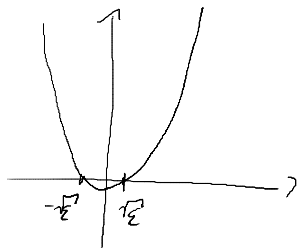
Nullstellen

$$f_0(x)=0 \qquad x\in\mathbb{R}^n$$
$$f_1(x)=0$$
$$\vdots$$

$$f(x)=0 \qquad x\in\mathbb{R}$$

Existenz ?

Bsp. $f(x)=x^2-\varepsilon$, $\varepsilon>0$ $\qquad$ $\sqrt{2}$ $\Rightarrow$ $x^2-2=0$

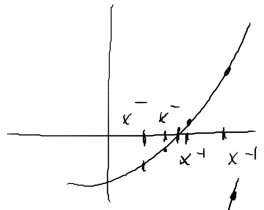


Annahme : x^-, x^+ : $f(x^-)<0$, $f(x^+)>0$

Bisektion $x\in[x^-,x^+]$ $\qquad$ $f(x)<0 \to x^-\leftarrow x$

$\qquad f(x)>0 \to x^+\leftarrow x$

$$x=\frac{x^-+x^+}{2}$$

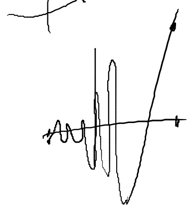


$f(x^*)=0$ $\qquad$ $f(x)=f(x^*)+f'(x^*)(x-x^*)+\dots$

$x=x^*\pm\varepsilon$ $\qquad$ $\boxed{f(x)\approx\varepsilon\cdot f'(x^*)}$

$\Rightarrow$ $\boxed{|f(x)|<\varepsilon}$ $\leftarrow$ Maschinengenauigkeit

reicht nicht!

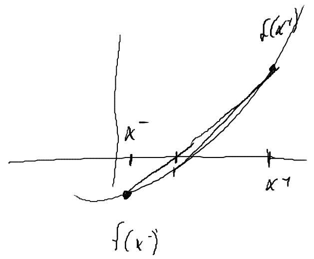


Abbruchkriterium

$$|x^+-x^-|<\varepsilon$$

Regula falsi

$$x=\frac{x^+f(x^-)-x^-f(x^+)}{f(x^-)-f(x^+)}$$



Abbruch $|x^+-x^-|<\varepsilon$ $\qquad$ $|f(x)|<\varepsilon\cdot\max\left(1,\frac{f(x^+)-f(x^-)}{x^+-x^-}\right)$

Newton-Verfahren

$$f(x) = f(x^*) + f'(x^*)(x - x^*) + \dots$$

$$0 = f(x^*) + f'(x^*)(x - x^*) \quad \to \quad x = \frac{-f(x^*) + f'(x^*)\,x^*}{f'(x^*)} = x^* - \frac{f(x^*)}{f'(x^*)}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$0 = f(x_i) + f'(x_i)(x - x_i) + \frac{1}{2} f''(r)(x - x_i)^2 \qquad r \in [x, x_i]$$

