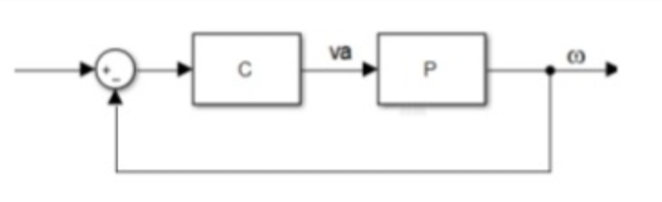
Marcos López López

A 22

Trabajo previo 1



$P(s) = \frac{b}{s^2+a_1 s+a_2}$

→ Análisis de estabilidad

a) $C(s) = \frac{K_p s + K_I}{s}$ determinar K_p y K_I que estabilizan el sistema

$$\boxed{\begin{array}{l} a_1 > 0 \\ a_2 > 0 \\ b > 0 \end{array}}$$

$$G(s) = \frac{\frac{K_p s + K_I}{s} \cdot \frac{b}{s^2+a_1 s+a_2}}{1 + \frac{b(K_p s + K_I)}{s(s^2+a_1 s+a_2)}}$$

→ Ec característica → $s(s^2+a_1 s+a_2) + b(K_p s + K_I) = 0$

$s^3 + a_1 s^2 + a_2 s + bK_p s + K_I \cdot b = 0 \rightarrow s^3 + a_1 s^2 + (a_2 + bK_p)s + bK_I = 0$

si $a_1, a_2, b > 0 \Rightarrow K_p, K_I > 0 \Rightarrow$ sistema estable (Cardano - Vietta)

→ Criterio de Routh

s^3	1	$a_2 + bK_p$
s^2	a_1	bK_I
s^1	A	
s^0	B	

$A = \frac{[a_1(a_2 + bK_p)] - bK_I}{a_1}$

$B = \frac{A \cdot bK_I}{A} = bK_I$

$1 > 0$

$a_1 > 0$

$A > 0 \rightarrow \frac{a_1 a_2 + a_1 bK_p - bK_I}{a_1} > 0 \rightarrow a_1 a_2 + a_1 bK_p - bK_I > 0 \rightarrow \boxed{K_p > \frac{bK_I - a_1 a_2}{a_1 b}}$

$K_p > \frac{K_I}{a_1} - \frac{a_2}{b}$

$B > 0 \rightarrow bK_I > 0 \rightarrow \boxed{K_I > 0}$

→ Ganancia crítica → $s^1 = 0$

$A = 0 \rightarrow \frac{a_1 a_2 + a_1 bK_p - bK_I}{a_1} = 0 \rightarrow a_1 a_2 + a_1 bK_p - bK_I = 0 \rightarrow a_1 bK_p = bK_I - a_1 a_2 \rightarrow \boxed{K_p = \frac{bK_I - a_1 a_2}{b \cdot a_1}}$

→ Las raíces se sitúan

$a_1 s^2 + bK_I = 0 \rightarrow a_1 j^2 \omega^2 + bK_I = 0 \rightarrow -a_1 \omega^2 + bK_I = 0 \rightarrow \omega^2 = \frac{bK_I}{a_1} \rightarrow \boxed{\omega = \pm\sqrt{\frac{bK_I}{a_1}}}$

b) $C(s) = K \rightarrow G_K(s) = \frac{\frac{Kb}{s^2+a_1 s+a_2}}{1 + \frac{Kb}{s^2+a_1 s+a_2}}$

→ Ecuación característica → $s^2 + a_1 s + a_2 + Kb = 0$ $K > 0$

s^2	1	$a_2 + Kb$
s^1	a_1	
s^0	A	

→ $A = \frac{a_1(a_2 + Kb)}{a_1} = a_2 + Kb > 0 \rightarrow Kb > a_2 \rightarrow \boxed{K > -\frac{a_2}{b}}$

