Set - 1 – Logistic Modeling of GDP and Trade Growth in Leading Economies.

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CS302, Modeling and Simulation

This study aims to predict the future trends of GDP and trade for the top six global economies using the logistic equation. To assess the accuracy of our predictions, we compare the model's results with actual data from the World Bank.

I. LOGISTIC MODEL

First-order autonomous dynamical systems have the general form of dx/dt = f (x) where x = x(t), with t being time.

$$\frac{dx}{dt} = ax - bx^2 \tag{1}$$

Eq. (1) represents the basic model of a nonlinear function, where a and b are fixed parameters. On solving this equation with initial condition x(0) = x0 and k = a/b we get the following result,

$$x(t) = \frac{kx_0 e^{at}}{k + x_0\left(e^{at} - 1\right)} \tag{2}$$

The time at which the non-linear effect starts asserting itself is given by:

$$t_{\text{nl}} = \frac{1}{a} \ln\left(\frac{k}{x_0} - 1\right) \tag{3}$$

The linearization of GDP and Trade is given by the following formula:

$$G(T) \sim T^{\alpha} \tag{4}$$

The time it takes for a country to reach any GDP in the future can be predicted by the following formula:

$$t = b + \frac{1}{a} \ln\left[\frac{x(k - x_0)}{x_0(k - x)}\right] \tag{5}$$

II. RESULTS

A. GDP and Trade of top economies of the world

The Fig. 1 shows the GDP and trade of USA using World Bank data.

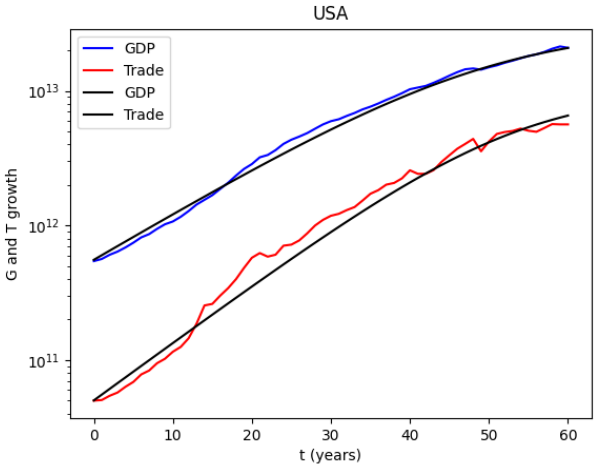

FIG. 1: Modeling the dynamics of GDP (top plot) and trade (bottom plot) for the USA using World Bank data. The black curves represent logistic equation fits based on parameter values from the referenced paper. The reference year for both plots is 1960, with the GDP plot extending to 2020 and the trade plot concluding in 2019. The parameter values used are $x_0 = 0.555 \times 10^{12}$ dollars, $a = 0.080$ per annum, and $k = 30$ trillion dollars.

The Fig. 2 shows the GDP and trade of China using World Bank data.

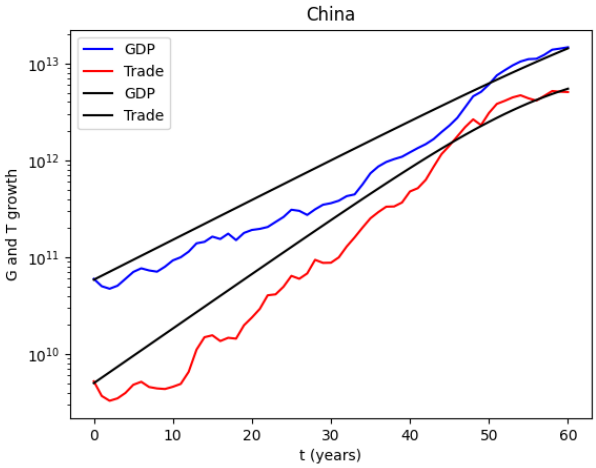

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FIG. 2: Modeling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for China. The black curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960, and both end in 2020.The parameter values used are $x_0 = 0.585 \times 10^{11}$ dollars, $a = 0.095$ per annum, and $k = 80$ trillion dollars.

