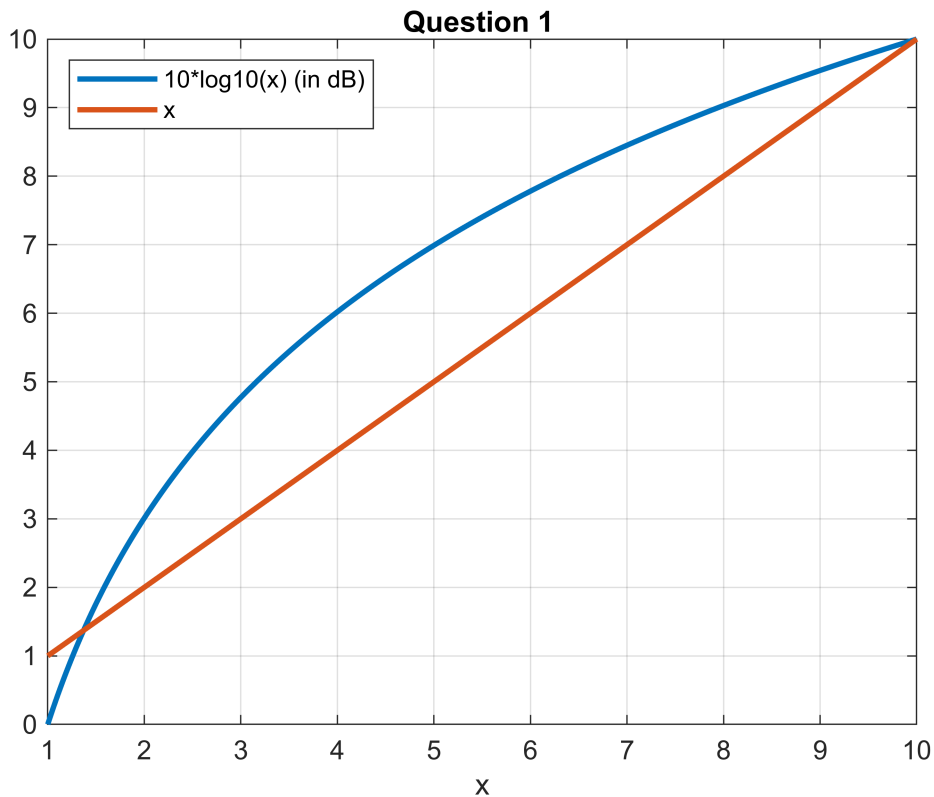


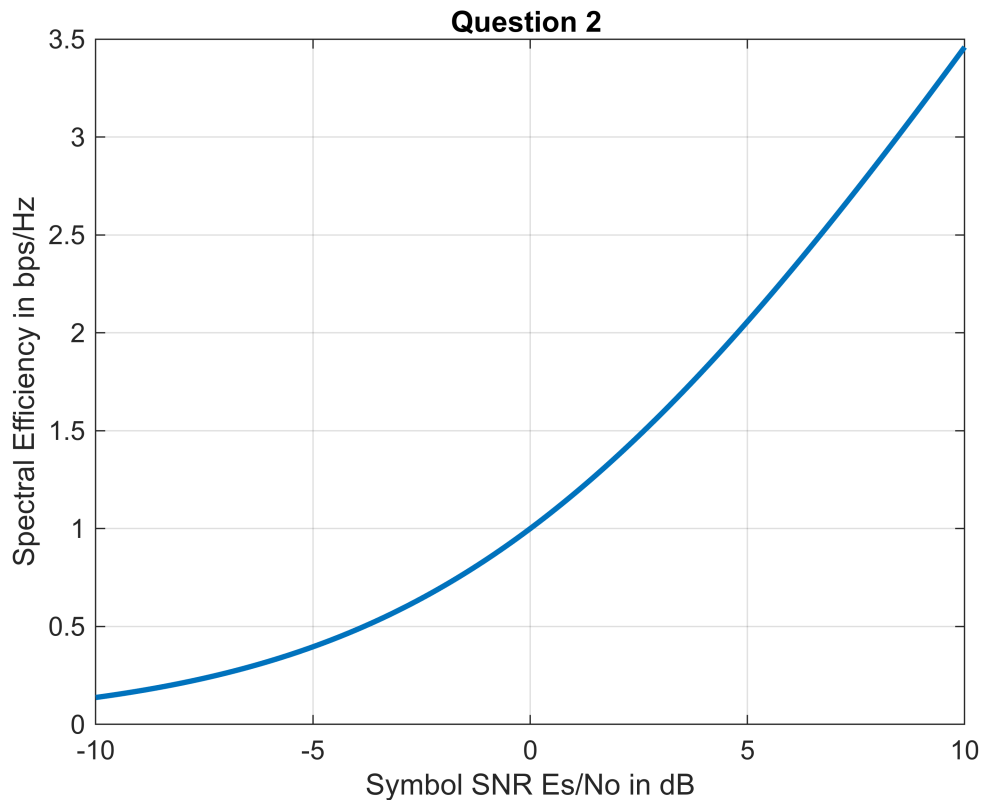
### %Question 1

```
x = 1:0.01:10;  
y = 10 * log10(x);  
plot(x, y, 'LineWidth',2);  
hold on;  
plot(x, x, 'LineWidth',2);  
legend({'10*log10(x) (in dB)', 'x'}, 'Location', 'NorthWest');  
xlabel('x');  
grid on;  
title('Question 1');
```



