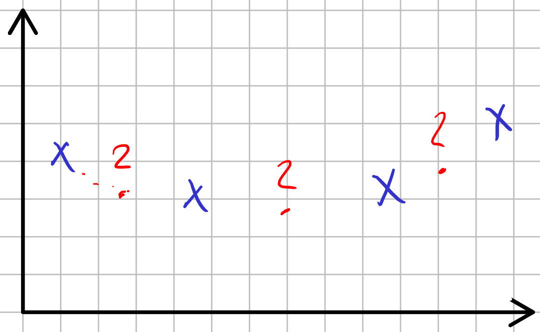
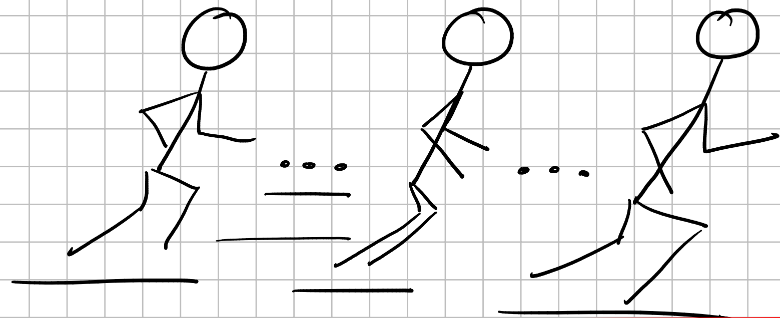
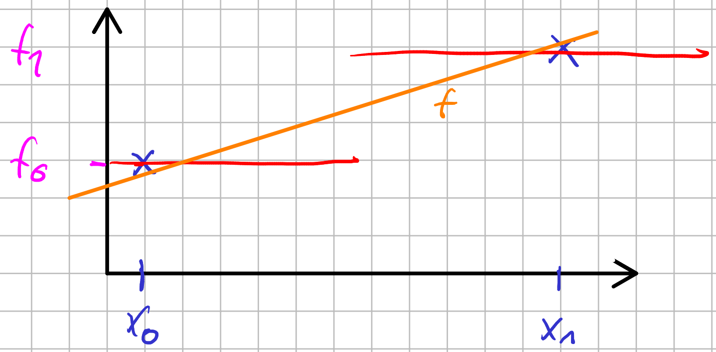
Interpolation

$$\begin{tikzcd} \boxed{\text{kontinuierlich}} \arrow[r, bend right, "\text{diskretisieren}"'] & \boxed{\text{diskret}} \arrow[l, bend right, "\text{Interpolation}"'] \end{tikzcd}$$







$$f(x_0) = f_0 = c_1 x_0 + c_0 \qquad f(x) = c_1 x + c_0$$

$$f(x_1) = f_1 = c_1 x_1 + c_0$$

$$\mathrm{Inv} \underline{\begin{pmatrix} 1 & x_0 \\ 1 & x_1 \end{pmatrix}} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \mathrm{Inv} \begin{pmatrix} f_0 \\ f_1 \end{pmatrix}$$

$\downarrow$ Inverse

$$\boxed{\frac{1}{x_1-x_0}\begin{pmatrix} x_1 & -x_0 \\ -1 & 1 \end{pmatrix}} \qquad \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \frac{1}{x_1-x_0}\begin{pmatrix} x_1 & -x_0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} f_0 \\ f_1 \end{pmatrix}$$

