

A type theory beyond animae

Michele Riva

Recently, a lot of work has been done towards the construction of a type theory for (∞) -categories. Martin-Löf dependent type theory (MLTT) is not suitable for this purpose, as its rules are, by default, the same in all contexts. This makes the framework of MLTT suitable to describe the theory of groupoids (where types in context X stand for isofibrations of groupoids over X , whose theory does not depend on X), but category theory heavily differs from groupoid theory under this aspect: on a category \mathbf{C} , objectwise properties clearly do not lift to functorial properties in general.

In order to create a suitable setting to develop the theory of categories, we introduce a new class of contexts, called animae, with the fundamental property of being those contexts over which the theory is the same as the theory in empty context. This allows for richer structures to be defined within the type theory whilst retaining some of the nice properties of MLTT at least over animae.

In this setting, we develop some of an axiomatic approach to (∞) -categories: we will introduce internal morphisms, functor categories, groupoid cores, and the characterisation of animae as groupoids, namely those types such that all morphisms are invertible. In particular, this will allow us to formalise the notion of objectwise statement within the theory, a key feature we did not have access to in MLTT.

This talk presents joint work in collaboration with Denis-Charles Cisinski and Tashi Walde.