



# Vicsek Model

A 2D model for modelling swarm behaviour

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- 1) Motivation and Theory.**
- 2) Periodic Boundaries.**
- 3) Reflecting Boundaries.**
- 4) Periodic Boundaries with Vision.**
- 5) Reflecting Boundaries with Vision.**
- 6) Conclusion. + discussion.**

# 1) Motivation and Theory

## **Q: Why is it interesting?**

Applications.<sup>1,2</sup>

- Living systems like the collective motion of birds, fish schools, bacteria colonies or cellular migrations
- Physical systems like ferromagnetism
- Simple model

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<sup>1</sup> T. Vicsek. *Novel Type of Phase Transition in a System of Self-Driven Particles*. VOLUME 75. NUMBER 6. PHYSICAL REVIEW LETTERS, 1995

<sup>2</sup> F. Ginelli. *The Physics of the Vicsek Model*. VOLUME 225. NUMBER 11–12. The European Physical Journal Special Topics, 2016

# 1) Motivation and Theory

## Q: Why is it so simple?

- Each particles direction is influenced by only its neighboring particles within a radius  $R$
- New position within one molecular dynamics step is given by equation of motion

$$\mathbf{r}_{t+\Delta t}^i = \mathbf{r}_t^i + \mathbf{v}_t^i \cdot \Delta t \quad (1)$$

with  $\mathbf{v}_t^i = v \cdot \mathbf{u}_t^i$ , where  $v = \text{const.}$  and  $\mathbf{u}_t^i = \begin{pmatrix} \cos(\theta_t^i) \\ \sin(\theta_t^i) \end{pmatrix}$

# 1) Motivation and Theory

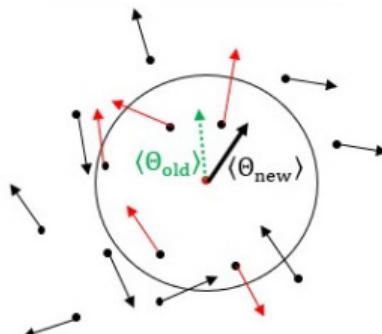
- Updating  $\theta_t^i$  using the expression

$$\theta_{t+\Delta t}^i = \langle \theta_t^i \rangle_{|r_i - r_j| < R} + \sqrt{2D_{\text{rot}} \Delta t} \cdot \eta_t, \quad (2)$$

with  $D_{\text{rot}}$  as rotational diffusion coefficient and  $\eta_t$  as Gaussian noise

- $\langle \theta_t^i \rangle_{|r_i - r_j| < R}$  describes an average direction obtained via

$$\langle \theta_t^i \rangle_{|r_i - r_j| < R} = \arctan \left( \frac{\langle \sin(\theta_t^i) \rangle_{|r_i - r_j| < R}}{\langle \cos(\theta_t^i) \rangle_{|r_i - r_j| < R}} \right) \quad (3)$$



# 1) Motivation and Theory

## Order Parameter.

- To study phase transitions we analyse the order parameter  $v_a$

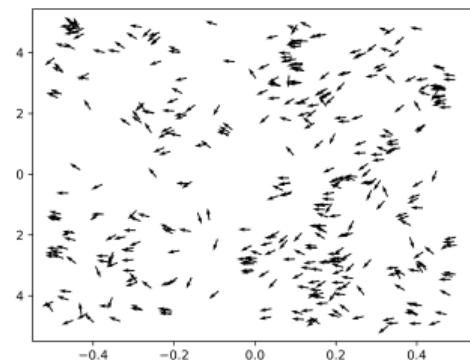
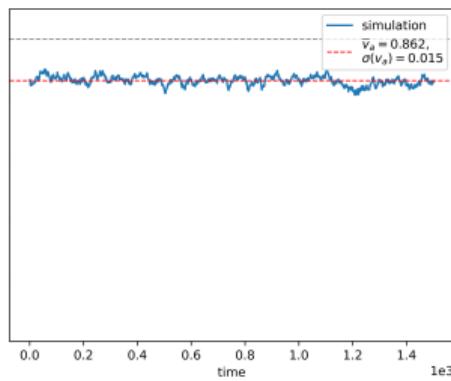
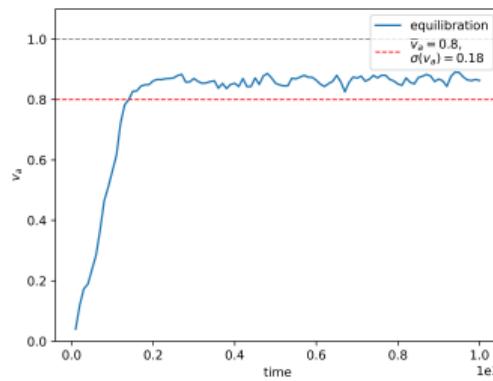
$$v_a = \frac{1}{Nv} \cdot \left| \sum_i^N \mathbf{v}^i \right| \in [0, 1] \quad (4)$$