$$c_0 = \frac{f_0 x_1 - f_1 x_0}{x_1 - x_0}$$

$$c_1 = \frac{-f_0 + f_1}{x_1 - x_0}$$

$$f(x) = c_0 + c_1 x$$

$$= \frac{f_0 x_1 - f_1 x_0}{x_1 - x_0} + \frac{-f_0 + f_1}{x_1 - x_0} x$$

$$= \frac{f_0 x_1 - f_1 x_0}{x_1 - x_0} + \frac{f_1 x - f_0 x}{x_1 - x_0}$$

$$= \frac{f_0 x_1 - f_1 x_0 + f_1 x - f_0 x}{x_1 - x_0}$$

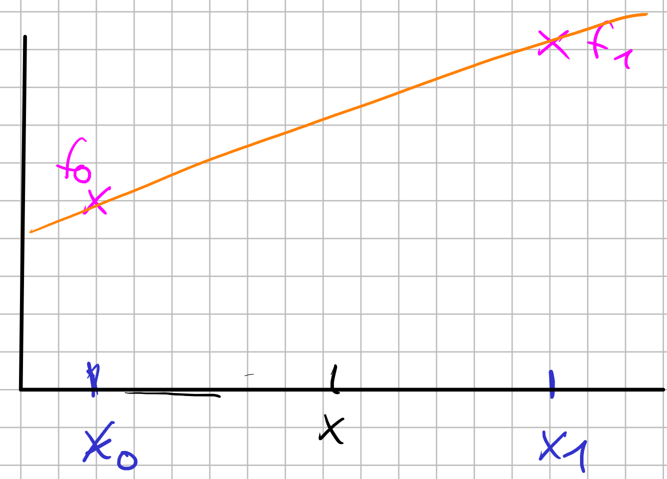
$$= \frac{f_0 (x_1 - x) + f_1 (x - x_0)}{x_1 - x_0} = \boxed{f_0 \left(\frac{x_1 - x}{x_1 - x_0} \right) + f_1 \left(\frac{x - x_0}{x_1 - x_0} \right)}$$

$$f_0 \left(\frac{x_0 - x_0 + x_1 - x}{x_1 - x_0} \right) + f_1 \left(\frac{x - x_0}{x_1 - x_0} \right)$$

Lineare Interpolation $\to$ $\boxed{f_0 \left(1 - \boxed{\frac{x - x_0}{x_1 - x_0}} \right) + f_1 \left(\boxed{\frac{x - x_0}{x_1 - x_0}} \right)}$

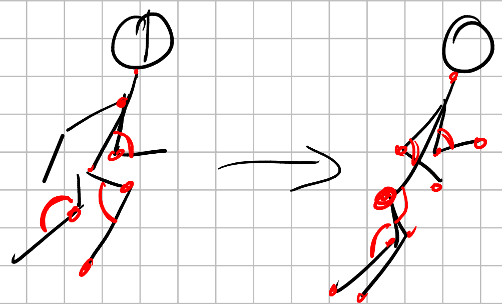
$$f_0(1-t)+f_1(t)$$

$$t\in[0,1]$$



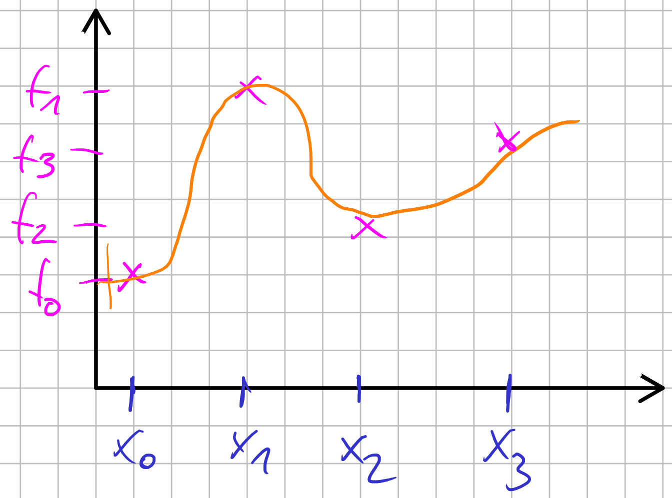
$$x\in[x_0,x_1] \qquad \frac{x-x_0}{x_1-x_0}\in[0,1]$$

Anwendung:



