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PRÁCTICA 3 – A22

→ Ejercicio previo 1

$$Y(s) = \frac{1}{s}\,\frac{\frac{10s}{c}+10}{(s+1)(s+10)} = \frac{A}{s} + \frac{B}{s+1} + \frac{D}{s+10}$$

$$Y(s) = \frac{1}{s} + \frac{\frac{-10/c+10}{-9}}{s+1} + \frac{\frac{-100/c+10}{90}}{s+10} \xrightarrow{\mathcal{L}^{-1}} y(t) = \overset{\infty}{1} - \overbrace{\left(\frac{-\frac{10}{c}+10}{9}\right)e^{-t} + \left(\frac{-\frac{100}{c}+10}{90}\right)e^{-10t}}^{\text{TRANSITORIO}}$$

$$\left.\begin{aligned} A &= Y(s)\cdot s\Big]_{s=0} = 1 \\ B &= Y(s)(s+1)\Big]_{s=-1} = \frac{-\frac{10}{c}+10}{-9} \\ D &= Y(s)(s+10)\Big]_{s=-10} = \frac{-\frac{100}{c}+10}{90} \end{aligned}\right\} \to \text{Como } y(t) = y_t + y_\infty \begin{cases} y_\infty = 1 \\ y_t = 0 = \lim_{t\to\infty}\left[-\left(\frac{-\frac{10}{c}+10}{9}\right)e^{-t} + \left(\frac{-\frac{100}{c}+10}{90}\right)e^{-10t}\right] \end{cases}$$

→ Si cogemos la respuesta $y_R(t) = 1 - e^{-10t}$ → Vemos que $\begin{cases} y_\infty = 1 \\ y_t = 0 = \lim_{t\to\infty}\left[-e^{-10t}\right] \end{cases}$

$\boxed{\to \text{Cuando } c = 1'1 \to y(t) = 1 - 0'101\, e^{-t} - 0'9\, e^{-10t}}$

→ Ejercicio previo 2

$$\left.\begin{aligned} U(s) &= 1/s \\ G(s) &= \frac{a^2}{(s+a)^2} \end{aligned}\right\} Y(s) = \frac{a^2}{s(s+a)^2} = \frac{A}{s} + \frac{B}{(s+a)^2} + \frac{C}{s+a}$$

$$\left.\begin{aligned} A &= Y(s)\cdot s\Big]_{s=0} = \frac{a^2}{a^2} = 1 \\ B &= Y(s)(s+a)^2\Big]_{s=-a} = \frac{a^2}{s}\Big]_{s=-a} = \frac{a^2}{-a} = -a \\ C &= \frac{dB}{ds}\Big]_{s=-a} = \frac{-a^2}{s^2} = \frac{-a^2}{a^2} = -1 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{a}{(s+a)^2} - \frac{1}{s+a}\right]$$

$$y(t) = 1 - at\cdot e^{-at} - e^{-at} \to \boxed{y(t) = 1 - (1+at)\,e^{-at}}$$

→ Ejercicio previo 3

$$G(s) = K_p + \frac{Kd \cdot s}{1+\varepsilon s} \rightarrow G(s) = \frac{K_p(1+\varepsilon s) + Kd\, s}{1+\varepsilon s} = \frac{\frac{(Kd + K_p\varepsilon)s + K_p}{\varepsilon}}{s + 1/\varepsilon}$$

$$U(s) = 1/s \qquad Y(s) = \frac{\frac{(Kd + K_p\varepsilon)s + K_p}{\varepsilon}}{(s + 1/\varepsilon)s} = \frac{A}{s} + \frac{B}{s + 1/\varepsilon}$$

$$A = Y(s) \cdot s \Big]_{s=0} = K_p$$

$$B = Y(s) \cdot (s + 1/\varepsilon) \Big]_{s=-1/\varepsilon} = \frac{(Kd + K_p\varepsilon)s + K_p}{\varepsilon \cdot s} \Big]_{s=-1/\varepsilon} = \frac{(Kd + K_p\varepsilon)(-1/\varepsilon) + K_p}{-1} = \frac{-Kd/\varepsilon - K_p + K_p}{-1} = \frac{Kd}{\varepsilon}$$

$$\Rightarrow Y(s) = \frac{K_p}{s} + \frac{Kd/\varepsilon}{s + 1/\varepsilon} \rightarrow \boxed{y(t) = K_p + \frac{Kd}{\varepsilon} e^{-t/\varepsilon}}$$

$$\boxed{\begin{aligned} y(0) &= K_p + \frac{Kd}{\varepsilon} \\ y(\infty) &= K_p \\ y(\varepsilon) &= K_p + \frac{Kd/\varepsilon}{e} \end{aligned}}$$