- Order parameter  $v_a$  inhibits (all) physics of the model

## 2) Periodic Boundaries

**Trajectory.** Example.

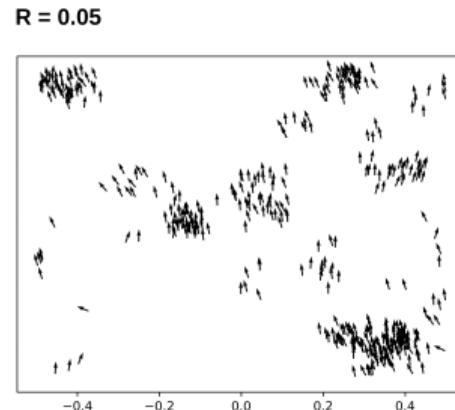
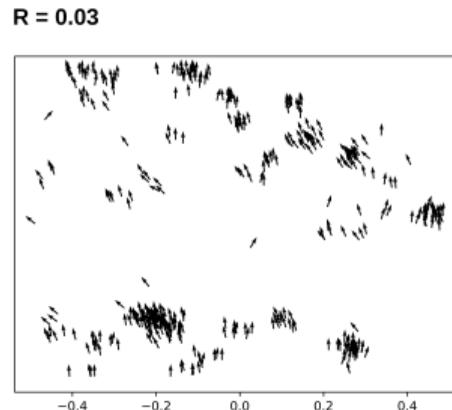
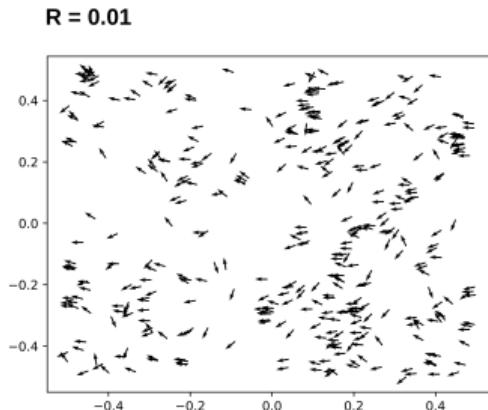
- $\rho = 400, v = 0.03, R = 0.01, D_{\text{rot}} = 0.01, \Delta t = 1.0$
- Representative trajectory is shown
- Equilibration in  $\approx 200$  steps



## 2) Periodic Boundaries

**Configurations.**  $R$ -dependence.

- $\rho = 400, v = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0$
- Bigger  $R \Rightarrow$  bigger flocks



## 2) Periodic Boundaries

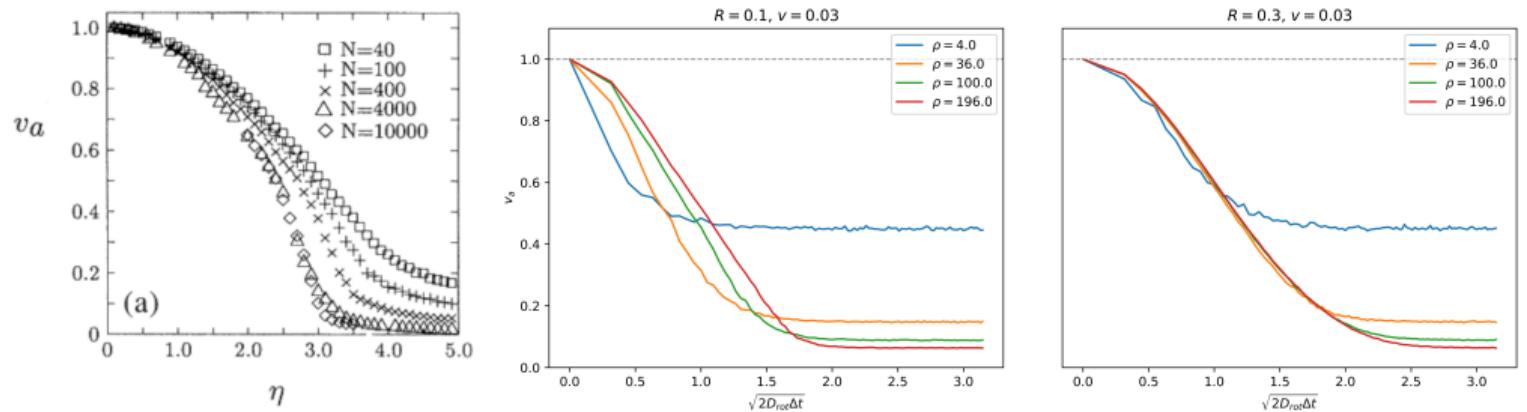
**Phase transitions.** Simulation settings.

- $N_{\text{sim}} = 3000, N_{\text{eq}} = 1000, N_{\text{save}} = 1$
- Fix  $v$ : vary
  - $R$  and  $D_{\text{rot}}$
  - $\rho$  and  $D_{\text{rot}}$
  - $\rho$  and  $R$
- Plot  $\bar{v}_a$  against varying parameters

## 2) Periodic Boundaries

**Phase transitions.** Comparing with reference.

- Reference:  $v = 0.03, R = 1.0, D_{\text{rot}} = 0.01, \Delta t = 1.0, (N, L) = (40, 3.1), (100, 5), (400, 10), (4000, 31.6), (10000, 50)$
- Our system:  $v = 0.03$