The Fig. 3 shows the GDP and trade of Japan using World Bank data.

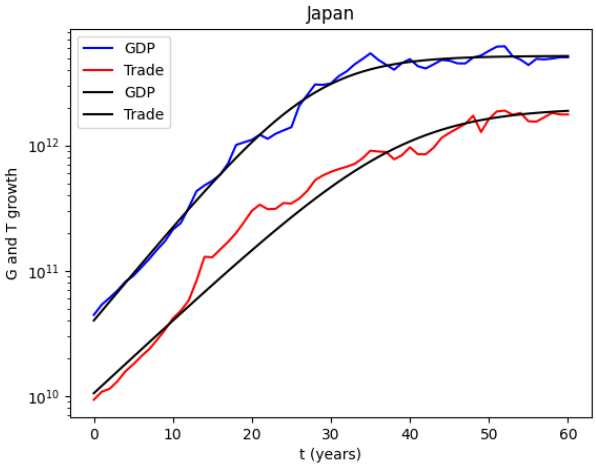

FIG. 3: Modeling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for Japan. The black curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960, and both end in 2019. The parameter values used are $x_0 = 0.4 \times 10^{11}$ dollars, $a = 0.175$ per annum, and $k = 5.2$ trillion dollars.

The Fig. 4 shows the GDP and trade of Germany using World Bank data.

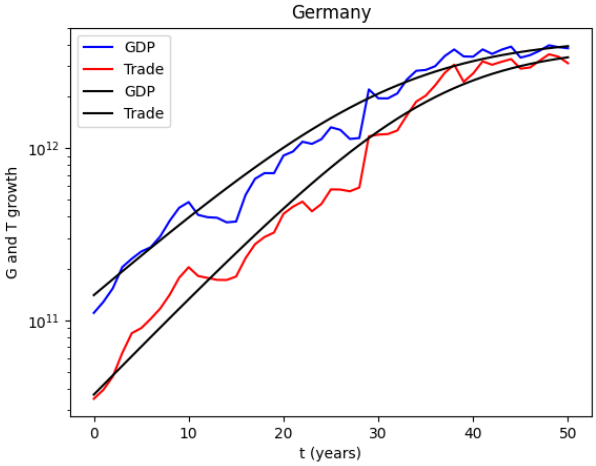

FIG. 4: Modeling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for Germany. The black curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1970, and both end in 2020. The parameter values used are $x_0 = 1.40 \times 10^{11}$ dollars, $a = 0.110$ per annum, and $k = 4.4$ trillion dollars.

The Fig. 5 shows the GDP and trade of UK using World Bank data.

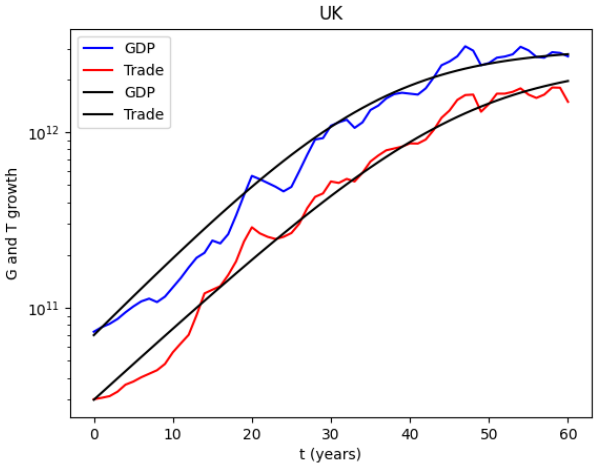

FIG. 5: Modeling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for UK. The black curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960, and both end in 2020. The parameter values used are $x_0 = 0.7 \times 10^{11}$ dollars, $a = 0.105$ per annum, and $k = 3$ trillion dollars.

The Fig. 6 shows the GDP and trade of India using World Bank data.

