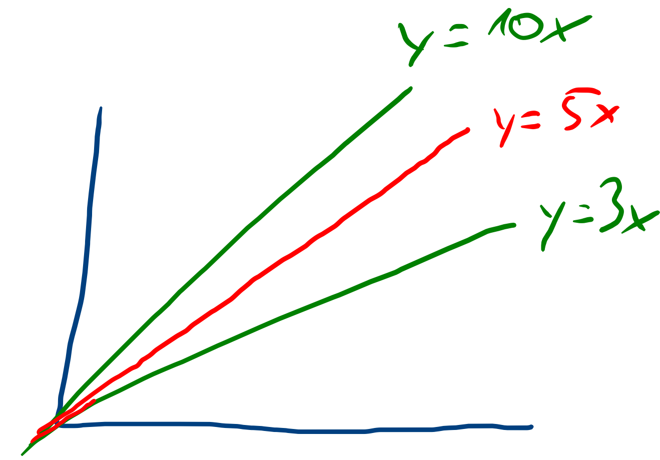
BeKo Modulkonferenz 05.01.23

$g: \mathbb{N} \to \mathbb{N}$

$$O(g(n)) = \{ f: \mathbb{N} \to \mathbb{N} \mid \underline{\exists c \in \mathbb{N}^+} \; \exists n_0 \in \mathbb{N} \; \forall n \geq n_0 \; f(n) \leq c \cdot g(n) \}$$

$$O(g(n)) = \{ f: \mathbb{N} \to \mathbb{N} \mid \lim_{n \to \infty} f(n)/g(n) < \infty \}$$



$$O(\log n) \ni 15 \log n$$

$$O(\log n) \ni 10 \log_2 n \qquad O(\log n) \ni 10 \log_{10} n$$

$$O(\log \log n)$$

$$O(\alpha(n))$$

inverse Ackermann

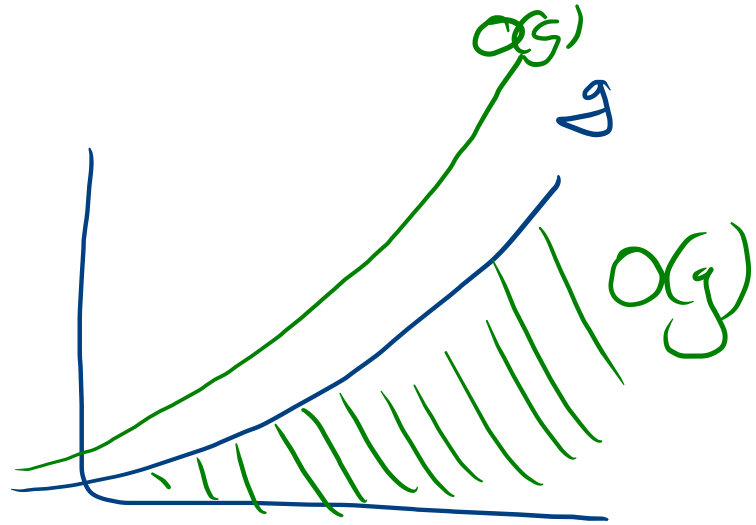
$$\alpha(n) = m \Leftrightarrow \mathrm{Ack}(m,m) = n$$

$$\underline{O(n)} \ni \underline{6n + \log n \leq 7n}$$

$$\Theta(n)$$

$$\lim_{n \to \infty} \frac{6n + \log n}{n} = \lim_{n \to \infty} \left(\left(\frac{6n}{n} \right) + \frac{\log n}{n} \right)$$

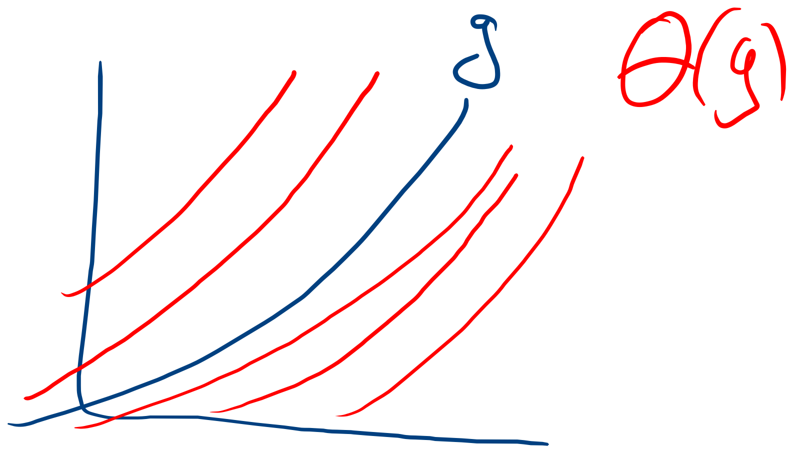
$$= 6 + 0 = 6 < \infty$$



Nachtrag!

