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*n*th root algorithm

The principal *n*th root $\sqrt[n]{A}$ of a positive real number *A*, is the positive real solution of the equation $x^n = A$. For a positive integer *n* there are *n* distinct complex solutions to this equation if $A > 0$, but only one is positive and real.

Using Newton's method

Newton's method is a method for finding a zero of a function *f(x)*. The general iteration scheme is:

1. Make an initial guess x_0
2. Set $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$
3. Repeat step 2 until the desired precision is reached.

The n^{th} root problem can be viewed as searching for a zero of the function

$$f(x) = x^n - A$$

So the derivative is

$$f'(x) = nx^{n-1}$$

and the iteration rule is

$$\begin{aligned} x_{k+1} &= x_k - \frac{f(x_k)}{f'(x_k)} \\ &= x_k - \frac{x_k^n - A}{nx_k^{n-1}} \\ &= x_k + \frac{1}{n}\left[-x_k + \frac{A}{x_k^{n-1}}\right] \\ &= \frac{1}{n}\left[(n-1)x_k + \frac{A}{x_k^{n-1}}\right]. \end{aligned}$$

See also

- Recurrence relation
- Shifting *n*th root algorithm
- Halley's method
- Householder's method

References

- Atkinson, Kendall E. (1989), *An introduction to numerical analysis* (2nd ed.), New York: Wiley, ISBN 0-471-62489-6.