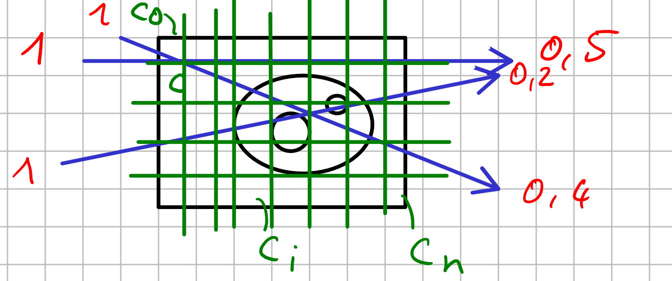
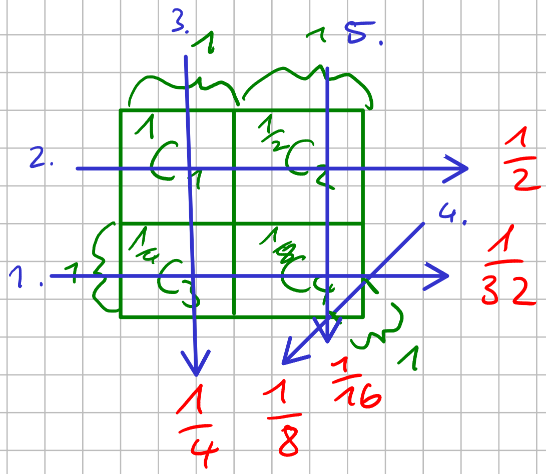
LGS

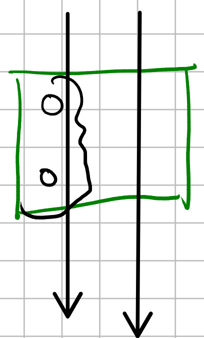




$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \log(c_1) \\ \log(c_2) \\ \log(c_3) \\ \log(c_4) \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ -2 \\ -3 \\ -4{,}01 \end{pmatrix}$$

unlösbar

$$\log(c_1) = 0$$
$$\log(c_2) = -1$$
$$\log(c_3) = -2$$
$$\log(c_4) = -3$$

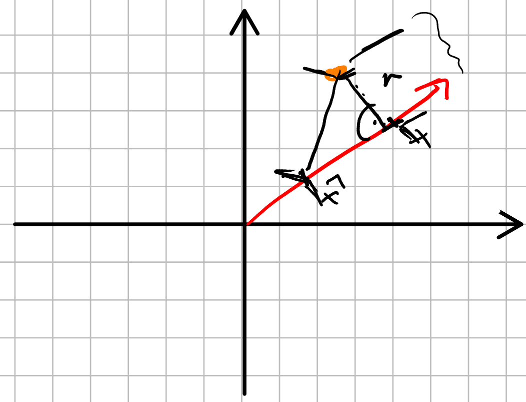


Ausgleichsrechnung

$$\boxed{\arg\min_{x} \|Ax - b\|_2^2}$$

$Ax = b$ ← unlösbar

← überbestimmt.



$$\|v\|^2 = v_1^2 + \dots + v_n^2$$

$$Ax = \begin{pmatrix} a_1 & a_2 \end{pmatrix} x = \begin{pmatrix} 2 & 3 \\ 2 & 2 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$$

$r := Ax - b$

$$\arg\min_{r} \|r\|^2 = \boxed{\arg\min_{x} \|Ax - b\|^2}$$

$$a_1^T r = 0$$
$$a_2^T r = 0$$

np.linalg.solve(A, b)

np.linalg.solve(A^TA, A^Tb)

$$a_1^T(Ax - b) = 0 \qquad\qquad a_1^T A x = a_1^T b$$
$$\Leftrightarrow$$
$$a_2^T(Ax - b) = 0 \qquad\qquad a_2^T A x = a_2^T b$$

$$\boxed{\Leftrightarrow A^T A x = A^T b} \Leftrightarrow \begin{pmatrix} a_1^T \\ a_2^T \end{pmatrix} A x = \begin{pmatrix} a_1^T \\ a_2^T \end{pmatrix} b$$

Methode der kleinsten Fehlerquadrate | Least squares

$$\underbrace{A^T A}\, x = A^T b$$

Eigenschaften:

0. Quadratisch
1. Symetrisch $\quad A^TA = (A^TA)^T = A^T A^{T^T} = A^T A$
2. Positiv (Semi)-Definit (PSD) $\quad x^T A^T A x \geq 0$