Formalisieren

Parametrisieren



$$(1, x, x^2, \dots, x^n)$$

$$f(x_i) = f_i$$

$$3x^2 + 2x + 7$$

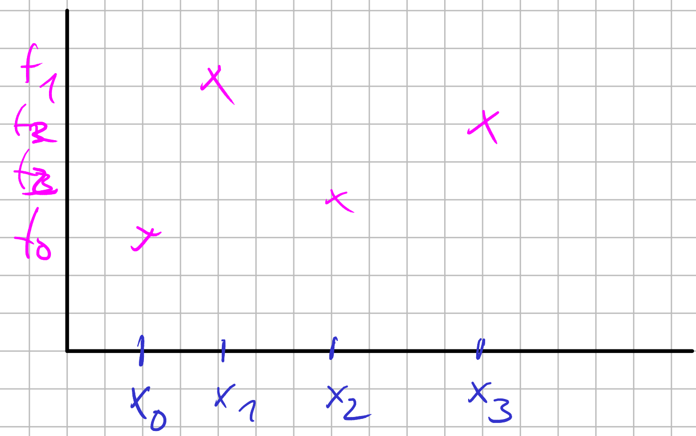
$$(1, x, x^2)\begin{pmatrix} 7 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & x_0 & x_0^2 & x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

Vandermonde Matrix

$$(1, x, x^2, x^3,)_{\text{Monom}}$$

$$(l_0, l_1, l_2, l_3)_{\text{Lagrange}}$$



$$l_0(x) = \frac{x_1 - x}{x_1 - x_0} \cdot \frac{x_2 - x}{x_2 - x_0} \cdot \frac{x_3 - x}{x_3 - x_0}$$

$$f(x_0) = 1 \cdot 1 \cdot 1$$

$$f(x_1) = 0 \qquad f(x_2) = 0 \qquad f(x_3) = 0$$

$$l_0(x_0) = 1$$

$$l_0(x_1) = 0$$

$$l_0(x_2) = 0$$

$$l_0(x_3) = 0$$

$$\boxed{l_i = \prod_{j \neq i}^{n} \frac{x_j - x}{x_j - x_i}}$$

$$f_0 l_0 + f_1 l_1 + f_2 l_2 + f_3 l_3$$

$$l_i(x_j) = 0 \qquad j \neq i$$

$$l_i(x_i) = 1$$

$$V_n \vec{c} = \vec{f}$$

$$(1, x, x^2, \dots, x^n)\,\vec{c} = f(x)$$

$$(l_0, l_1, \dots, l_n)\,\vec{f}$$

$$(1 \quad , \quad x)_M$$

$$\left(\frac{x_1 - x}{x_1 - x_0} \,,\, \frac{x_0 - x}{x_0 - x_1} \right)_L$$

L

$$\begin{pmatrix} \frac{x_1}{x_1 - x_0} & \frac{x_0}{x_0 - x_1} \\ \frac{-1}{x_1 - x_0} & \frac{-1}{x_0 - x_1} \end{pmatrix}$$

$$\underline{V_n}$$

$$\vec{f} = \vec{c} \quad \Big| \quad Lf = c$$

$$\vec{c} = \vec{f}$$

Unbekannte

$$V^{-1} V c = V^{-1} f$$

$$c = V^{-1} f \Rightarrow$$

$$L = V^{-1}$$

$\boxed{p_1(x)}, \boxed{p_2(x)} \in$ Polynome 2 Grades

$$\boxed{p_1 p_2 = p_1(x) \cdot p_2(x)} \in \text{Polynome 4. Grades}$$

$$\vec{c}_{p_1} = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ 0 \\ 0 \end{pmatrix} \qquad \vec{d}_{p_2} = \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ 0 \\ 0 \end{pmatrix}$$

$$(1, x, x^2, x^3, x^4)$$

$$(l_0, l_1, l_2, l_3, l_4) \begin{pmatrix} f_{p_1 0} \\ f_{p_1 1} \\ f_{p_2 2} \\ f_{p_2 3} \\ f_{p_2 4} \end{pmatrix}$$

$$\vec{e}_{p_1 p_2} = \begin{pmatrix} ? \end{pmatrix}$$

$$\begin{pmatrix} f_{p_2 0} \\ f_{p_2 1} \\ f_{p_2 2} \\ f_{p_2 3} \\ f_{p_2 4} \end{pmatrix}$$

multiplikation

$$\begin{pmatrix} f_{p_1 0} \cdot f_{p_2 0} \\ f_{p_1 1} \cdot f_{p_2 1} \\ \vdots \end{pmatrix}$$