### %Question 2

```
figure;  
x=-10:0.1:10;  
z = 10.^(x/10);  
y = log2(1+z);  
plot(x,y,'LineWidth',2);  
xlabel('Symbol SNR Es/No in dB');  
ylabel('Spectral Efficiency in bps/Hz');  
grid on;  
title('Question 2');
```



```
%{
(a)The mismatch we see is that when  $\lambda \rightarrow 0$  the value for  $\eta_b$  on the graph is 1 but in our analysis, we got the value for  $\eta_b$  as 0.
The reason for this mismatch is the graph we plotted is in linear scale whereas the values of  $\lambda$  are in decibel scale.
One way to remove this mismatch is to plot the graph against a logarithmic scale.
%}
```

%For reference of the above the code goes like:-

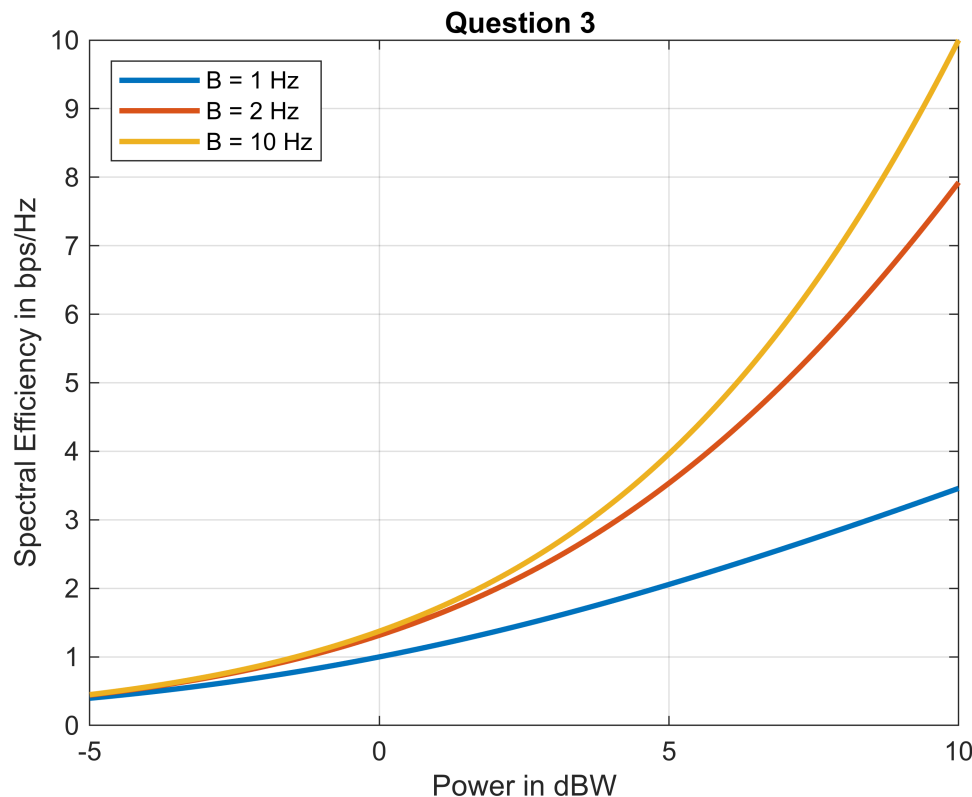
```
%x=-10:0.1:10;
%z = 10.^(x/10);
%y = log2(1+z);
%plot(z,y,'LineWidth',2);
%xlabel('Symbol SNR Es/No in dB');
%ylabel('Spectral Efficiency in bps/Hz');
%grid on;
%title('Question 2 z y ');
```