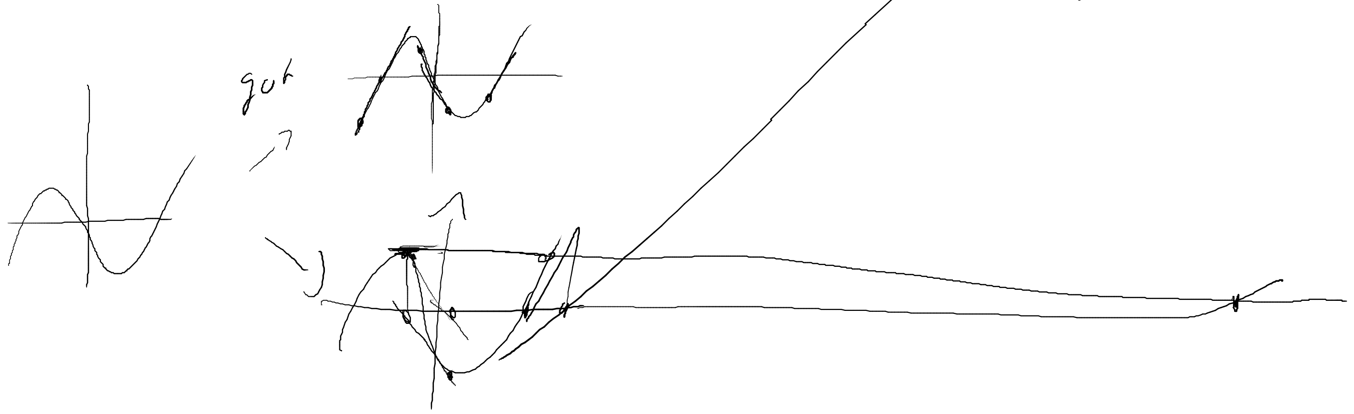
$$x_i = x_{i+1} + \frac{f(x_i)}{f'(x_i)}$$

$$0 = f(x_i) + f'(x_i)\left(x - x_{i+1} - \frac{f(x_i)}{f'(x_i)}\right) + \frac{1}{2} f''(r)(x - x_i)^2$$

$$= f'(x_i)(x - x_{i+1}) + \frac{1}{2} f''(x - x_i)^2$$

$$|x^* - x_{i+1}| = \frac{f''(x_i)}{2f'(x_i)}(x^* - x_i)^2$$

↖ konvergiert quadratisch



Attraktor NSt x^* $\Omega(x^*) \in \mathbb{R}$

nicht zusammenhängend

Fraktal

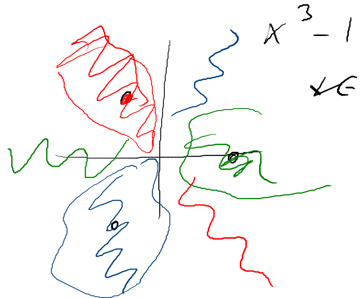
$x_0 \in \Omega$

x_1

x_2

$\vdots$

x^*



$x^3 - 1 = 0$

$x \in \mathbb{C}$

Nullstellen $\quad x \in \mathbb{R}^n \quad f : \mathbb{R}^n \to \mathbb{R}^m$

$\frac{\partial f_c}{\partial x_i}$ partielle Ableitung

$f(x_0, x_1) = x_0^2 + x_1 x_0 + x_1^2$

$\frac{\partial f}{\partial x_0} = 2x_0 + x_1$

$\frac{\partial f}{\partial x_1} = x_0 + 2x_1$

$$J_f = \begin{pmatrix} \frac{\partial f_0}{\partial x_0} & \frac{\partial f_0}{\partial x_1} & \frac{\partial f_0}{\partial x_2} & \cdots \\ \frac{\partial f_1}{\partial x_0} & \frac{\partial f_1}{\partial x_1} & & \\ \vdots & & & \end{pmatrix} \in \mathbb{R}^{m \times n}$$

Jacobi Matrix

$x^i \in \mathbb{R}^n$

x im i-ten Iterationsschr.

$$x^{i+1} = x^i - J_f^{-1}(x^i) \cdot f(x^i)$$

$J^{-1} \in \mathbb{R}^{n \times m}$

$$\Delta x = x^{i+1} - x^i$$

$$J_f(x^i) \cdot \Delta x = -f(x^i)$$

$f : \mathbb{C} \to \mathbb{C}$

$$z_{i+1} = z_i - \frac{f(z_i) \cdot \overline{f'(z_i)}}{(f'(z_i))^2}$$

$f : \mathbb{R}^n \to \mathbb{R}$

$$\nabla f = \left(\frac{\partial f}{\partial x_0}, \frac{\partial f}{\partial x_1}, \ldots\right)^T$$

$$x^{i+1} = x^i - \frac{f(x_i)}{\|\nabla f(x_i)\|} \cdot \nabla f(x_i)$$