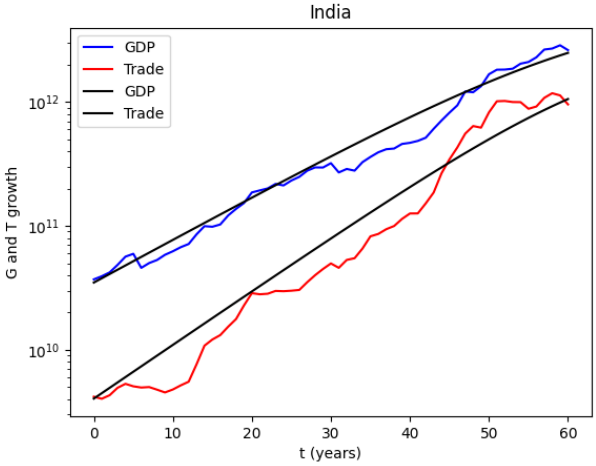

FIG. 6: Modeling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for India. The black curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960, and both end in 2020. The parameter values used are $x_0 = 0.349 \times 10^{11}$ dollars, $a = 0.080$ per annum, and $k = 6$ trillion dollars.

B. GDP Versus Trade of Top Economies

The Fig. 7 shows GDP against trade using World Bank data for USA.

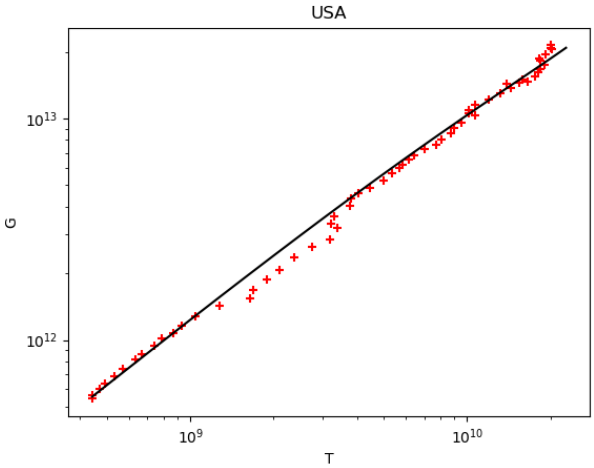

FIG. 7: Plotting GDP against trade using World Bank data for USA. The dotted line follows Eq.(4) with $\alpha = 0.75$. The plot begins in 1960 and ends in 2020. The parameter values used are $x_0 = 0.499 \times 10^{11}$ dollars, $a = 0.099$ per annum, and $k = 10$ trillion dollars.

The Fig. 8 shows the GDP against trade of China using World Bank data.

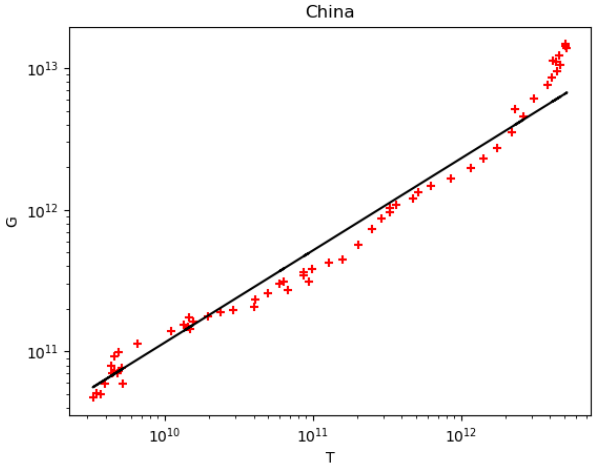

FIG. 8: Plotting GDP against trade using World Bank data for China. The dotted line follows Eq.(4) with $\alpha = 0.65$. The plot begins in 1960 and ends in 2020. The parameter values used are $x_0 = 0.5 \times 10^{10}$ dollars, $a = 0.130$ per annum, and $k = 10$ trillion dollars.

The Fig. 9 shows the GDP against trade of Japan using World Bank data.

