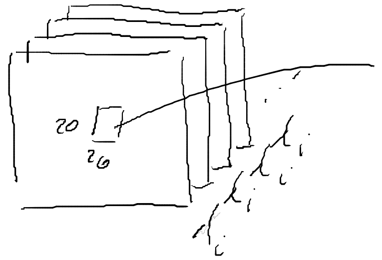
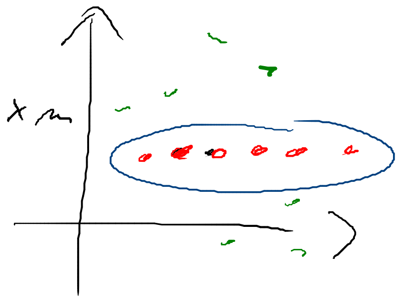


$g \in \mathbb{R}^n$

Daten $g_i \in \mathbb{R}^n$

$G = (g_0, g_1, g_2 \dots) \in \mathbb{R}^{n \times m}$



$\arg\min_i \underbrace{\|g - g_i\|}$ Aufwendig weil $\underset{m}{n}$ groß ist

$f: \mathbb{R}^o \to \mathbb{R}^n$, $o \ll n$

$$f(x) = \begin{pmatrix} c_{00}x_0 + c_{01}x_1 + \dots & + s_0 \\ c_{10}x_0 + c_{11}x_1 + \dots & + s_1 \end{pmatrix} = Cx + s$$

→ linearen Unterraum

Idee: $C^TC = I$

$\|f(x) - f(x_i)\| = \|Cx + s - Cx_i - s\| = \|C(x - x_i)\| = \|x - x_i\|$

$C^TCx + C^Ts = C^Tg$ $\qquad x = \underbrace{(C^TC)}_{I}^{-1} C^T(g - s)$

$\|C(x - x_i)\|^2 = (C(x - x_i))^T C(x - x_i)$

$(x - x_i)\underbrace{C^TC}_{I}(x - x_i) = \|x - x_i\|^2$

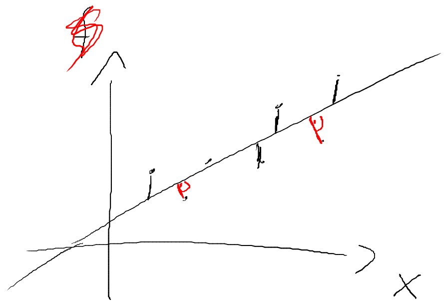
$$r_i = f(x_i) - g_i = C x_i + s - g_i = \underbrace{C\, \underbrace{C^T(g_i - s)}_{x_i}}_{} + s - g_i = (CC^T - I)(g_i - s)$$

$$\boxed{\underset{s,\, C^TC = I}{\operatorname{argmin}} \sum_i} r_i^T r_i = \sum_i (g_i - s)^T (CC^T - I)^T (CC^T - I)(g_i - s)$$

$$(C\underbrace{C^T C}_{I}C^T - CC^T - CC^T + I)$$

$$\underset{s,\, C^TC = I}{\operatorname{argmin}} \sum_i (g_i - s)^T (I - CC^T)(g_i - s)$$

Orthogonale Regression (total least squares)



$I - CC^T = M$ PSD

$$\underset{s,\, C^TC = I}{\operatorname{argmin}} \sum_i (g_i - s)^T M (g_i - s)$$

Annahme $\sum g_i = 0$

„M. Helwert befreit"

$$\sum_i g_i^T M g_i - g_i^T M s - s^T M g_i + s^T M s$$

$$\underbrace{(\sum g_i)}_{0} M s - s^T M \underbrace{(\sum g_i)}_{0} + s^T M s$$

$$\underbrace{\sum g_i^T M g_i}_{\text{Konstant}} + s^T M s \rightarrow s^T M s \geq 0 \rightarrow s = 0$$

$$\sum (g_i - s) = 0 \qquad s = \frac{1}{m} \sum g_i$$

Mittelwert von g_i

$y_i = g_i - S_F$ Mittelwert

$$\underset{C^TC=I}{\arg\min} \sum_i y_i^T(I - CC^T)y_i = \sum_i \underbrace{y_i^T y_i}_{\text{konstant}} - y_i^T CC^T y_i$$

$$\underset{C^TC=I}{\arg\max} \sum_i y_i^T CC^T y_i$$

Einfacher: suche $c \in \mathbb{R}^o \rightarrow \|c\| = 1$

$$\underset{c^Tc=1}{\arg\max} \sum_i y_i^T c \, c^T y_i = \sum_i c^T y_i \, y_i^T c = c^T \left(\sum y_i y_i^T\right) c$$

$$= c^T (YY^T) c$$

$$= c^T Q \Lambda \underbrace{Q^T c}_{d} \qquad c = Qd$$

$$\underset{d^Td=1}{\arg\max} \quad d^T \Lambda d = d_0^2 \lambda_0 + d_1^2 \lambda_1 + \dots$$

$(\lambda_0, \lambda_1, \lambda_2, \dots)$, $\lambda_0 \geq \lambda_1 \dots$; $d_0 \rightarrow 1$, $d_1 \rightarrow 0$

$$1 = c^T c = d^T Q^T Q d = d^T d$$

$$\Rightarrow d = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix} = e_0 \qquad \Rightarrow c = Qd = (q_0, q_1, \dots) d = q_0$$

$c_0 = c \qquad c_1 \,? \qquad c_1^T c_1 = 1 \qquad c_1^T c_0 = 0$

$$\underset{d_1^T d_1 = 1 \;\; d_1^T d_0 = 0}{\arg\max} \quad d^T \Lambda d \Rightarrow d_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix} = e_1$$

$$\Rightarrow c_1 = q_1$$

Zusammengefasst:

Daten $g_i \in \mathbb{R}^n$

1. Bestimme Mittelpunkt $b = \frac{1}{m} \sum_i g_i$

2. $y_i = g_i - b$ $\left(\sum_i y_i = 0\right)$

3. $Y = (y_0, y_1, y_2, \dots) \longrightarrow YY^T$

4. $\boxed{\underbrace{YY}Y^T = Q \Lambda Q^T}$ $C = (q_0, q_1, q_2 \dots)$

???

Eigenzerlegung, „Kernel Trick" $\boxed{C^TC \quad CC^T}$

$M^T = M \qquad M = Q \Lambda Q^T \qquad Q^T M Q = \Lambda = \begin{pmatrix} \lambda_0 & & & \\ & \lambda_1 & & \\ & & \lambda_2 & \\ & & & \ddots \end{pmatrix}$

$x^T \underbrace{A^T A}\, x \geq 0 \qquad x^T \underbrace{Q \Lambda Q^T}\, \underbrace{Q^T x}_{y} = y^T \Lambda y \geq 0 \qquad \lambda_0 \geq \lambda_1 \geq \dots$

$\Rightarrow \lambda_i \geq 0$

$y_0^2 \lambda_0 + y_1^2 \lambda_1 \dots \geq 0$

$\lambda_i > 0 \qquad A^T A \qquad A A^T$

$A^T A \cdot x = \lambda x \qquad A A^T \underbrace{A x} = A \lambda x = \lambda \underbrace{A x}$

$C^T C = I \qquad \lambda_i = 1 \qquad C C^T \qquad \lambda_i = 1, 0$

$I - CC^T = I - Q \Lambda Q^T = Q I Q^T - Q \Lambda Q^T = Q(I - \Lambda) Q^T$

$(I - CC^T)$ PSD