Problem 1

Total: 20 points (10 points each)
Convert the following regular expressions into equivalent NFAs.

1. a*(b ∪ c)*c
2. ((b ∪ a)* ∪ (c ∪ a))*(cb)*

Solution

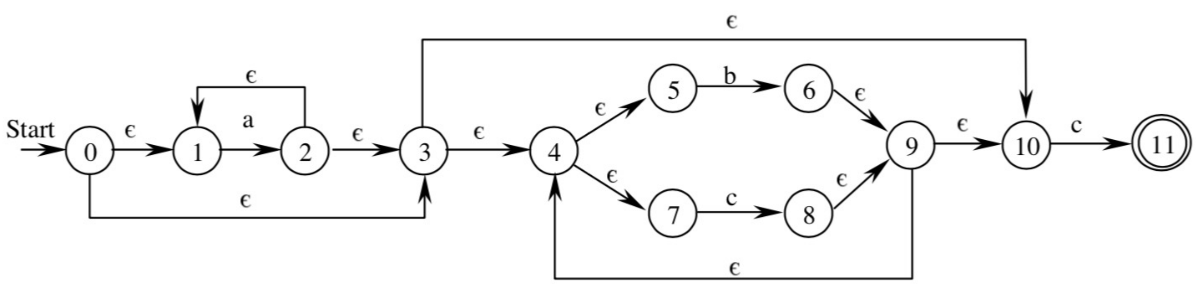

Figure 1: NFA for question 1.

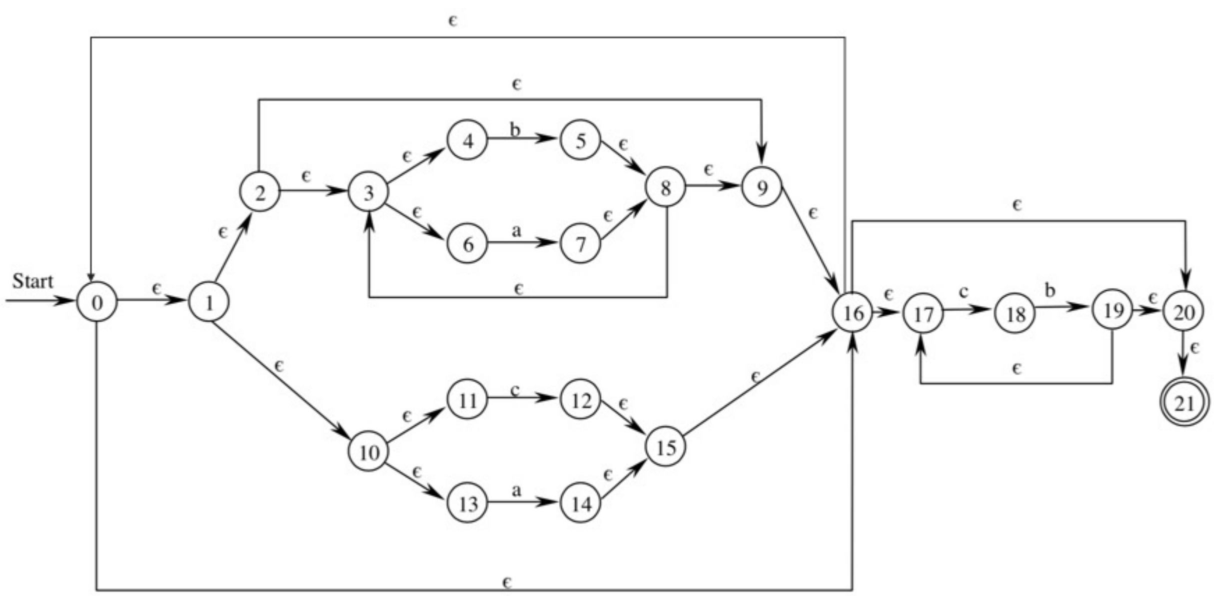

Figure 2: NFA for question 2.

Problem 2

Total: 20 points (10 points each)
Convert the following NFAs into equivalent regular expressions. Show **all the intermediate steps** (i.e. GNFAs) and make appropriate comments to help graders understand your steps.
For example, step 1: remove state "q_0"; step 2: remove state "q_3", etc.