%Question 3

```

figure;
Ps = -5:0.1:10;
B=1;
Pslinear=10.^(Ps/10);
C=B*log2(1+Pslinear./B);
plot(Ps,C,'LineWidth',2);
hold on;
B=5;
Pslinear=10.^(Ps/10);
C=B*log2(1+Pslinear./B);
plot(Ps,C,'LineWidth',2);
hold on;
B=10;
Pslinear=10.^(Ps/10);
C=B*log2(1+Pslinear./B);
plot(Ps,C,'LineWidth',2);
legend({'B = 1 Hz','B = 2 Hz', 'B = 10 Hz'}, 'Location', 'NorthWest');
xlabel('Power in dBW');
ylabel('Spectral Efficiency in bps/Hz');
grid on;
title('Question 3 ');

```



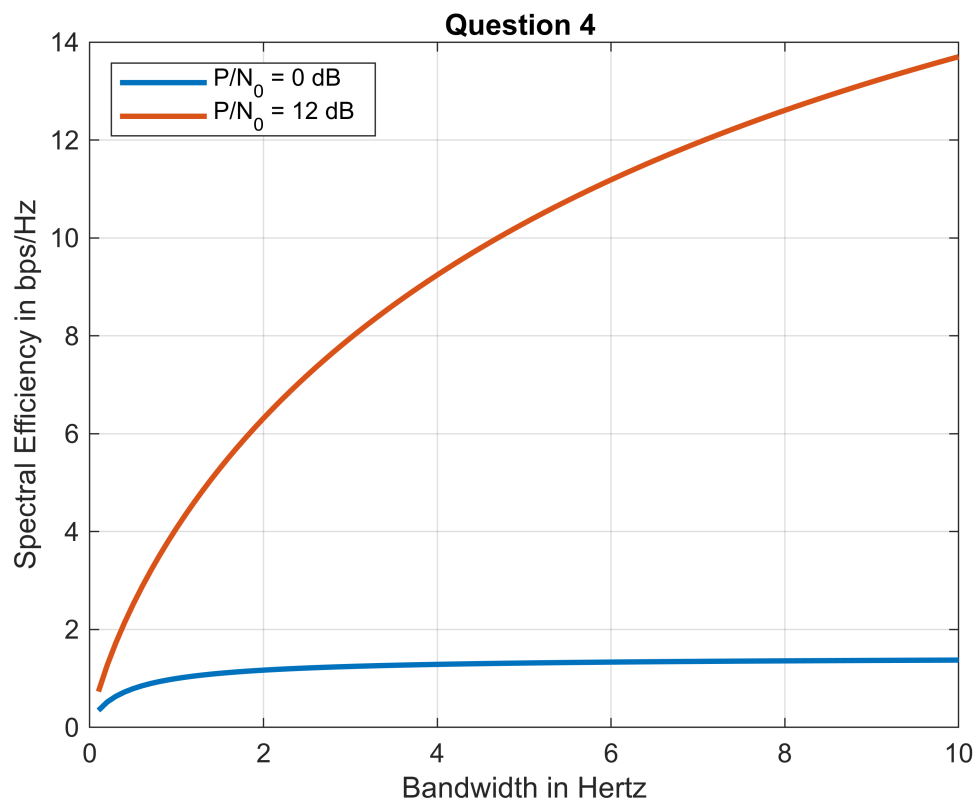
%{  
The graph between  $\eta_b$  and power looks similar to the graph between  $\eta_b$  because both the graphs are plotting the same equation with a different scale.