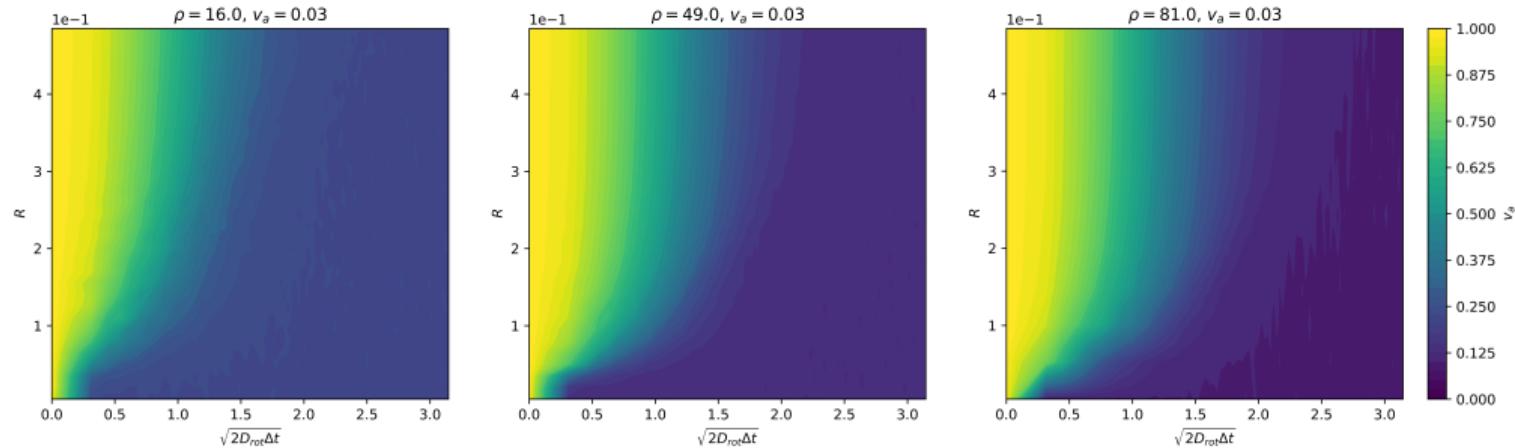


(left) Tamás Vicsek, András Czirók, Eshel Ben-Jacob, Inon Cohen, and Ofer Shochet. Novel Type of Phase Transition in a System of Self-Driven Particles. Phys. Rev. Lett. 75, 1226 – Published 7 August 1995

## 2) Periodic Boundaries

**Phase transitions.** 2D Levels in parameter space.

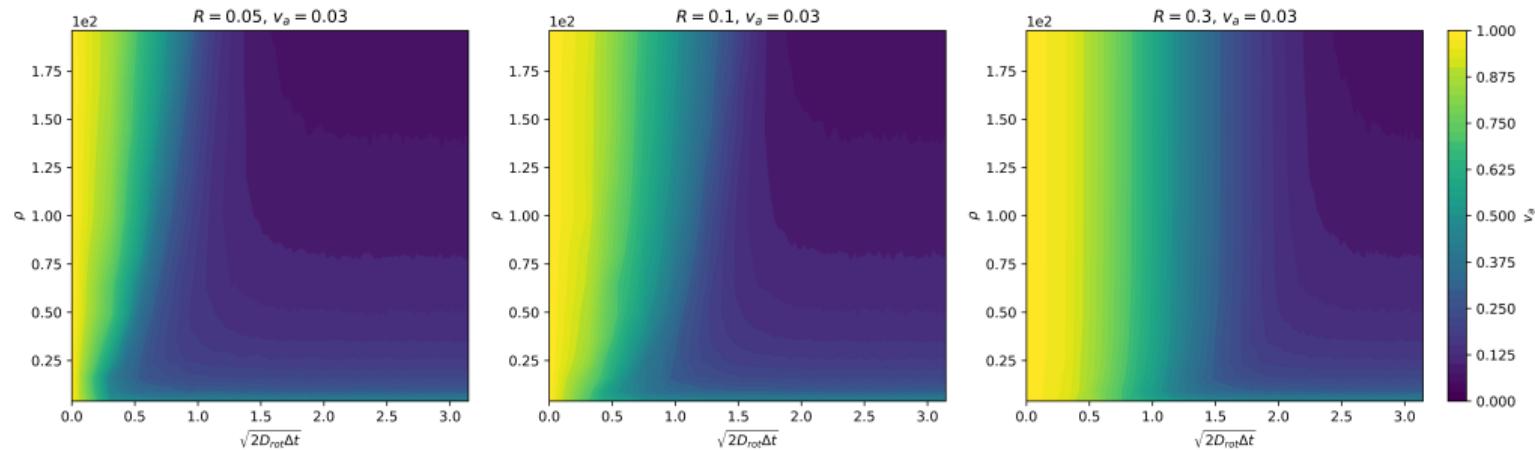
- $R$  against  $\sqrt{2D_{\text{rot}}\Delta t}$



## 2) Periodic Boundaries

**Phase transitions.** 2D Levels in parameter space.

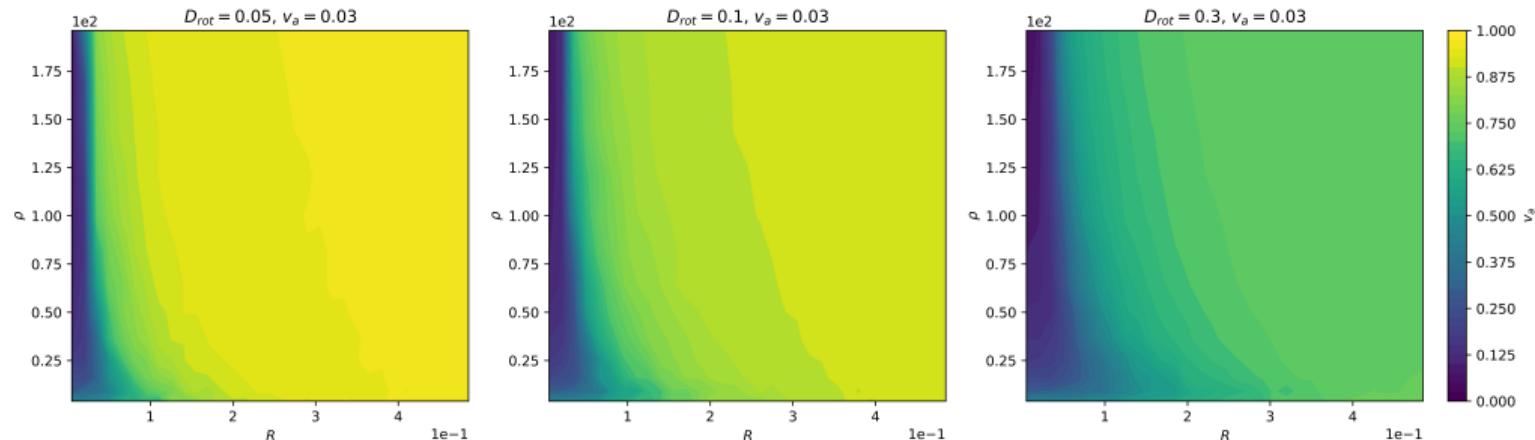
- $\rho$  against  $\sqrt{2D_{\text{rot}}\Delta t}$



