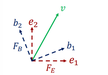
车辆运动建模 → 为定义其运动的几何约束（运动学建模）
→ 为施作用在车辆上的所有力和力矩（动态建模）

地心地坐标系

- Right handed by convention
- Inertial frame
  - Fixed, usually relative to earth
- Body frame
  - Attached to vehicle, origin at vehicle center of gravity, or center of rotation
- Sensor frame
  - Attached to sensor, convenient for expressing sensor measurements

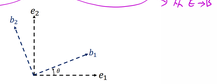
车身坐标系

传感器框架是一个坐标系



$C_{EB} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ 从B→E

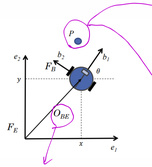
$C_{BE} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ 从E→B



坐标系E和坐标系B有相同的固定原点

B相对于E为其转动角度

点P在车身坐标系FB和惯性坐标系FE有关系式



- Location of point (P) in Body Frame (B)

$P_B = C_{EB}(\theta)\, P_E + O_{EB}$ ← Translation term, expressed in body frame

- Location of point (P) in Inertial Frame (E)

$P_E = C_{BE}(\theta)\, P_B + O_{BE}$ ← Translation term, expressed in inertial frame

原点的平移

增广矩阵向量时展到四维 x,y,0,1

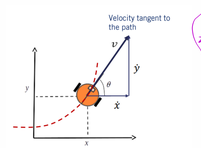
- A 2D vector in homogeneous form

$P = \begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \bar{P} = \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$

- Transforming a point from body to inertial coordinates with homogeneous coordinates

$\bar{P_E} = \begin{bmatrix} C_{EB}(\theta) & O_{EB} \end{bmatrix} \bar{P_B}$

↳ 车身视为惯性坐标系



机器人的运动被限制向前运动 → 因为它的轮子指向这个方向

排完整性约束

限制了机器人位置的变化率

y方向的速度

$\frac{dy}{dx} = \tan\theta = \frac{\sin\theta}{\cos\theta}$ → 速度由路径的切向量定义

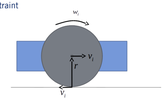
x方向的速度

$\dot{y}\cos\theta - \dot{x}\sin\theta = 0 \Rightarrow \begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \end{aligned}$

轮子速度等于车体旋转速度

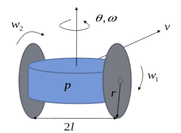
Kinematic constraint

$v_i = rw_i$



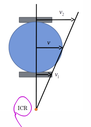
Assume control inputs are wheel speeds

- Center: p
- Wheel to center: l
- Wheel radius: r
- Wheel rotation rates: w1, w2 （左右车轮的角速度）



Velocity is the average of the two wheel velocities

$v = \frac{v_1 + v_2}{2} = \frac{rw_1 + rw_2}{2}$



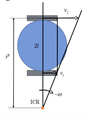
瞬时转动中心

机器人速度为以用两个轮子的平均速度计算

- Use the instantaneous center of rotation (ICR)
- Equivalent triangles give the angular rate of rotation

$\omega = \frac{-v_2}{\rho} = \frac{-(v_2 - v_1)}{2l}$

$\omega = \frac{rw_1 - rw_2}{2l}$



- Continuous time model:

$\dot{x} = \left[\left(\frac{rw_1 + rw_2}{2}\right)\cos\theta\right]$

$\dot{y} = \left[\left(\frac{rw_1 + rw_2}{2}\right)\sin\theta\right]$

$\dot{\theta} = \left(\frac{rw_1 - rw_2}{2l}\right)$

- Discrete time model:

$x_{k+1} = x_k + \left[\left(\frac{rw_{1,k} + rw_{2,k}}{2}\right)\cos\theta_k\right]\Delta t$

$y_{k+1} = y_k + \left[\left(\frac{rw_{1,k} + rw_{2,k}}{2}\right)\sin\theta_k\right]\Delta t$

$\theta_{k+1} = \theta_k + \left(\frac{rw_{1,k} - rw_{2,k}}{2l}\right)\Delta t$