

Esercizi deduzioni naturali Logica Proposizionale

1 Esercizi svolti

1. $\vdash (\varphi \vee \psi) \rightarrow (\psi \vee \varphi)$

$$\frac{\frac{[\varphi \vee \psi]^1 \quad \frac{[\varphi]^2}{\psi \vee \varphi} \vee Il \quad \frac{[\psi]^3}{\psi \vee \varphi} \vee Ir}{\psi \vee \varphi} \vee E^{2,3}}{(\varphi \vee \psi) \rightarrow (\psi \vee \varphi) \rightarrow I^1}$$

2. $\vdash \perp \leftrightarrow (\varphi \wedge \neg \varphi)$

$$\frac{\frac{\frac{[\varphi \wedge \neg \varphi]^2}{\neg \varphi} \wedge Er \quad \frac{[\varphi \wedge \neg \varphi]^2}{\varphi} \rightarrow E \wedge El \quad \frac{[\perp]^1}{\varphi \wedge \neg \varphi} E \perp}{\perp} \perp}{\perp \leftrightarrow (\varphi \wedge \neg \varphi) \leftrightarrow I^{1,2}}$$

3. $\vdash \neg(\varphi \wedge \psi) \rightarrow (\neg \varphi \vee \neg \psi)$

$$\frac{\frac{\frac{[\neg(\varphi \wedge \psi)]^1 \quad \frac{[\varphi]^2}{\varphi \vee \psi} \vee Ir}{\perp} \rightarrow E \quad \frac{\perp}{\neg \varphi} \neg I^2}{\neg \varphi \vee \neg \psi} \vee Ir}{\neg(\varphi \wedge \psi) \rightarrow (\neg \varphi \vee \neg \psi) \rightarrow I^1}$$

4. $\neg \varphi \vdash \varphi \rightarrow \psi$

$$\frac{\frac{\neg \varphi \quad [\varphi]^1}{\perp} \rightarrow E \quad \frac{\perp}{\psi} E \perp}{\varphi \rightarrow \psi} \rightarrow I^1$$

5. $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$

$$\frac{\neg(\varphi \wedge \neg\psi) \quad \frac{\varphi \quad [\neg\psi]^1}{\varphi \wedge \neg\psi} \wedge I}{\perp} \text{RAA}^1 \rightarrow E$$

6. $[\varphi \rightarrow (\psi \rightarrow \sigma)] \leftrightarrow [(\varphi \wedge \psi) \rightarrow \sigma]$

$$\frac{\frac{[(\varphi \wedge \psi) \rightarrow \sigma]^1 \quad \frac{[\varphi]^3 \quad [\psi]^4}{\varphi \wedge \psi} \wedge I}{\frac{\sigma}{\psi \rightarrow \sigma} \rightarrow I^4} \rightarrow E \quad \frac{[\varphi \rightarrow (\psi \rightarrow \sigma)]^2 \quad \frac{[\varphi \wedge \psi]^5}{\varphi} \wedge El}{\psi \rightarrow \sigma} \rightarrow E \quad \frac{[\varphi \wedge \psi]^5}{\psi} \wedge Er}{\frac{\sigma}{(\varphi \wedge \psi) \rightarrow \sigma} \rightarrow I^5} \rightarrow E \quad \frac{\frac{\sigma}{\psi \rightarrow \sigma} \rightarrow I^4 \quad \frac{(\varphi \wedge \psi) \rightarrow \sigma}{\psi \rightarrow \sigma} \rightarrow I^5}{[\varphi \rightarrow (\psi \rightarrow \sigma)] \leftrightarrow [(\varphi \wedge \psi) \rightarrow \sigma]} \leftrightarrow I^{1,2}$$

2 Esercizi aggiuntivi

1. $\vdash (\varphi \rightarrow \psi) \vee (\psi \rightarrow \varphi)$
2. $\vdash \varphi \rightarrow \varphi$
3. $\vdash \perp \rightarrow \varphi$
4. $\neg(\varphi \wedge \neg\psi), \varphi \vdash \psi$
5. $\vdash \varphi \vee \varphi \leftrightarrow \varphi$
6. $\vdash (\varphi \vee \psi) \leftrightarrow ((\varphi \rightarrow \psi) \rightarrow \psi)$
7. $\vdash \neg\neg\varphi \leftrightarrow \varphi$
8. $(\varphi \rightarrow \psi) \leftrightarrow (\neg\psi \rightarrow \neg\varphi)$