$\boxed{K > 0}$

→ Ganancia crítica $s^1 = 0 \rightarrow a_1 = 0$

→ Raíces $s^2 + a_2 + Kb = 0$, $s = j\omega$ → $\omega^2 = a_2 + Kb \rightarrow \boxed{\omega = \pm\sqrt{\frac{a_2}{Kb}}}$

c) $C(s) = \frac{K_p \cdot s + K_I}{s}$; $P(s) = \frac{b}{s^2 + a_1 s + a_2}$; $\omega_{ref}(s) = 1/s$ (Escalón)

$$\omega(\infty) = \lim_{s \to 0} s\, G_{LC}(s) \cdot \omega_{ref}(s)$$

$$G_{LC}(s) = \frac{b K_p s + b K_I}{s^3 + a_1 s^2 + (a_2 + b K_p) + b K_I}$$

$$\boxed{\omega(\infty) = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot G_{LC}(s) = 1}$$

$$\boxed{e(\infty) = \omega_{ref}(\infty) - \omega(\infty) = \lim_{s \to 0} s \cdot \frac{1}{s} - 1 = 0}$$

$$K_p = \lim_{s \to 0} G_{LA}(s)$$

$$e(\infty) = \frac{1}{1 + K_p}$$

$$G_{LA}(s) = P(s) \cdot C(s) \cdot \overset{=1}{H(s)} = \frac{b}{s^2 + a_1 s + a_2} \cdot \frac{K_p s + K_I}{s}$$

$$K_p = \lim_{s \to 0} \left(\frac{b}{s^2 + a_1 s + a_2} \cdot \frac{K_p s + K_I}{s} \right) = \frac{b \cdot K_I}{0} = \infty$$

$$\boxed{e(\infty) = \frac{1}{\infty} = 0}$$

d)

$C(s) = K$

$$G_{LC}(s) = \frac{\frac{K \cdot b}{s^2 + a_1 s + a_2}}{1 + \frac{K \cdot b}{s^2 + a_1 s + a_2}} = \frac{K \cdot b}{s^2 + a_1 s + a_2 + K \cdot b}$$

$$\omega(\infty) = \lim_{s \to 0} s \cdot G_{LC}(s) \cdot \omega_{ref}(s) = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot G_{LC}(s) \rightarrow \boxed{\omega(\infty) = \frac{K \cdot b}{a_2 + K \cdot b}}$$

$$e(\infty) = \omega_{ref}(\infty) - \omega(\infty) = \lim_{s \to 0} s \frac{1}{s} - \frac{K \cdot b}{a_2 + K \cdot b} \Rightarrow e(\infty) = 1 - \frac{K \cdot b}{a_2 + K b} = \frac{a_2 + \cancel{Kb} - \cancel{Kb}}{a_2 + Kb} \rightarrow \boxed{e(\infty) = \frac{a_2}{a_2 + K \cdot b}}$$

→ Comprobación

$$K_p = \lim_{s \to 0} G_{LA}(s)$$

$$e(\infty) = \frac{1}{1 + K_p}$$

$$G_{LA}(s) = P(s) \cdot C(s) \cdot \overset{=1}{H(s)} = \frac{K \cdot b}{s^2 + a_1 s + a_2}$$

$$K_p = \lim_{s \to 0} \frac{K \cdot b}{s^2 + a_1 s + a_2} = \frac{K \cdot b}{a_2}$$

$$e(\infty) = \frac{1}{1 + K_p} = \frac{1}{1 + \frac{K \cdot b}{a_2}} \Rightarrow \boxed{e(\infty) = \frac{a_2}{a_2 + K \cdot b}}$$

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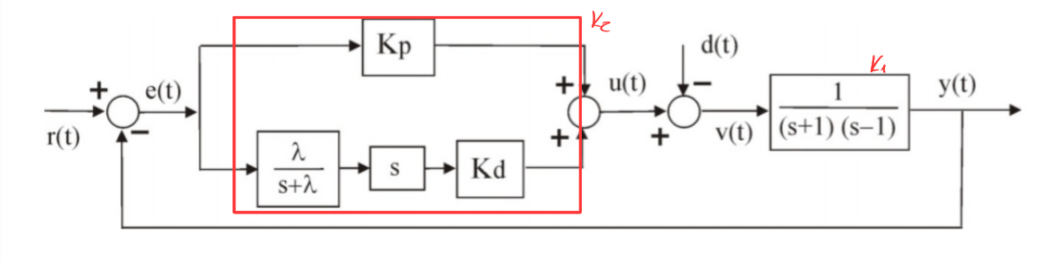
<u>Trabajo Previo 2</u>



$e = r - y \;;\; y = K_1[u-d] = K_1[K_2 e - d]$

$e = r - K_1[K_2 e - d] \rightarrow e = r - K_1K_2 e + d \rightarrow e(1+K_1K_2) = r + d \rightarrow e = \frac{1}{1+K_1K_2} r + \frac{1}{1+K_1K_2} d$

