

# Overview and examples of braided monoidal bicategories

The notion of a braided monoidal category arises naturally from a higher-categorical point of view, as a tricategory with a single object and a single morphism. This process is the reverse of the so-called “horizontal categorification” or delooping. Proceeding “vertically” in the categorification leads us to braided monoidal bicategories.

The braided structure plays a central role in understanding the limitations of strictification for tricategories. Indeed, it is, in some sense, precisely the presence of the braiding that prevents a complete strictification in this context. In this talk, I will discuss more classical ([GPS95]) and more recent ([Gur11]) semi-strictification results for both monoidal and braided monoidal bicategories, emphasizing the main conceptual and technical obstacles.

A key tool in studying these structures is the string diagrammatic formalism, and I will explain how we can use it and why it works. The use of string diagrams for computations in 2-dimensional category theory has proved extremely powerful, as illustrated in the work of Garner and Shulman [GS16], where enrichments in monoidal bicategories are carefully developed. The braiding also plays an essential role in defining several fundamental constructions in this theory, such as the opposite and the tensor product of enriched bicategories.

As a motivating case study, we will consider the 2-category  $\text{Add}$  of additive categories, and we will prove that it carries a braided (indeed, symmetric) monoidal structure. This fact enables the study of bicategories enriched over  $\text{Add}$ , providing a rich setting for higher-dimensional algebraic structures, which, time permitting, will also be mentioned.

The talk will be based on [Car25] and on a work in progress for the main applications.

## References

- [Car25] Nicola Carissimi. “Enriched bi(co)ends”. In: (Sept. 2025). DOI: [10.48550/arXiv.2509.05070](https://doi.org/10.48550/arXiv.2509.05070). URL: <https://arxiv.org/pdf/2509.05070.pdf>.
- [GPS95] R. Gordon, A. J. Power, and Ross Street. *Coherence for tricategories*. English. 558th ed. Vol. 117. United States: American Mathematical Society, Sept. 1995. ISBN: 9780821803448. DOI: [10.1090/memo/0558](https://doi.org/10.1090/memo/0558).
- [GS16] Richard Garner and Michael Shulman. *Enriched categories as a free cocompletion*. 2016. DOI: <https://doi.org/10.1016/j.aim.2015.11.012>. URL: <https://www.sciencedirect.com/science/article/pii/S0001870815004715>.
- [Gur11] Nick Gurski. *Loop spaces, and coherence for monoidal and braided monoidal bicategories*. 2011. arXiv: 1102.0981 [math.CT]. URL: <https://arxiv.org/abs/1102.0981>.