Since both the variations of  $\eta_b$  are derived from the same equation : Shannon's Channel Capacity Equation.

```
%}
```

#### %Question 4

```
figure;
B = 0:0.1:10;
Ps = 0;
Ps = 10.^(Ps./10);
C = B.*(log2(1+(Ps)./B));
plot(B,C,'LineWidth',2);
hold on;
Ps = 12;
Ps = 10.^(Ps./10);
C = B.*(log2(1+(Ps)./B));
plot(B,C,'LineWidth',2);
legend({'P/N_0 = 0 dB', 'P/N_0 = 12 dB'}, 'Location', 'NorthWest');
xlabel('Bandwidth in Hertz');
ylabel('Spectral Efficiency in bps/Hz');
grid on;
title('Question 4 ');
```



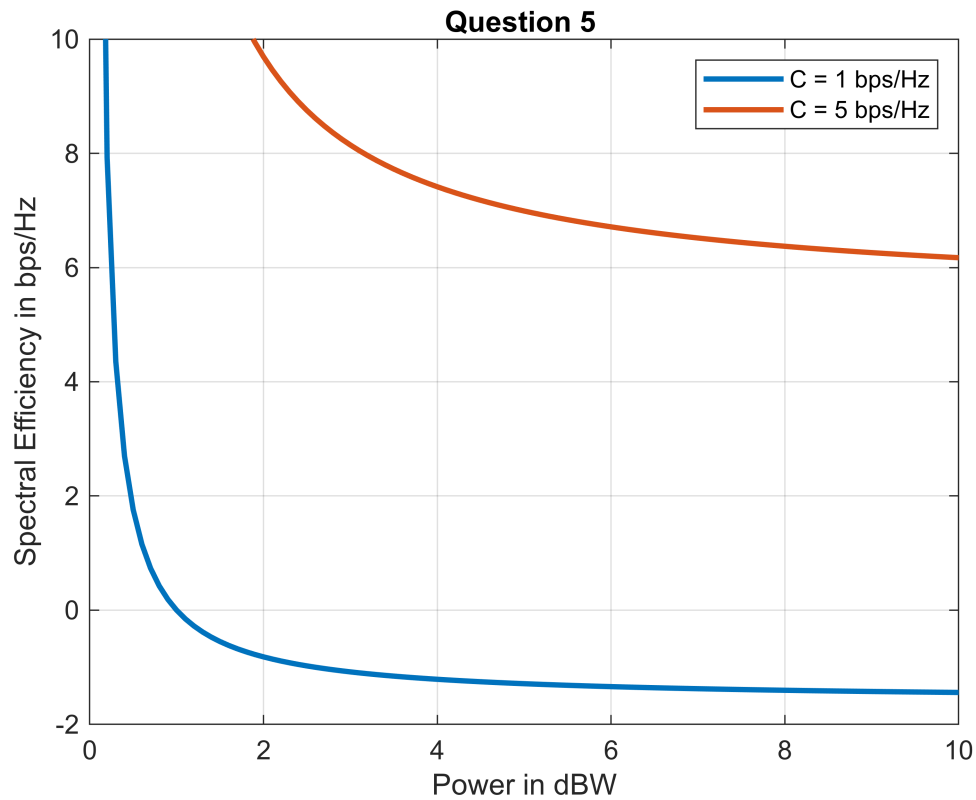
```
%{
```

The graph shows us how channel capacity varies with B for some fixed received power level. This fixed value of data rate/ channel capacity is different with different values of B