$$3n^2 + n + \log n \in \Theta(n^2)$$

$$3n^2 \cdot \log n \notin \Theta(n^2)$$
$$\notin O(n^2)$$



$$f(n) \in \Theta(g(n)) \iff f(n) \in O(g(n)) \ \& \ g(n) \in O(f(n))$$

$$10 \cdot n^{Ack(10,10)} \in O(2^n)$$
$$\in n^{O(1)} = \bigcup_{i>0} O(n^i)$$

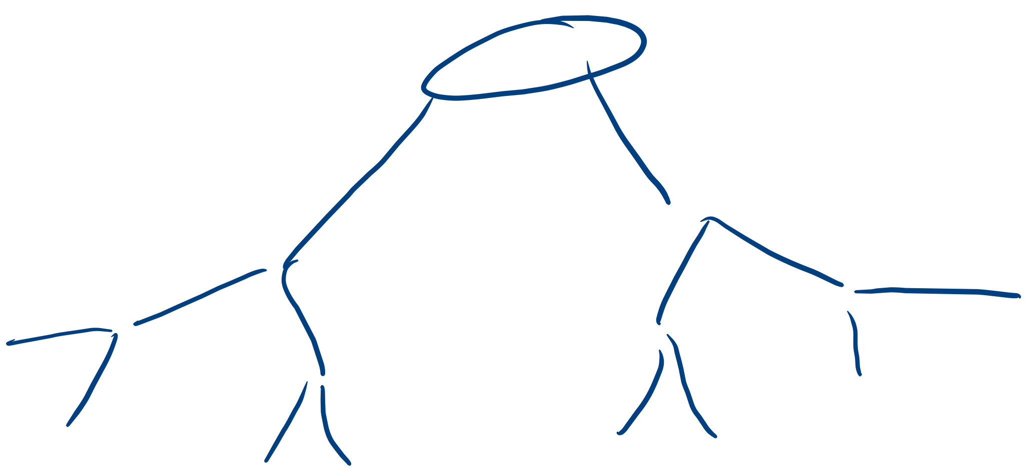
$$\boxed{\Theta(g) \subseteq O(g)}$$

$$f(x) = \begin{cases} 10x & \text{falls } x \text{ prim} \\ \perp & \text{sonst} \end{cases}$$

$$f(w) = \begin{cases} w+1 & \text{falls } T(M_w) = \emptyset \\ \perp & \end{cases}$$

