Homework 2

1.3: 5, 6, 11, 12, 17, 18, 25, 26 1.4: 11, 12, 21, 22, 25, 26 1.5: 1, 2, 5, 6, 11, 12, 15, 16, 19 ## Davis Davalos-DeLosh ### 1.3: 5, 6, 11, 12, 17, 18, 25, 26 In Exercises 5 and 6, write a system of equations that is equivalent to the given vector equation. 5. $x_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}$

$6x_1 - 3x_2 = 1$ $-x_1 + 4x_2 = -7$ $5x_1 = -5$

6. $x_1 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-2x_1 + 8x_2 + x_3 = 0$ $3x_1 + 5x_2 - 6x_3 = 0$

In Exercises 11 and 12 , determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1, \mathbf{a}_2$, and $\mathbf{a}_3$.

11. $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \overset{R_2+2R_1}{\Rightarrow} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 2 & 8 & 6 \end{array}\right] \overset{R_3+((-2)R_2)}{\Rightarrow} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 12 \end{array}\right]$$

The linear system is inconsistent, so $\mathbf{b}$ is not a linear combination

$$12.\ \mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{array}\right] \overset{R_2+2R_1}{\to} \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 2 & 5 & 8 & -7 \end{array}\right]$$
$$\overset{R_3+((-2)R_1)}{\to} \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{array}\right]$$

The system is inconsistent since $5c_2 + 4c_3 = 1$ and $5c_2 + 4c_3 = 3$ are mutually exdusive statements, therefore, b is not a linear combination.

17. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$. For what value(s) of h is $\mathbf{b}$ in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$?

$$
\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{array}\right] \xrightarrow{R_2+((-4)R_1)} \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 5 & -15 \\ -2 & 7 & h \end{array}\right]
$$

$$
\xrightarrow{R_3+2R_1} \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{array}\right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{array}\right]
$$

$$
\xrightarrow{R_3+((-3)R_2)} \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h+17 \end{array}\right] t
$$

The system is consistent when $h+17=0$, or $h=-17$

18. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$. For what value(s) of h is $\mathbf{y}$ in the plane generated by $\mathbf{v}_1$ and $\mathbf{v}_2$?

$$
\left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{array}\right] \xrightarrow{R_3+2R_1} \left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 2 & 2h-3 \end{array}\right]
$$

$$
\xrightarrow{R_3+((-2)R_2)} \left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & 2h+7 \end{array}\right]
$$

y is in the plane generated by V_1 & V_2 when $2h+7=0$, or when $h=-7/2$

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1.4: 11, 12, 21, 22, 25, 26

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1.5: 1, 2, 5, 6, 11, 12, 15, 16, 19

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25. Let $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$. Denote the columns of A by $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$, and let $W = \text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

a. Is $\mathbf{b}$ in $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$? How many vectors are in $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$?

b. Is $\mathbf{b}$ in W? How many vectors are in W?

c. Show that $\mathbf{a}_1$ is in W. [*Hint:* Row operations are unnecessary.]

$3 + -8$
$-4 + 8$

a) No. There are 3 vectors

b) $\left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{array}\right] \xrightarrow{R_3 + 2R_1} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{array}\right]$

Yes! There are infinitely many vectors in W

$\xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 1 & -2/3 & 1/3 \\ 0 & 6 & -5 & 4 \end{array}\right] \xrightarrow[(8 - 1R_3)]{R_3 + ((-6)R_2)} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 1 & -2/3 & 1/3 \\ 0 & 0 & 1 & -2 \end{array}\right]$

c) $\left[\begin{array}{ccc|c} 1 & 0 & -4 & 1 \\ 0 & 3 & -2 & 0 \\ -2 & 6 & 3 & -2 \end{array}\right] \xrightarrow{R_3 + 2R_1} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 1 \\ 0 & 3 & -2 & 0 \\ 0 & 6 & -5 & 0 \end{array}\right]$

Figure 1: original image

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