TALLER PROGCOMP: TRACK GRAFOS

LOWEST COMMON ANCESTOR

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Part I

Ordered Tree

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Definición

Es un árbol, pero que tiene un nodo designado como ráiz y existe un orden de los hijos de cada nodo.

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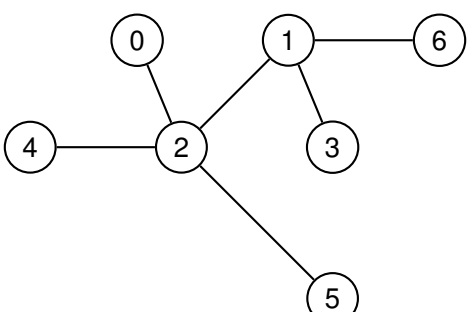

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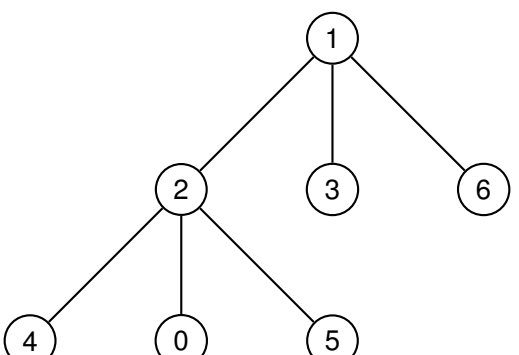


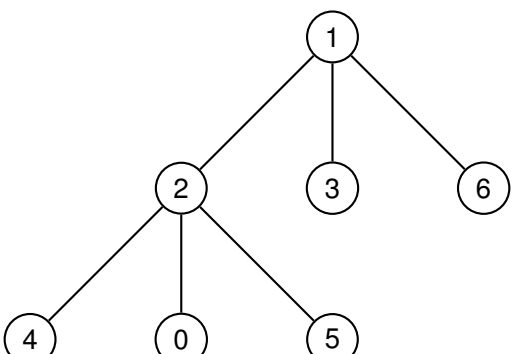
Part II

Lowest Common Ancestor

Lowest Common Ancestor

Definición

Dado dos vértices *a* y *b* de un *ordered tree*, se quiere determinar el menor ancestro en común.



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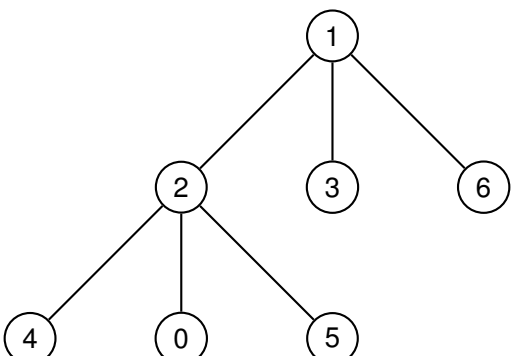

- $LCA(4, 5) = 2$

Lowest Common Ancestor

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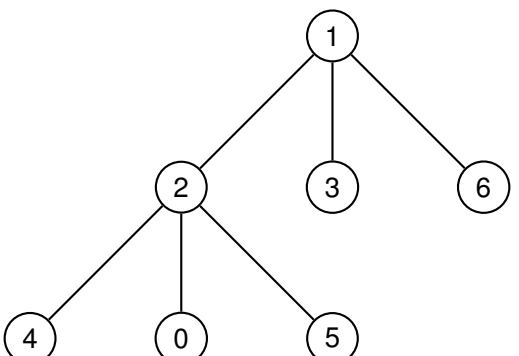

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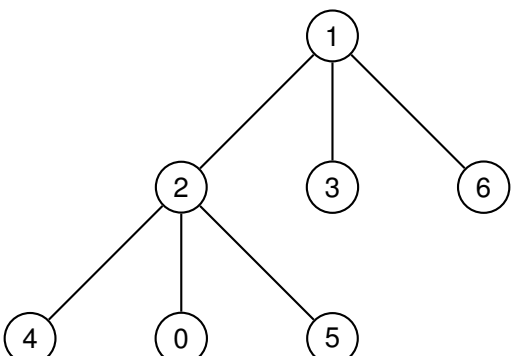

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¿Cómo resolverlo eficientemente?

