The universal property of the coKleisli-Kleisli adjunction

Fosco Loregian
Tallinna Tehnikaülikool

November 5, 2025

Every adjunction $F : \mathcal{C} \rightleftarrows \mathcal{D} : G$ induces an adjunction

$$\hat{G} : \mathrm{coKl}(FG) \rightleftarrows \mathrm{Kl}(GF) : \hat{F},$$

between the coKleisli category of the comonad FG, and the Kleisli category of the monad GF where, intriguingly, the *left* adjoint $\hat{G}$ acts as the *right* adjoint G. There exists, as a consequence, a certain canonical square of adjunctions

Why does this happen? What is this sorcery? Does it have a universal property—and if so, which one?

j/w Nathanael Arkor (Tallinna Tehnikaülikool) and Ülo Reimaa (Tartu Ülikool).