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Vicsek Model

A 2D model for modelling swarm behaviour

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Content

1) Motivation and Theory

Q: Why is it interesting?

Applications.[1,2]

- Living systems like the collective motion of birds, fish schools, bacteria colonies or cellular migrations
- Physical systems like ferromagnetism
- Simple model

[1] T. Vicsek. *Novel Type of Phase Transition in a System of Self-Driven Particles*. VOLUME 75. NUMBER 6. PHYSICAL REVIEW LETTERS, 1995

[2] F. Ginelli. *The Physics of the Vicsek Model*. VOLUME 225. NUMBER 11–12. The European Physical Journal Special Topics, 2016

1) Motivation and Theory

Q: Why is it so simple?

- Each particles direction is influenced by only it's neighboring particles within a radius R
- New position within one molecular dynamics step is given by equation of motion

$$\mathbf{r}^i_{t+\Delta t} = \mathbf{r}^i_t + \mathbf{v}^i_t \cdot \Delta t \tag{1}$$

with $\mathbf{v}^i_t = v \cdot \mathbf{u}^i_t$, where $v = \mathrm{const.}$ and $\mathbf{u}^i_t = \begin{pmatrix} \cos\left(\theta^i_t\right) \\ \sin\left(\theta^i_t\right) \end{pmatrix}$

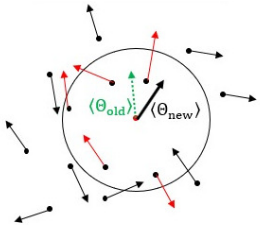
1) Motivation and Theory

- Updating θ_t^i using the expression

$$\theta_{t+\Delta t}^i = \langle \theta_t^j \rangle_{|r_i - r_j| < R} + \sqrt{2 D_{\mathrm{rot}} \Delta t} \cdot \eta_t, \tag{2}$$

  with D_{rot} as rotational diffusion coefficient and η_t as Gaussian noise
- $\langle \theta_t^j \rangle_{|r_i - r_j| < R}$ describes an average direction obtained via

$$\langle \theta_t^j \rangle_{|r_i - r_j| < R} = \arctan \left(\frac{\langle \sin(\theta_t^j) \rangle_{|r_i - r_j| < R}}{\langle \cos(\theta_t^j) \rangle_{|r_i - r_j| < R}} \right) \tag{3}$$



1) Motivation and Theory

Order Parameter.

- To study phase transitions we analyse the order parameter v_{a}

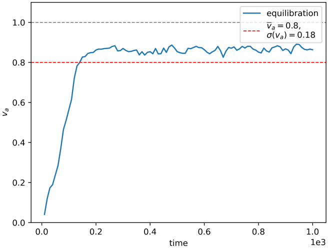
$$v_{\mathrm{a}} = \frac{1}{Nv} \cdot \left| \sum_{i}^{N} \mathbf{v}^{i} \right| \in [0, 1] \tag{4}$$

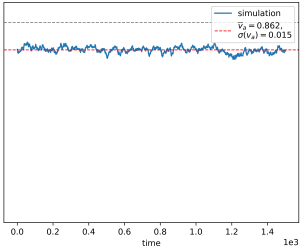
- Order parameter v_{a} inhibits (all) physics of the model

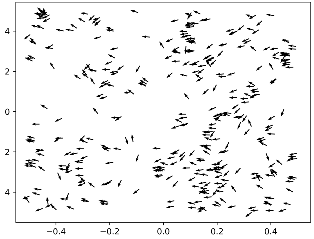
2) Periodic Boundaries

Trajectory. Example.

- $\rho = 400, v = 0.03, R = 0.01, D_{\text{rot}} = 0.01, \Delta t = 1.0$
- Representative trajectory is shown
- Equilibration in $\approx$ 200 steps



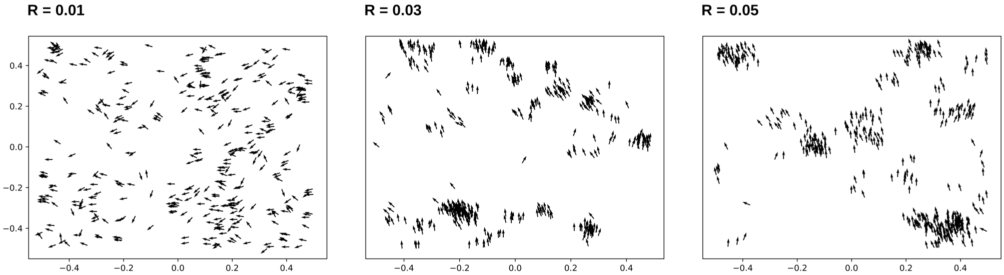




2) Periodic Boundaries

Configurations. *R*-dependence.

