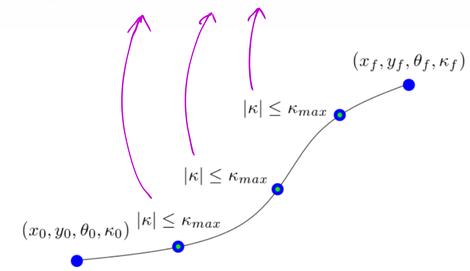
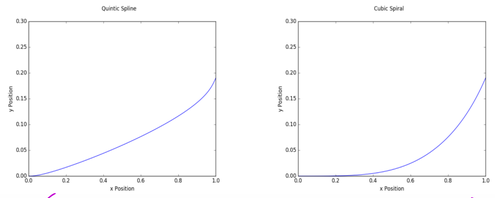
对于路径规划器 唯一的运动学约束就是限制路径上的最大曲率




对于自动驾驶 → 通常要求路径是参数曲线 → 可以被描述为一组具有特定参数的方程

两种常见的路径参数化

- Two common parameterized curves are quintic splines and cubic spirals
- Both allow us to satisfy boundary conditions, and can be optimized parametrically




五阶多项式

三次曲率方程

quintic splines（五次样条） x和y的五阶多项式函数

cubic spirals（多项式累旋） 多项式曲率函数给出相对纵长

## Quintic Splines

- $x$ and $y$ are defined by 5th order splines
- Closed form solution available for $(x, y, \theta, \kappa)$ boundary conditions

$$x(u) = \alpha_5 u^5 + \alpha_4 u^4 + \alpha_3 u^3 + \alpha_2 u^2 + \alpha_1 u + \alpha_0$$

$$y(u) = \beta_5 u^5 + \beta_4 u^4 + \beta_3 u^3 + \beta_2 u^2 + \beta_1 u + \beta_0$$

$$u \in [0,1]$$

## Quintic Splines Curvature

- Challenging to constrain curvature due to nature of spline's curvature
  - Due to potential discontinuities in curvature or its derivatives

$$\kappa(u) = \frac{x'(u)y''(u) - y'(u)x''(u)}{(x'(u)^2 + y'(u)^2)^{\frac{3}{2}}}$$

很难将曲率限制在一定的范围内

## Polynomial Spirals

- Spirals are defined by their curvature as a function of arc length
- Closed form curvature definition allows for simple curvature constraint checking
  - Curvature is well-behaved between sampled points as well due to polynomial formulation

$$\kappa(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$\theta(s) = \theta_0 + \int_0^s a_3 s'^3 + a_2 s'^2 + a_1 s' + a_0 ds'$$

$$= \theta_0 + a_3 \frac{s^4}{4} + a_2 \frac{s^3}{3} + a_1 \frac{s^2}{2} + a_0 s$$

$$x(s) = x_0 + \int_0^s \cos(\theta(s'))ds'$$

$$y(s) = y_0 + \int_0^s \sin(\theta(s'))ds'$$

## Polynomial Spiral Position

- Spiral position does not have a closed form solution
- Fresnel integrals need to be evaluated numerically
  - This can be done using Simpson's rule

$$x(s) = x_0 + \int_0^s \cos(\theta(s'))ds'$$

$$y(s) = y_0 + \int_0^s \sin(\theta(s'))ds'$$

$$\int_0^s f(s')ds' \approx \frac{s}{3n}\left(f(0) + 4f\left(\frac{s}{n}\right) + 2f\left(\frac{2s}{n}\right) + \cdots + f(s)\right)$$

Simpson's Rule

⇐ 没有曲率约束位置和方向的封闭形式解

Simpson's Rule ?

辛普森法则

生成速度剖面 → 第一步是确定最终的零速度

一个好的参考值是行为规划者的参考速度

在处理动态障碍物时 → 在计算中最好留出空间和时间缓冲