Euler Tour

1. Partiendo de la raiz, se realiza un DFS en el árbol, se agrega a un arreglo el nodo al momento de “entrar al nodo” en el DFS.

Euler Tour

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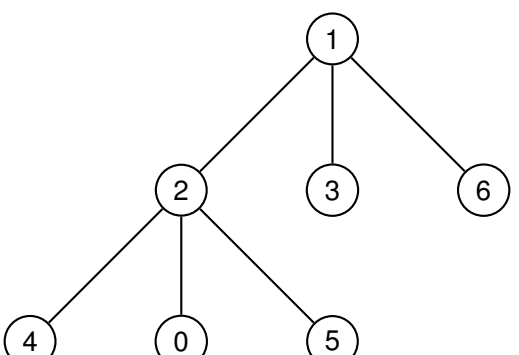
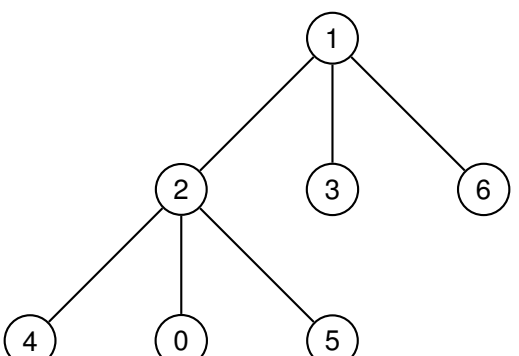
2. Se guarda la altura del nodo al momento de “entrar al nodo” en el DFS.



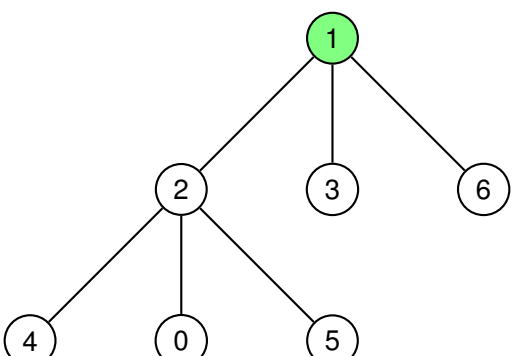
Euler Tour

Euler Tour	[]
Altura	[]



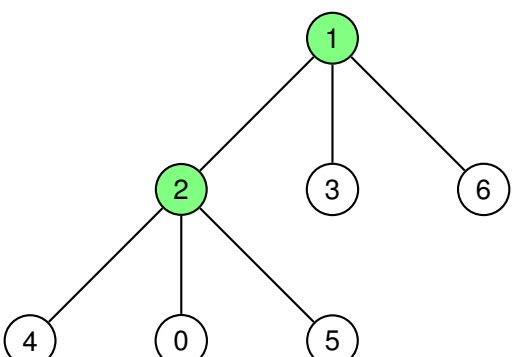
Euler Tour

Euler Tour	[1]
Altura	[0]



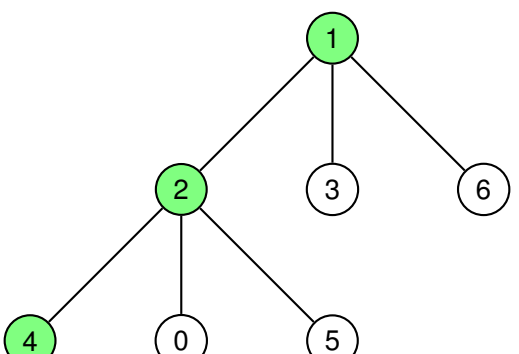
EULER TOUR

Euler Tour	[1, 2]
Altura	[0, 1]



