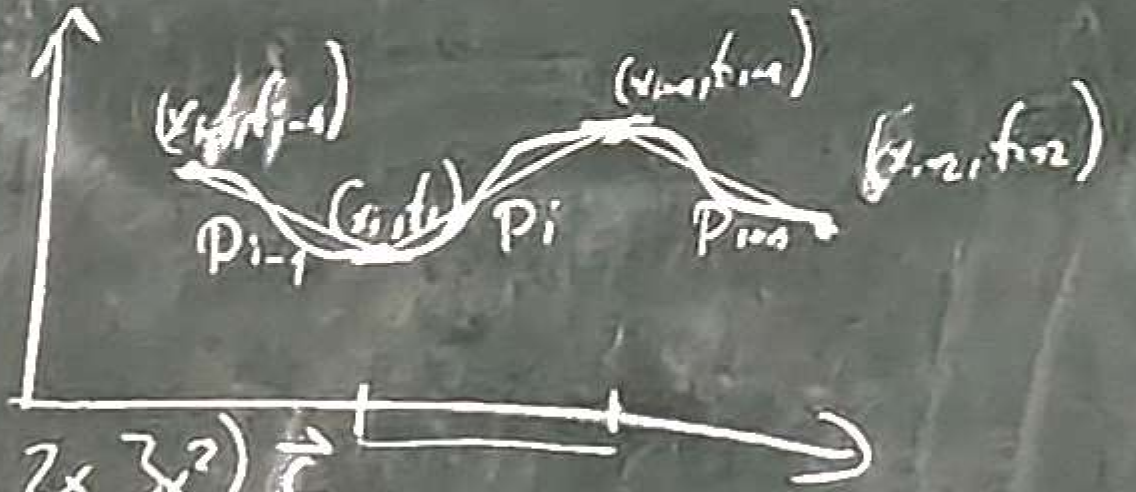
Lokale Polynominterpolation (Hermiteinterpolation)



Gesucht: $p_i(x) = (1 \ x \ x^2 \ x^3)\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = c_0 + c_1 x + c_2 x^2 + c_3 x^3$

Bedingung:

$\cancel{p_i(x_i) = f_i}$ $\quad p_i'(x_i) = 0 \Leftrightarrow p_i'(x_i) = c_1 + c_2 2x + c_3 3x^2 = (0 \ 1 \ 2x \ 3x^2)\vec{c}$

$p_i(x_{i+1}) = f_{i+1}$ $\quad \boxed{p_i'(x_{i+1}) = 0}$

$$\begin{pmatrix} \cancel{1} & \cancel{x_i} & \cancel{x_i^2} & \cancel{x_i^3} \\ 1 & x_{i+1} & x_{i+1}^2 & x_{i+1}^3 \\ 0 & 1 & 2x_i & 3x_i^2 \\ 0 & 1 & 2x_{i+1} & 3x_{i+1}^2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} f_i \\ f_{i+1} \\ 0 \\ 0 \end{pmatrix}$$

