Set 8: Battles and War games: Iwo Jima (land) and Trafalgar (naval)

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CS302, Modeling and Simulation

This study applies Lanchester's model to analyze the Battle of Iwo Jima and the Battle of Trafalgar. For Iwo Jima, numerical simulations using Euler's method predict outcomes based on combat effectiveness parameters. In Trafalgar, Lord Nelson's three-stage battle plan is simulated, highlighting the impact of strategy on fleet dynamics. The study bridges mathematical modeling, historical analysis, and strategic decision-making in warfare.

I. BATTLE OF IWO JIMA:

A. Model Equations

The Lanchester model equations for the Battle of Iwo Jima are given by:

$$\frac{dJ}{dt} = -aA \tag{1}$$

$$\frac{dA}{dt} = -jJ \tag{2}$$

Where:

A : Strength of Force 1
J : Strength of Force 2
a : Effectiveness coefficient for Force 1
j : Effectiveness coefficient for Force 2

Here, $a = 0.0106$ and $j = 0.0544$ are the combat effectiveness parameters for the American and Japanese troops, respectively. The initial troop numbers are $J(0) = J_0 = 18274$ and $A(0) = A_0 = 66454$.

Lanchester's square law

$$jJ^2 - aA^2 = k \tag{3}$$

B. Solution

1. Prediction by Lanchester's Square Law

By applying Lanchester's square law, we can clearly see that American forces will win the battle.

2. Integration by Euler's Method

The battle lasted for 32 days. After 32 days, 51527 American troops were left and no Japanese troops were left. Thus casualties for American forces were 14927 and for Japan was 18274.

We stop the integration when troops fall below 1 for two main reasons: Mathematically, we can't have a fraction of a person in reality. Militarily, an army with less than 1 soldier is effectively defeated and no longer combat-capable.

The high American casualty rate despite overwhelming numerical superiority demonstrates the effectiveness of the Japanese defenders and their high combat effectiveness parameter.

3. Graphs

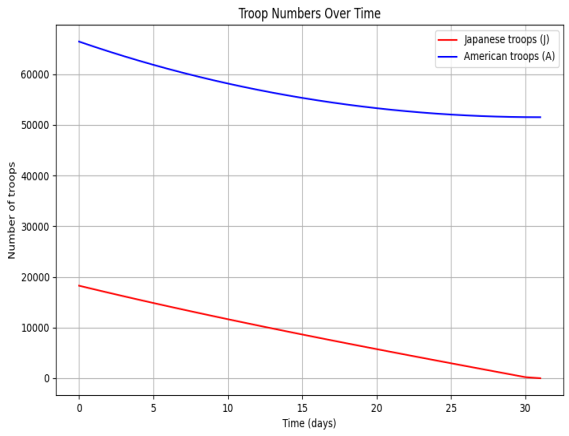

FIG. 1: Plotting J and A along the vertical axis and t along the horizontal axis

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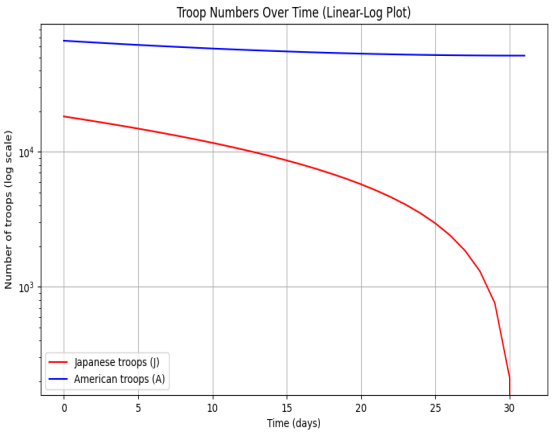

FIG. 2: Logarithmic Scale of J and A along the vertical axis and t along the horizontal axis

Given the computed values:

$$|\text{slope}_J| \approx 0.1004, \quad |\text{slope}_A| \approx 0.00858, \quad \sqrt{aj} \approx 0.0239$$

Comparison:

- $|\text{slope}_A| = 0.00858$ is significantly smaller than $\sqrt{aj} = 0.0239$.
- $|\text{slope}_J| = 0.1004$ is much larger than $\sqrt{aj}$.

Conclusion: The slopes deviate considerably from $\sqrt{aj}$. The Japanese troops (J) are depleting much faster, while the American troops (A) are depleting much slower than expected. This suggests an imbalance in the assumed attrition rates.

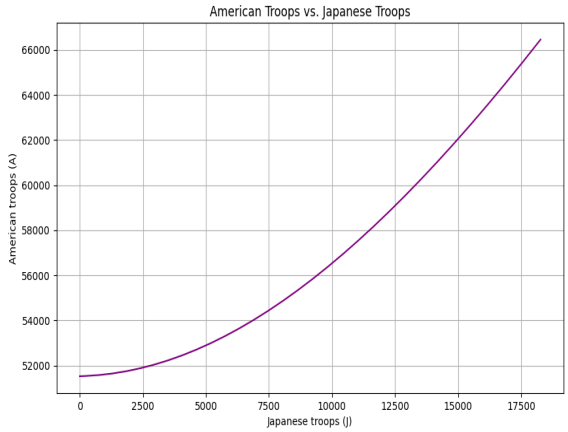

FIG. 3: Graph of A vs J (Phase Plot)

II. BATTLE OF TRAFALGAR:

A. Model Equations

The Lanchester model equations for the Battle of Trafalgar are given by:

$$\frac{dF}{dt} = -bB \tag{4}$$

$$\frac{dB}{dt} = -fF \tag{5}$$

Where:

B : Strength of Force 1
F : Strength of Force 2
b : Effectiveness coefficient for Force 1
f : Effectiveness coefficient for Force 2

Here, $f = b = 0.05$ are the combat effectiveness parameters for the French and British fleets, respectively. The initial fleet numbers are $F(0) = F_0 = 33$ and $B(0) = B_0 = 27$.