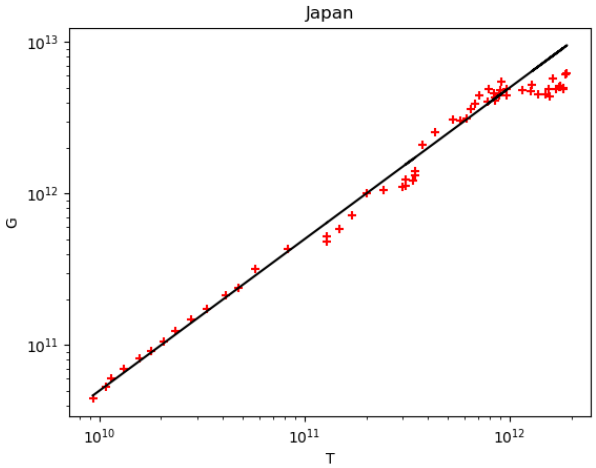

FIG. 9: Plotting GDP against trade using World Bank data for Japan. The dotted line follows Eq.(4) with $\alpha = 1.00$. The plot begins in 1960 and ends in 2020. The parameter values used are $x_0 = 1.05 \times 10^{10}$ dollars, $a = 0.135$ per annum, and $k = 2$ trillion dollars.

The Fig. 10 shows the GDP against trade of Germany using World Bank data.

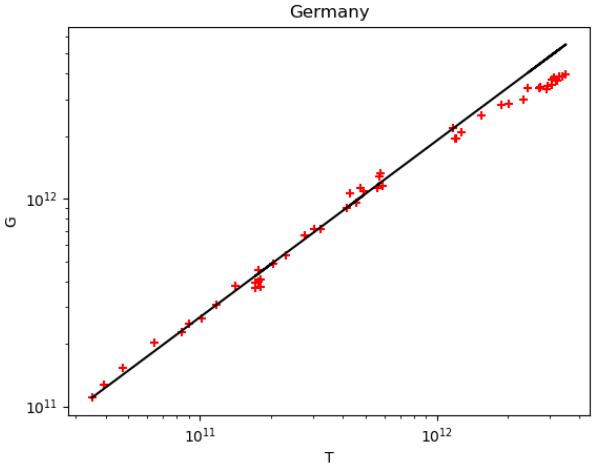

FIG. 10: Plotting GDP against trade using World Bank data for Germany. The dotted line follows Eq.(4) with $\alpha = 0.85$. The plot begins in 1970 and ends in 2020. The parameter values used are $x_0 = 0.37 \times 10^{11}$ dollars, $a = 0.130$ per annum, and $k = 3.9$ trillion dollars.

The Fig. 11 shows the GDP against trade of UK using World Bank data.

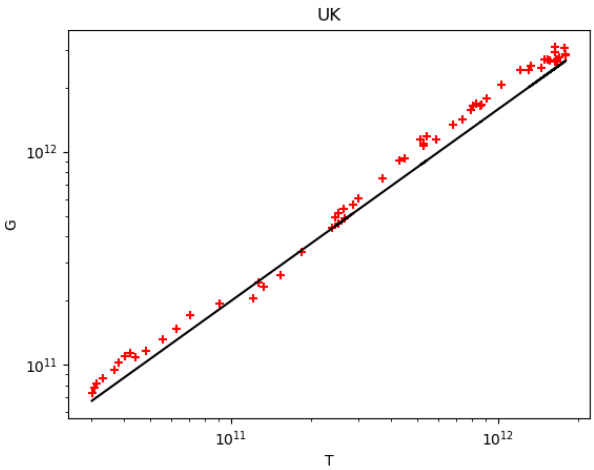

FIG. 11: Plotting GDP against trade using World Bank data for UK. The dotted line follows Eq.(4) with $\alpha = 0.90$. The plot begins in 1960 and ends in 2020. The parameter values used are $x_0 = 0.300 \times 10^{11}$ dollars, $a = 0.095$ per annum, and $k = 2.5$ trillion dollars.

The Fig. 12 shows the GDP against trade of India using World Bank data.

