

# Esercizi su valutazioni e tautologie

## 1 Valutazioni

1.  $\llbracket \varphi \rightarrow (\psi \rightarrow \varphi) \rrbracket_v = 1$

Per ogni valutazione  $v$ :

$$\begin{aligned}\llbracket \varphi \rightarrow (\psi \rightarrow \varphi) \rrbracket_v = 1 &\iff \llbracket \varphi \rrbracket_v = 0 \quad \text{oppure} \quad \llbracket \psi \rightarrow \varphi \rrbracket_v = 1 \\ &\iff \llbracket \varphi \rrbracket_v = 0 \quad \text{oppure} \quad \llbracket \psi \rrbracket_v = 0 \quad \text{oppure} \quad \llbracket \varphi \rrbracket_v = 1 \\ &\iff \llbracket \varphi \rrbracket_v = 0 \quad \text{oppure} \quad \llbracket \varphi \rrbracket_v = 1\end{aligned}$$

2.  $\varphi \models \varphi \vee \psi$

Per ogni valutazione  $v$ :

$$\begin{aligned}\varphi \models \varphi \vee \psi &\iff \forall v (\llbracket \varphi \rrbracket_v = 1 \rightarrow \llbracket \varphi \vee \psi \rrbracket_v = 1) \\ &\iff \forall v (\llbracket \varphi \rrbracket_v = 1 \rightarrow (\llbracket \varphi \rrbracket_v = 1 \quad \text{oppure} \quad \llbracket \psi \rrbracket_v = 1))\end{aligned}$$

3.  $\varphi \vee \psi \not\models \varphi$

Quindi:

$$\begin{aligned}\varphi \vee \psi \not\models \varphi &\iff \llbracket \varphi \vee \psi \rrbracket_v = 1 \quad \text{e} \quad \llbracket \varphi \rrbracket_v = 0 \\ &\iff \llbracket \psi \rrbracket_v = 1 \quad \text{e} \quad \llbracket \varphi \rrbracket_v = 0\end{aligned}$$

4.  $\models \neg(\varphi \wedge \psi) \leftrightarrow \neg(\varphi) \vee \neg(\psi)$

Per ogni valutazione  $v$ :

$$\begin{aligned}\models \neg(\varphi \wedge \psi) \leftrightarrow \neg(\varphi) \vee \neg(\psi) &\iff \llbracket \neg(\varphi \wedge \psi) \rrbracket_v = 1 \quad \text{e} \quad \llbracket \neg(\varphi) \vee \neg(\psi) \rrbracket_v = 1 \\ &\iff (\llbracket \varphi \rrbracket_v = 0 \quad \text{oppure} \quad \llbracket \psi \rrbracket_v = 0) \quad \text{e} \\ &\quad (\llbracket \neg\varphi \rrbracket_v = 1 \quad \text{oppure} \quad \llbracket \neg\psi \rrbracket_v = 1) \\ &\iff \llbracket \varphi \rrbracket_v = 0 \quad \text{oppure} \quad \llbracket \psi \rrbracket_v = 0\end{aligned}$$

$$5. \models \perp \leftrightarrow \varphi \wedge \neg \varphi$$

Per ogni valutazione  $v$ :

$$\begin{aligned} \models \perp \leftrightarrow \varphi \wedge \neg \varphi &\iff \llbracket \perp \rrbracket_v = 0 \quad \text{e} \quad \llbracket \varphi \wedge \neg \varphi \rrbracket_v = 0 \\ &\iff \llbracket \perp \rrbracket_v = 0 \quad \text{e} \quad \llbracket \varphi \rrbracket_v = 0 \quad \text{e} \quad \llbracket \neg \varphi \rrbracket_v = 0 \\ &\iff \llbracket \varphi \rrbracket_v = 0 \quad \text{oppure} \quad \llbracket \varphi \rrbracket_v = 1 \end{aligned}$$

## 2 Tabelle di verità

1.  $p \rightarrow (q \rightarrow p)$  , è una tautologia.

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
0	0	1	1
0	1	0	1
1	0	1	1
1	1	1	1

2.  $\underbrace{(p \rightarrow (q \rightarrow r))}_{\alpha} \rightarrow \underbrace{((p \rightarrow q) \rightarrow (p \rightarrow r))}_{\beta}$  , è una tautologia.

p	q	r	$q \rightarrow r$	$p \rightarrow q$	$p \rightarrow r$	$\alpha$	$\beta$	$\alpha \rightarrow \beta$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	0	0	1	1
1	0	1	1	0	1	1	1	1
1	1	0	0	1	0	0	0	1
1	1	1	1	1	1	1	1	1

3.  $p \vee q \rightarrow p$  , non è una tautologia.

p	q	$p \vee q$	$p \vee q \rightarrow p$
0	0	0	1
0	1	1	0
1	0	1	1
1	1	1	1