- $\rho = 400, v = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0$
- Bigger $R \Rightarrow$ bigger flocks



2) Periodic Boundaries

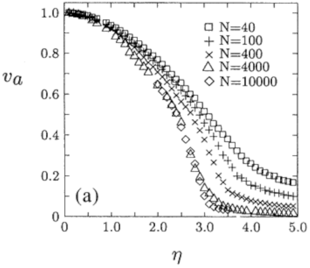
Phase transitions. Simulation settings.

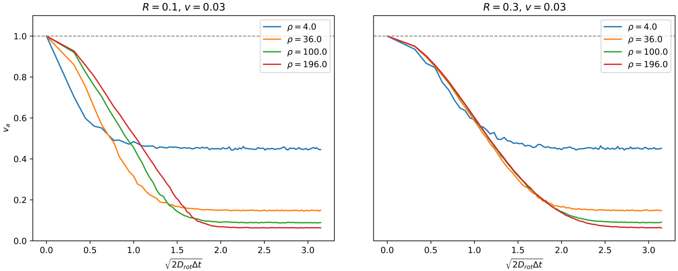
- $N_{\text{sim}} = 3000, N_{\text{eq}} = 1000, N_{\text{save}} = 1$
- Fix v: vary
  - R and D_{rot}
  - ρ and D_{rot}
  - ρ and R
- Plot $\overline{v}_a$ against varying parameters

2) Periodic Boundaries

Phase transitions. Comparing with reference.

- Reference: $v = 0.03, R = 1.0, D_{\text{rot}} = 0.01, \Delta t = 1.0, (N, L) = (40, 3.1), (100, 5), (400, 10), (4000, 31.6), (10000, 50)$
- Our system: $v = 0.03$



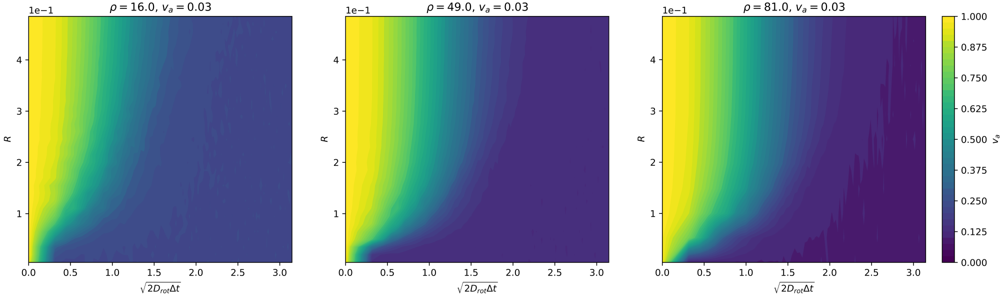


(left) Tamás Vicsek, András Czirók, Eshel Ben-Jacob, Inon Cohen, and Ofer Shochet. Novel Type of Phase Transition in a System of Self-Driven Particles. Phys. Rev. Lett. 75, 1226 – Published 7 August 1995

2) Periodic Boundaries

Phase transitions. 2D Levels in parameter space.

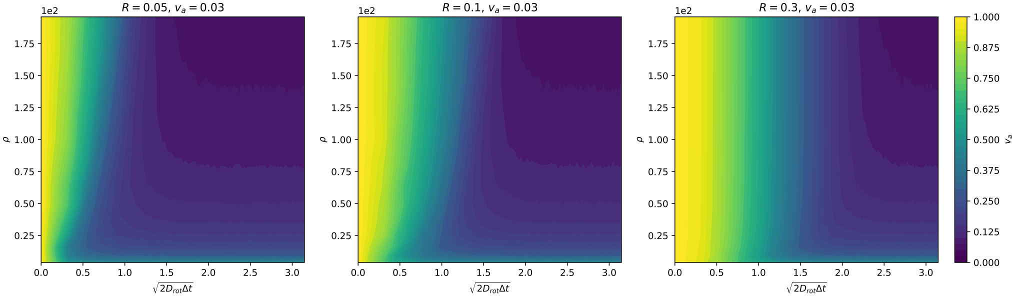
- R against $\sqrt{2D_{\text{rot}}\Delta t}$



2) Periodic Boundaries

Phase transitions. 2D Levels in parameter space.