## 2) Periodic Boundaries

**Phase transitions.** 2D Levels in parameter space.

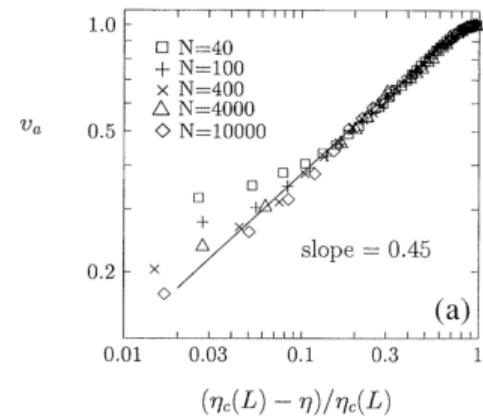
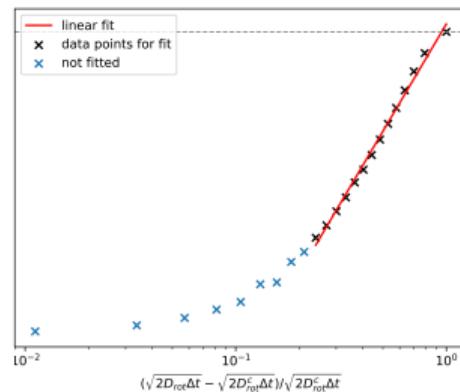
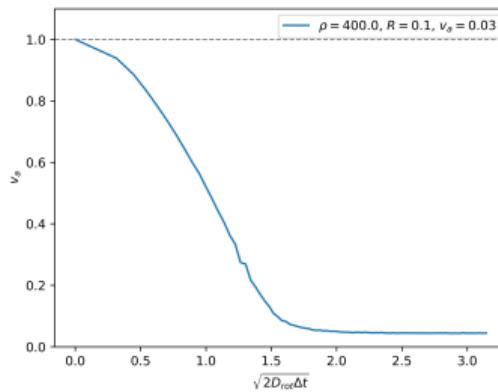
- $\rho$  against  $R$



## 2) Periodic Boundaries

**Phase transitions.** Example for critical exponent.

- $v_a \approx (\sqrt{2D_{\text{rot}}^c \Delta t} - \sqrt{2D_{\text{rot}} \Delta t})^\beta$
- Here:  $\sqrt{2D_{\text{rot}}^c \Delta t} \approx 1.7$  (see figure on the left)
- Linear fit: slope = 0.449(10), offset = 1.023(9)  $\Rightarrow \beta = 0.449(10)$

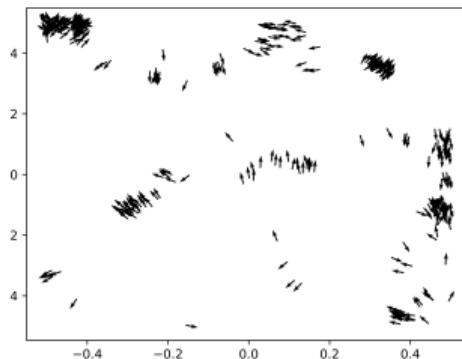
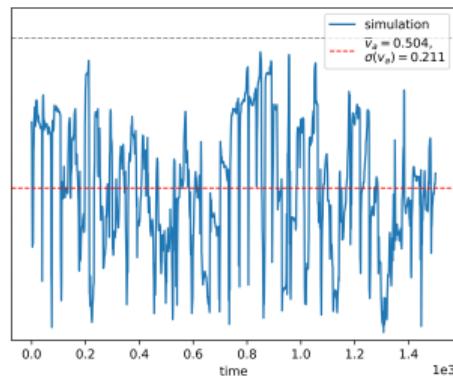
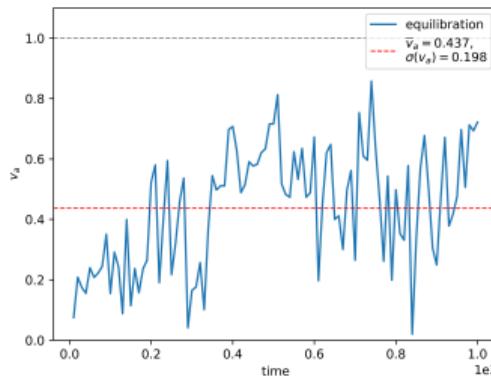


(right) Tamás Vicsek, András Czirók, Eshel Ben-Jacob, Inon Cohen, and Ofer Shochet. Novel Type of Phase Transition in a System of Self-Driven Particles. Phys. Rev. Lett. 75, 1226 – Published 7 August 1995