$\longrightarrow$ nicht berechenbar :(

NTM



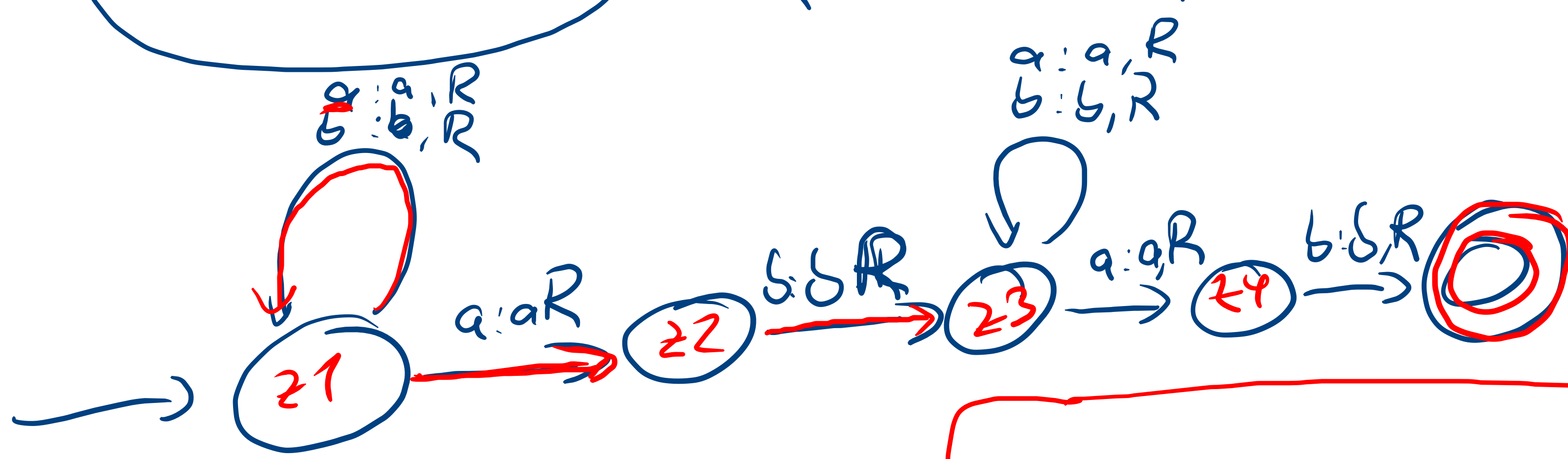
Speedup Theorem

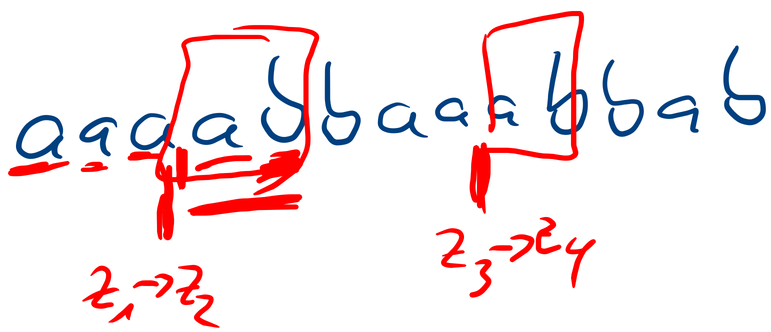
Sei $c > 0$, sei $f: \mathbb{N} \to \mathbb{N}$ & M eine (Mehrband) (N)TM M die $f(n)$-zeitbeschränkt.

