Multivariable Calc 2

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1. shit

2. (a)

$$T(x,y,z) = x^2 + 2y^2 - 3z + 1$$
$$\Delta T =< 2x, 4y, -3 >$$
$$< \frac{2}{\sqrt{13}}, 0, -\frac{3}{\sqrt{13}} >$$

   (b)

$$T(3,2,1) = 15 \quad 17 = x^2 + y^2$$
$$T(\sqrt{15}, 1, 1) = 15$$
$$< \frac{3-\sqrt{15}}{\sqrt{1+(3-\sqrt{15})^2}}, \frac{1}{\sqrt{1+(3-\sqrt{15})^2}}, 0 >$$

3.

$$f(x,y) = e^{2x-y-2} + y + \sin(x-1) \quad x(t) = \cos(5t), y(t) = \sin(5t)$$
$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$
$$\frac{\partial f}{\partial x} = 2e^{2x-y-2} + \cos(x-1) \quad \frac{dx}{dt} = -5\sin(t)$$
$$\frac{\partial f}{\partial y} = -e^{2x-y-2} + 1 \quad \frac{dy}{dt} = 5\cos(t)$$
$$\left(2e^{2x-y-2} + \cos(x-1)\right) \cdot -5\sin(t) + \left(-e^{2x-y-2} + 1\right) \cdot 5\cos(t) = \frac{df}{dt}$$

4.

$$f(x,y) = xy + x + 2y \quad g(x,y) = xy - 4$$
$$\Delta f =< y+1, x+2 > \quad \Delta g =< y, x >$$
$$\frac{y+1}{y} = \frac{x+2}{x} \rightarrow x = 2y$$
$$2y^2 = 4 \rightarrow y = \sqrt{2}, x = 2\sqrt{2}$$

5. s

6.

$$f : x^2 + y^2 + z^2 - 9 = 0 \quad g : z - x^2 - y^2 + 3 = 0$$
$$\nabla f =< 2x, 2y, 2z > \quad \nabla g =< -2x, -2y, 1 >$$

Tangent plane of f and g respectively: $4(x-2) - 2(y+1) + 4(z-2) = 0$

$$-4(x-2) + 2(y+1) + z - 2 = 0$$
$$\vec{n_1} =< 4, -2, 4 > \quad \vec{n_2} =< -4, 2, 1 >$$
$$\cos(\theta) = \frac{\vec{n_1} \cdot \vec{n_2}}{\| \vec{n_1} \| \cdot \| \vec{n_2} \|}$$
$$\cos(\theta) = \frac{-16}{6\sqrt{21}}$$

7. (a)

$$f\,(x,y,z) = x^3 + y^3 - z^3 \quad f\,(9,10,12) = 1$$
$$f_x = 3x^2 \quad f_y = 3y^2 \quad f_z = -3z^2$$
$$L(x,y,z) = f(9,10,12) + f_x(x-9) + f_y(y-10) + f_z(z-12)$$
$$L(x,y,z) = 1 + 243(x-9) + 300(y-10) - 432(z-12)$$
$$L(9.001, 10.02, 12.001) = 1+243*.001+300*.02-432*.001 = 6.811$$
$$f(9.001, 10.02, 12.001) = 6.822999$$

(b)

$$f(x,y) = x\sqrt{y}f(3.141, 163) = 3.141\sqrt{163}$$
$$f_x = \sqrt{y} = \sqrt{163} \quad f_y = \frac{x}{2\sqrt{y}} = \frac{3.141}{2\sqrt{163}}$$
$$L(x,y) = 3.141\sqrt{163} + \sqrt{163}(x-3.141) + \frac{3.141}{2\sqrt{163}}(y-163)$$
$$L(3,169) = 39.0395 \quad f(3,169) = 39$$

8.

$$< \frac{3}{5}, \frac{4}{5} > \cdot \nabla f = 2 \quad < \frac{-4}{5}, \frac{3}{5} > \cdot \nabla f = 2$$

$$\frac{3}{5} f_x + \frac{4}{5} f_y = 2 \quad \frac{-4}{5} f_x + \frac{3}{5} f_y = -1$$

$$f_x(0,0) = 2 \quad f_y(0,0) = 1$$

$$2x + y + 1 = L(x,y) \quad f(0.06, .08) = 1.2$$

9. 9

10. 10

11. (a)

$$f(x,y) = 2x^2 + y^4 - 4xy$$

$$f_x = 4x - 4y = 0 \quad f_y = 4y^3 - 4x = 0$$

$$(-1,-1), (0,0), (1,1)$$

(b)

$$\begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 12y^2 \end{bmatrix}$$

(c)

$$\mathbf{D} = f_{xx} f_{yy} - f_{xy}^2 = 48y^2 - 16$$

$$D(-1,-1) > 0, f_{xx} > 4 \to \text{local minimum}$$

$$D(0,0) < 0 \to \text{saddle point}$$

$$D(1,1) > 0, f_{xx} > 4 \to \text{local minimum}$$

(d)

$$-\nabla f = - < 4x - 4y, 4y^3 - 4x > \quad -\nabla f(3,2) = - < 4, 20 >$$