$y := Ax \qquad y^T y \geq 0$

$$x = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$Ax = \vec{a}_1$

$x^T A x = a_{00}$

A^TA ist PD $\Leftrightarrow$ A vollen Spaltenrang

$x^T A^T A x > 0$

$A^TA = L L^T \leftarrow$ Cholesky-Zerlegung

$$\begin{pmatrix} a_{00} & a_{10} & a_{20} & \dots & a_{n0} \\ a_{10} & a_{11} & a_{21} & \dots & a_{n1} \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & a_{nn} \\ a_{n0} & \dots & & & \end{pmatrix} = \begin{pmatrix} l_{00} & 0 & \dots & & 0 \\ l_{10} & l_{11} & 0 & \dots & 0 \\ l_{20} & l_{21} & l_{22} & 0 \dots & 0 \\ \vdots & & & \ddots & 0 \\ l_{n0} & \dots & & & l_{nn} \end{pmatrix} \cdot \begin{pmatrix} l_{00} & l_{10} & l_{20} & \dots & l_{n0} \\ 0 & l_{11} & & \dots & l_{n1} \\ \vdots & & & \ddots & \vdots \\ 0 & - & & \dots & l_{nn} \end{pmatrix}$$

$a_{00} = l_{00}^2 \Leftrightarrow l_{00} = \sqrt{a_{00}}$

$a_{10} = l_{10} \cdot l_{00} \Leftrightarrow l_{10} = \frac{a_{10}}{l_{00}}$

$a_{11} = l_{10}^2 + l_{11}^2 \Leftrightarrow l_{11} = \sqrt{a_{11} - l_{10}^2}$

$$l_{ij} = \sqrt{a_{ii} - \sum_{k=0}^{i-1} l_{ik}^2}$$

$$l_{ij} = \frac{a_{ij} - \sum_{k=0}^{i-1} l_{ik} l_{jk}}{a_{jj}}$$

$A^T A = L L^T$

$L L^T x = b$

$L^T x =: y$

$L y = b \leftarrow$ Vorwärtseinsetzen. $O(n^2)$

$L^T x = y \leftarrow$ Rückwärtseinsetzen $O(n^2)$

LGS überbestimmt

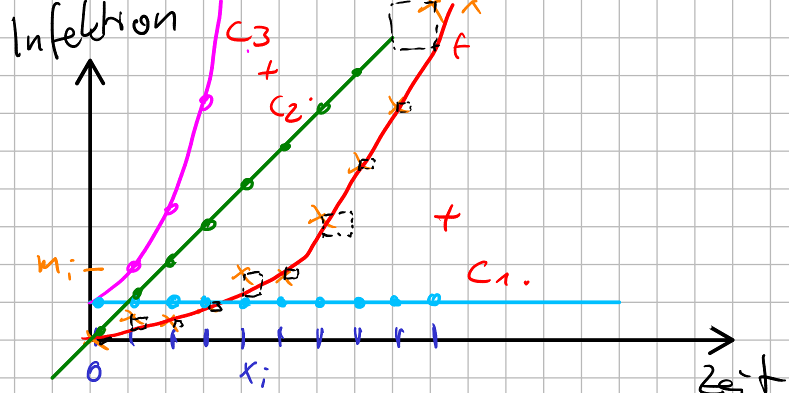
$\Rightarrow$ Ausgleichsrechnung

$A^T A x = A^T b$

$\Rightarrow$ Cholesky - Zerlegung.

Lineare Regression

$$f(x_i) = m_i$$



$\mathcal{B} =$

$f(x) = 1 \qquad f(x) = x \qquad f(x) = e^x \qquad \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

$$\boxed{f(x) = c_1 1 + c_2 x + c_3 e^x}$$

$$\begin{pmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_n) \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & x_1 & e^{x_1} \\ 1 & x_2 & e^{x_2} \\ \vdots & \vdots & \vdots \\ 1 & x_n & e^{x_n} \end{pmatrix}}_{X \in \mathbb{R}^{n \times 3}} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$n > 3$

$$X \vec{c} = \vec{m}$$

$$(X\vec{c})_i$$

$$\sum_i \left(f(x_i) - m_i\right)^2$$

$$\underline{f(x) = x^2}$$

$$\mathbb{R} \to \mathbb{R}$$

$$x \longmapsto x^2$$

$$x \in \mathbb{R}$$

$$\left\{ (0, 0), (0{,}5, 0{,}25), (1, 1) \ldots \right.$$
$$\left. (1{,}5, \tfrac{9}{4}), (2, 4) \ldots \right\}$$

$$\downarrow$$

$$\begin{pmatrix} 0 \\ 1 \\ 4 \\ 9 \\ \vdots \\ n^2 \end{pmatrix}$$