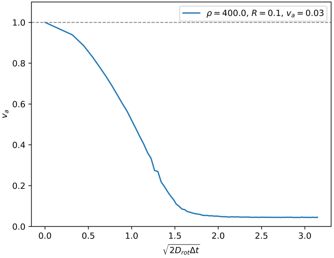
- ρ against $\sqrt{2D_{\text{rot}}\Delta t}$

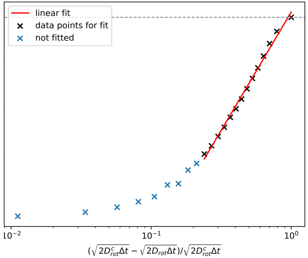


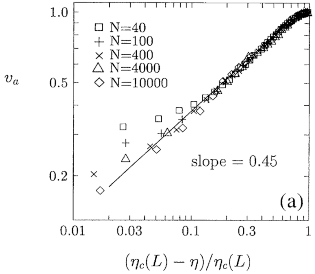
2) Periodic Boundaries

Phase transitions. Example for critical exponent.

- $v_a \approx \left(\sqrt{2D^c_{\text{rot}}\Delta t} - \sqrt{2D_{\text{rot}}\Delta t}\right)^\beta$
- Here: $\sqrt{2D^c_{\text{rot}}\Delta t} \approx 1.7$ (see figure on the left)
- Linear fit: slope $= 0.449(10)$, offset $= 1.023(9) \Rightarrow \beta = 0.449(10)$







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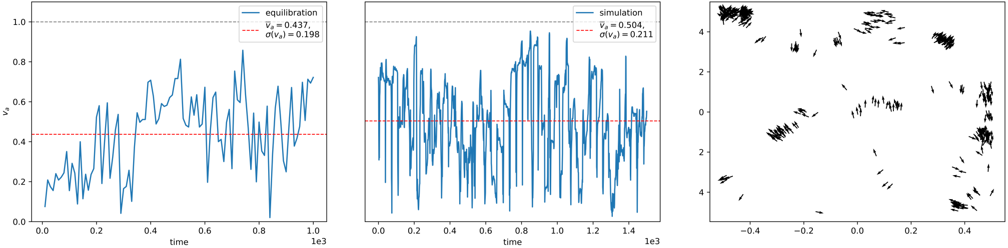
3) Reflecting Boundaries

Trajectory. Example.

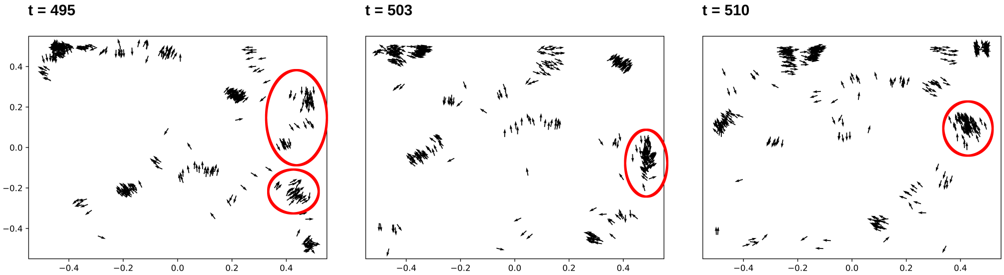
- $\rho = 400, v = 0.03, R = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- Representative trajectory is shown
- Large fluctuations (collisions with boundary)



3) Reflecting Boundaries

Trajectory. Interesting Collisions.