$x_i = 0$

$x_{i+1} = 1$

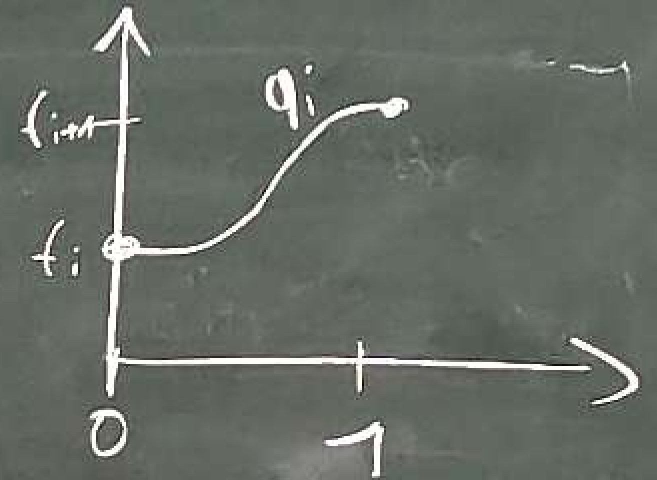
$q_i(0) = f_i$

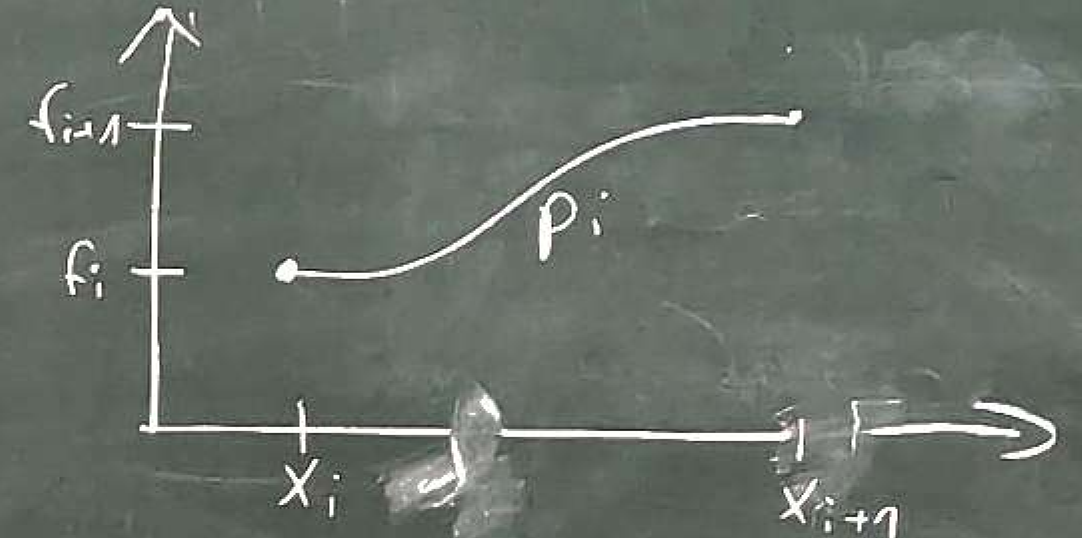
$q_i(1) = f_{i+1}$

$q_i'(0) = f_i' \cdot (x_{i+1} - x_i)$

$q_i'(1) = f_{i+1}' \cdot (x_{i+1} - x_i)$

$$p_i(x) = q_i\left(\frac{(x - x_i)}{x_{i+1} - x_i}\right)$$





$$H \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} f_i \\ f_{i+1} \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow q_i(x) = (1 \ x \ x^2 \ x^3) \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$p_i'(x) = \frac{1}{x_{i+1} - x_i} q'\left(\frac{x - x_i}{x_{i+1} - x_i}\right)$$

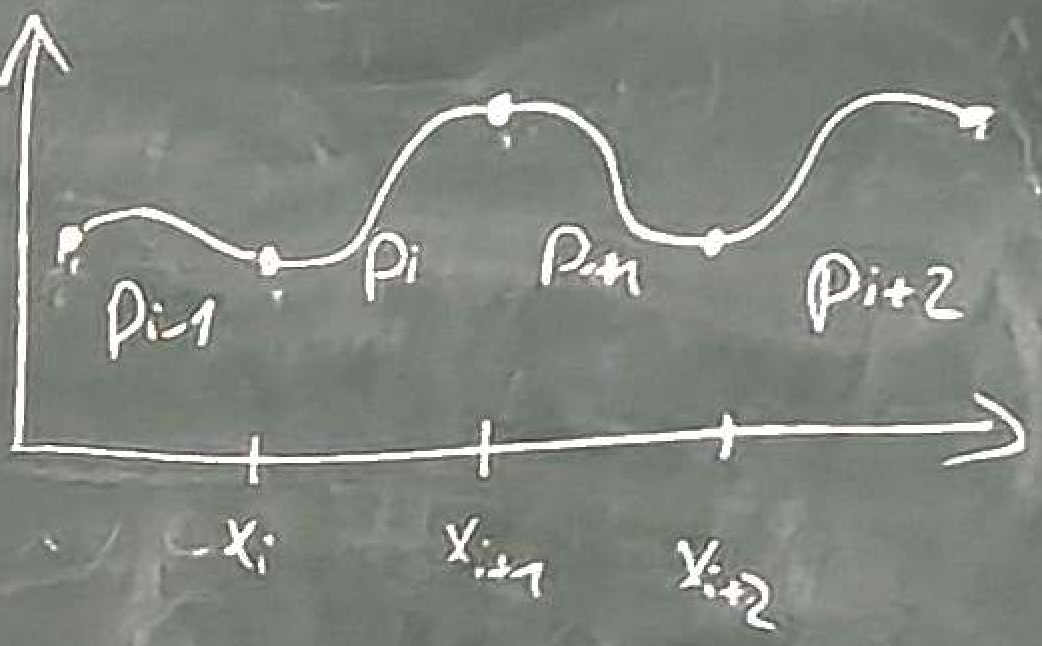
$$H^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow q_i(x) = (1 \ x \ x^2 \ x^3) \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \underbrace{(1 \ x \ x^2 \ x^3) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{pmatrix}}_{\text{Hermite basis}} \begin{pmatrix} f_i \\ f_{i+1} \\ 0 \\ 0 \end{pmatrix}$$

