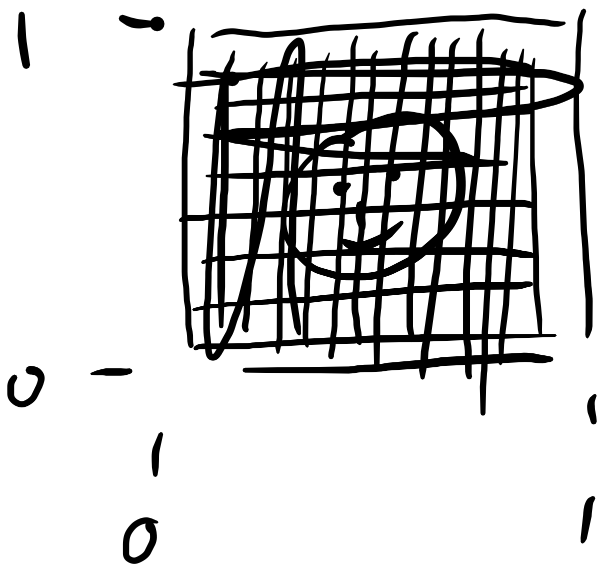
Zahlen

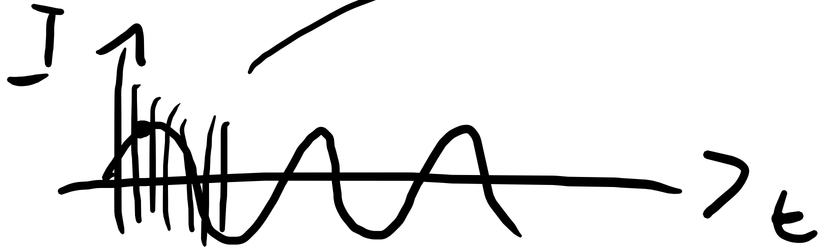
Bild



$f: [0,1]^2 \to [0,1]$

$v \in \mathbb{R}^n$ $v \in \mathbb{N}^n$

Audio



$\to v \in \mathbb{R}^n$

$f: \mathbb{R} \to \mathbb{R}$

$f = \sum_i^n \alpha_i B_i(x)$

$B_i(x) =$ $\sin(x), \sin(2x), \sin(3x),$
$\cos(x), \cos(2x), \cos(3x),$

Natürliche Zahlen $\quad \mathbb{N} = \{0, 1, 2, \dots\}$

$a, b \in \mathbb{N} \qquad a+b\,, \quad a \cdot b \in \mathbb{N}$

$a + 0 = a\,, \quad a \cdot 1 = a$

$a + b = b + a$

$ab = ba$

$a(b+c) = ab + ac$

$a \leq b \qquad \exists\, x : \quad a + x = b$

Reflexiv $\quad a \leq a$

Antisymmetrie $\quad a \leq b \wedge b \leq a \implies a = b$

Transitiv $\quad a \leq b \wedge b \leq c \implies a \leq c$

Total $\quad a \leq b \vee b \leq a$

Repräsentation von $\mathbb{N}$

$$x_{n-1}\, x_{n-2} \dots x_1\, x_0 = \sum_{i=0}^{n-1} x_i\, b^i$$

$$y_{n-1}\, y_{n-2} \dots y_1\, y_0$$

$\leq$

$\rightarrow \rightarrow$

$b=2$, n fest $n = 8 \rightarrow [0, 255]$

Unär $\cancel{||||}\ |||$

V X L C D M

IV VI

$b=10 \rightarrow 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

$$\boxed{a + x = b}$$

Ganze Zahlen

$$(a,b) \sim (c,d) \iff a+d = b+c$$

$$\mathbb{Z} = \{[a,b],\ a,b \in \mathbb{N}\}$$

$$[a,b] + [c,d] = [a+c,\ b+d]$$

$$[a,b] \cdot [c,d] = [ad+bc,\ ac+bd]$$

$$[a,b] \le [c,d] \iff b+c \le a+d$$

$$-[a,b] = [b,a]$$

$(a,s) \quad a + x = s$

Repräsentation von $\mathbb{Z}$

≥ 0 $\quad\quad$ ≤ 0

$(0, x)$ $\quad\quad$ $(y, 0)$ $\quad\quad$ (Vorzeichendarst.)