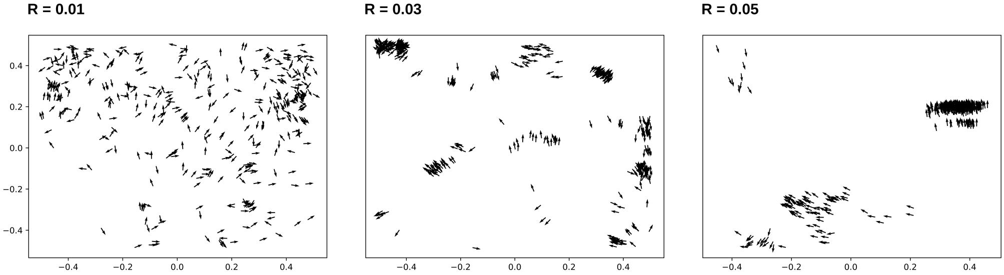
- $\rho = 400, v = 0.03, R = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- Two incoming flocks $\rightarrow$ one outgoing flock



3) Reflecting Boundaries

Configurations. R-dependence.

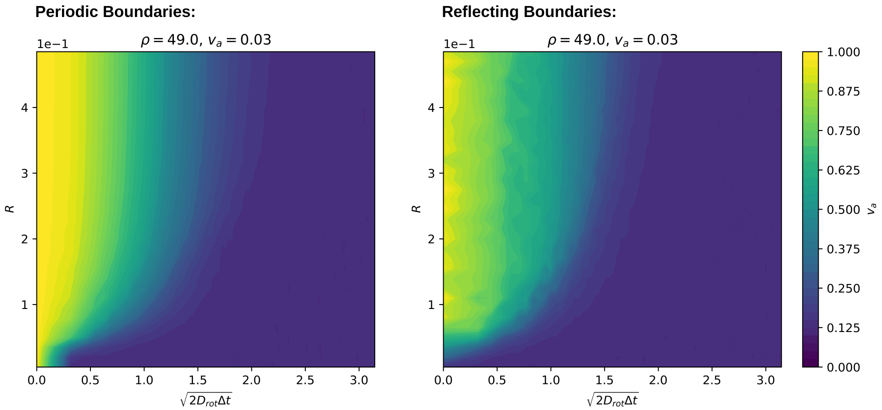
- $\rho = 400, v = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- Bigger $R \Rightarrow$ bigger flocks
- Flocking stronger compared to periodic boundaries



3) Reflecting Boundaries

Phase transitions. 2D Levels in parameter space.

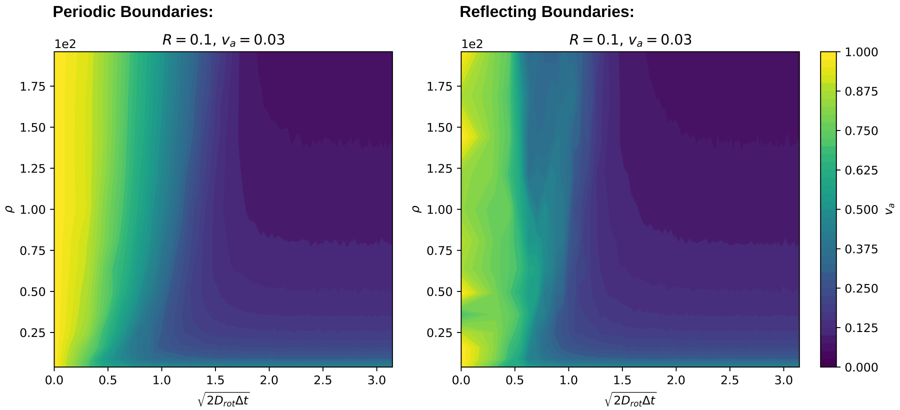
- R against $\sqrt{2D_{\text{rot}}\Delta t}$



3) Reflecting Boundaries

Phase transitions. 2D Levels in parameter space.

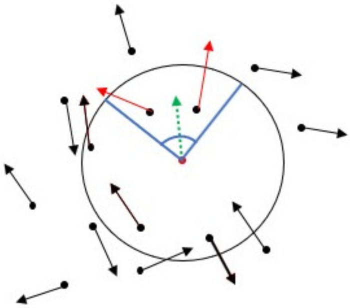
- ρ against $\sqrt{2D_{\text{rot}}\Delta t}$



4) Periodic Boundaries with Vision

Implementation.