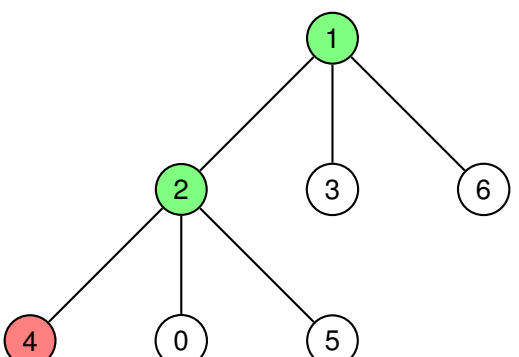
Euler Tour

Euler Tour	$[1, 2, 4]$
Altura	$[0, 1, 2]$



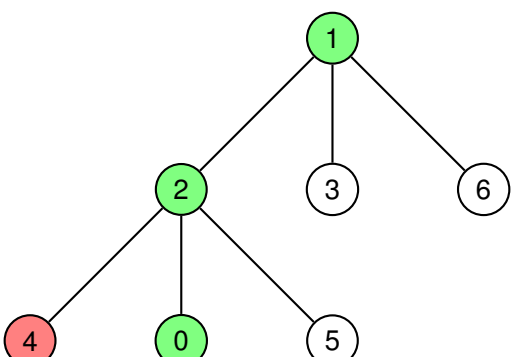
Euler Tour

Euler Tour	$[1, 2, 4, 2]$
Altura	$[0, 1, 2, 1]$



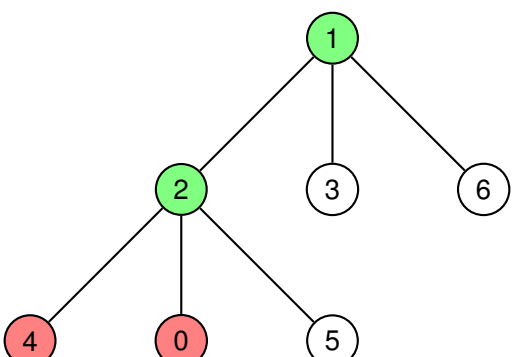
Euler Tour

Euler Tour	$[1, 2, 4, 2, 0]$
Altura	$[0, 1, 2, 1, 2]$



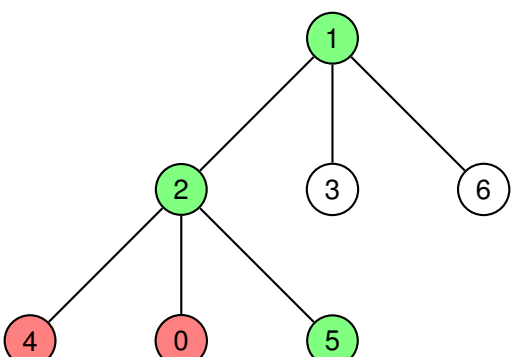
Euler Tour

Euler Tour	$[1, 2, 4, 2, 0, 2]$
Altura	$[0, 1, 2, 1, 2, 1]$



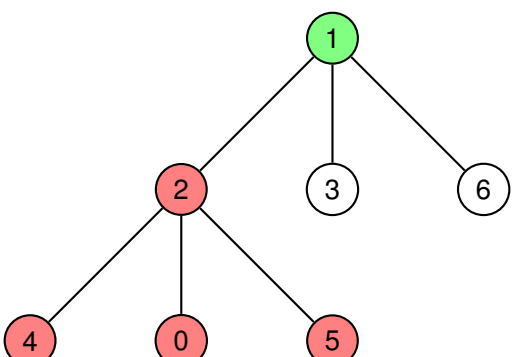
Euler Tour

Euler Tour	$[1, 2, 4, 2, 0, 2, 5]$
Altura	$[0, 1, 2, 1, 2, 1, 2]$



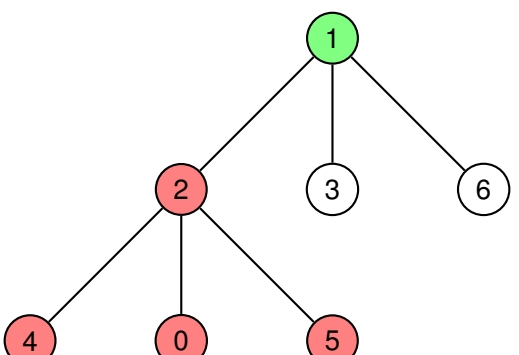
Euler Tour

Euler Tour	$[1, 2, 4, 2, 0, 2, 5, 2]$
Altura	$[0, 1, 2, 1, 2, 1, 2, 1]$



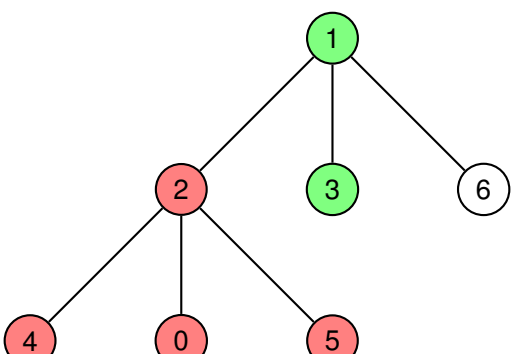
Euler Tour

Euler Tour	$[1, 2, 4, 2, 0, 2, 5, 2, 1]$
