

Name: \_\_\_\_\_Blake Williams\_\_\_\_\_

### INEG 33103 - Probability and Statistics

#### Week 6 Homework (100 points)

For each problem, you can solve by hand or use Excel to help. You need to show all work in either case.

**Question 1 (p197, 4-53).** An article in *Computers in Electrical Engineering* ("Parallel Simulation of Cellular Neural Networks", 1996, Vol. 22, pp.61-84) considered the speed-up of cellular neural networks for a parallel general-purpose computing architecture. The data follow.

3.775302	3.350679	4.217981	4.030324	4.639692	4.139665
4.395575	4.824257	4.268119	4.584193	4.930027	4.315973
4.600101					

(a) Is there sufficient evidence to reject the claim that the mean speed-up exceeds 4.0? Assume  $\alpha = 0.05$ .

- a. Null hypothesis =  $\mu = 4.0$
- b. Alternative Hypothesis =  $\mu > 4.0$
- c. Sample Mean = 4.31322
- d. Sample Standard Deviation = 0.432852
- e.  $(4.31322 - 4.0) / (0.432852 / \sqrt{13}) = 2.607$
- f.  $T_{0.05,12} = 1.782$
- g.  $2.607 > 1.782$
- h. We reject the Null hypothesis and replace it with the Alternative Hypothesis. So yes there is sufficient evidence.

(b) Find a 95% two-sided CI on the mean speed-up time.

- a.  $E = 2.179 * (0.432852 / \sqrt{13}) = 0.26157$
- b. Upper Limit = Sample Mean + E =  $4.31322 + 0.26157 = 4.57479$
- c. Lower Limit = Sample Mean - E =  $4.31322 - 0.26157 = 4.05165$
- d.  $4.05165 \leq \mu \leq 4.57479$

**Question 2 (p204, 4-68).** The sugar content of the syrup in canned peaches is normally distributed, and the variance is thought to be  $\sigma^2 = 18$  (mg)<sup>2</sup>.

(a) Test the hypothesis that the variance is not 18 (mg)<sup>2</sup> if a random sample of 10 cans yields a sample standard deviation of 4 mg, using a fixed-level test with  $\alpha = 0.05$ .

- a. Null Hypothesis
- b.  $H_0: \sigma^2 = 18$
- c. Alternative Hypothesis
- d.  $H_1: \sigma^2 \neq 18$
- e.  $\chi^2 = ((n-1)s^2) / 18$
- f.  $\chi^2 = ((10 - 1) (16)) / 18 = 8$
- g.  $2.7004 \leq 8 \leq 19.0228$

- h. We fail to disprove the Null hypothesis as there is not sufficient evidence to conclude that the variance differs from 18
- (b) What is the  $P$ -value for this test?
  - a.  $X^2 \leq 8 = 0.4009$
  - b.  $X^2 > 8 = 1 - 0.4009 = 0.5991$
  - c.  $\text{Min} = 0.4009$
  - d.  $P\text{-value} = 2 * 0.4009 = 0.8018$
- (c) Find a 95% two-sided CI for  $\sigma$ .
  - a. Lower Limit =  $\sqrt{144/19.0228} = 2.752$
  - b. Upper Limit =  $\sqrt{144/2.7004} = 7.300$
- (d) Use the CI in part © to test the hypothesis.
  - a.  $\sqrt{18} = 4.243$
  - b. Since the Hypothesized standard deviation lies within the confidence interval, we fail to reject the null hypothesis.

**Question 3 (p214, 4-75).** Large passenger vans are thought to have a high propensity of rollover accidents when fully loaded. Thirty accidents of these vans were examined, and 11 vans had rolled over.

- (a) Test the claim that the proportion of rollovers exceeds 0.25 with  $\alpha = 0.1$ 
  - a. Null Hypothesis  $p > 0.25$
  - b. Alternative Hypothesis  $p > 0.25$
  - c. Sample Proportion =  $11/30 = 0.3667$
  - d. Standard Error =  $\sqrt{0.1875/30} = 0.0791$
  - e. Z-test =  $((0.3667 - 0.25) / 0.0791) = 1.476$
  - f.  $Z_{0.10} = 1.2816$
  - g.  $Z = 1.476 > z_{\alpha} = 1.2816$
  - h. The Null Hypothesis is rejected and there is significant evidence to support the claim that rollovers exceeds 0.25
- (b) Suppose that the true  $p = 0.35$  and  $\alpha = 0.1$ . How large a sample would be required if we want  $\beta = 0.1$ ?
  - a. Significance level =  $z_{0.90} = 1.2816$
  - b. Power = 1.2816
  - c. Calculating using sample size formula
  - d.  $N = (1.1684/0.10)^2 = 136.51$
  - e. Round up to 137
  - f. We need 137 vans
- (c) Find a 90% traditional lower confidence bound on the rollover rate of these vans.
  - a.  $SE = \sqrt{0.2321/30} = 0.0879$
  - b.  $L = 0.3667 - 1.2816 * 0.0879 = 0.2540$
  - c. 25.4%