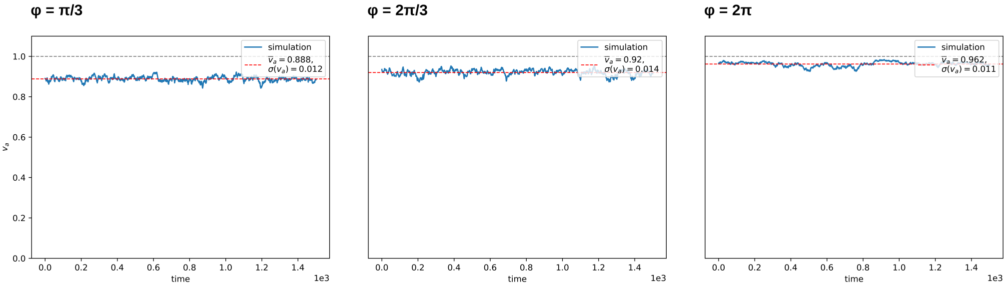
- Just particles in circle segment defined by vision angle φ are considered



4) Periodic Boundaries with Vision

Trajectory. Comparing Angles.

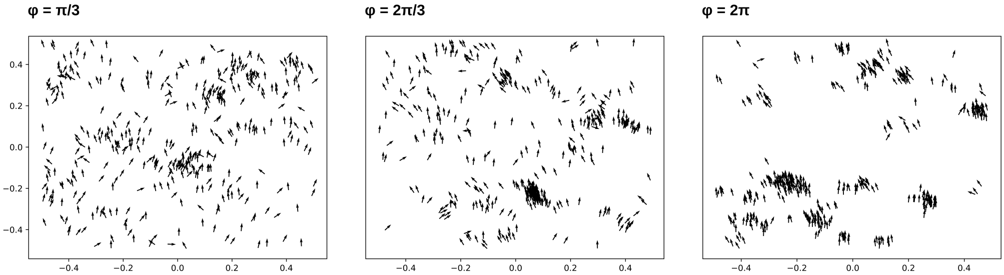
- $\rho = 400, v = 0.03, R = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0$
- v_a grows with vision angle



4) Periodic Boundaries with Vision

Configurations. Comparing Angles.

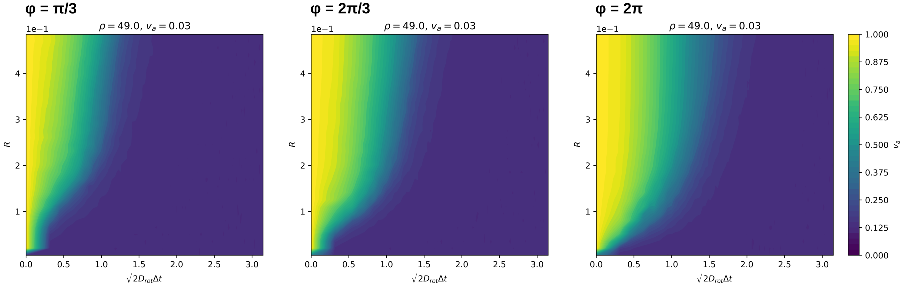
- $\rho = 400, v = 0.03, R = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0$
- Flock size grows with vision angle



4) Periodic Boundaries with Vision

Phase transitions. 2D Levels in parameter space.

- R against $\sqrt{2D_{\mathrm{rot}}\Delta t}$
- Region of ordered phase increases with vision angle



5) Reflecting Boundaries with Vision

Order parameter. Comparing with pbc.

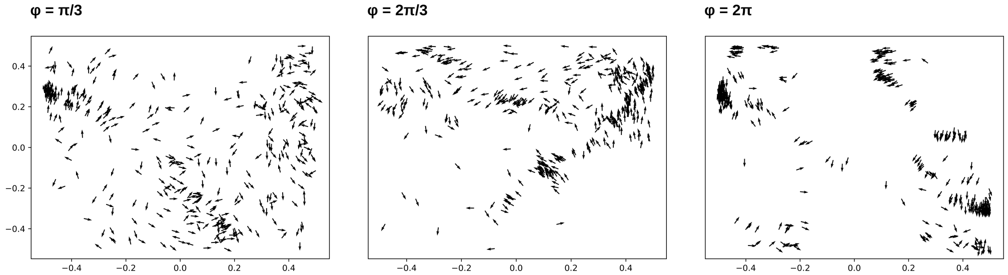
- $\rho = 400, v = 0.03, R = 0.025, D_{\mathrm{rot}} = 0.01, \Delta t = 1.0, N_{\mathrm{sim}} = 1500, N_{\mathrm{eq}} = 1000$
- v_a grows with vision angle
- Less order and higher fluctuations for rbc