Altura	$[0, 1, 2, 1, 2, 1, 2, 1, 0]$



Euler Tour

Euler Tour	$[1, 2, 4, 2, 0, 2, 5, 2, 1, 3]$
Altura	$[0, 1, 2, 1, 2, 1, 2, 1, 0, 1]$



Euler Tour

Euler Tour	$[1, 2, 4, 2, 0, 2, 5, 2, 1, 3, 1]$
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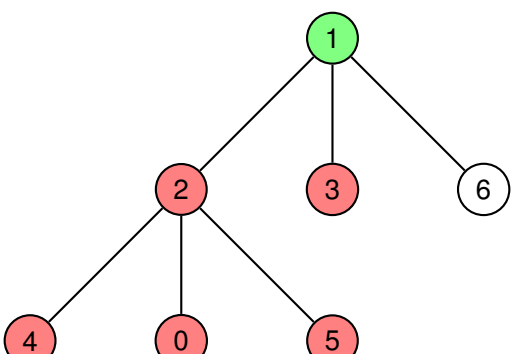
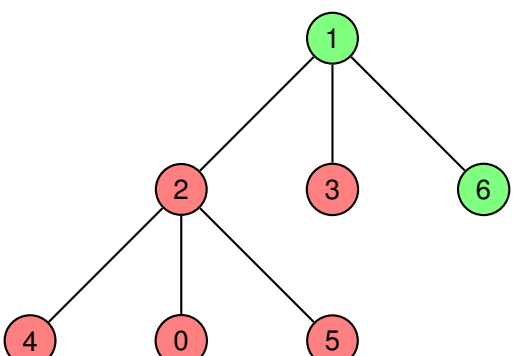


Euler Tour

Euler Tour	$[1, 2, 4, 2, 0, 2, 5, 2, 1, 3, 1, 6]$
Altura	$[0, 1, 2, 1, 2, 1, 2, 1, 0, 1, 0, 1]$



Euler Tour

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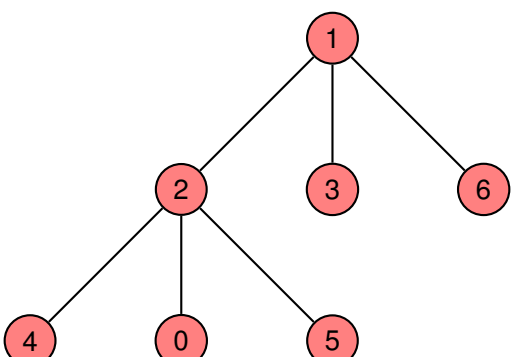
Altura	$[0, 1, 2, 1, 2, 1, 2, 1, 0, 1, 0, 1, 0]$



Query sobre Euler Tour

¿$LCA(0, 3)$?

Euler Tour	$[1, 2, 4, 2, 0, 2, 5, 2, 1, 3, 1, 6, 1]$
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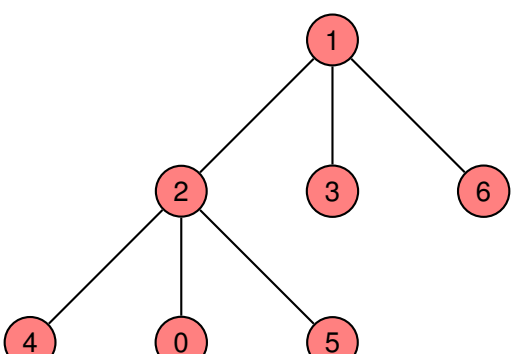

