# Set 10: Modeling the Dynamics of Endemic Diseases and Epidemics in Populations

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This study models the spread of infectious diseases, distinguishing between endemic diseases, which persist in a population, and epidemics, which spread rapidly. Using coupled equations, we analyze the dynamics of infected, susceptible, and recovered groups. These models help understand disease behavior and support public health planning.

## I. EPIDEMIC DISEASES

### A. Model Equations

Consider a population size of $N$, through which an infection spreads. The population is divided into three classes — the infected class $x(t)$, the susceptible class $y(t)$, and the recovered class $z(t)$, so that $x(t) + y(t) + z(t) = N$ (constant). The coupled dynamics of these variables is given by:

$$\dot{x} = Axy - Bx \tag{1}$$
$$\dot{y} = -Axy \tag{2}$$
$$\dot{z} = Bx \tag{3}$$

in which $A$ is the infection rate and $B$ is the removal rate $(A, B > 0)$. At $t = 0$, $x(0) = x_0$ and $z(0) = 0$. Hence, $y(0) = y_0 = N - x_0$, where $x_0$ is small $(x_0 \ll N)$.

The Euler Solutions to above equations are given as:

$$x_{n+1} = x_n + (Ax_ny_n - Bx_n)\Delta t \tag{4}$$
$$y_{n+1} = y_n + (-Ax_ny_n)\Delta t \tag{5}$$
$$z_{n+1} = z_n + (Bx_n)\Delta t \tag{6}$$

The total number of students in a boarding school is 763. Initially, a single student introduces an infectious disease in this population. Taking $A = 2.18\times10^{-3}$ day$^{-1}$ and $B = 0.44$ day$^{-1}$ in all plots.

### B. Results

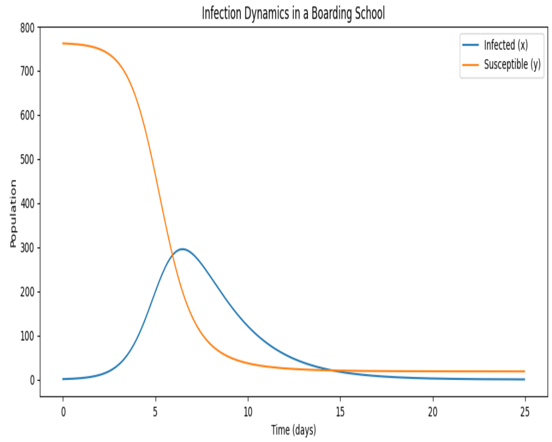


FIG. 1: Where $\Delta t = 1$ hour, $x_0 = 1$, and $y_0 = 762$. The time when $x$ reaches its maximum value is 6.46 days.

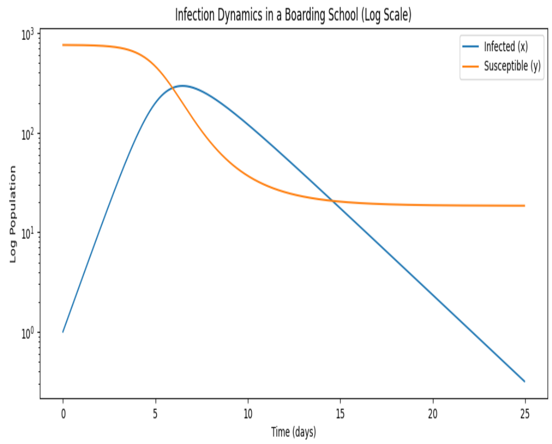


FIG. 2: Log plot

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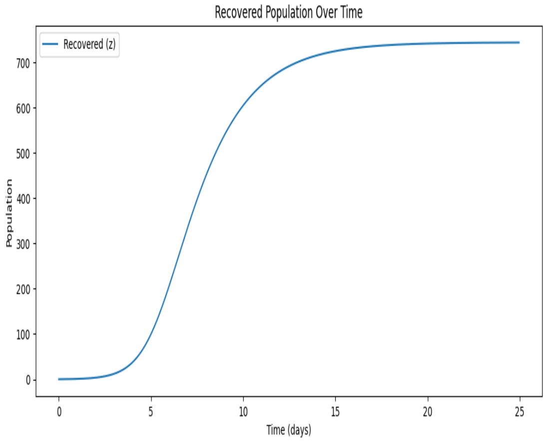


FIG. 3: Recovered Population, Where $\Delta t = 1$ hour, $z_0 = 0$.