$(0, x)$ $\quad 0 \leq x \leq 2^{n-1}-1$
$\quad y$ $\quad\quad$ $\boxed{(2^{n-1}-1, y)}$

Bit reversal

$\overset{+}{\rightarrow} (2^{n-1}-1, x+y) \overset{-}{\rightarrow} (2^{n-1}-1, x+y)$ $\quad\quad$ (1er Komplement)

$\overset{+}{\rightarrow} (0, x+y-2^{n-1}+1)$

$(0, x)$ $\quad\quad$ $(2^{n-1}, y)$

$\overset{+}{\rightarrow} (2^{n-1}, x+y) \overset{-}{\rightarrow} (2^{n-1}, x+y)$

$\overset{+}{\rightarrow} (0, x+y-2^{n-1})$

$(2^{n-1}, x)$ $\quad 0 \leq x, y \leq 2^{n}-1$

$(2^{n-1}, y)$

$+ \; (2^{n}, x+y) \rightarrow (2^{n-1}, x+y-2^{n-1})$

Rationale Zahlen $\mathbb{Q}$

$(a,b) \sim (c,d) \iff ad = bc$

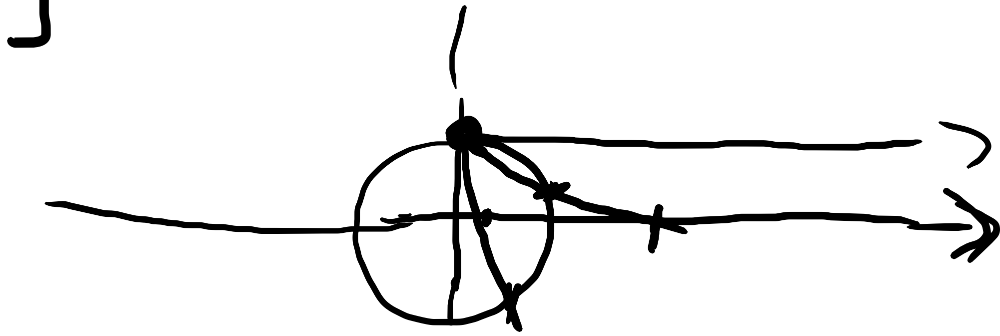
$ax = b$

$\left(x = \frac{b}{a}\right)$

$\mathbb{Q} = \{ [a,b] \,,\; a,b \in \mathbb{Z} \}$

$[a,b] + [c,d] := [ac, ad + bc]$

$a \neq 0 \quad [a,b]^{-1} := [b,a]$



fest

Darstellung von $\mathbb{Q}$

$(b^{e}, x) \hat{=} \frac{x}{b^e}$ Festkommadarst.

$x_{n-1} x_{n-2} \cdots x_e , x_{e-1} \cdots x_1 x_0 = \sum_{i=0}^{n-1} x_i \cdot b^{i-e}$

variabel!

$(b^{-e}, m) \hat{=} m \cdot b^e$ Gleitkommazahl

↑ Mantisse

$0,1 = \cancel{0},1 \cdot 10^0 = 10 \cdot 10^{-2} = \boxed{1,0 \cdot 10^{-1}}$

$\pm \underbrace{x_{n_m+n_e-1} \cdots x_{n_m}}_{e,\text{ Exponent}} \underbrace{x_{n_m-1} \cdots x_0}_{\text{Mantisse}} = x_{n_m-1} \cdots x_0 \cdot b^e$

Excess-Codierung

$e = \underbrace{\sum_{i=0}^{n_e-1} x_{n_m+i} \cdot b^i}_{=0} - B$ → kleinste positive Zahl b^{-B}

$e = b^{n_e} - 1 - B$ → gr. Zahl $b^{b^{n_e}-1-B}$

$1,111\ldots1 \cdot b^{b^{n_e}-1-B}$

$b^{b^{n_e}-B} - b^{b^{n_e}-1-B-n_m}$