Due Date February 2, 2023
Name **Your Name**
Student ID **Your Student ID**
Collaborators **List Your Collaborators Here**

Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.

- You should submit your work through the **class Gradescope page** only (linked from Canvas). Please submit one PDF file, compiled using this LaTeX template.

- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).

- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

Honor Code (Make Sure to Virtually Sign)

- My submission is in my own words and reflects my understanding of the material.

- Any collaborations and external sources have been clearly cited in this document.

- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.

- I have neither copied nor provided others solutions they can copy.

Agreed (signature here)..

Standard 1: Proof by Induction

Problem 1 (1 point)

Problem 1. A student is trying to prove by induction that $4^n < n!$ for $n \geq 9$.

Student's Proof. The proof is by induction on $n \geq 9$.

- **Base Case:** When $n = 9$, we have that:

$$4^9 = 262,144$$
$$\leq 362,880$$
$$= 9!$$

- **Inductive Hypothesis:** Now suppose that for all $k \geq 9$ we have that $4^k < k!$.

- **Inductive Step:** We now consider the $k+1$ case. As $k+1 > 9$, we have from the inductive hypothesis that $4^{k+1} < (k+1)!$. The result follows by induction.

There are two errors in this proof.

1. The Inductive Hypothesis is not correct. Write an explanation to the student explaining why their Inductive Hypothesis is not correct. [**Note:** You are being asked to explain why the Inductive Hypothesis is wrong, and **not** to rewrite a corrected Inductive Hypothesis.]

   Answer.

2. The Inductive Step is not correct. Write an explanation to the student explaining why their Inductive Step is not correct. [**Note:** You are being asked to explain why the Inductive Step is wrong, and **not** to rewrite a corrected Inductive Step.]

   Answer.

Problem 2 (1 point)

Problem 2. Consider the recurrence relation, defined as follows:

$$T_n = \begin{cases} 1 & : n = 0, \\ 11 & : n = 1, \\ T_{n-1} + 12T_{n-2} & : n \geq 2. \end{cases}$$

Prove by induction that $T_n = (-1) \cdot (-3)^n + 2 \cdot (4)^n$, for all integers $n \in \mathbb{N}$. [**Recall:** $\mathbb{N} = \{0, 1, 2, ...\}$ is the set of non-negative integers.]

Proof.

Problem 3 (2 points)

Problem 3. The complete, balanced ternary tree of depth d, denoted $\mathcal{T}(d)$, is defined as follows.

- $\mathcal{T}(0)$ consists of a single vertex.

- For $d > 0$, $\mathcal{T}(d)$ is obtained by starting with a single vertex and setting each of its three children to be copies of $\mathcal{T}(d-1)$.

Prove by induction that $\mathcal{T}(d)$ has 3^d leaf nodes. To help clarify the definition of $\mathcal{T}(d)$, illustrations of $\mathcal{T}(0), \mathcal{T}(1),$ and $\mathcal{T}(2)$ are on the next page. [**Note:** $\mathcal{T}(d)$ is a tree and **not** the number of leaves on the tree. Avoid writing $\mathcal{T}(d) = 3^d$, as these data types are incomparable: a tree is not a number.]

Proof.

Example 1. We have the following:

for tree= circle, draw, fill, minimum width=2pt, inner sep=0pt, parent anchor=center, child anchor=center, s sep+=25pt, [ ]

$\mathcal{T}(0)$.

for tree= circle, draw, fill, minimum width=2pt, inner sep=0pt, parent anchor=center, child anchor=center, s sep+=25pt, [[] [] []]

$\mathcal{T}(1)$.

for tree= circle, draw, fill, minimum width=2pt, inner sep=0pt, parent anchor=center, child anchor=center, s sep+=25pt, [[[] [][]] [[] [][]] [[] [][]]]

$\mathcal{T}(2)$.