Representability in Categorical Quantum Probability

Tobias Fritz, Antonio Lorenzin

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Categorical probability, and in particular Markov categories, has attracted much interest in recent years. This synthetic approach has proven fruitful in several areas, such as statistics [1], graphical models [2], ergodic theory [3], and more. Regarding the quantum perspective, Parzygnat introduced the concept of Quantum Markov Categories in [4], where he focused on finite-dimensional C*-algebras. His work explains that, in the quantum perspective, probability is encoded by well-behaved subcategories rather than by the whole Quantum Markov Categories.

This talk offers an alternative description of Quantum Markov Categories and their subcategories of interest, which we call *pictures*. This view has the advantage of avoiding any distinction between odd and even morphisms. We then discuss a particular example which comprises infinite-dimensional (pre)-C*-algebras, and argue that the theory of such algebras can benefit from this new synthetic perspective.

More precisely, we provide three different universal descriptions of the (algebra of continuous functions on the) state space of a C*-algebra. All three arise from a quantum generalization of *representability* in the sense of [1]. In particular, representability implies that the category under consideration is a Kleisli category [1, Theorem 3.19]). The most interesting version of representability is given by the quantum de Finetti theorem. We conclude by noting that this viewpoint allows us to understand the deterministic behavior of the state space with respect to any object.

References

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