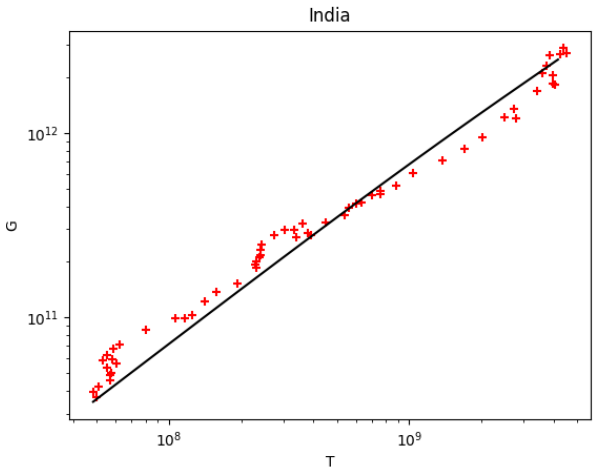

FIG. 12: Plotting GDP against trade using World Bank data for India. The dotted line follows Eq.(4) with $\alpha = 0.60$. The plot begins in 1960 and ends in 2020. The parameter values used are $x_0 = 0.404 \times 10^{10}$ dollars, $a = 0.1$ per annum, and $k = 3$ trillion dollars.

C. Plot a comparison of the GDP of the world's top economies.

The Fig. 13 shows comparison of GDP growth of the USA and China.

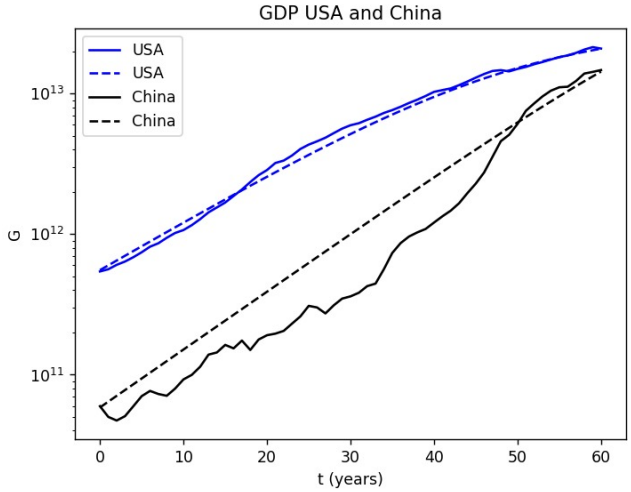

FIG. 13: The comparison of GDP growth between the USA and China, the world's largest and second-largest economies, can be visualized using smooth dotted curves. These curves model GDP growth based on Eq. (2), incorporating specific values of parameters a and k.

The Fig. 14 shows comparison of GDP growth of the India and Japan.

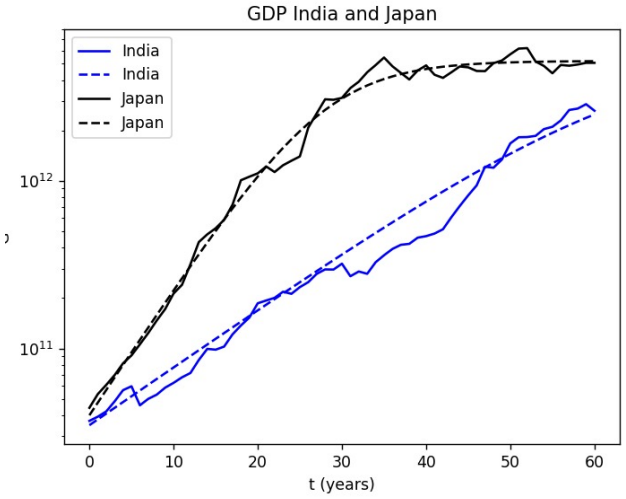

FIG. 14: A comparison of GDP growth between Japan and India—respectively the second and third largest economies in the Indo-Pacific region after China—reveals distinct trends. Japan's GDP initially grew rapidly but stagnated around the year 2000, a pattern closely modeled by the logistic function, represented by the smooth dotted curve. In contrast, India's GDP growth has been steady but slow, eventually accelerating. By 2020, India's GDP was rising at a steeper rate than Japan's, suggesting that if this trend continues, India's GDP will eventually surpass Japan's.