$\frac{e}{r+d} =$

$\frac{Y(s)}{R(s)} = \frac{K_1K_2}{1+K_1K_2}$

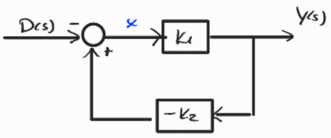
$K_2 = K_p + \frac{\lambda K_d \cdot s}{s+\lambda} = \frac{K_p s + \lambda K_p + \lambda K_d s}{s+\lambda} = \frac{(K_p+\lambda K_d)s + \lambda K_p}{s+\lambda}$

$K_1 \cdot K_2 = \frac{(K_p+\lambda K_d)s + \lambda K_p}{s+\lambda} \cdot \frac{1}{(s+1)(s-1)}$

$1 + K_1K_2 = \frac{(s+\lambda)(s+1)(s-1) + (K_p+\lambda K_d)s + \lambda K_p}{(s+\lambda)(s+1)(s-1)}$

$$\boxed{G(s) = \frac{(K_p+\lambda K_d)s + \lambda K_p}{(s+\lambda)(s-1)(s+1) + (K_p+\lambda K_d)s + \lambda K_p}}$$

b)



$x = -d - K_2 y$; $y = K_1 x$ | $x = -d - K_2K_1 x \rightarrow (1+K_1K_2)x = -d$; $y = \frac{-K_1 d}{1+K_1K_2} \rightarrow \frac{y}{d} = \frac{-K_1}{1+K_1K_2}$

$$\boxed{G(s) = \frac{-(s+\lambda)}{(s+\lambda)(s+1)(s-1) + (K_p+\lambda K_d)s + \lambda K_p}}$$

c)

$y(\infty) = \lim_{s\to 0} s\,G(s)\cdot D(s) = \lim_{s\to 0} s \cdot \frac{20}{s} \cdot \frac{-(s+\lambda)}{(s+\lambda)(s+1)(s-1) + (K_p+\lambda K_d)s + \lambda K_p} = \frac{-20\lambda}{-\lambda + \lambda K_p} = \frac{-\lambda\, 20}{-\lambda(1-K_p)} \Rightarrow \boxed{y(\infty) = \frac{20}{1-K_p}}$

d)

$\left|\frac{20}{1-K_p}\right| < 1 \rightarrow 20 < 1 - K_p \rightarrow 20 - 1 < -K_p \rightarrow \boxed{K_p > -19}$ / $-20 < 1 - K_p \rightarrow -K_p > -20 - 1 \rightarrow -K_p > -21$ $\boxed{K_p > 21}$

Ecuación característica $(s+\lambda)(s^2-1) + K_p s + \lambda K_d s + \lambda K_p = 0$

$s^3 - s + \lambda s^2 - \lambda + K_p s + \lambda K_d s + \lambda K_p = 0$

$s^3 + \lambda s^2 + (K_p + \lambda K_d - 1)s + \lambda K_p - \lambda = 0$

→ Cardano - Vietta

$\lambda > 0$

$\lambda K_p - \lambda > 0 \rightarrow K_p > 1$

$K_p + \lambda K_d - 1 > 0 \rightarrow$ como $K_p > 1$ y $\lambda > 0 \Rightarrow K_d > 0$

Routh - Hurwitz

s^3	1	$K_p+\lambda K_d-1$
s^2	λ	$\lambda K_p - \lambda$
s^1	A	
s^0	B	

$A = \frac{\lambda(K_p + \lambda K_d - 1) - \lambda(K_p - 1)}{\lambda} = \cancel{K_p} + \lambda K_d \cancel{-1} \cancel{-K_p} \cancel{+1} = \lambda K_d$