### 3) Reflecting Boundaries

**Trajectory.** Example.

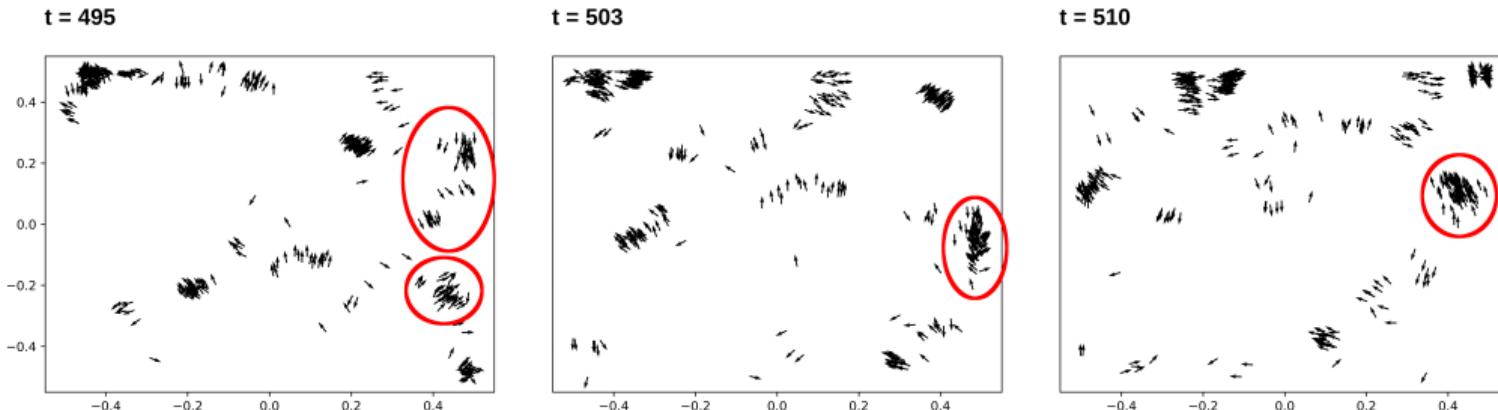
- $\rho = 400, v = 0.03, R = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- Representative trajectory is shown
- Large fluctuations (collisions with boundary)



### 3) Reflecting Boundaries

**Trajectory.** Interesting Collisions.

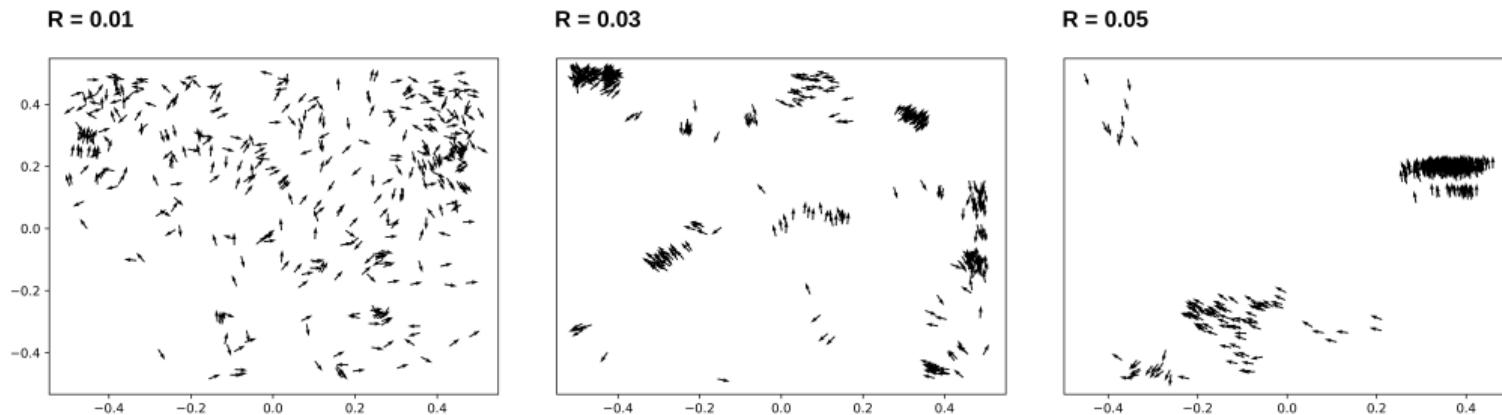
- $\rho = 400, v = 0.03, R = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- Two incoming flocks  $\rightarrow$  one outgoing flock



### 3) Reflecting Boundaries

**Configurations.**  $R$ -dependence.

- $\rho = 400, v = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- Bigger  $R \Rightarrow$  bigger flocks
- Flocking stronger compared to periodic boundaries