Lanchester's square law

$$fF^2 - bB^2 = k \tag{6}$$

B. Solution

1. *First Stage of Lord Nelson's Battle Plan*

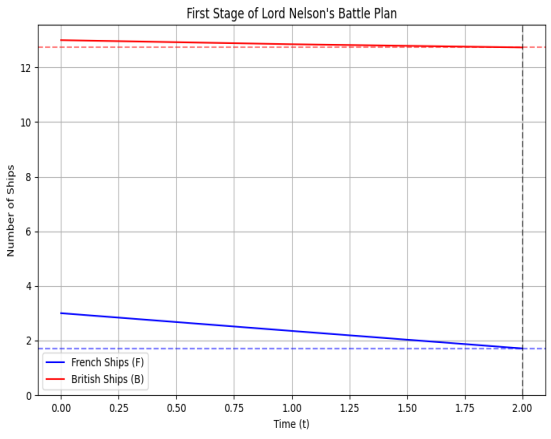

FIG. 4: Graph of B vs F with $B_0 = 13$ & $F_0 = 3$ & $\Delta t = 1day$

In the first stage, Lord Nelson engaged a small French force with 13 ships, keeping 14 ships in reserve. Thus initial British force was 13 and French force was 3.
After 2 days of war 12 full and 1 damaged (12.7325 remaining) British ships were left and 1 full and 1 damaged (1.7075 remaining) French ship was left.

2. *Second Stage of Lord Nelson's Battle Plan*

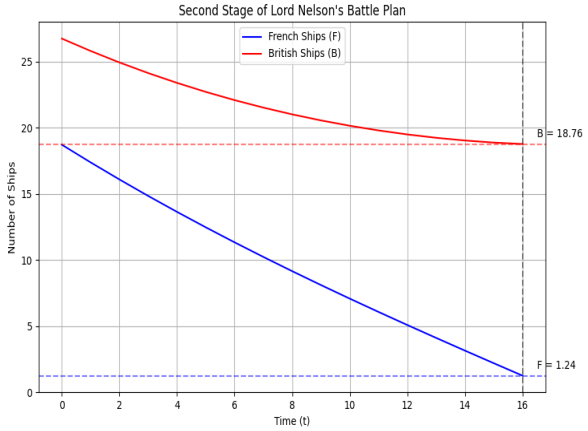

FIG. 5: Graph of B vs F with $B_0 = 26.7325$ & $F_0 = 18.7075$ & $\Delta t = 1day$

In the second stage, surviving British ships from the first stage join the reserve fleet, and surviving French ships join a larger force. Thus initial British force was 26 full ships and 1 damaged ship and French force was 18 full ship and 1 damaged ship.
After 16 days of war 18 full and 1 damaged (18.7585 remaining) British ships were left and 1 full and 1 damaged (1.2409 remaining) French ship was left.

3. *Third Stage of Lord Nelson's Battle Plan*

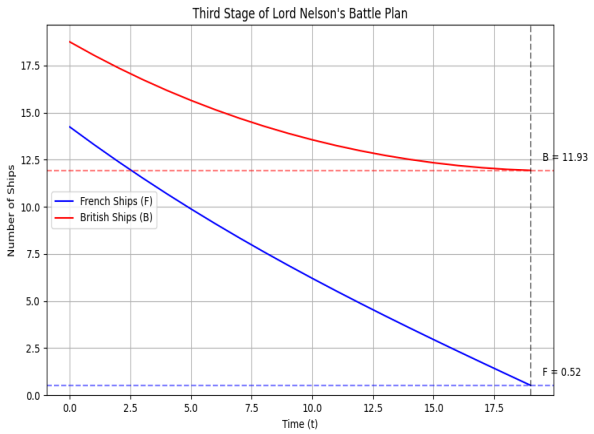

FIG. 6: Graph of B vs F with $B_0 = 18.7585$ & $F_0 = 14.2409$ & $\Delta t = 1day$

In the third stage, surviving French ships from the second stage join the last group of 13 French ships, engaging with the remaining British ships. Thus initial British force was 18 full ships and 1 damaged ship and French force was 14 full ship and 1 damaged ship.
After 19 days of war 11 full and 1 damaged (11.934 remaining) British ships were left and 1 damaged (0.5183 remaining) French ship was left Thus French lost the war.

4. *War Game Analysis*

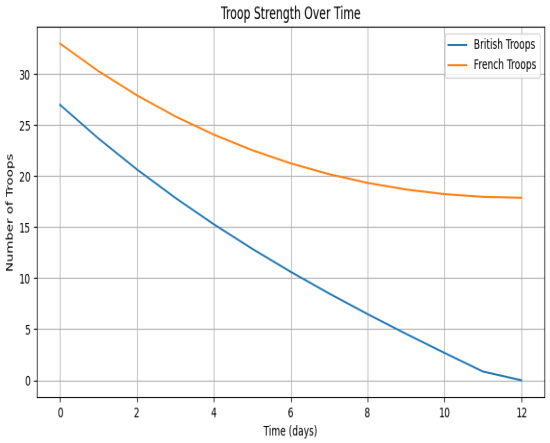

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FIG. 7: Graph of B vs F with $B_0 = 27$ & $F_0 = 33$ & $\Delta t = 1day$

After full on war with 27 British ships and 33 French ships, the British lost the war.