UNIVERSITY OF ARKANSAS

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INEG 33103 - Probability and Statistics

Week 10 Homework (100 points)

For each problem, you can solve by hand or use Excel to help. You need to show all work in either case.

Question 1 (adapted from p238, 5-6&5-7). Two machines are used to fill plastic bottles with dishwashing detergent. The standard deviations of fill volume are known to be $\sigma_1 = 0.10$ and $\sigma_2 = 0.15$ fluid ounces for the two machines, respectively. Two random samples of $n_1 = 12$ bottles from machine 1 and $n_2 = 10$ bottles from machine 2 are selected, and the sample mean fill volumes are 30.61 and 30.34 fluid ounces. Assume normality.

(a) Test the hypothesis that both machines fill to the same mean volume. Use the P-value approach.
   a. Standard Error = sqrt((0.10^2/12) + (0.15^2/10)) = 0.0555
   b. Z = (30.61 – 30.34) / 0.0555 = 4.86
   c. Assuming significance level of 0.05 the P value calculated is bellow 0.05 meaning we reject the null hypothesis

(b) Construct a 90% two-sided CI on the mean difference in fill volume. Interpret this interval.
   a. Z = 1.645
   b. Margin of Error = 1.645 * 0.0555 = 0.0913
   c. CI = (0.27 – Margin of Error,0.27 + Margin of Error)
   d. CI = (0.179,0.361)

(c) If $\alpha = 0.05$ and the β-error of the test when the true difference in fill volume is 0.2 fluid ounces should not exceed 0.1, what sample size must be used?
   a. Significance Level = 0.05
   b. Z = 1.96
   c. β = 0.1
   d. Z_b=1.28
   e. Calculating for n using n =((1.96 + 1.28)^2 * (0.10^2 + 0.15^2)) / (0.2^2) = 8.26
   f. Round up gives 9 for the sample size
   g.

Question 2 (adapted from p250, 5-17). An article in *Electronic Components and Technology Conference* (Vol. 52, 2001, pp.1167-1171) describes a study comparing single versus dual spindle saw processes for copper metalized wafers. A total of 15 devices of each type were measured for the width of the backside chipouts, $\bar{x}_{single} = 66.385$, $s_{single} = 7.895$, $\bar{x}_{double} = 45.278$, $s_{double} = 8.612$.

(a) Do the sample data support the claim that both processes have the same chip outputs? Assume that both populations are normally distributed and have the same variance.
   a. Given Nsingle = 15, Ndouble = 15, Ssingle = 7.895, and Sdouble = 8.612
   b. Calculate the Sp = sqrt(((Nsingle – 1)S^2single + (Ndouble – 1)S^2double) / (Nsignle + Ndouble -2)
   c. This gives aproximitly 8.261
   d. Calculating the Tstat= (66.385 - 45.278) / (8.26 * SQRT(1/15 + 1/15)) = 6.997

e. Calculating Degrees of Freedom = 15 + 15 – 2 = 28
f. Critical t-value for a two-tailed test at the 5% significance level with 28 degrees of freedom is approximately 2.048.
g. Since 6.997 > 2.048 we reject the null hypothesis and conclude that there is a significant difference Chip outputs \

(b) Construct a 95% confidence interval on the mean difference in spindle saw process. Compare this interval to the results in part (a).
   a. Mean Difference = 66.385 – 45.278 = 21.107
   b. Margin Error = 2.048 * 8.261 * sqrt((1/15) + (1/15)) = 6.1778
   c. Confidence Interval = (MeanDiff – Margin Error, MeanDiff + Margin Error) = (14.93,27.28)

Question 3 (p250, 5-18). An article in *IEEE International Symposium on Electromagnet Compatibility* (Vol. 2, 2002, pp.667-670) describes the quantification of the absorption of electromagnetic energy and the resulting thermal effect from cellular phones. The experimental results were obtained from in vivo experiments conducted on rats. The arterial blood pressure values (mmHg) for the control group (8 rats) during the experiment are $\bar{x}_1 = 90$, $s_1 = 5$ and for the test group (9 rats) are $\bar{x}_2 = 115$, $s_2 = 10$.

(a) Is there evidence to support the claim that the test group has higher mean blood pressure? Assume that both populations are normally distributed but the variances are not equal.
   a. T = (115 – 90) / (sqrt (5^2/8 + 10^2/9) = 6.626
   b. Degree of Freedom = (5^2/8 + 10^2/9)^2 / ((5^2/8)^2/8-1) + ((10^2/9)^2/9-1) = 12.044
   c. Calculated P Value is 1.2221×10^{-5}
   d. P value is less than the significance level, therefore we reject the null hypothesis.

(b) Calculate a confidence interval to answer the question in part (a).
   a. SE = sqrt((5^2/8 + 10^2/9)) = 3.7731
   b. Critical Value t = 2.1788
   c. Margin of Error = 2.1788 * 3.7731 = 8.221
   d. Confidence Interval = (16.78,33.22)