Multivariable Calc 2

Ford Smith

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1. shit
2. (a)

$$T(x, y, z) = x^2 + 2y^2 - 3z + 1$$
$$\Delta T =< 2x, 4y, -3 >$$
$$< \frac{2}{\sqrt{13}}, 0, -\frac{3}{\sqrt{13}} >$$

   (b)

$$T(3, 2, 1) = 15 \quad 17 = x^2 + y^2$$
$$T(\sqrt{15}, 1, 1) = 15$$
$$< \frac{3 - \sqrt{15}}{\sqrt{1 + (3 - \sqrt{15})^2}}, \frac{1}{\sqrt{1 + (3 - \sqrt{15})^2}}, 0 >$$

3.

$$f(x, y) = e^{2x-y-2} + y + \sin(x - 1) \quad x(t) = \cos(5t), y(t) = \sin(5t)$$
$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$
$$\frac{\partial f}{\partial x} = 2e^{2x-y-2} + \cos(x - 1) \quad \frac{dx}{dt} = -5\sin(t)$$
$$\frac{\partial f}{\partial y} = -e^{2x-y-2} + 1 \quad \frac{dy}{dt} = 5\cos(t)$$
$$\left(2e^{2x-y-2} + \cos(x - 1)\right) \cdot -5\sin(t) + \left(-e^{2x-y-2} + 1\right) \cdot 5\cos(t) = \frac{df}{dt}$$

4.

$$f(x,y) = xy + x + 2y \quad g(x,y) = xy - 4$$
$$\Delta f =< y+1, x+2 > \quad \Delta g =< y, x >$$
$$\frac{y+1}{y} = \frac{x+2}{x} \rightarrow x = 2y$$
$$2y^2 = 4 \rightarrow y = \sqrt{2}, x = 2\sqrt{2}$$

5. s

6.

$$f: x^2 + y^2 + z^2 - 9 = 0 \quad g: z - x^2 - y^2 + 3 = 0$$
$$\nabla f =< 2x, 2y, 2z > \quad \nabla g =< -2x, -2y, 1 >$$

Tangent plane of f and g respectively: $4(x-2)-2(y+1)+4(z-2)=0$

$$-4(x-2) + 2(y+1) + z - 2 = 0$$
$$\vec{n_1} =< 4, -2, 4 > \quad \vec{n_2} =< -4, 2, 1 >$$
$$\cos(\theta) = \frac{\vec{n_1} \cdot \vec{n_2}}{\| \vec{n_1} \| \cdot \| \vec{n_2} \|}$$
$$\cos(\theta) = \frac{-16}{6\sqrt{21}}$$

7.

$$f\,(x,y,z) = x^3 + y^3 - z^3 \quad f\,(9,10,12) = 1$$
$$f_x = 3x^2 \quad f_y = 3y^2 \quad f_z = -3z^2$$
$$L(x,y,z) = f(9,10,12) + f_x(x-9) + f_y(y-10) + f_z(z-12)$$
$$L(x,y,z) = 1 + 243(x-9) + 300(y-10) - 432(z-12)$$
$$L(9.001, 10.02, 12.001) = 1 + 243 * .001 + 300 * .02 - 432 * .001 = 6.811$$
$$f(9.001, 10.02, 12.001) = 6.822999$$

8.

$$< \frac{3}{5}, \frac{4}{5} > \cdot \nabla f = 2 \quad < \frac{-4}{5}, \frac{3}{5} > \cdot \nabla f = 2$$
$$\frac{3}{5} f_x + \frac{4}{5} f_y = 2 \quad \frac{-4}{5} f_x + \frac{3}{5} f_y = -1$$
$$f_x(0,0) = 2 \quad f_y(0,0) = 1$$
$$2x + y + 1 = L(x,y) \quad f(0.06, .08) = 1.2$$

9. 9

10. 10

11. (a)

$$f(x,y) = 2x^2 + y^4 - 4xy$$

$$f_x = 4x - 4y = 0 \quad f_y = 4y^3 - 4x = 0$$

$$(-1,-1), (0,0), (1,1)$$

(b)

$$\begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 12y^2 \end{bmatrix}$$

(c)

$$\mathbf{D} = f_{xx}f_{yy} - f_{xy}^2 = 48y^2 - 16$$

$$D(-1,-1) > 0, f_{xx} > 4 \rightarrow \text{local minimum}$$

$$D(0,0) < 0 \rightarrow \text{saddle point}$$

$$D(1,1) > 0, f_{xx} > 4 \rightarrow \text{local minimum}$$

(d)

$$-\nabla f = - < 4x - 4y, 4y^3 - 4x > \quad -\nabla f(3,2) = - < 4, 20 >$$