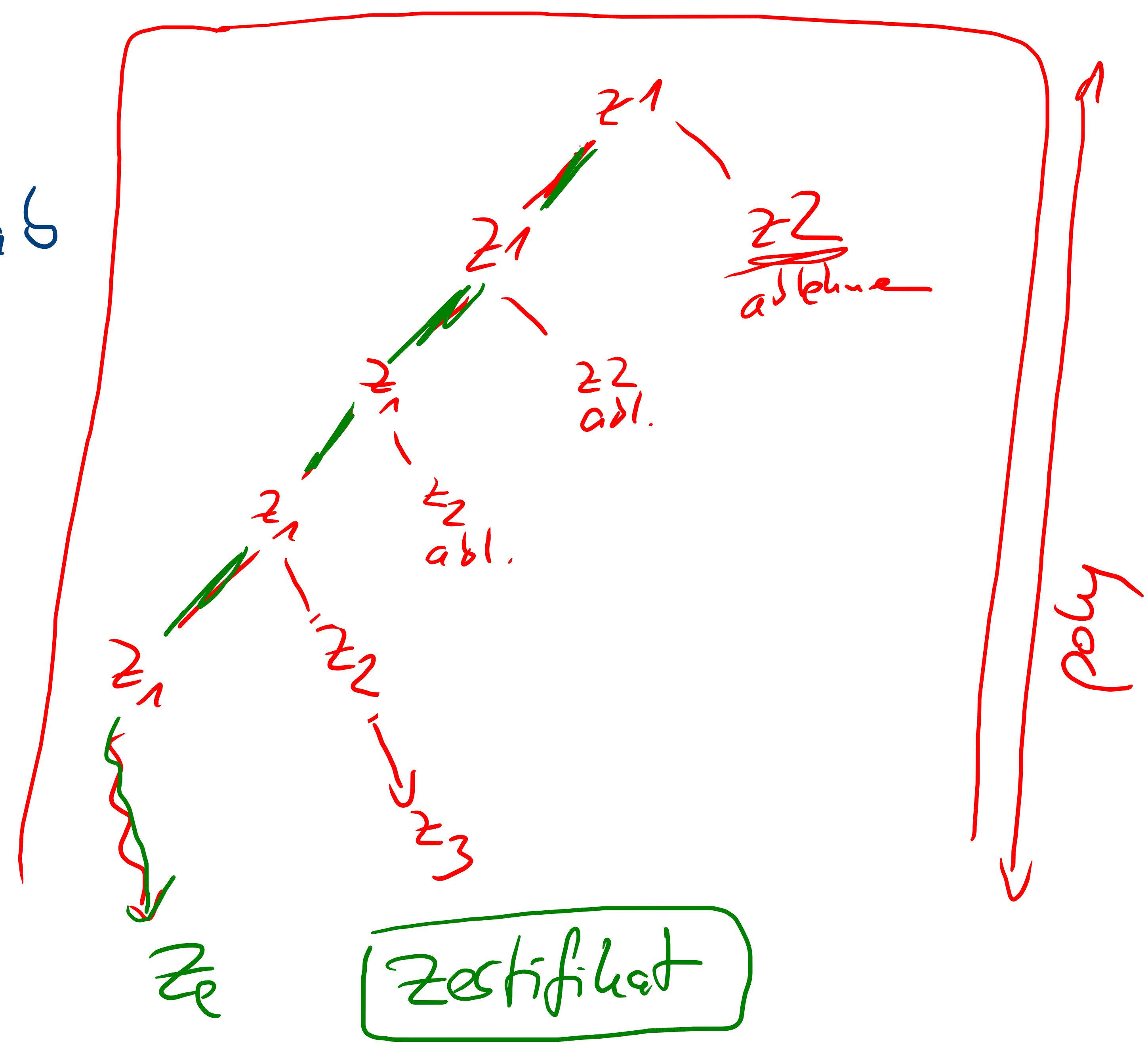
Dann existiert eine (Mehrband) (N)TM M' sodass $T(M') = T(M)$ & M' ist $f(n)/c + n + 2$-zeitbeschr.