$y(\infty) = 2 \rightarrow \boxed{K_p = 2}$

$y(0) = 12 = K_p + \frac{Kd}{\varepsilon} \rightarrow \frac{Kd}{\varepsilon} = 10$

$y(\varepsilon) = 2 + \frac{10}{e} \rightarrow y(\varepsilon) = 5'679 \leftarrow$ Lo busco en la gráfica

$\rightarrow \boxed{\varepsilon \approx 0'1}$

$\boxed{Kd = 1}$

→ Ejercicio previo 4

$$y(t) = y_0 \cdot e^{-pt}$$

$$x(t) = 1 - \frac{y(t)}{y_0}$$

a)

$$x(t) = 1 - \frac{\cancel{y_0} e^{-pt}}{\cancel{y_0}} = 1 - e^{-pt}$$

→ Como la transformada inversa de Laplace de un sistema de primer orden del tipo $G(s) = \frac{1}{s+p}$ es e^{-pt}, se verifica que $x(t)$ es la respuesta a escalón unitario de un sistema de primer orden

b) $td = 5570$ $\quad$ ¿p?
$x(td) = 0'5$

$$x(t) = 1 - e^{-pt} \rightarrow 0'5 = 1 - e^{-p \cdot 5570} \rightarrow 1 - 0'5 = \frac{1}{e^{5570p}} \rightarrow p = \frac{\ln 2}{5570} \rightarrow p = 1'24 \times 10^{-4}$$

c)

$$x(td) = 0'25 = 1 - e^{-p \cdot td} \rightarrow 1 - 0'25 = \frac{1}{e^{p\,td}} \rightarrow \ln(4/3) = p \cdot td \rightarrow td = 2311'76 \text{ (años)}$$

→ Ejercicio previo 5

$$M\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Ky(t) = U(t)$$

$$S.O. = e^{-\frac{\pi}{tg\theta}} = 0'1$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0'5\,s$$

$y(t)$ = posición → $y(\infty) = 0'01m$

$u(t)$ = fuerza → Escalón unitario

$$0'1 = e^{-\pi/tg\theta} \rightarrow \ln 0'1 = -\pi/tg\theta$$

$$tg\theta = -\pi/\ln 0'1 = 1'3644 \rightarrow \theta = 56'761$$

$$\zeta = \cos\theta = 0'5911$$

$$\omega_n = \frac{\pi}{0'5\sqrt{1-\zeta^2}} = 7'7897$$

$$\mathcal{L}[\text{función impedancia}] = M\cdot s^2 Y(s) + Bs\,Y(s) + K\,Y(s) = U(s) \rightarrow$$

$$\rightarrow Y(s) = \frac{U(s)}{Ms^2 + Bs + K}$$

$$\boxed{\rightarrow G_G(s) = \frac{Y(s)}{U(s)} = \frac{1}{Ms^2+Bs+K} = \frac{1/M}{s^2+\frac{B}{M}s+\frac{K}{M}}}$$

→ Ec característica sist 2° orden (con ganancia)

$$G(s) = \frac{K_p\,\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$ → Igualando términos con G_G

$$\left.\begin{aligned} \frac{B}{M} &= 2\zeta\omega_n = 9'20898 \\ \frac{K}{M} &= \omega_n^2 = 60'679 \\ K_p\,\omega_n^2 &= \frac{1}{M} \end{aligned}\right\}$$

→ sabemos que $y(\infty) = 0'01$

$$y(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y(s)$$

$$\lim_{s\to 0} s\,Y(s) = \lim_{s\to 0}\frac{1/s\cdot s}{Ms^2+Bs+K} = \frac{1}{K} = 0'01$$

$$\boxed{\begin{aligned} &\rightarrow K = 100 \\ &\rightarrow M = 1'648 \\ &\rightarrow B = 15'1765 \\ &\rightarrow K_p = 0'01 \end{aligned}}$$

$$\boxed{G_G(s) = \frac{1/1'648}{s^2+9'20898s+60'679}}$$