In the case of  $P/N_0 = 0\text{dB}$ , the limiting value is around 1.44 which is in agreement with our analytical answer in 4(b) which is  $1/\ln(2) = 1/(0.69) = 1.44$   
%}

#### %Question 5

```
figure;
B=0:0.1:10;
C=1;
Ps = ((2.^(C./B))-1).*B;
Ps = 10*log10(Ps);
ylim([-2 10]);
plot(B,Ps,'LineWidth',2);
C=5;
hold on;
Ps = ((2.^(C./B))-1).*B;
Ps = 10*log10(Ps);
ylim([-2 10]);
plot(B,Ps,'LineWidth',2);
legend({'C = 1 bps/Hz','C = 5 bps/Hz'});
xlabel('Power in dBW');
ylabel('Spectral Efficiency in bps/Hz');
grid on;
title('Question 5');
```



**%Question 6**

**%Method 1**

```
figure;
nb=0:0.1:6;

ratio=((2.^nb)-1)./(nb);
ratio = 10.*log10(ratio);
plot(ratio,nb,'LineWidth',2);
hold on;
xaxis = 0:0.1:10;

yaxis = 0:0.2:6;
val = zeros(length(xaxis));
tolerance=0.1;
for i = 1:length(xaxis)
    left = 1;
    right = length(yaxis);

    while left <= right
        mid = floor((left + right) / 2);
        x = yaxis(mid);
        alpha = (2^x - 1) / x;

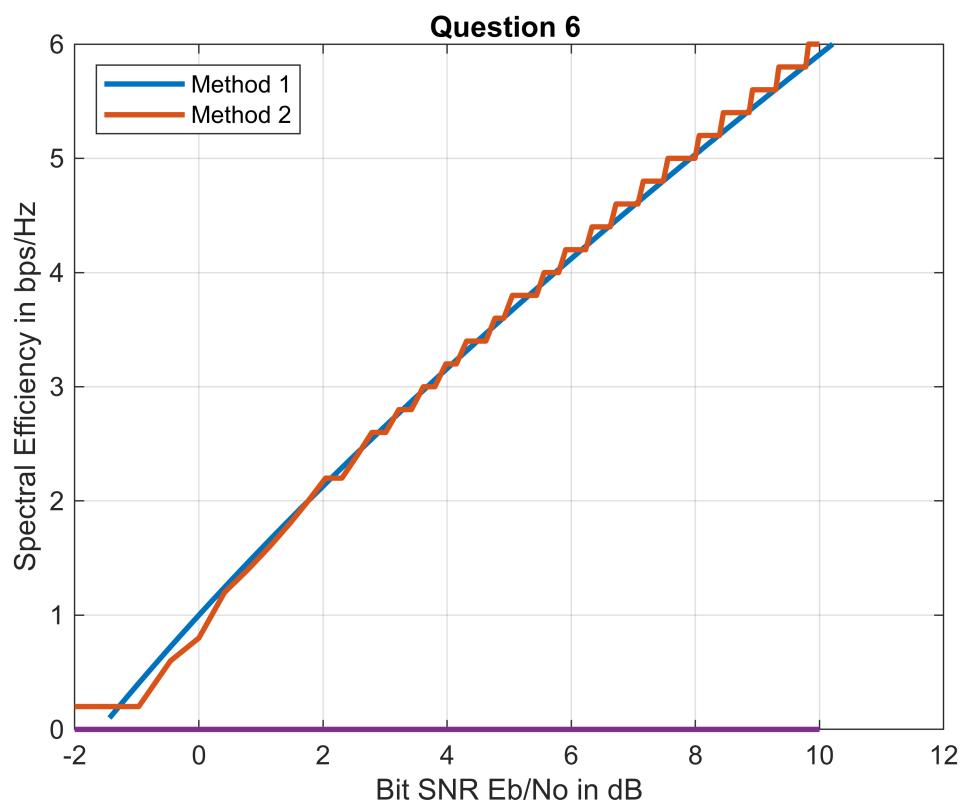
        if alpha >= xaxis(i)
```

```

        val(i) = x;
        right = mid - 1;
    else
        if (xaxis(i)-alpha)<=tolerance
            val(i)=x;
            break;
        end
        left = mid + 1;
    end
end
end

plot(10.*log10(xaxis), val,'LineWidth',2);
xlim([-2,12]);
legend({'Method 1','Method 2'}, 'Location', 'NorthWest');
xlabel('Bit SNR Eb/No in dB');
ylabel('Spectral Efficiency in bps/Hz');
grid on;
title('Question 6 ');

```



%{  
 The value of  $E_b/N_0$  when  $\eta_B$  becomes zero is -1.59 in dB which is equal to the analytical answer  $\ln(2)$  in linear scale.