NTM

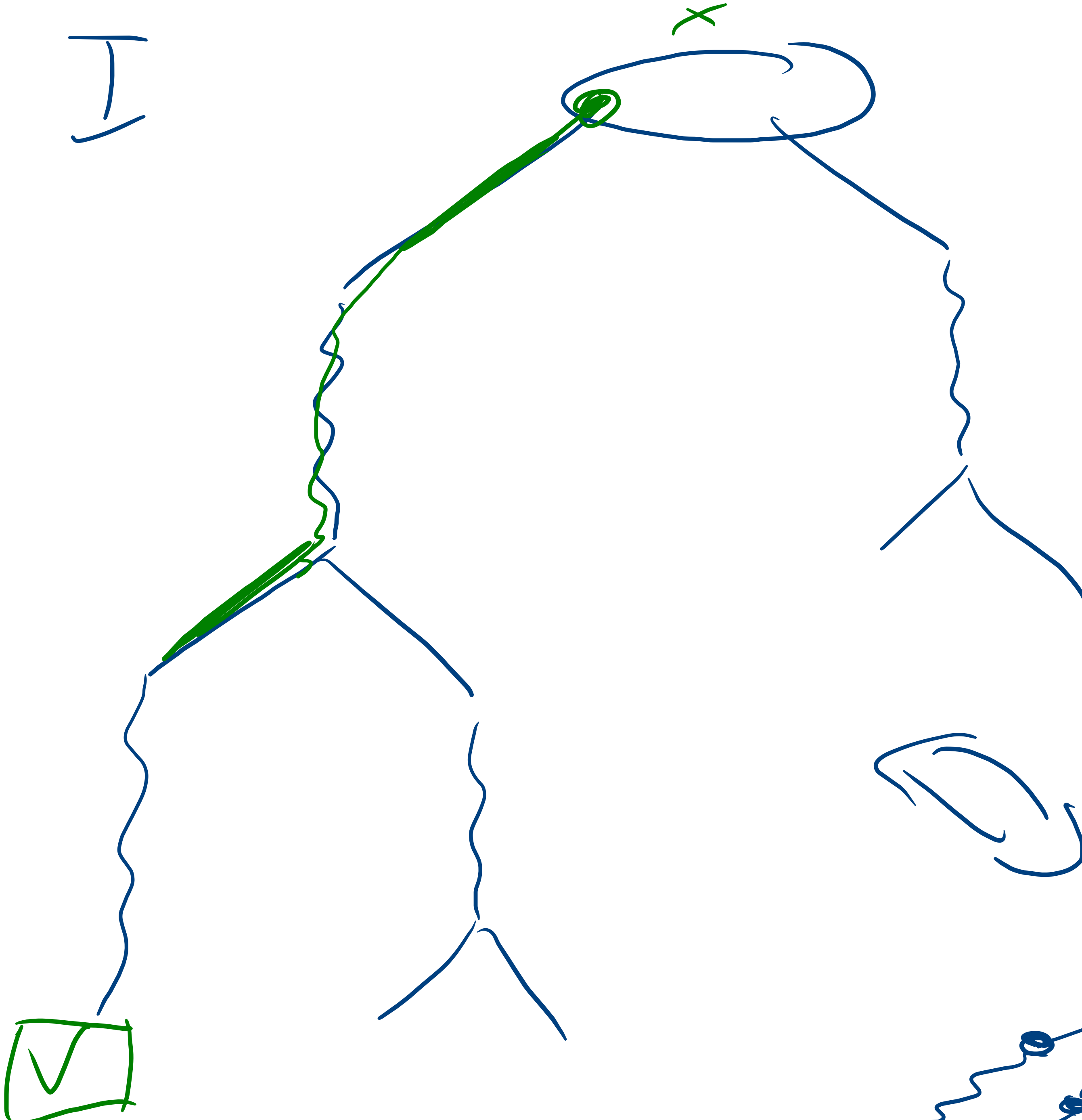
$\{\omega \in \{a,b\}^* \mid \omega$ enthält $2\times$ das Teilwort $ab\}$