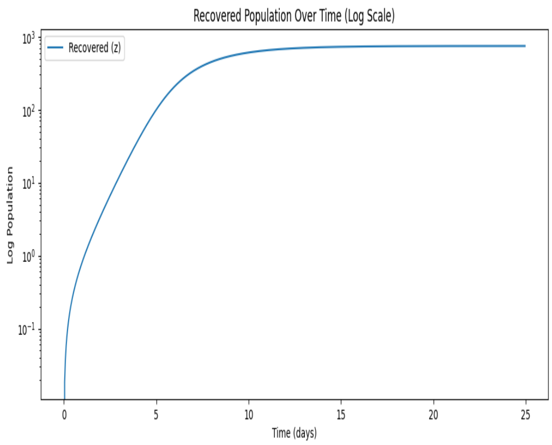


FIG. 4: Log plot

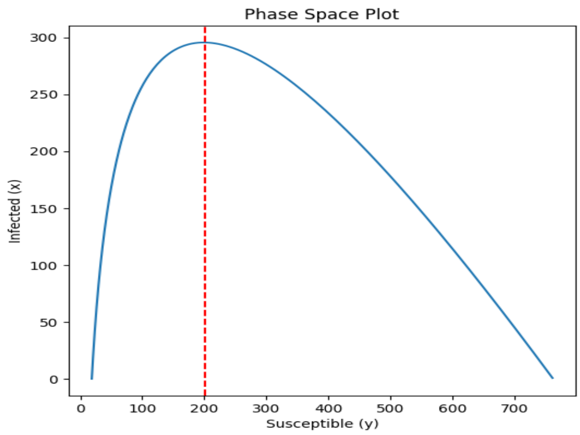


FIG. 5: Infected vs. Susceptible, Where $\Delta t = 1$ hour, $x_0 = 1$, $y_0 = 762$. The threshold value is $\rho = 201.8349$ and the Reproduction Number is $R = 3.7754$.

## II. ENDEMIC DISEASES

### A. Model Equation

In this scenario, the total population size, denoted by $N$, fluctuates over time. When considering per capita death and birth rates, represented by $a$ and $b$ respectively (where both are greater than 0), the corresponding system of equations becomes

$$\dot{x} = Axy - Bx - ax \tag{7}$$
$$\dot{y} = bN - Axy - ay \tag{8}$$
$$\dot{z} = Bx - az \tag{9}$$
$$\dot{N} = (b - a)N \tag{10}$$

Assuming $a = b = 0.02$ year$^{-1}$, leading to a fixed $N$, we take $A = 10^{-6}$ year$^{-1}$, $B = 0.333$ year$^{-1}$, $N = 10^6$, $x_0 = 10^5$, and $y_0 = 9 \times 10^5$.

The Euler Solutions to above equations are given as:

$$x_{n+1} = x_n + (Ax_ny_n - Bx_n - ax_n)\Delta t \tag{11}$$
$$y_{n+1} = y_n + (bN_n - Ax_ny_n - ay_n)\Delta t \tag{12}$$
$$z_{n+1} = z_n + (Bx_n - az_n)\Delta t \tag{13}$$

### B. Results

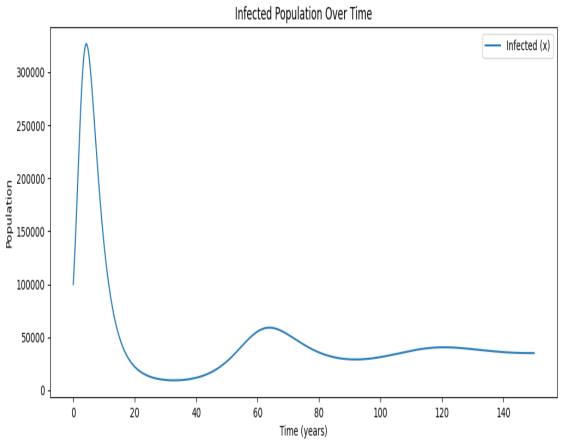


FIG. 6: Where $\Delta t = 1$ day and $x_0 = 10^5$

- Time when $x$ reaches peak 1 is 4.2410 years.
- Time when $x$ reaches peak 2 is 63.8410 years.
- Time when $x$ reaches peak 3 is 120.6986 years.

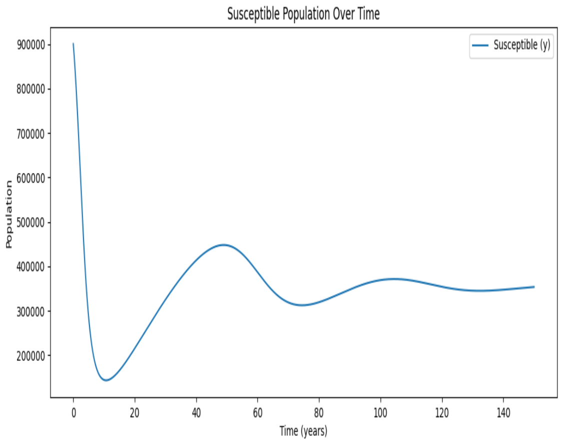


FIG. 7: Where $\Delta t = 1$ day and $y_0 = 9 \times 10^5$

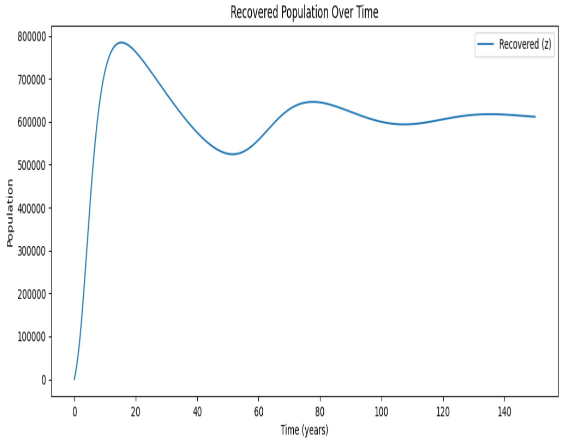


FIG. 8: Where $\Delta t = 1$ day and $z_0 = 0$