Polynominterpolation

$(n-1)\cdot f$

$i+2$



Gesucht: $p_i(x)$

$p_{i+1}(x)$

Bedingung: $p_i(x_i) = f_i$ $p_i'(x_i) = f_i'$

$p_i(x_{i+1}) = f_{i+1}$

$$\boxed{c_0^i + x_{i+1} c_1^i + x_{i+1}^2 c_2^i + x_{i+1}^3 c_3^i}$$

$$p_i'(x_{i+1}) = p_{i+1}'(x_{i+1}) \Leftrightarrow (c_1^i) + 2x_{i+1}(c_2^i) + 3x_{i+1}^2(c_3^i) - (c_1^{i+1}) - 2x_{i+1}(c_2^{i+1}) - 3x_{i+1}^2(c_3^{i+1}) = 0$$

$$p_i''(x_{i+1}) = p_{i+1}''(x_{i+1}) \Leftrightarrow 2c_2^i + 6x_{i+1}(c_3^i) - 2c_2^{i+1} - 6x_{i+1}c_3^{i+1} = 0$$

$p_{i+1}(x_{i+1}) = f_{i+1}$

$p_{i+1}(x_{i+2}) = f_{i+2}$ $p_{i+1}'(x_{i+2}) = f_{i+2}'$

$$\begin{bmatrix} 1 & x_i & x_i^2 & x_i^3 & 0 & 0 & 0 & 0 \\ 1 & x_{i+1} & x_{i+1}^2 & x_{i-1}^3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2x_{i+1} & 3x_{i+1}^2 & 0 & -1 & -2x_{i+1} & -3x_{i+1}^2 \\ 0 & 0 & 2 & 6x_{i+1} & 0 & 0 & -2 & -6x_{i+1} \\ 0 & 0 & 0 & 0 & 1 & x_{i+1} & x_{i+1}^2 & x_{i+1}^3 \\ 0 & 0 & 0 & 0 & 1 & x_{i+2} & x_{i+2}^2 & x_{i+2}^3 \\ 0 & 1 & 2x_i & 3x_i^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2x_{i+2} & 3x_{i+2}^2 \end{bmatrix} \begin{bmatrix} c_0^i \\ c_1^i \\ c_2^i \\ c_3^i \\ c_0^{i+1} \\ c_1^{i+1} \\ c_2^{i+1} \\ c_3^{i+1} \end{bmatrix} = \begin{bmatrix} f_i \\ f_{i+1} \\ 0 \\ 0 \\ f_{i+1} \\ f_{i+2} \\ f_i' \\ f_{i+2}' \end{bmatrix}$$

Randbedingungen:

1. Vorgegeben: $p_0'(x_0) = f_0'$
   $p_{n-2}(x_{n-1}) = f_{n-1}'$
2. Periodisch:
   $p_0'(x_0) = p_{n-2}'(x_{n-1})$
   $p_0''(x_0) = p_{n-2}''(x_{n-1})$
3. Natürlich:
   $p_0''(x_0) = 0$
   $p_{n-2}''(x_{n-1}) = 0$