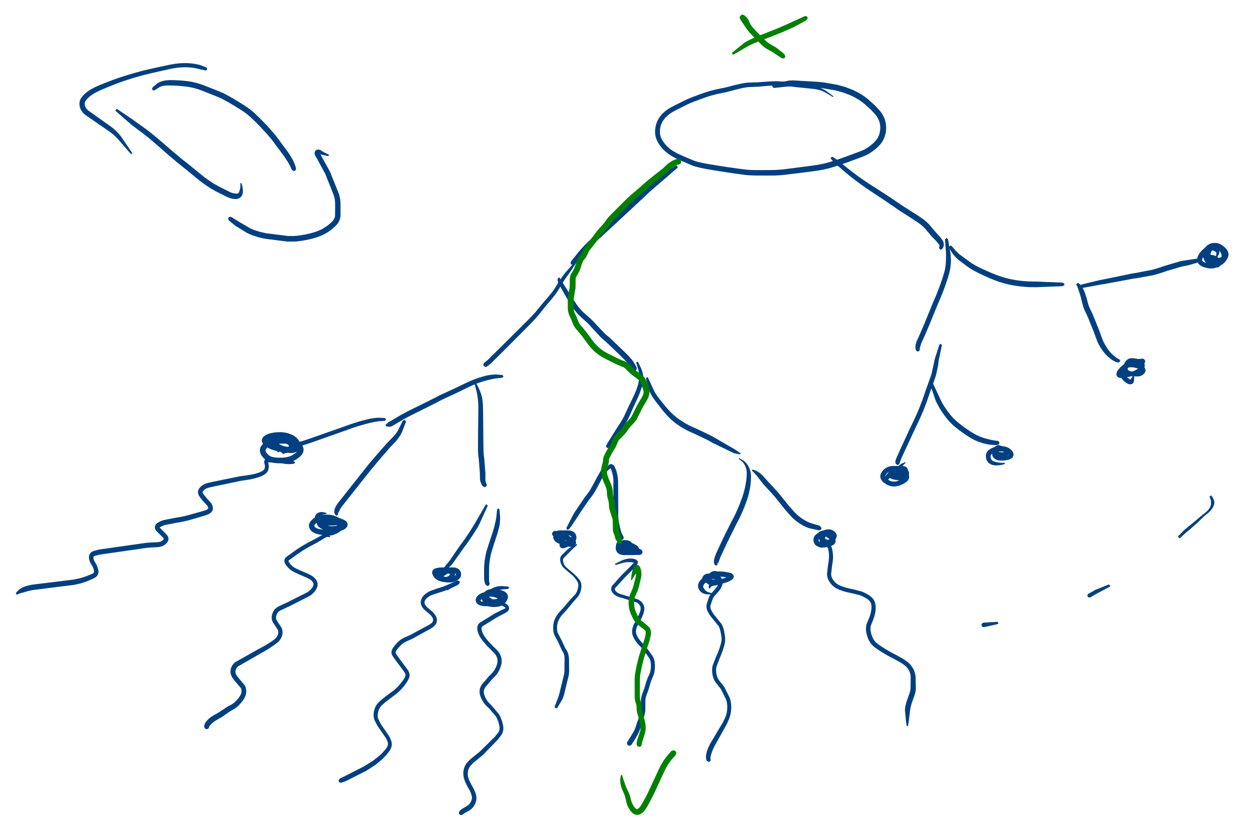
I



guess & check

⇒ könne annehme dass NTMs alle „nichtdet." Schritte am Anfang ausführen

II



Sprache in NP

Subset Sum

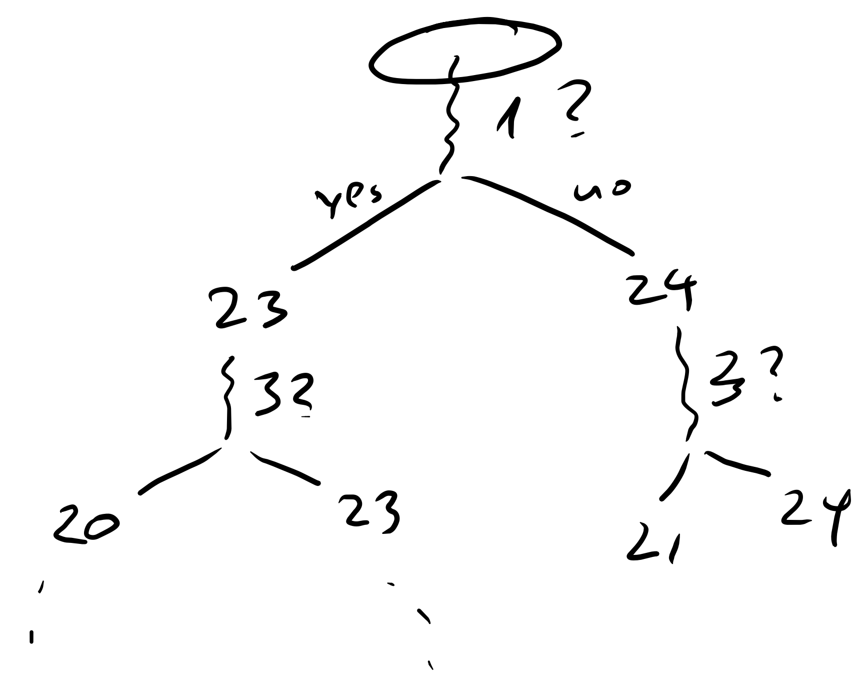
In: n Zahlen $a_i \in \mathbb{N}$ mit $i \in \{1,\dots,n\}$, Zahl $k \in \mathbb{N}$

Q: gibt es $X \subseteq \{1,\dots,n\}$ s.d. $k = \sum_{i \in X} a_i$

$\langle \{1, 3, 15, 6, 7\}, 24 \rangle \in$ Subset Sum $\in NP$?