$B = \frac{A(\lambda K_p - \lambda)}{A} = \lambda(K_p - 1)$

$\boxed{\lambda > 0}$

$A > 0 \Rightarrow \lambda K_d > 0 \Rightarrow \boxed{K_d > 0}$

$B > 0 \Rightarrow \lambda(K_p - 1) > 0 \rightarrow \boxed{K_p > 1}$

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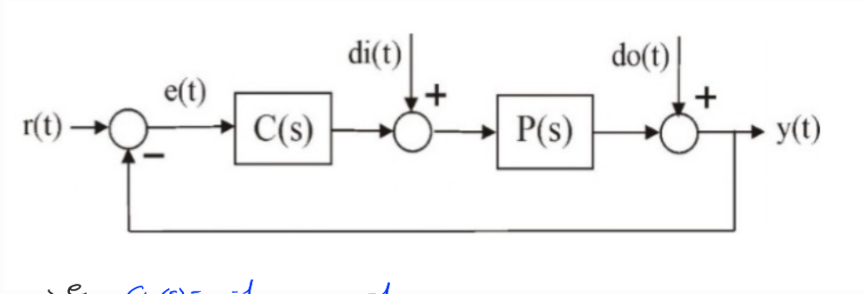
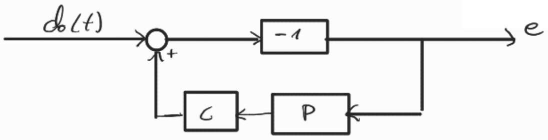
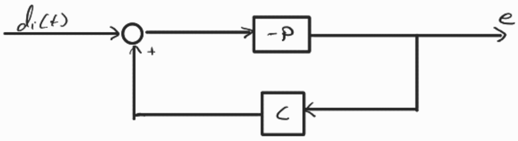
Trabajo previo 3





$$G_{lc}(s) = \frac{-1}{1 - C \cdot P(-1)} = \frac{-1}{1+PC}$$



$$G_{lc} = \frac{-P}{1-C(-P)} = \frac{-P}{1+CP}$$

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{1+s/N} = K_p + \frac{K_i}{s} + \frac{NK_d \cdot s}{N+s} = \frac{K_p \cdot s(s+N) + K_i(s+N) + NK_d \cdot s^2}{s(s+N)}$$

a) $d_i(t)=0$, $r(t)=0$ $\Big\{$ valor permanente e_∞

$$e_\infty = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{s\cancel{}} \cdot G_{lc}(s)$$

$$1 + C\cdot P = 1 + \frac{1}{s^2} \frac{K_p \cdot s(s+N) + K_i(s+N) + NK_d \cdot s^2}{s(s+N)} = \frac{s^3(s+N) + K_p \cdot s(s+N) + K_i(s+N) + NK_d \cdot s^2}{s^3(s+N)}$$

$$e_\infty = \lim_{s\to 0} \frac{1}{\cancel{s}} \cdot \frac{-s^{\cancel{3}}(s+N)}{s^3(s+N) + K_p \cdot s(s+N) + K_i(s+N) + NK_d \cdot s^2} = \frac{0}{+K_i \cdot N} = 0$$

b) $d_o(t)=0$, $r(t)=0$ $\Big\{$ valor permanente e_∞

$$e_\infty = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{s\cancel{}} \cdot \frac{-P}{1+PC}$$

$C(s) = K_p + \frac{K_i}{s} + \frac{K_d \cdot s}{1+s/N}$

$P = 1/s^2$

$$1 + PC = \frac{s^3(s+N) + K_p \cdot s(s+N) + K_i(s+N) + NK_d \cdot s^2}{s^3(s+N)}$$

$$e_\infty = \lim_{s\to 0} \frac{1}{\cancel{s}} \cdot \frac{-\frac{1}{\cancel{s^2}} \cancel{s^3}(s+N)}{s^3(s+N) + K_p \cdot s(s+N) + K_i(s+N) + NK_d \cdot s^2} = \frac{N}{K_i \cdot N} = \frac{1}{K_i}$$

c) $d_i(t)=0$, $r(t)=0$ $\Big\{$ valor permanente e_∞

$$C(s) = K_p + \frac{K_d \cdot s}{1+s/N} = K_p + \frac{NK_d \cdot s}{s+N} = \frac{K_p(s+N) + NK_d \cdot s}{s+N}$$

$$G_{lc}(s) = \frac{-1}{1+CP}$$

$$e_\infty = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} G_{lc}(s)$$

$$1 + CP = 1 + \frac{1}{s^2} \frac{K_p(s+N) + NK_d \cdot s}{s+N} = \frac{s^2(s+N) + K_p(s+N) + NK_d \cdot s}{s^2(s+N)}$$

$$e_\infty = \lim_{s\to 0} \frac{1}{\cancel{s}} \frac{-s^{\cancel{2}}(s+N)}{s^2(s+N) + K_p(s+N) + NK_d \cdot s} = \frac{0}{K_p \cdot N} = 0$$

d) $d_o(t)=0$, $r(t)=0$ $\Big\{$ valor permanente e_∞

$$G_{lc} = \frac{-P}{1+PC}$$

$$1 + PC = \frac{s^2(s+N) + K_p(s+N) + NK_d \cdot s}{s^2(s+N)}$$

$$e_\infty = \lim_{s\to 0} \frac{1}{s} \frac{-\frac{1}{\cancel{s^2}} \cdot \cancel{s^2}(s+N)}{s^2(s+N) + K_p(s+N) + NK_d \cdot s} = \frac{-N}{0} = \infty$$