1. Convert the NFA in Figure 3 into an equivalent regular expression.

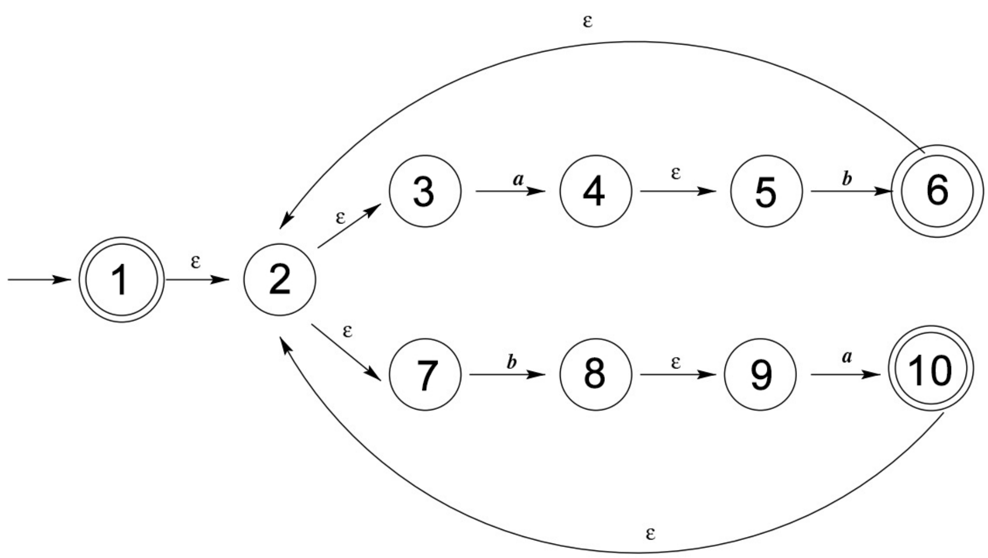

Figure 3:

Solution:
A regular expression for the above NFA is $(ab \cup ba)^*$.

2. Convert the NFA in Figure 4 into an equivalent regular expression.

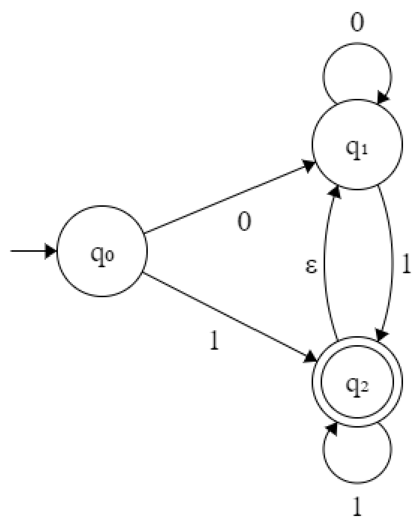

Figure 4:

Solution:
By removing the states in the order of q_0, q_1, q_2: $(1 \cup 00^*1)(0^* \cup 1)^*$
Your solution may be different if the states were removed in a different order.

Problem 3

Total: 20 points (10 points each)
Alphabet $\Sigma = \{0, 1\}$
For each provided language (X and Y), please answer these two questions:
(a) Is the language regular or non-regular?
(b) If your answer is regular, please provide either a DFA, NFA or regular expression (choose only one) that recognizes the language. If your answer is non-regular, please prove it by contradiction using Pumping Lemma. The presentation format of the proof is expected to follow those examples in the book.

1. $X = \{0^m 1^n \mid m > n \geq 0\}$
Solution:

X is not a regular language. We will prove it by contradiction using the Pumping Lemma.

Suppose X is regular. Then by the Pumping Lemma, there is the pumping length p. Let $s = 0^p 1^{p-1}$. Since $2p + 1 > p$, $s \in Y$ and $|s| \geq p$, s can be divided into three pieces $s = xyz$ satisfying the following conditions:

1. for each $i > 0$, $xy^i z \in X$
2. $|xy| \leq p$
3. $|y| > 0$

The problem occurs when we try to pump "down" s as y can only consist of 0s by the second condition. Since s only has one more 0 than 1, pumping down any number of 0s will result in a string with less than or equal number of 0s than 1s, which can't belong to X. Since this is a contradiction, X must be a non-regular language. □

2. $Y = \{0^n \mid n \text{ is a prime}\}$
Solution:

Y is not a regular language. We will prove it by contradiction using the Pumping Lemma.

Suppose Y is regular. Then by the Pumping Lemma, there is the pumping length p. Let $s = 0^q$, where q is a prime larger than p. Then $s \in Y$ and s can be divided into three pieces $s = xyz$ satisfying the following conditions:

1. for each $i > 0$, $xy^iz \in Y$
2. $|xy| \leq p$
3. $|y| > 0$

Suppose $|y| = k$. Then $|xy^{q+1}z| = |xyz| + |y^q| = q + kq = q(1 + k)$. Since $k > 0$ by the third condition, $k + 1 \geq 2$, which means $q(1 + k)$ is a composite number and cannot be a prime. Hence, xy^qz does not belong to Y, which contradicts the Pumping Lemma. Thus, Y must be non-regular. *square*