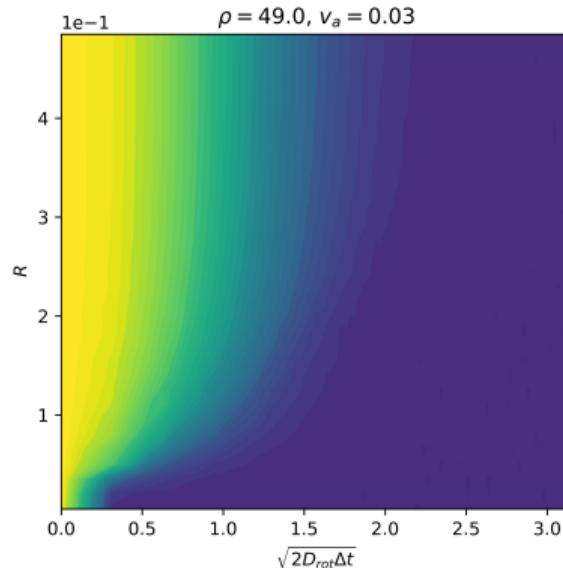


### 3) Reflecting Boundaries

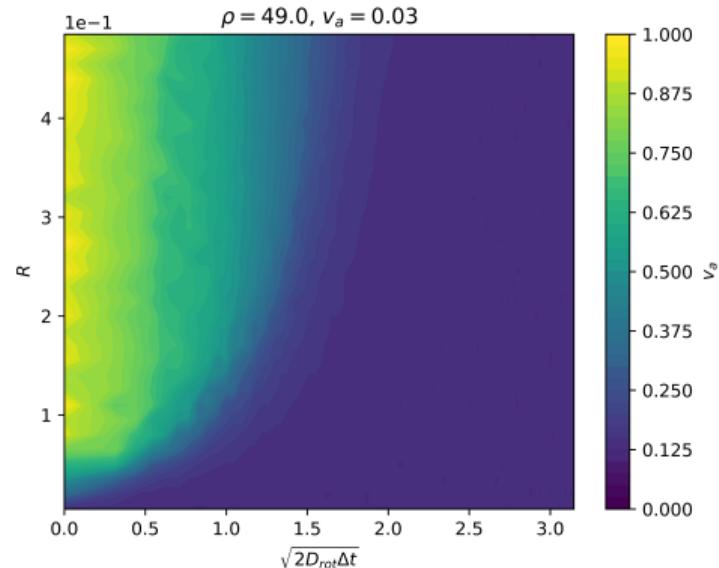
**Phase transitions.** 2D Levels in parameter space.

- $R$  against  $\sqrt{2D_{\text{rot}}\Delta t}$

Periodic Boundaries:



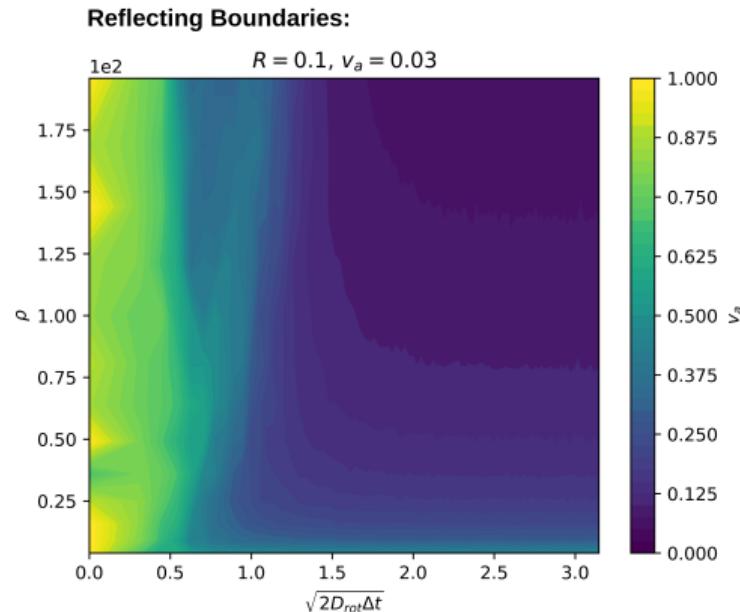
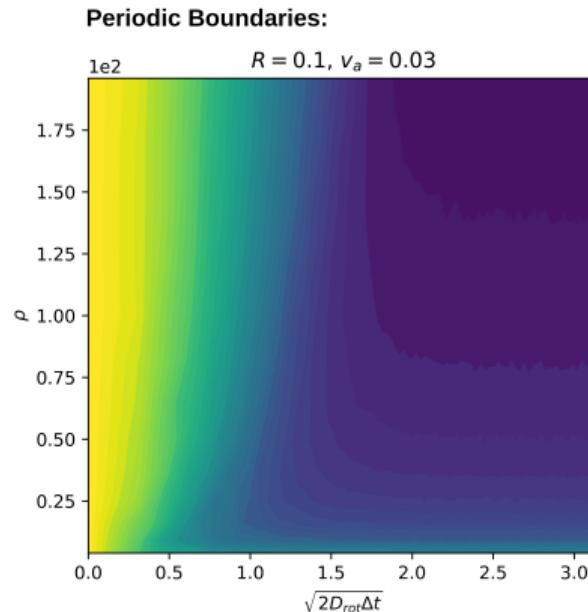
Reflecting Boundaries:



### 3) Reflecting Boundaries

**Phase transitions.** 2D Levels in parameter space.

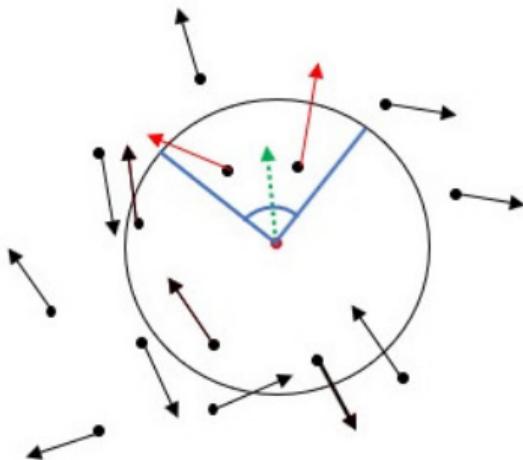
- $\rho$  against  $\sqrt{2D_{\text{rot}}\Delta t}$



