$$ack(0, y) = y + 1$$
$$ack(x, 0) = ack(x-1, 1)$$
$$ack(x, y) = ack(x-1, \underline{ack(x, y-1)})$$
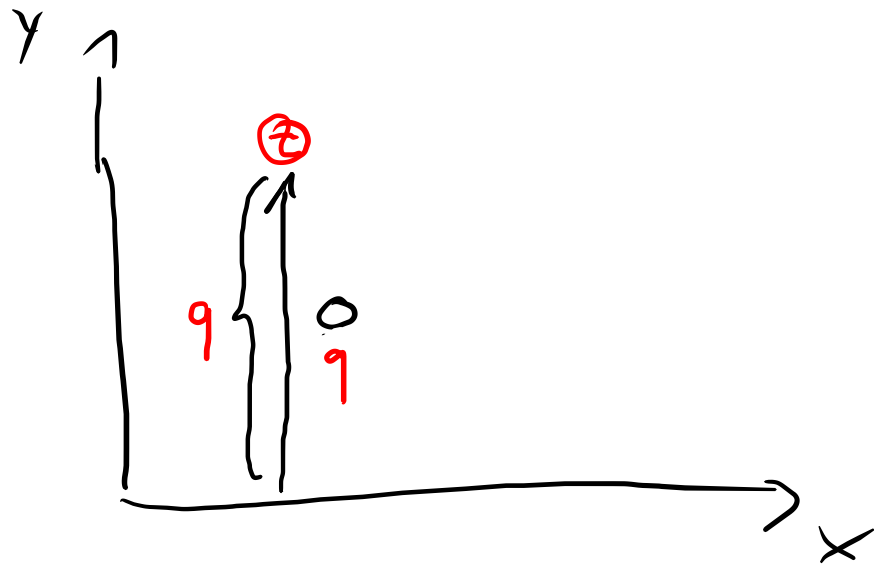
$\rightarrow$ verallgemeinerte Potenzfunktion

x	f4t
0	add
1	mult
2	pot
3	exp.-Form
4	
$\vdots$	

$x+1$

$$P_4 \left[\quad P_3 \left[\quad P_2 \left[\quad x_2 := x_1 + 5; \right. \right. \right.$$

LOOP x_1 DO

$P_1 \left[\quad x_3 := x_2 - 2; \right.$

END

$\exists \ell : \dots$

$\exists \ell : \forall_{n \geq \ell} \, f_{P_1}(n) < a(\ell, n)$

$$x_1 := x_2 \cdot x_5;$$

$$C(x,y) = \frac{2^{x+y} + x}{-2^{x+y}}$$

$$\overbrace{}^{x+y}$$
1 0 0 0 0 0 0 0 0

$x,y \rightsquigarrow \dfrac{C(x,y)}{x_1 \, x_2}$

1 0 1 1 0 0

$+$

x

x_1: 1 0 0 0 0 0   1 1 0 1 0

x_2: 1 0 0 0   0 0 0 0 0 0 0 0 0

$x_0 := 1;$

LOOP x_1 DO $x_0 := x_0 \cdot 2$ END;

LOOP x_2 DO $x_0 := x_0 \cdot 2$ END;   // $x_0 = 2^{x+y}$

$x_0 := x_0 + x_1$

$x_2 := 1$

$x_3 := 0;$

LOOP x_1 DO

    $x_2 := x_2 \cdot 2$;   //

    IF $x_2 \leq x_1$ THEN   //

        $x_3 := x_3 + 1$;

        $x_4 := x_2$;

    END

END // $x_4 = 2^{x+y}$ $x_3 = x+y$

$x_2 := x_1 - x_4$ // $x_2 = x$

$x_3 := x_3 - x_2$ // $x_3 = y$

Bemerkung: slide 46: $\ell_3 \geq 2$
gar nicht nötig weil $\ell_3 \geq 1$
(reichtschon) garantiert ist

Monotonie: ① $\underline{ack(x+1, y+1) \geq ack(x, y+1)}$

② $\qquad\qquad\qquad\qquad \geq \underline{ack(x+1, y)}$

Lemma: $ack(x,y) \geq y+1$

Ind: $x=0 \quad ack(0,y) = y+1$

$\boxed{x>0} \quad y=0: \quad ack(x,0) = ack(x-1,1) \geq 2$

$\qquad \boxed{y>0} \quad ack(x,y) = ack(x-1, \underset{IV(y)}{\underline{ack(x, y-1)}})$

$\qquad\qquad\qquad ack(x-1, \overset{IV(x)}{\underset{\geq y}{\underline{\geq y}}}) \overset{IV(x)}{\geq} y+1 \qquad \square$

$B: x=0: \quad ack(1, y+1) = ack(0, \underline{ack(1,y)}) = ack(1,y)+1 \overset{②}{\geq} ack(1,y)$

$\qquad\qquad\qquad \underset{①}{\geq} ack(0, \overset{\geq}{y+1})$

$\boxed{x>0} \quad y=0: \quad ack(x+1,1) = \underline{ack}(\underline{x}, \underset{①\, IV^{Lem}}{\underline{ack(x+1,0)}}) \overset{Lem}{\geq} ack(x+1,0)+1 \overset{②}{\underset{\|}{\geq}} ack(x+1,0)$

$\qquad\qquad\qquad\qquad \underline{ack(x, 1)}$

$\qquad y>0: \quad ack(x+1, y+1) = ack(x, \underline{ack(x+1,y)}) \overset{Lem}{\underset{①}{\geq}} ack(x, y+1)$

$\qquad\qquad\qquad\qquad\qquad IV\, IV(y)$

$\qquad\qquad\qquad ack(x, ack(x+1, \underline{y-1})) \overset{②}{=} ack(x+1, y)$