We get the analytical answer by finding  $\lim_{\eta_b \rightarrow 0} (2^{\eta_b} - 1)/\eta_b$  and getting value for  $E_b/N_0$

```
(L'Hospital Rule)
%}
```



# Communication Systems

## \* Analysis Exercises

1.  $1W \rightarrow 10^3 \text{ mW}(x)$   
 $x_d = 10 \log_{10}(x)$

$$x_d = 10 \log_{10}(10^3) = 30 \text{ dBm}$$

Now:

10 dBm power in W

$$\therefore 10 = 10 \log_{10}(x)$$

$$1x = 10 \text{ mW} = 0.01 \text{ W}$$

2. Small-SNR  $\rightarrow$  ~~very~~ very small  
Large-SNR  $\rightarrow$  very large

$$r_b = \log_2(1+\lambda)$$

$$\text{Small SNR} \Rightarrow r_b = \log_2(1+\lambda) \approx \lambda$$

$$\text{Large SNR} \Rightarrow r_b = \log_2(1+\lambda) \approx \log_2(\lambda)$$

$$\text{SNR} = k \text{ (fixed)}$$

$$\rightarrow r_b = \log_2(1+\lambda) \\ = \log_2(1+k)$$

$$r_b = \text{const.} = \frac{R_b}{B}$$

$$R_b = cB \Rightarrow \boxed{r_b \propto B \text{ Linearly}}$$



4. (a) SNR or  $\lambda = \frac{P_s}{N_0}$   $\frac{P_s}{N_0} = k(\text{fixed}), \text{ as } B \rightarrow \infty$

$$\lambda = \frac{P_s}{N_0} = \frac{P_s}{P_n} \times P_n \cdot N_0 \cdot B$$

$$\lambda = \frac{P_s}{N_0 \cdot B} = \boxed{\frac{k}{B}} \quad \text{as } B \rightarrow \infty$$

$$\therefore \boxed{\lambda \rightarrow 0}$$

(b)  $R_B = \log_2(1 + \lambda)$

$$\lambda = \frac{P_s}{P_n}$$

$$P_s = B \log_2 \left( 1 + \frac{P_s}{N_0 B} \right)$$

$$R_B = \log_2 \left( 1 + \frac{P_s}{N_0 B} \right)^B$$

$$\text{As } \left( 1 + \frac{P_s}{N_0 B} \right)^B \Rightarrow 1^\infty \quad \text{Taking } \lim_{B \rightarrow \infty}$$

$$\therefore \lim_{B \rightarrow \infty} \left( 1 + \frac{P_s}{N_0 B} \right)^B \Rightarrow e^{P_s/N_0}$$

$$R_B = \frac{\log_e e^{P_s/N_0}}{\log_e 2} \Rightarrow \boxed{R = \frac{1}{\ln 2} \frac{P_s}{N_0}}$$

