

mod. Vorgänger $\underline{v(x)} = \begin{cases} x-1 & \text{falls } x \geq 1 \\ 0 & \text{sonst} \end{cases}$

\mathbb{N}^0

$\underline{k=0}$

ρ

$v(0) = 0 = \underline{0_0} \quad \leftarrow 8$
 $\underline{v(n+1) = n = \pi_1^2} \quad \leftarrow 4$

$\leadsto pr(\pi_1^2, 0_0)$

$v: \mathbb{N}^1 \rightarrow \mathbb{N} \leadsto k+1=1$

mod sub $s(x, y) = \begin{cases} x-y & \text{falls } x \geq y \\ 0 & \text{sonst} \end{cases}$

$s(x, \underline{y+1}) = \begin{cases} x-(y+1) & \text{falls } x \geq y+1 \\ 0 & \text{sonst} \end{cases} = \begin{cases} \underline{(x-y)-1} \\ 0 \text{ sonst} \end{cases}$

$s'(x, y) = s(y, x) = s_0(\pi_2^2, \pi_1^2)$

$\underline{s(x, y)-1 = s(x, \underline{y+1})}$

$\underline{s(x, 0) = x-0 = x}$

$\underline{g(x) = x}$

$pr(v \circ \pi_2^3, \pi_1^1)$

$\underline{h(y, \underline{s(x, y)}, x) = (x-y)-1 = v(\underline{s(x, y)}) = \underline{v \circ \pi_2^3}}$

$$\text{sgn}(x) = \begin{cases} \underline{0} & \text{falls } x = 0 \\ \underline{1} & \text{sonst} \end{cases}$$

$\mathbb{N}^1 \rightarrow \mathbb{N}$

Basis: $\underline{g = 0_0}$
 Rekursion: $\underline{h = 1_2}$

$$pr(1_2, 0_0)$$

$$\underline{pr(0_2, 1_0)} \rightsquigarrow \begin{aligned} f(0) &= g = 1 \\ f(n+1) &= h(n, f(n)) = 0_2 = 0 \end{aligned}$$

$$\left. \begin{array}{c} \vdots \\ \vdots \end{array} \right\} f(1) = h(0, f(0)) = 0_2 = 0$$

$$f = a \circ (s, s \circ (\underline{\pi_2}, \underline{\pi_1}))$$

$$f(x,y) = \underline{add} \left(s(x,y), s(y,x) \right)$$

$$= \underline{add} \left(\max\{x-y, 0\}, \max\{\underline{y-x}, 0\} \right)$$

$$y \geq x$$

$$0$$

$$y < x$$

$$x-y$$

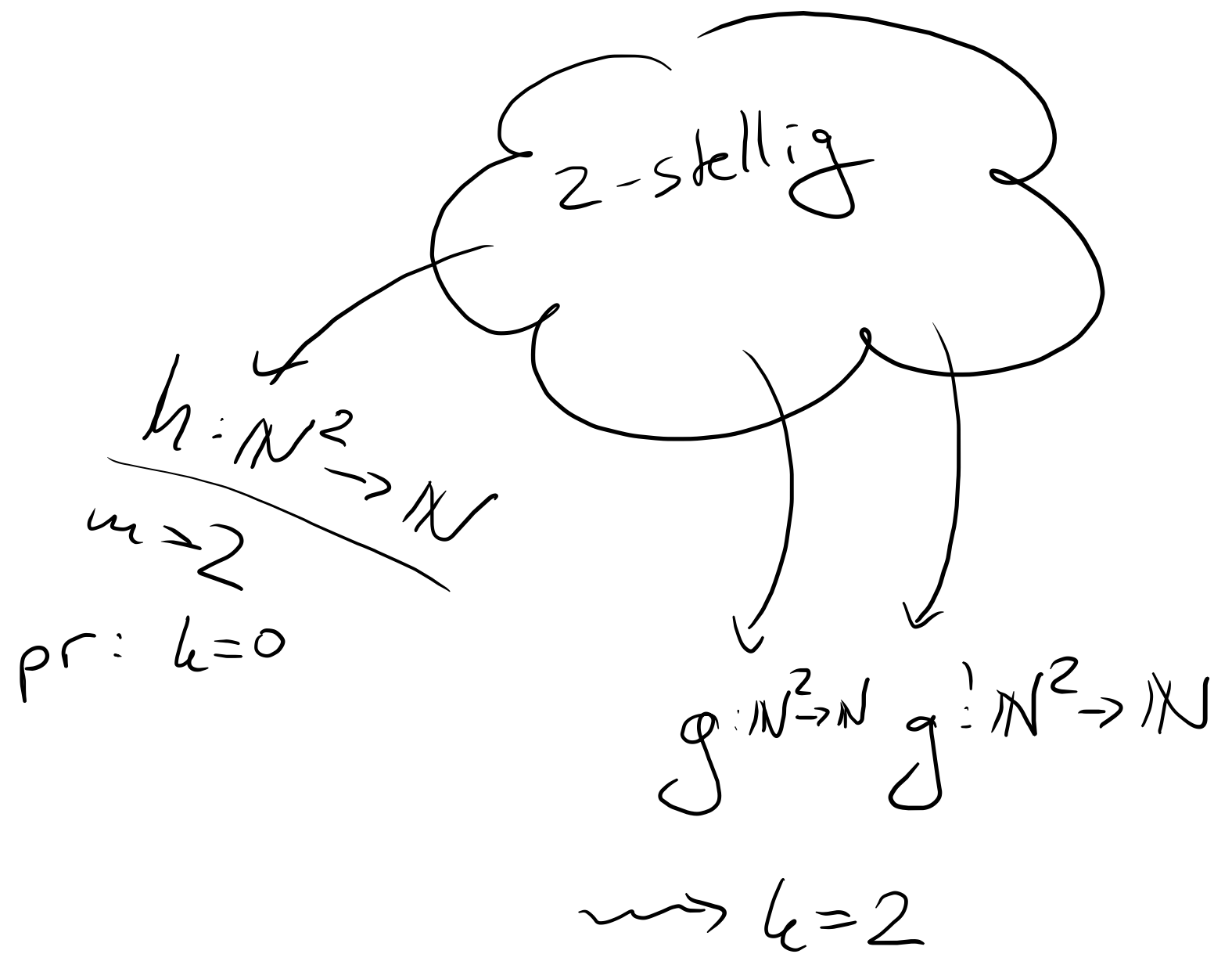
$$y-x$$

$$0$$

$$= |x-y|$$

$$p(0_2, 1_0) \rightarrow 1 \text{ stelling}$$

$$1 - \text{sqn}(x)$$



$$f = h \circ (g, g') : \mathbb{N}^2 \rightarrow \mathbb{N}$$

$$f \in \{2\text{-stellig}\}$$

$$\underline{\text{SQRT}(x)} = \min \{q \in \mathbb{N} \mid q^2 = x\}$$

\Rightarrow suchen Nullstelle von $\underbrace{|q^2 - x|}_{\text{primitiv rekursiv}}$

$$g(q, x) = |q^2 - x|$$

$$f = \mu g$$

$$f(x) = \min \{ \underline{u} \mid g(u, x) = 0 \}$$

falls $\leq x$ min nicht existiert

$$= \min \{ \underline{u} \mid \underbrace{|u^2 - x| = 0}_{\iff}$$

$$= \min \{ \underline{u} \mid \underline{u^2 = x} \}$$

$$= \begin{cases} \sqrt{x} & \text{falls } \sqrt{x} \in \mathbb{N} \\ \perp & \text{sonst} \end{cases}$$

primitiv
rekursiv