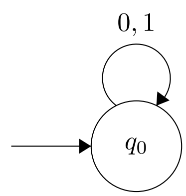
Problem 1

Total: 40 points (10 points each)
Draw the state diagram of DFAs recognizing the following languages. Note: $\Sigma = \{0, 1\}$

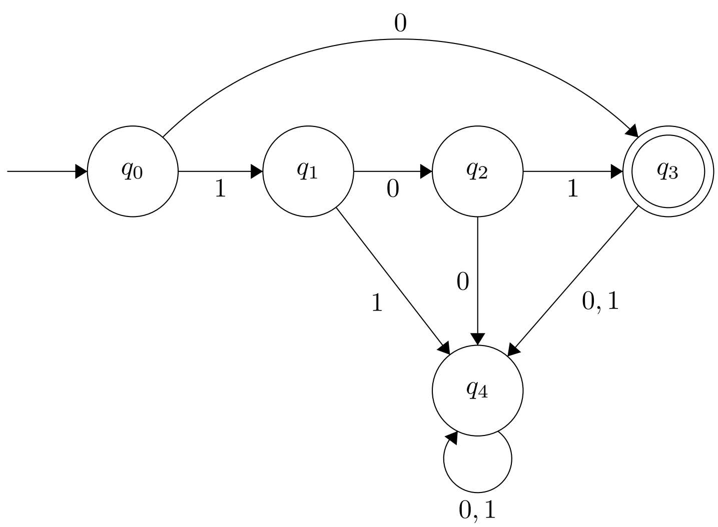
1. $A = \{w \mid \text{length of } w, |w|, \text{is an even prime number greater than } 2\}$
2. $B = \{0, 101\}$
3. $C = \{w \mid w \text{ contains an even number of 0's and an odd number of 1's}\}$
4. $D = \{w \mid \text{every third position of } w \text{ is } 1\}$

Solution:

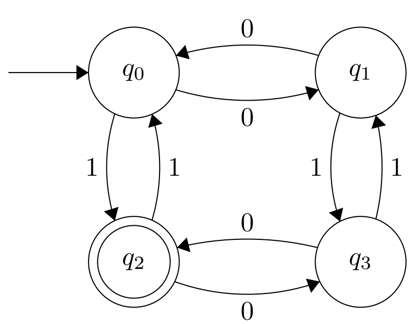
1. There are no even primes greater than 2. $A = \varnothing$



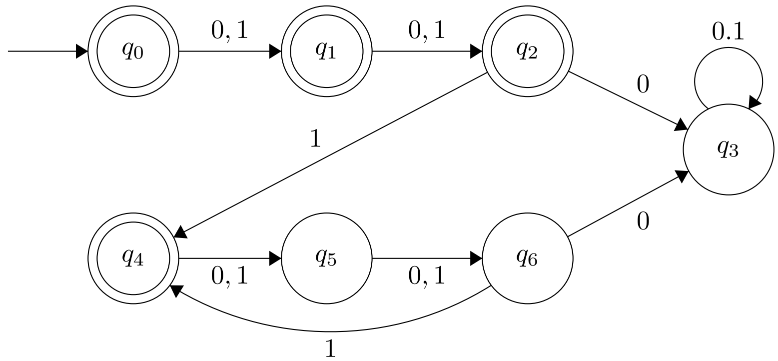
2.



3.



4. Note that any string with a length of 2 or less does not have a third position and thus *vacuously* satisfies the membership requirement of D.



Problem 2

Total: 20 points
Example of set difference: $A = \{0, 01\}$, and $B = \{0, 11\}$. Then, $A - B = \{01\}$.

Prove that regular languages are closed under the set *difference* operation. That is, if A and B are regular languages, then $A - B$ is also a regular language.

Hint: One can prove the statement above by either (1) contradiction or (2) construction. For the proof, you may make use of the theorems that regular languages are closed under *union, intersection,* and *complement.*

Solution 1 - Proof by contradiction
First, we know that $A - B = A \cap B^C$. And because B is regular, then $\Sigma^* - B = B^C$ is also regular via the closure of regular languages under the *complement* operation.
Assumption: We assume that $A \cap B^C$ is not regular.
Because regular languages are closed under intersection, therefore, the assumption implies at least A or B^C must not be a regular language. However, this contradicts the known facts that both A and B^C are regular. Therefore, $A \cap B^C$ is regular. $\square$

Solution 2 - Proof by construction
Similar to the proof in the **footnote** of page 46 in the textbook.