# Set - 3 : Logistic Equation and its Modifications

Rakshit Pandhi (202201426)* and Kalp Shah (202201457)†
*Dhirubhai Ambani Institute of Information & Communication Technology,*
*Gandhinagar, Gujarat 382007, India*
*CS302, Modelling and Simulation*

## I. MODEL

### A. Human Population

The human population grows according to the logistic equation

$$\frac{dx}{dt} = ax - bx^2 \tag{1}$$

where $k = \frac{a}{b}$ represents the carrying capacity. On solving this equation with initial condition x(0) = x0 and k = a/b we get the following result,

$$x(t) = \frac{kx_0e^{at}}{k + x_0\left(e^{at} - 1\right)} \tag{2}$$

### B. Spread of Agricultural Innovation

The x(t) for the spread of agricultural innovations among farmers through personal communications is given by

$$x = \frac{Ne^{CNt}}{(N-1) + e^{CNt}} \tag{4}$$

The dynamical equation for the spread of agricultural innovations among farmers through impersonal communications is given by

$$\dot{x} = (Cx + C')(n - x) \tag{5}$$

The x(t) for the spread of agricultural innovations among farmers through impersonal communications is given by

$$x = \frac{NC'[1 - e^{-(CN+C')t}]}{C' + CNe^{-(CN+C')t}} \tag{6}$$

Defining X = x/N, T = cNt, A = C/CN and X˙ dX/dT,

Recast dynamical equation for the spread of agricultural innovations among farmers through personal communications is given by

$$\dot{X} = X(1 - X) \tag{7}$$

X(T) for the spread of agricultural innovations among farmers through personal communications is given by

$$X = \frac{1}{1 + A^{-1}e^{-T}} \tag{8}$$

Recast dynamical equation for the spread of agricultural innovations among farmers through impersonal communications is given by

$$\dot{X} = (X + A)(1 - X) \tag{9}$$

X(T) for the spread of agricultural innovations among farmers through impersonal communications is given by

$$X = \frac{1 - e^{-(1+A)T}}{1 + A^{-1}e^{-(1+A)T}} \tag{10}$$

### C. Harvesting Model

The logistic equation is modified as

$$\dot{x} = f(x) = rx[1 - (x/k)] - h \tag{1}$$

Where h is the “harvesting” rate. For h=0 this equations is same as the logistic equation with $a = r$ and $b = \frac{r}{k}$.

For h=0 the solution becomes

$$x(t) = \frac{x_0e^{rt}}{1 + \frac{x_0}{k}(e^{rt} - 1)}$$

*Electronic address: 202201426@daiict.ac.in
†Electronic address: 202201457@daiict.ac.in

## II. RESULTS

### A. Human Population

Fig. 9 shows the plot of analytical solution for Eq. 1.

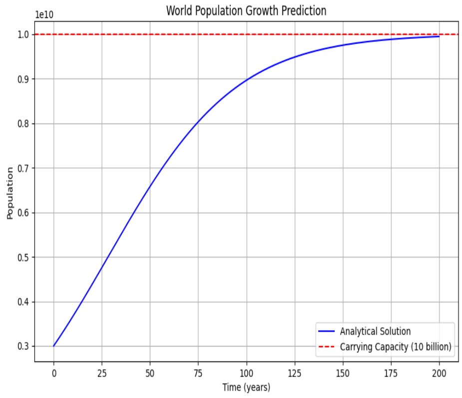


FIG. 1: a=0.03 per annum and b= $3 * 10^{-12}$ and x0= $3 * 10^{9}$

The limiting value of the world population from the plot and by using $k = \frac{a}{b}$ is **10 billion**. As per the current data, it is **8 billion** which is very close .

Fig. 2 shows x (by Euler's method) versus t with timestep of 0.01.