## 4) Periodic Boundaries with Vision

### Implementation.

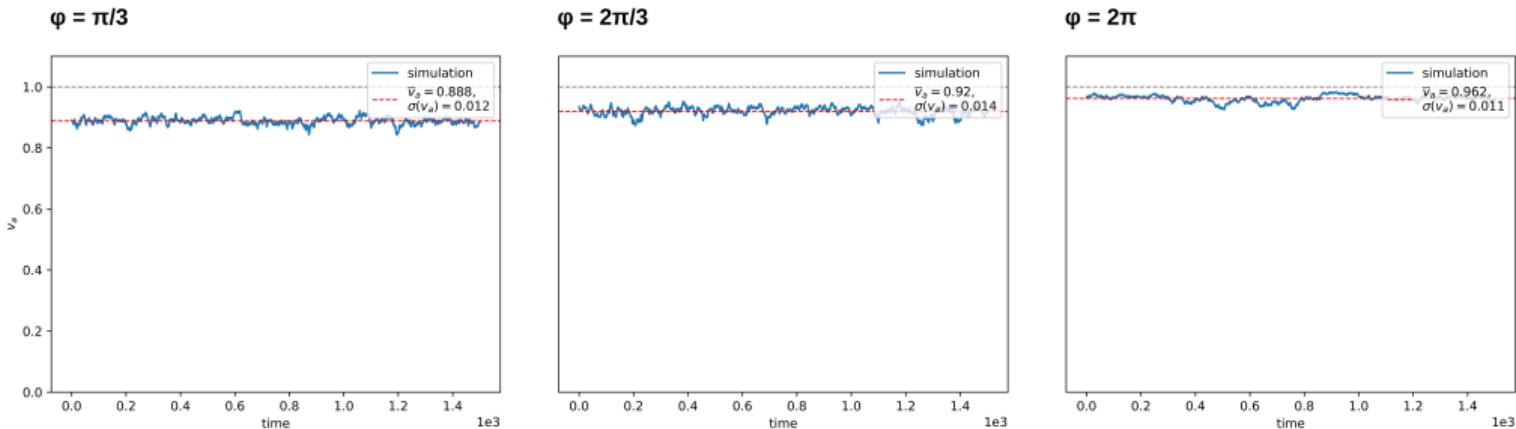
- Just particles in circle segment defined by vision angle  $\varphi$  are considered



## 4) Periodic Boundaries with Vision

**Trajectory.** Comparing Angles.

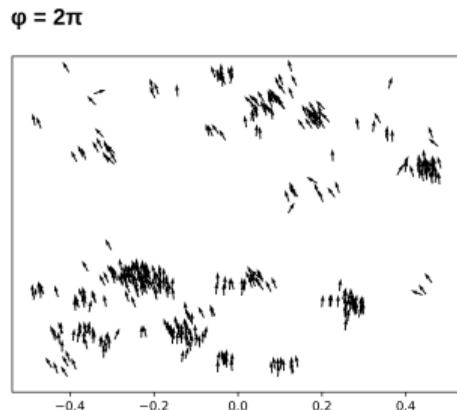
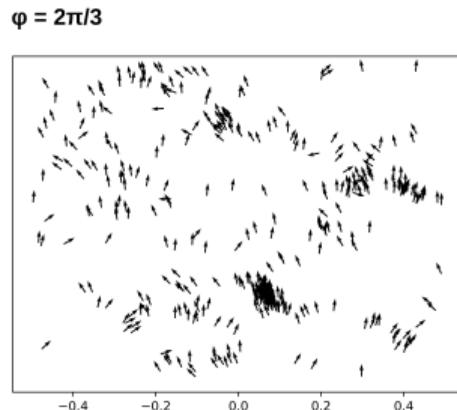
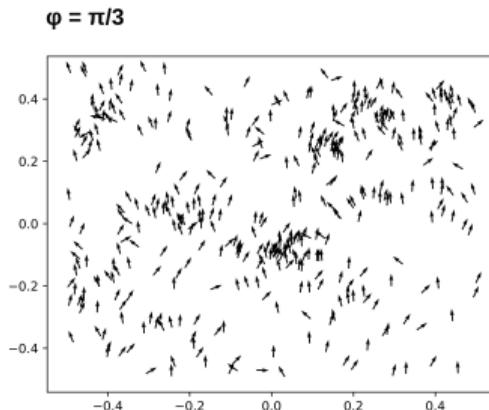
- $\rho = 400, v = 0.03, R = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0$
- $v_a$  grows with vision angle



