebras, sets the foundations of such theory and motivated a deeper understanding of the structure of the category of coalgebras. Analogous results have been obtained in *positive* modal logic, when one replaces Stone spaces with Priestley spaces (or, equivalently, spectral spaces), and the Vietoris functor with appropriate versions of it in the Priestley/spectral setting.

Via categorical methods (rather than algebraic/equational ones) we show that dualities between category of coalgebras for Vietoris functors and equational classes of algebras also occur when we remove the hypothesis that the spaces are totally disconnected.

Theorem 1. *The opposite of the category of coalgebras for the Vietoris functor on compact Hausdorff spaces is monadic over* Set.

Moreover, we prove analogous results where one replaces compact Hausdorff spaces with Nachbin's compact ordered spaces (or, equivalently, stably compact spaces) and replaces the Vietoris functor with appropriate Vietoris-like functors on these categories.

Let $\mathsf{Alg}(T)$ denote the category of T-algebras and let $U\colon \mathsf{Alg}(T) \to \mathsf{A}$ be the forgetful functor. Our proofs build on the following fact: (while composition of monadic functors in general fails to be monadic) under certain conditions on an endofunctor $T\colon \mathsf{C} \to \mathsf{C}$, the composition of $U\colon \mathsf{Alg}(T) \to \mathsf{C}$ with every monadic functor $\mathsf{C} \to \mathsf{D}$ is still monadic.

*Speaker.