Trabajo Previo 1

a) $1 + P \cdot C = 0$

$P(s) = \dfrac{b}{s^2 + a_1 s + a_2}$ | $C(s) = \dfrac{K_p \cdot s + K_I}{s}$

$$1 + \frac{b(K_p s + K_I)}{s(s^2 + a_1 s + a_2)} = 0$$

$$\frac{s(s^2 + a_1 s + a_2) + b(K_p s + K_I)}{s(s^2 + a_1 s + a_2)} = 0$$

$$\boxed{\begin{aligned} &s(s^2 + a_1 s + a_2) + b(K_p s + K_I) = 0 \\ &s^3 + a_1 s^2 + (a_2 + bK_p)s + bK_I = 0 \end{aligned}}$$

$s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + \alpha s^2 + 2\zeta\omega_n\alpha s + \omega_n^2\alpha$

$s^3 + s^2(2\zeta\omega_n + \alpha) + s(\omega_n^2 + 2\zeta\omega_n\alpha) + \omega_n^2\alpha$

b)

(1) $a_1 = 2\zeta\omega_n + \alpha$

(2) $a_2 + bK_p = \omega_n^2 + 2\zeta\omega_n\alpha$

(3) $bK_I = \omega_n^2\alpha$

$\overset{(3)}{\Rightarrow} K_I = \dfrac{\omega_n^2\alpha}{b}$

$\overset{(2)}{\Rightarrow} K_p = \dfrac{\omega_n^2 + 2\zeta\omega_n\alpha - a_2}{b}$

Comprobación (2) $(s+\alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2)$

$$S_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2\cdot 1} = \frac{-2\zeta\omega_n \pm \sqrt{4\omega_n^2(\zeta^2 - 1)}}{2\cdot 1} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$S_3 = -\alpha$

$$\underbrace{-\alpha}_{S_3} \underbrace{-\zeta\omega_n + \cancel{j\omega_n\sqrt{1-\zeta^2}}}_{S_2} \underbrace{-\zeta\omega_n - \cancel{j\omega_n\sqrt{1-\zeta^2}}}_{S_1} = \boxed{-\alpha - 2\zeta\omega_n = -a_1}$$

b (2)

c)

$\alpha^2 - a_1\alpha + a_2 = 0$ (3)

$2\alpha\zeta\omega_n = a_2$ (4)

(3) $\alpha = \dfrac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2}$

$\dfrac{a_1 + \sqrt{a_1^2 - 4a_2}}{2}$

$\dfrac{a_1 - \sqrt{a_1^2 - 4a_2}}{2} = \alpha \leftarrow$ Valor mínimo

$-\sqrt{a_1^2 - 4a_2} = 2\alpha - a_1 \Rightarrow \cancel{a_1^2} - 4a_2 = 4\alpha^2 - 4\alpha a_1 + \cancel{a_1^2}$

$-\cancel{4}a_2 = \cancel{4}\alpha^2 - \cancel{4}\alpha a_1$

$-a_2 = \alpha^2 + \alpha(-a_1)$

$-a_2 = \alpha^2 + \alpha(-\alpha - 2\zeta\omega_n)$

$-a_2 = \cancel{\alpha^2} - \cancel{\alpha^2} - 2\zeta\omega_n\alpha$

$$\boxed{a_2 = 2\zeta\omega_n\alpha}$$

d)

$2\alpha\zeta\omega_n = a_2$ (4)

$K_I = \alpha K_p$

(i) $K_I = \dfrac{\omega_n^2\alpha}{b}$

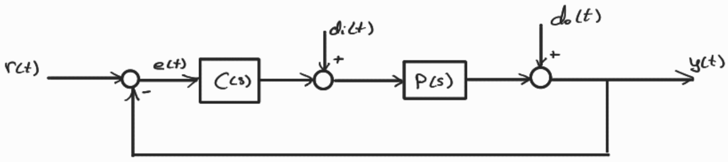
(ii) $K_p = \dfrac{\omega_n^2 + \cancel{2\zeta\omega_n\alpha} - \cancel{2\zeta\omega_n\alpha}}{b}$

Divido (i) ÷ (ii)

$$\Rightarrow \frac{K_I}{K_p} = \frac{\omega_n^2\alpha / \cancel{b}}{\omega_n^2 / \cancel{b}} \Rightarrow \boxed{K_I = \alpha K_p}$$

Trabajo Previo 2

$$C(s) = K_p + \frac{K_d\, s}{1 + s/N} = \frac{As + B}{s + N}$$



$$P(s) = 1/s$$

→ Se obtiene un lazo cerrado de segundo orden de formato:

$$T(s) = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)} = \frac{\left(\frac{P_1 P_2}{c}\right)(s + c)}{(s + P_1)(s + P_2)}$$

a)

$$C(s) = K_p + \frac{K_d\, s}{1 + s/N} = \frac{K_p + \frac{K_p s}{N} + K_d s}{1 + s/N} = \frac{K_p N + K_p s + K_d N s}{N + s} = \frac{(K_p + K_d N)s + K_p N}{N + s} = \frac{As + B}{s + N} \quad \begin{cases} A = K_p + K_d N \\ B = K_p N \end{cases}$$

b)

$$T(s) = \frac{\frac{As + B}{s + N}\,\frac{1}{s}}{1 + \frac{As + B}{s + N}\,\frac{1}{s}} = \frac{As + B}{s(s + N) + As + B} = \frac{As + B}{s^2 + (N + A)s + B}$$

$$T(s) = \frac{\frac{P_1 P_2}{c}\, s + P_1 P_2}{s^2 + (P_2 + P_1)s + P_1 P_2}$$

$$\begin{cases} A = \frac{P_1 P_2}{c} = \frac{1 \cdot 11}{10} \rightarrow \boxed{A = 1'1} \\ N + A = P_2 + P_1 \rightarrow N = 11 + 1 - 1'1 \rightarrow \boxed{N = 10'9} \\ B = P_1 P_2 \rightarrow \boxed{B = 11} \end{cases}$$