LDA Whitening

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whitening transformation : $z = \Lambda^{-\frac{1}{2}} U^T x$

NCC : $w_{NCC} = w_o - w_\Delta$

where $w_o = \frac{1}{N_o} \sum_{n=1}^{N_o} x_n$

$w_\Delta = \frac{1}{N_\Delta} \sum_{n=1}^{N_\Delta} x_n$

LDA : $w_{LDA} = S_W^{-1} (w_o - w_\Delta)$

where $S_W = U \Lambda U^T$ eigenvalue decomposition

Auxiliary calculations:

$$S_W^{-1} = (U \Lambda U^T)^{-1} = U^{T^{-1}} \Lambda^{-1} U^{-1} \overset{\text{U orthogonal, } U^{-1} = U^T}{=} U \Lambda^{-1} U^T$$

centers of the whitened data:

$$w_o'' = \frac{1}{N_o} \sum_{n=1}^{N_o} z_n \overset{\text{def } z}{=} \frac{1}{N_o} \sum_n \Lambda^{-\frac{1}{2}} U^T x_n \overset{\Lambda^{-1/2},\, U \text{ ind. of } n}{=} \Lambda^{-\frac{1}{2}} U^T \left(\frac{1}{N_o} \sum_n x_n \right)$$

$$\overset{\text{def } w_o}{=} \Lambda^{-\frac{1}{2}} U^T w_o$$

same holds for class Δ:

$$w_\Delta'' = \Lambda^{-\frac{1}{2}} U^T w_\Delta$$

Apply LDA on a data point x:

$$w_{LDA}^T x = (S_W^{-1} (w_o - w_\Delta))^T x$$

$$\overset{(AB)^T = B^T A^T}{=} (w_o - w_\Delta)^T S_W^{-1^T} x$$

$$\overset{S_W^{-1^T} = S_W^{T^{-1}} = S_W^{-1},\ S_W \text{ sym.}}{=} (w_o - w_\Delta)^T S_W^{-1} x$$

$$\overset{\text{def } S_W^{-1}}{=} (w_o - w_\Delta)^T U \Lambda^{-1} U^T x$$

$$\overset{\Lambda^{-1} = \Lambda^{-1/2} \Lambda^{-1/2}}{=} (w_o - w_\Delta)^T U \Lambda^{-\frac{1}{2}} \underbrace{\Lambda^{-\frac{1}{2}} U^T x}$$

def z' $= (w_0 - w_\Delta)^T U \Lambda^{-\frac{1}{2}} z'$

$(C^T B^T A^T) = (ABC)^T$ $= \left(\Lambda^{-\frac{1}{2}T} U^T (w_0 - w_\Delta)\right)^T z$

$\Lambda^{-1/2}$ sym. $= \left(\Lambda^{-\frac{1}{2}} U^T (w_0 - w_\Delta)\right)^T z$

$X(v-w)^T = Xv^T - Xw^T$ $= \left(\Lambda^{-\frac{1}{2}} U^T w_0 - \Lambda^{-\frac{1}{2}} U^T w_\Delta\right)^T z$

def w_0', w_Δ' $= (w_0' - w_\Delta')^T z$

$= w_{NCC}'^T z$

$\Rightarrow$ applying LDA on some data point is the same as applying NCC on whitened data