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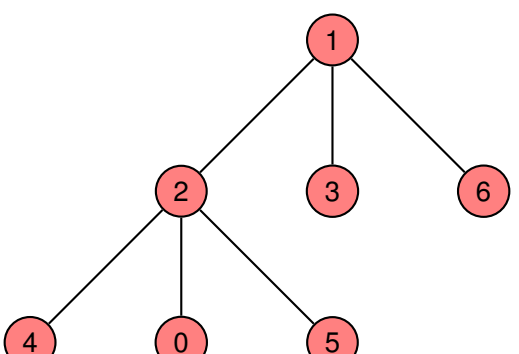

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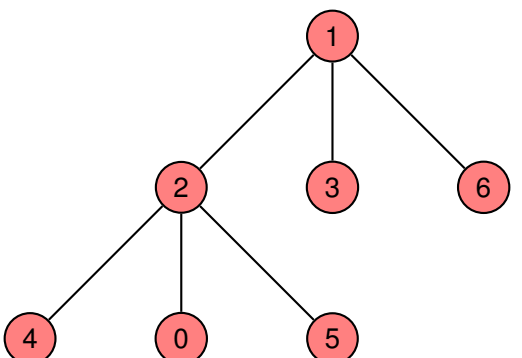

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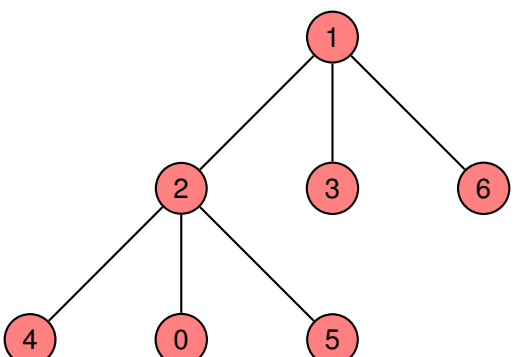

Complejidad: $O(n + a) + O(lca(a, b))$

QUERY SOBRE EULER TOUR

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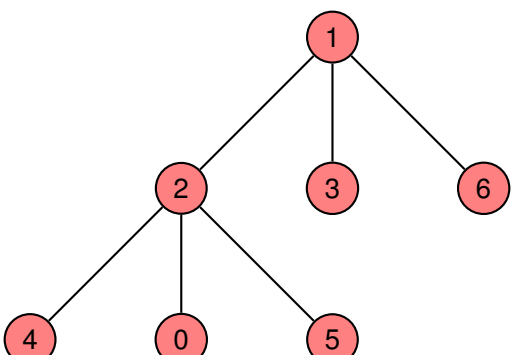

Complejidad: $O(n + a) + O(lca(a, b))$

- SQRT Decompositon: $O(\sqrt{n})$ con $O(n)$ en construcción
- Segment Tree/ Fenwick Tree: $O(\log n)$ con $O(n)$ en construcción
- Sparse Table: $O(1)$ con $O(n \log n)$ en construcción

Código

```cpp
int n, v;
vector< bool > visitado;
vector< vector < int > > lista;
// cuando aparece por primera vez el nodo i
vector< int > first;
// euler tour
vector< int > alts;
vector< int > euler;

void dfs(int u, int alt) {
  visitado[u] = true;
  alts.push_back(alt);
  euler.push_back(u);
  first[u] = euler.size() - 1;
  for(int i = 0; i < lista[u].size(); i++) {
    int v = lista[u][i];
    if(visitado[v] == false){
      dfs(v, alt + 1);
      alts.push_back(alt);
      euler.push_back(u);
    }
  }
}
```

Código usando Segment Tree

```cpp
// nodo de segment tree
// first -> cual nodo del arbol tiene minima altura en el rango
// second -> la altura de ese nodo del arbol
pair< int, int > merge(pair< int, int > a, pair< int, int > b) {
  if(a.second < b.second) return a;
  return b;
}

int main() {
  // se asume que generaron el arbol correctamente

  dfs(root); // root es el nodo raiz del arbol
  vector< pair< int, int > > euler_alts;
  for(int i = 0; i < euler.size(); i++) {
    euler_alt.push_back({euler[i], alts[i]});
  }
  segment_tree< pair< int, int >, merge > st(euler_alts);

  int a, b;
  cin >> a >> b;
  if(first[a] > first[b]) swap(a, b);
  cout << st.query(first[a], first[b]).first << "\n";
}
```

REFERENCES I