## 4) Periodic Boundaries with Vision

**Configurations.** Comparing Angles.

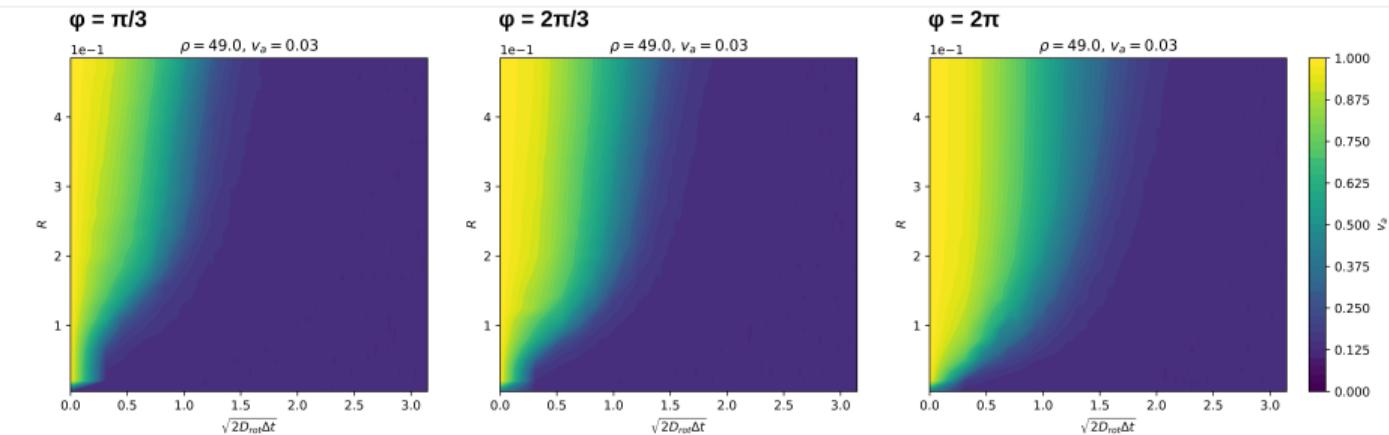
- $\rho = 400, v = 0.03, R = 0.03, D_{\text{rot}} = 0.01, \Delta t = 1.0$
- Flock size grows with vision angle



## 4) Periodic Boundaries with Vision

**Phase transitions.** 2D Levels in parameter space.

- $R$  against  $\sqrt{2D_{\text{rot}}\Delta t}$
- Region of ordered phase increases with vision angle



## 5) Reflecting Boundaries with Vision

**Order parameter.** Comparing with pbc.

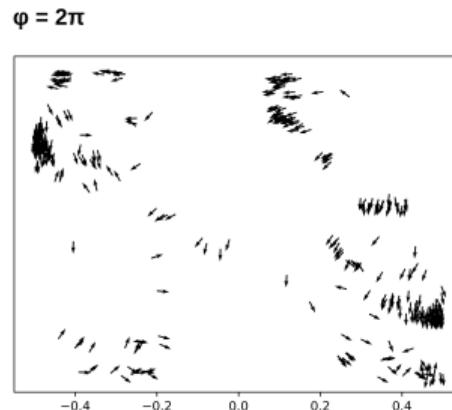
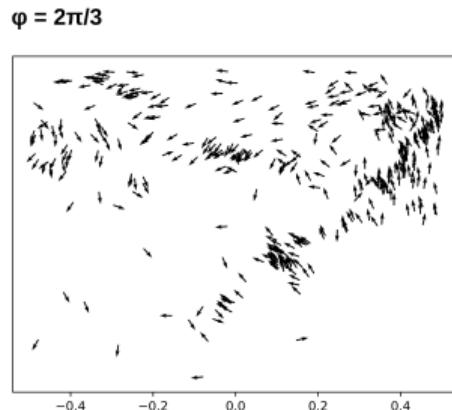
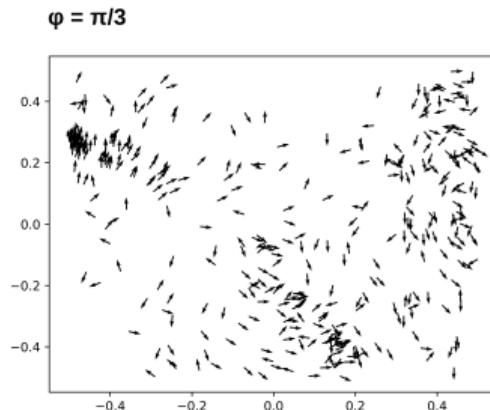
- $\rho = 400, v = 0.03, R = 0.025, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- $v_a$  grows with vision angle
- Less order and higher fluctuations for rbc

	$\bar{v}_a (\varphi = \pi/3) \pm \sigma (\bar{v}_a)$	$\bar{v}_a (\varphi = 2\pi/3) \pm \sigma (\bar{v}_a)$	$\bar{v}_a (\varphi = 2\pi) \pm \sigma (\bar{v}_a)$
pbc	$0.855 \pm 0.014$	$0.907 \pm 0.013$	$0.953 \pm 0.01$
rbc	$0.397 \pm 0.139$	$0.461 \pm 0.136$	$0.496 \pm 0.191$

## 5) Reflecting Boundaries with Vision

**Configurations.** Comparing Angles.

- $\rho = 400, v = 0.03, R = 0.025, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- Flock size grows with vision angle



## 6) Conclusion

### Summary.

- Successful implementation of the 2D Vicsek model
- Demonstration of flocking behaviour
- Analysis of phases and their dependence on controlling parameters
  - Results of Vicsek *et al.* have been reproduced
  - Reproduced critical exponents
- Implementation of reflecting boundaries
  - Slight influence on phase transitions
  - Larger fluctuations of  $v_a$
- Implementation of vision

## 6) Conclusion

### **Outlook.**

- Investigate other order parameters? e. g. average size of flocks
- Investigate dependence on  $v$
- Also update  $v$  in every time step
- Optimize simulation for higher particle numbers/densities
- Higher dimensions



Thank you for your attention!

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# A-5) Reflecting Boundaries with Vision

**Trajectories.** Dependence on Vision Angle .

- $\rho = 400, v = 0.03, R = 0.025, D_{\text{rot}} = 0.01, \Delta t = 1.0, N_{\text{sim}} = 1500, N_{\text{eq}} = 1000$
- $v_a$  grows with vision angle
- Less order and higher fluctuations for rbc