The Fig. 15 shows comparison of GDP growth of the UK and Germany.

FIG. 15: A comparison of GDP growth between Germany and the UK—ranked after the USA—shows closely matched trends until 1999-2000. According to World Bank data, the UK's annual GDP records begin in 1960 ($t = 0$) and extend to 2020, while Germany's GDP data starts from 1970 ($t = 10$ years). Despite this difference in data timelines, both countries exhibited similar GDP growth trajectories up until the turn of the century.

D. Prediction

The Fig. 16 shows comparison of GDP growth of the Japan, Germany, UK and India.

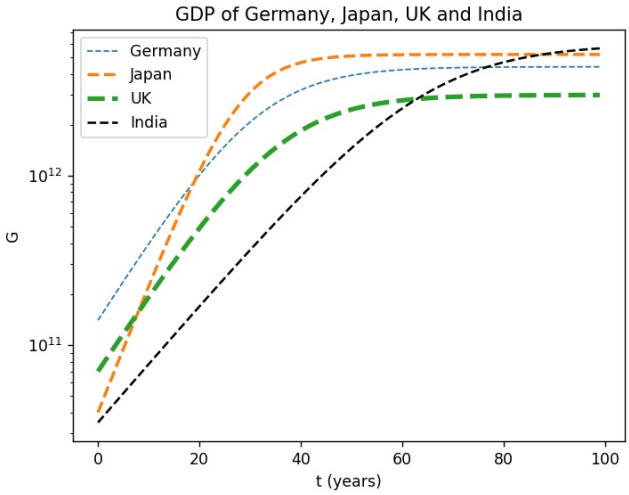

FIG. 16: The logistic function curves intersect twice before 2020—first in 1966, when Japan's GDP surpassed that of the UK, and again in 2000. Three additional intersections are projected to occur after 2020. Specifically, India's GDP is expected to surpass Germany's by 2035 and Japan's by 2047.

III. FORECASTING INDIA'S TIMELINE TO REACH A $4 AND $5 TRILLION ECONOMY

Using Equation (5), we can predict when India will become a $4 and $5 trillion economy, respectively.

According to our predictions using the given model and stated equation, India will become a **$4 trillion economy in 2032** and a **$5 trillion economy in 2044.**

IV. PREDICTING INDIA'S GDP IN 2047 USING EXPONENTIAL GROWTH

Assuming that India's GDP grows exponentially as

$$x = x_0 e^{at}$$

and using the given parameter values, we calculate the GDP in 2047.

The predicted GDP of India in 2047 is **$36.7718 trillion**.

V. STATISTICAL ANALYSIS

A. Statistical analysis of GDP

Country Name	Mean	Standard deviation
USA	0.0317	0.0853
China	0.0877	0.2317
Japan	0.0125	0.1241
Germany	0.0512	0.1616
UK	-0.0742	0.1560
India	-0.0901	0.1790

B. Statistical analysis of Trade

Country Name	Mean	Standard deviation
USA	0.1120	0.2032
China	-0.3569	0.3365
Japan	0.1467	0.3406
Germany	0.0701	0.1616
UK	0.0051	0.1665
India	-0.1521	0.3541

VI. CONCLUSIONS

- The results indicate a strong correlation between a nation's GDP and trade revenue, where an increase in GDP is accompanied by a rise in trade.

- A significant discrepancy exists between actual GDP data and the simulation results, particularly for China and India. Furthermore, the recent overtaking of Japan by Germany in GDP growth, which was not predicted by our model, raises concerns about the reliability of its projections.

- The growth patterns of Trade and GDP follow the logistic equation, which predicts a saturation point over long timescales. This saturation effect is evident in both GDP and Trade graphs.

[1] Arnab K. Ray, Logistic modelling of economic dynamics, DA-IICT (2023).