Multivariable Calc 2

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1. By saying $\nabla f = \lambda \nabla g$, we are saying that ∇g is a constant multiple of ∇f. Since they are both vectors, they can have a cross product. This cross product then must become 0 because they are constant multiples.

$$f(x,y) = xy \quad \nabla f = <y, x>$$

$$g(x,y) = x^2 + y^2 - 9 \quad \nabla g = <2x, 2y>$$

$$y = \lambda 2x \quad x = \lambda 2y \rightarrow \frac{y}{2x} = \frac{x}{2y}$$

$$x = y \quad 2x^2 = 9 \rightarrow x = y = \pm \frac{3}{\sqrt{2}}$$

2. (a)
$$T(x,y,z) = x^2 + 2y^2 - 3z + 1$$
$$\Delta T = <2x, 4y, -3>$$
$$< \frac{2}{\sqrt{13}}, 0, -\frac{3}{\sqrt{13}} >$$

   (b)
$$T(3,2,1) = 15 \quad 17 = x^2 + y^2$$
$$T(\sqrt{15}, 1, 1) = 15$$
$$< \frac{3 - \sqrt{15}}{\sqrt{1 + (3 - \sqrt{15})^2}}, \frac{1}{\sqrt{1 + (3 - \sqrt{15})^2}}, 0 >$$

3.

$$f(x,y) = e^{2x-y-2} + y + \sin(x-1) \quad x(t) = \cos(5t), y(t) = \sin(5t)$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$\frac{\partial f}{\partial x} = 2e^{2x-y-2} + \cos(x-1) \quad \frac{dx}{dt} = -5\sin(t)$$

$$\frac{\partial f}{\partial y} = -e^{2x-y-2} + 1 \quad \frac{dy}{dt} = 5\cos(t)$$

$$\left(2e^{2x-y-2} + \cos(x-1)\right) \cdot -5\sin(t) + \left(-e^{2x-y-2} + 1\right) \cdot 5\cos(t) = \frac{df}{dt}$$

4.

$$f(x,y) = xy + x + 2y \quad g(x,y) = xy - 4$$

$$\Delta f = <y+1, x+2> \quad \Delta g = <y, x>$$

$$\frac{y+1}{y} = \frac{x+2}{x} \rightarrow x = 2y$$

$$2y^2 = 4 \rightarrow y = \sqrt{2}, x = 2\sqrt{2}$$

5. s

6.

$$f : x^2 + y^2 + z^2 - 9 = 0 \quad g : z - x^2 - y^2 + 3 = 0$$

$$\nabla f = <2x, 2y, 2z> \quad \nabla g = <-2x, -2y, 1>$$

Tangent plane of f and g respectively: $4(x-2) - 2(y+1) + 4(z-2) = 0$

$$-4(x-2) + 2(y+1) + z - 2 = 0$$

$$\vec{n_1} = <4, -2, 4> \quad \vec{n_2} = <-4, 2, 1>$$

$$\cos(\theta) = \frac{\vec{n_1} \cdot \vec{n_2}}{\| \vec{n_1} \| \cdot \| \vec{n_2} \|}$$

$$\cos(\theta) = \frac{-16}{6\sqrt{21}}$$

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7. (a)
$$f(x, y, z) = x^3 + y^3 - z^3 \quad f(9, 10, 12) = 1$$
$$f_x = 3x^2 \quad f_y = 3y^2 \quad f_z = -3z^2$$
$$L(x, y, z) = f(9, 10, 12) + f_x(x - 9) + f_y(y - 10) + f_z(z - 12)$$
$$L(x, y, z) = 1 + 243(x - 9) + 300(y - 10) - 432(z - 12)$$
$$L(9.001, 10.02, 12.001) = 1 + 243*.001 + 300*.02 - 432*.001 = 6.811$$
$$f(9.001, 10.02, 12.001) = 6.822999$$

(b)
$$f(x, y) = x\sqrt{y} \quad f(3, 169) = 3\sqrt{169}$$
$$f_x = \sqrt{y} = \sqrt{169} \quad f_y = \frac{x}{2\sqrt{y}} = \frac{3}{2\sqrt{169}}$$
$$L(x, y) = 3\sqrt{169} + \sqrt{169}(x - 3) + \frac{3}{2\sqrt{169}}(y - 169)$$
$$L(3.141, 163) = 40.1407 \quad f(3, 169) = 40.1016$$

8.
$$<\frac{3}{5}, \frac{4}{5}> \cdot \nabla f = 2 \quad <\frac{-4}{5}, \frac{3}{5}> \cdot \nabla f = 2$$
$$\frac{3}{5}f_x + \frac{4}{5}f_y = 2 \quad \frac{-4}{5}f_x + \frac{3}{5}f_y = -1$$
$$f_x(0, 0) = 2 \quad f_y(0, 0) = 1$$
$$2x + y + 1 = L(x, y) \quad L(0.06, .08) = 1.2$$

9. 9

10.
$$f_x = \frac{1}{2} \quad f_y = 1 \quad f_{xx} = -\frac{1}{4} \quad f_{yy} = -1$$

$$Q_f = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) +$$
$$\frac{1}{2}f_{xx}(x_0, y_0)(x - x_0)^2 +$$
$$f_{xy}(x_0, y_0)(x - x_0)(y - y_0) +$$
$$\frac{1}{2}f_{yy}(x_0, y_0)(y - y_0)^2$$

$$Q_f = 1 + \frac{1}{2}(x-1) + y - \frac{1}{8}(x-1)^2 - \frac{1}{2}(x-1)y - \frac{1}{2}(y)^2$$

$$Q_f(1,0) = 1\checkmark$$

11. (a)

$$f(x,y) = 2x^2 + y^4 - 4xy$$

$$f_x = 4x - 4y = 0 \quad f_y = 4y^3 - 4x = 0$$

$$(-1,-1), (0,0), (1,1)$$

(b)

$$\begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 12y^2 \end{bmatrix}$$

(c)

$$\mathbf{D} = f_{xx}f_{yy} - f_{xy}^2 = 48y^2 - 16$$

$$D(-1,-1) > 0, f_{xx} > 4 \to \text{local minimum}$$

$$D(0,0) < 0 \to \text{saddle point}$$

$$D(1,1) > 0, f_{xx} > 4 \to \text{local minimum}$$

(d)

$$-\nabla f = -<4x - 4y, 4y^3 - 4x> \quad -\nabla f(3,2) = -<4, 20>$$

12.

$$f(x,y) = e^{\frac{x}{y}} \quad f(tx, yx) = e^{\frac{tx}{ty}} = e^{\frac{x}{y}} = t^0 f(x,y) \quad n = 0$$

$$f_x(x,y) = \frac{e^{\frac{x}{y}}}{y} \quad f_y(x,y) = \frac{-xe^{\frac{x}{y}}}{y^2}$$

$$\frac{xe^{\frac{x}{y}}}{y} - \frac{xe^{\frac{x}{y}}}{y^2} = (0)f(x,y)$$

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