Set - 4 :Constrained growth beyond the logistic model

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In this lab, we analyzed the Gompertz equation for the modelling of tumour growth. We also analyzed the Allee effect which models high growth rate of population when the initial size of the population has an intermediate value.

I. MODEL

A. Gompertz law

The Gompertz equation models tumour growth as

$$\dot{x} = -axln(bx) \tag{1}$$

Where $a, b > 0$.
Rescale $X = x/b^{-1}$ and T = at,

$$\dot{X} = -XlnX \tag{2}$$

The integral solution of above equation is,

$$X = e^{[ln(X_0)e^{-T}]} \tag{3}$$

B. Allee Effect

The Allee effect models high growth rate of a population when the initial population size has an intermediate value.The model equation is,

$$\dot{x} = x[r - a(x - b)^2] \tag{4}$$

Where $a, b, r > 0$.The model is effective only when $r < ab^2$.

II. RESULTS

A. Gompertz Law

Fig. 1 Plot of $\dot{x}$ vs x.

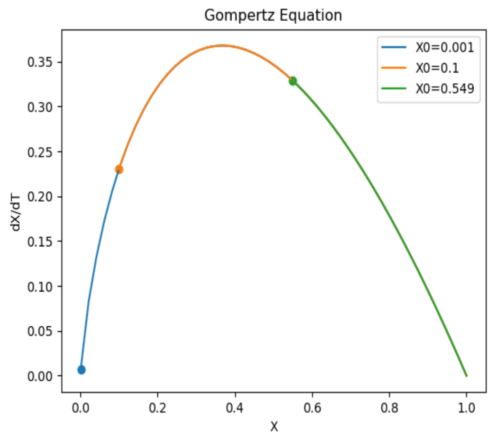

FIG. 1: Here the initial values of x are 0.001, 0.1 and 0.549 $\Delta x = 0.01unit$

Fig. 2 Integral solution of $X(t)$.

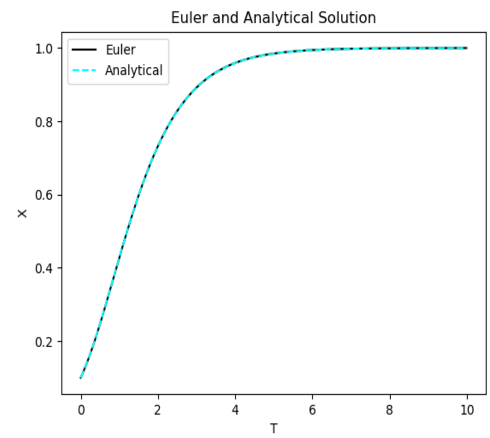

FIG. 2: Here the initial value of x(0) = 0.1. $\Delta t = 0.01unit$

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Fig. 3 shows comparison between the analytical solution and the numerical solution.

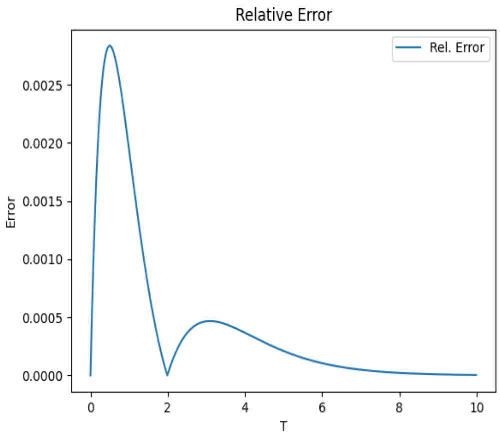

FIG. 3: relative error between the analytical solution and the numerical solution . The graph has zero value at t=0 and 1.175.

B. Allee Effect

Fig. 4 Plot of $\dot{x}$ vs x.

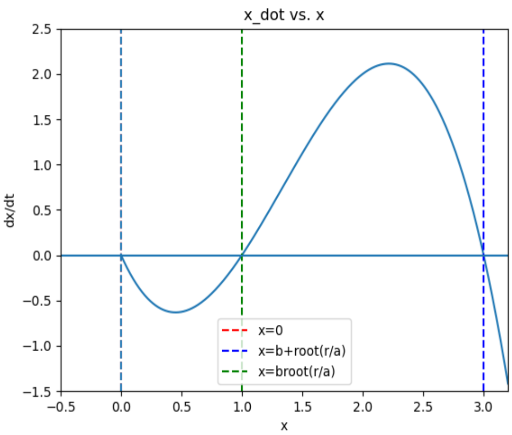

FIG. 4: Here a=1,b=2,r=1. Graph value becomes zero at t=0,1 and 3 units. $\Delta t = 0.01 unit$

Fig. 5 Integral solution of $x(t)$.

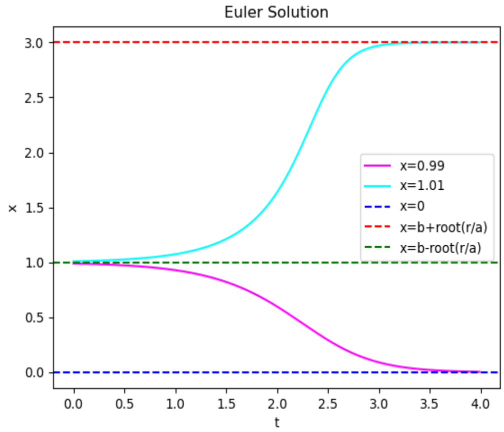

FIG. 5: The two initial values taken are 0.99 and 1.01. $\Delta t = 0.01 unit$

III. CONCLUSION

- The rescaled Gompertz equation reduces the tumor growth model to a differential equation without parameters, simplifying its analysis and interpretation.
- In the Gompertz model, the plot of $\dot{X}$ versus X reveals the growth rate behavior of the tumor for various initial conditions, offering insights into the dynamics of the system.
- The Allee effect is evident in the population growth model, where an intermediate initial population size results in higher growth rates. The asymptotic values of x as $t \to \infty$ provide valuable information about the long-term behavior of the population under the influence of the Allee effect.