Comma 2-comonad and its cousins

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by

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The operation which associates to any pair of categories $\mathscr{A}$ and $\mathscr{B}$ a category $(\mathscr{A}, \mathscr{B})$ which came to be known by the name "comma category" was introduced by Lawvere in his thesis for the purpose of a foundational clarification, in particular of the notion of *adjointness*. Since then the comma construction turned out to be fundamental in many areas of category theory from computing Kan extensions to the general calculus of adjoints and limits. Despite its importance, there are only few places in the literature until today which deal with the comma construction from the perspective of the formal category theory. In my talk I describe a comma 2-comonad on the 2-category whose objects are functors, 1-cell are colax squares and 2-cells are their transformations. After describing its Eilenberg-Moore 2-category of colax coalgebras, colax morphisms between them and their transformations I show how many fundamental constructions in category theory, homotopy theory, algebraic geometry and theoretical computer science like (co)reflections, fully faithful adjoint triples, distributive adjoint strings, distributive laws, comprehension structures, Frobenius functors, cohesive toposes, universal covering spaces, schemes etc. naturally fit in this context. Then I proceed by describing the cousin of the comma 2-comonad - the associated fibration 2-monad, and I show that it is admissible in a sense of Bunge and Funk. Finally, I show that there exists a canonical distributive law between the comma 2-comonad and the associated split fibration 2-monad and I describe its corresponding two-sided Kleisli 2-category.