5. (a)  $(1+p) \rightarrow 4p \text{ dB}$

$$1 \leq (1+p) \leq 1.25$$

$$\boxed{0 \leq p \leq 0.25} \quad \text{--- (i)}$$

$$x_d = 10 \log_{10}(x)$$

$$\Rightarrow x_d = 10 \log_{10}(1+p)$$

$$0 \leq x_d \leq 10 \log_{10}(1.25)$$

$$0 \leq x_d \leq 0.9691$$

as  $10 \log_{10}(1+p)$  is approximated as  $4p \text{ dB}$

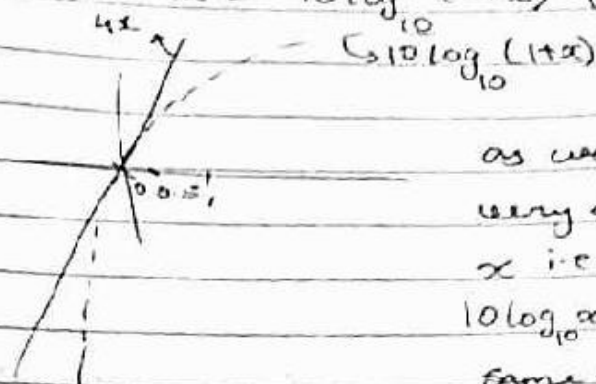
$$0 \leq 4p \leq 0.9691 \text{ (dB)}$$

$$\boxed{0 \leq p \leq 0.24227} \quad \text{--- (ii)}$$

From (i) & (ii)

we can say that they both are very close to each other  $\therefore$  it is valid approx.

Also we can see this as  $4x \approx 10 \log_{10} (1+x)$  (through graph)



as we can see for very small value of  $x$  i.e.  $p$  (here)  $4x \approx 10 \log_{10} x$  are approx. same.

$$4x \approx 10 \log_{10} (1+x)$$

$$\text{for } 0 < x < 0.55$$

Hence a valid approx.

(b) Ps in quest<sup>n</sup> → money is taken into consideration & dB based scaling is asked. So let us check what does doubling money mean in dB scale.

$$10 \log_{10} (x) = X_d$$

$$10 \log_{10} (2x) = 10 \log_{10} (2) + 10 \log_{10} (x) \approx 3 + X_d$$

(approx)

So to check as whether  $(1+p) \rightarrow 4p$  dB corresponds to "75 Rule".

Verification Compound Int.  $\rightarrow P \left(1 + \frac{r}{100}\right)^t$  Assuming this to be our I/P in eq. (10).

$$10 \log_{10} \left( P \left(1 + \frac{r}{100}\right)^t \right) \approx 4 \left( 2P \left(1 + \frac{r}{100}\right)^t - 1 \right)$$

twice the money

Let  $P = 1$   
 $r = 5\%$  (as in Q.)

$$\therefore 10 \log_{10} (1.05)^t \approx 4 (2(1.05)^t - 1)$$

If  $t = 15 \text{ yrs}$   
 $3.17 \approx 12.63$

→  $\therefore 12.63$  is approx. equal to 4 times  $3.17$ .  
 $\therefore t = 15 \text{ yrs}$  at  $5\%$  rate is valid approx. which nearly corresponds to Rule of 75.

For doubling money at  $r = 5\%$   
 $\frac{69.12}{5} \approx 13.824 \text{ yrs}$

(c) Using C.I. formula of  $t$   
 $P \left( 1 + \frac{r}{n} \right)^{nt} \quad (P = 1 R_0)$

$t = \text{time (yrs)}$

$n$  is no. of times

$d$  gets compounded

(or is actually in  $\frac{r}{100}$  form)

$$\therefore \lim_{n \rightarrow \infty} \left( 1 + \frac{r}{n} \right)^{nt} \rightarrow 1^{\infty} \text{ form}$$

$$\lim_{n \rightarrow \infty} \left[ \left( 1 + \frac{r}{n} \right)^{nt} \right] \rightarrow e^{rt} \quad (\text{Applying Lim form})$$

$$r = 100\%$$

$$t = 1 \text{ yr}$$

$$P = 1 R_0$$

$e^{\frac{100}{100}} = e$ . In comparison to  $e$  it is  
~~or~~ ~~very~~ ~~close~~ equal to it

(d)  $P \left( 1 + \frac{r}{100} \right)^t = 2P$

$$(1.05)^t = 2$$

~~$t \approx 15 \text{ yrs}$~~  Taking log

$$\rightarrow t \log_{10} (1.05) = \log_{10} 2$$

$$t \approx 14.15 \text{ yrs}$$

which was closest in the case of Rule of 72.