

Multivariable Calc 2

Ford Smith

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1. shit

2. (a)

$$T(x, y, z) = x^2 + 2y^2 - 3z + 1$$

$$\Delta T = \langle 2x, 4y, -3 \rangle$$

$$\langle \frac{2}{\sqrt{13}}, 0, -\frac{3}{\sqrt{13}} \rangle$$

(b)

$$T(3, 2, 1) = 15 \quad 17 = x^2 + y^2$$

$$T(\sqrt{15}, 1, 1) = 15$$

$$\langle \frac{3 - \sqrt{15}}{\sqrt{1 + (3 - \sqrt{15})^2}}, \frac{1}{\sqrt{1 + (3 - \sqrt{15})^2}}, 0 \rangle$$

3.

$$f(x, y) = e^{2x-y-2} + y + \sin(x-1) \quad x(t) = \cos(5t), y(t) = \sin(5t)$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

$$\frac{\partial f}{\partial x} = 2e^{2x-y-2} + \cos(x-1) \quad \frac{dx}{dt} = -5\sin(t)$$

$$\frac{\partial f}{\partial y} = -e^{2x-y-2} + 1 \quad \frac{dy}{dt} = 5\cos(t)$$

$$(2e^{2x-y-2} + \cos(x-1)) \cdot -5\sin(t) + (-e^{2x-y-2} + 1) \cdot 5\cos(t) = \frac{df}{dt}$$

4.

$$f(x, y) = xy + x + 2y \quad g(x, y) = xy - 4$$

$$\Delta f = \langle y + 1, x + 2 \rangle \quad \Delta g = \langle y, x \rangle$$

$$\frac{y + 1}{y} = \frac{x + 2}{x} \rightarrow x = 2y$$

$$2y^2 = 4 \rightarrow y = \sqrt{2}, x = 2\sqrt{2}$$

5. s

6.

$$f : x^2 + y^2 + z^2 - 9 = 0 \quad g : z - x^2 - y^2 + 3 = 0$$

$$\Delta f = \langle 2x, 2y, 2z \rangle \quad \Delta g = \langle -2x, -2y, 1 \rangle$$

Tangent plane of f and g respectively: $4(x - 2) - 2(y + 1) + 4(z - 2) = 0$

$$-4(x - 2) + 2(y + 1) + z - 2 = 0$$

$$\vec{n}_1 = \langle 4, -2, 4 \rangle \quad \vec{n}_2 = \langle -4, 2, 1 \rangle$$

$$\cos(\theta) = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|}$$

$$\cos(\theta) = \frac{-16}{6\sqrt{21}}$$