falls a_i in unär gegeben

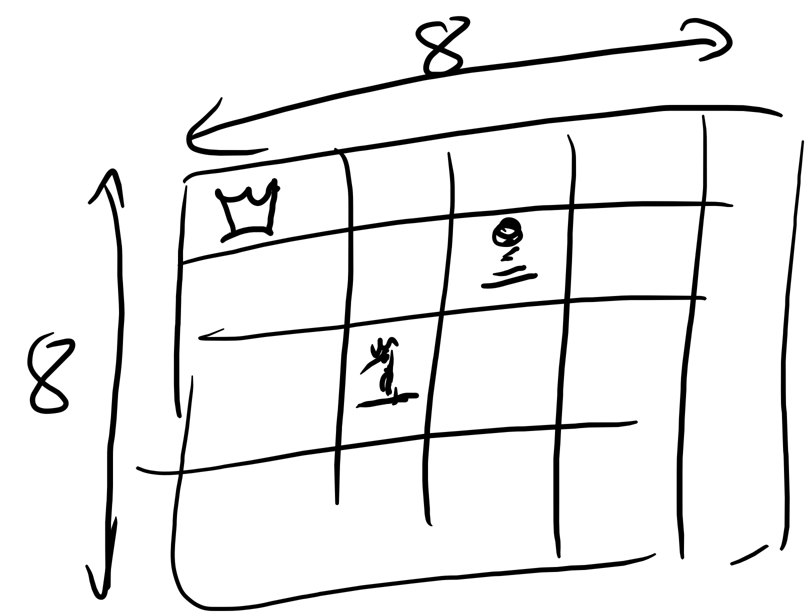
$\leadsto$ dann auch in P



Gewinnstrategie

In: Schachbrettkonfiguration auf $n \times n$ Feld

Q: Kann Schwarz einen Sieg erzwingen?



vermutlich $\notin NP$

$\exists_{\text{Zugfolge}}$

$\exists_{\text{Zug}} \forall_{\text{Züge}} \exists_{\text{Zug}} \forall_{\text{Züge}} \dots$

SAT $\exists_{\text{Beleg-g}}$

vs.

QBFSAT

Subset Sum falls alle Eingaben unär

Dynamic Programming

$\{1, 3, 6, 7, 15\}$ k

$q =$ Eingabegröße $= \boxed{\sum_i a_i}$

$k = 59$

1 ? ✓

2 ? no

3 ? ✓

(+1) 4 ? ✓

18 ?

21 ?

17 ?

9 ? ✓ $\boxed{3+6}$

$q \geq n$

23 ?

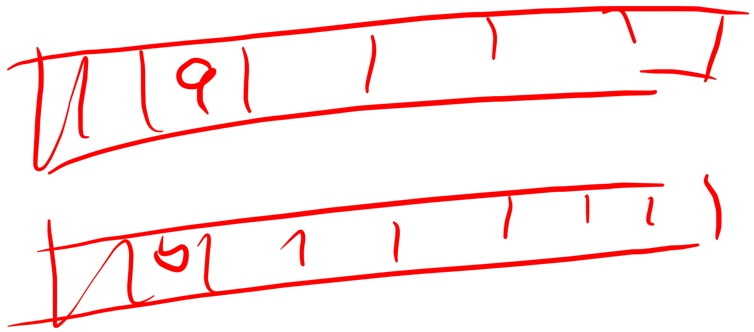
$\boxed{24}$: $\boxed{3+6} + 15$

$O(n)$ Zeit pro Zelle $\times$ k Zellen $\to$ $O(k \cdot n)$ Zeit

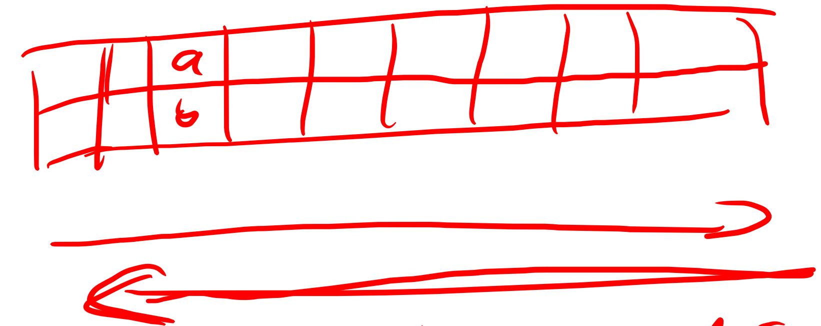
$\leq O(\sum a_i \cdot n)$ Zeit

$[= O(q^2)$ Zeit$]$

Mehrband $f(n)$



Einband



$f(n)$ Schritte um 1 Schritt der Mehrband TM zu simulieren

$$\Longrightarrow (f(n))^2$$

$$f(n) \in poly(n) \iff (f(n)^2) \in poly(n)$$