	$\overline{v}_a\left(\varphi = \pi/3\right) \pm \sigma\left(\overline{v}_a\right)$	$\overline{v}_a\left(\varphi = 2\pi/3\right) \pm \sigma\left(\overline{v}_a\right)$	$\overline{v}_a\left(\varphi = 2\pi\right) \pm \sigma\left(\overline{v}_a\right)$
pbc	0.855±0.014	0.907± 0.013	0.953±0.01
rbc	0.397±0.139	0.461±0.136	0.496±0.191

5) Reflecting Boundaries with Vision

Configurations. Comparing Angles.

- $\rho = 400, v = 0.03, R = 0.025, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- Flock size grows with vision angle



6) Conclusion

Summary.

- Successful implementation of the 2D Vicsek model
- Demonstration of flocking behaviour
- Analysis of phases and their dependence on controlling parameters
  - Results of *Vicsek et al.* have been reproduced
  - Reproduced critical exponents
- Implementation of reflecting boundaries
  - Slight influence on phase transitions
  - Larger fluctuations of v_a
- Implementation of vision

6) Conclusion

Outlook.

- Investigate other order parameters? e. g. average size of flocks
- Investigate dependence on v
- Also update v in every time step
- Optimize simulation for higher particle numbers/densities
- Higher dimensions

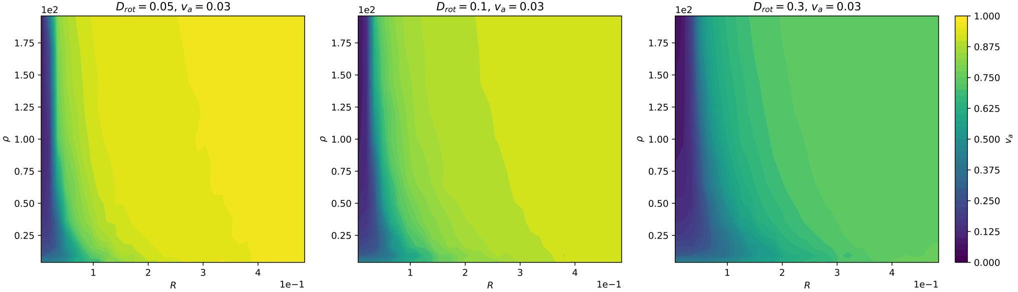
Thank you for your attention!

Florian KLUIBENSCHEDL, René SCHWARZ and Ariel HARNIK

A-2) Periodic Boundaries

Phase transitions. 2D Levels in parameter space.

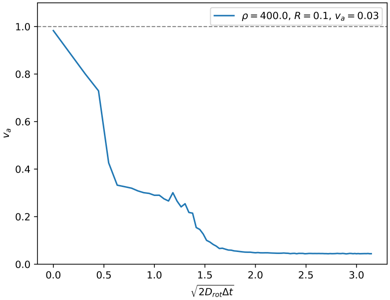
- ρ against R

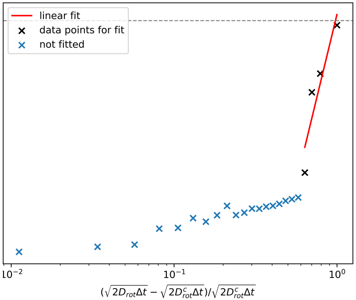


A-3) Reflecting Boundaries

Phase transitions. Example for critical exponent.

- $v_a \approx \left(\sqrt{2D^c_{\mathrm{rot}}\Delta t} - \sqrt{2D_{\mathrm{rot}}\Delta t}\right)^\beta$
- Here: $\sqrt{2D^c_{\mathrm{rot}}\Delta t} \approx 1.7$ (see figure on the left)
- Linear fit: slope $= 1.1(3)$, offset $= 1.0(1) \Rightarrow \beta = 1.1(3)$???





A-5) Reflecting Boundaries with Vision

Trajectories. Dependence on Vision Angle .

- $\rho = 400, v = 0.03, R = 0.025, D_{\mathrm{rot}} = 0.01, \Delta t = 1.0, N_{\mathrm{sim}} = 1500, N_{\mathrm{eq}} = 1000$
- v_a grows with vision angle
- Less order and higher fluctuations for rbc