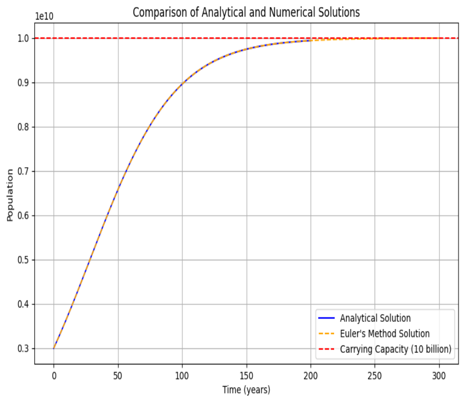


FIG. 2: Euler Solution

Fig. 3 shows relative error between Euler method and Analytical solution

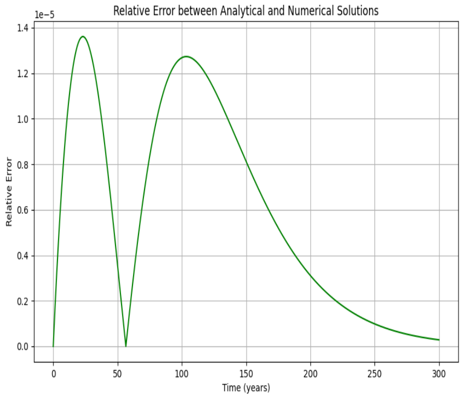


FIG. 3: shows the Relative error between Analytical and Euler Solution

### B. Agricultural Innovation

Fig. 4 $\frac{dX}{dT}$ versus X plot for different A values

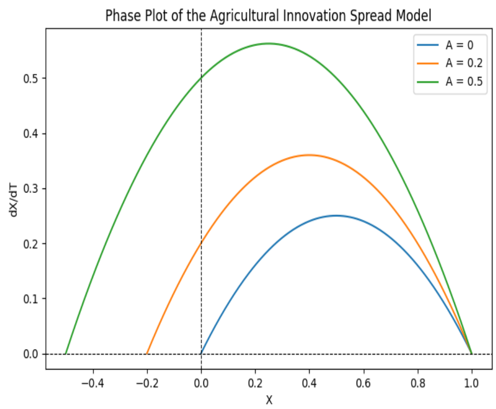


FIG. 4: Here shows the plot of $\dot{X}$ versus X for $A = 0, 0.20, 0.50$

Fig. 5 X(T) verus T for different A values

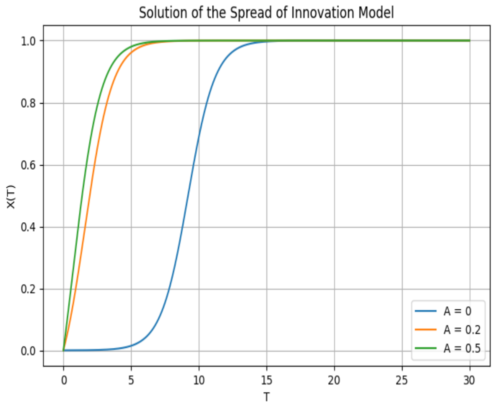


FIG. 5: Here shows the plot of X versus T for $A = 0, 0.2, 0.5$ and X(0)=0.0001

## C. Harvesting Model

Fig. 6 $\frac{dx}{dt}$ versus x for different h values

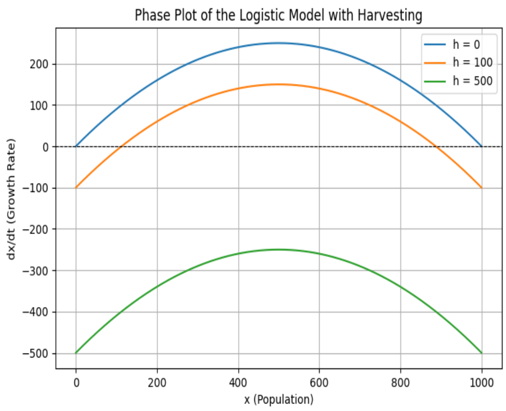


FIG. 6: Here shows the plot of $\dot{x}$ versus x for $h = 0, 100, 500$ and r=1,k=1000

Fig. 7 Numerical Solution for different h values

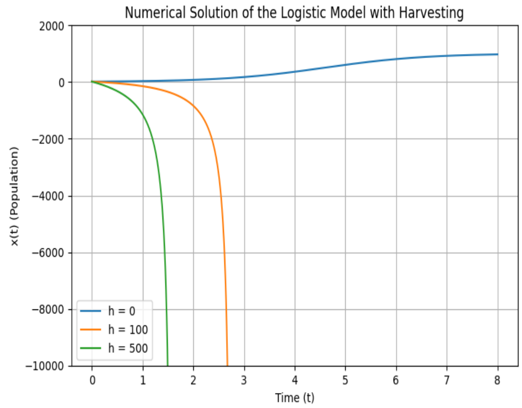


FIG. 7: Here shows the plot of Euler solution for $h = 0, 100, 500$ and x(0)=10 and r=1,k=1000

Fig. 8 Comparison of Analytical and Euler

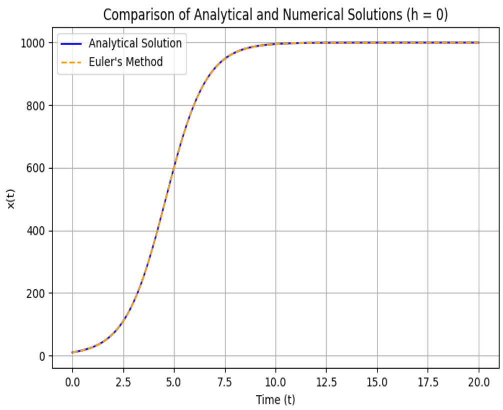


FIG. 8: Here shows the euler and analytical solution for $h = 0$ and r=1,k=1000

Fig. 9 Relative error between the two graphs

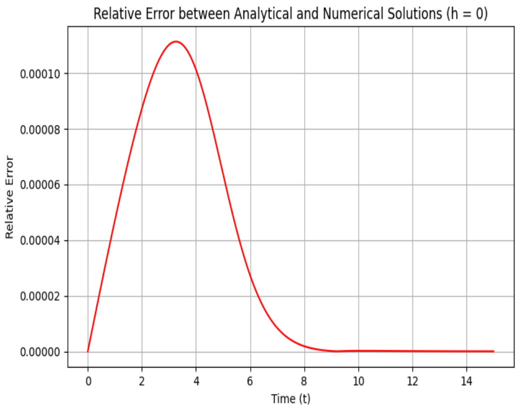


FIG. 9: Here shows the Relative error for $h = 0$