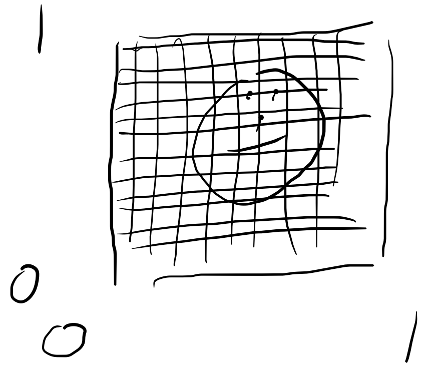
Zahlen

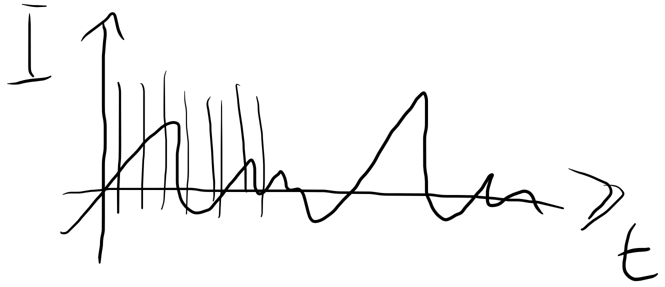
Bild

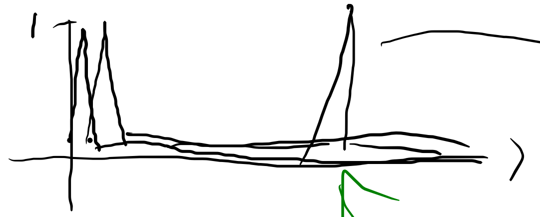


$$f : [0,1]^2 \to [0,1]$$

$$v \in \mathbb{N}^n$$

Audio





$$v \in \mathbb{R}^n$$

$$f : \mathbb{R} \to \mathbb{R}$$

$$f = \sum \boxed{a_i} B_i(t)$$

B_i: $\sin(t), \sin(2t), \sin(3t), \ldots$
$\cos(t), \cos(2t), \cos(3t), \ldots$

DFT

Natürliche Zahlen

$\mathbb{N} = \{0, 1, 2, \dots\}$

$a, b \in \mathbb{N}$ $\quad a+b \in \mathbb{N}$ $\quad a \cdot b \in \mathbb{N}$ $\quad a+b = b+a$ $\quad ab = ba$

$a + 0 = a$ $\quad a \cdot 1 = a$ $\quad a(b+c) = ab + ac$

$a \leq b$ $\quad \exists\, x \in \mathbb{N}$ $\quad a + x = b$

Reflexiv $\quad a \leq a$

Antisymmetrisch $\quad a \leq b \wedge b \leq a \Rightarrow a = b$

Transitiv $\quad a \leq b \wedge b \leq c \Rightarrow a \leq c$

Totalordnung $\quad a \leq b \vee b \leq a$

Repräsentation

unär $\quad$ 𝍸 𝍸 III

I V X L C M

4 = IV $\quad$ VI = 6

Stellenwertsysteme

$$\underbrace{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}_{b=10}$$

$$x_{n-1}\, x_{n-2} \dots x_2\, x_1\, x_0 = \sum_{i=0}^{n-1} x_i b^i$$

$b=2,\ n=8$ $\quad [0, 255] \subset \mathbb{N}$

x		1	0	1	0	1	1
y		0	1	1	0	1	0
z	1	0	0	0	1	0	1

Übertrag

$z_{i+1} = x_i \wedge y_i \vee y_i \wedge z_i \vee z_i \wedge x_i$

$z_i = x_i \oplus y_i \oplus z_i$

Ganze Zahlen

$a + x = b$ $(x = b - a)$

$(a,b) \sim (c,d) \iff a+d = b+c$

$\mathbb{Z} = \{ [a,b] , \ a,b \in \mathbb{N} \}$

"-1" $= (1,0) = (2,1) = \ldots$

Äquivalenzkl.

$$\boxed{\begin{aligned} [a,b] + [c,d] &= [a+c,\ b+d] \\ [a,b] \cdot [c,d] &= [ad+bc,\ ac+bd] \\ [a,b] \le [c,d] &\iff b+c \le a+d \end{aligned}} \quad \in \mathbb{N}$$

$-[a,b] = [b,a]$ $\qquad [a,b] - [c,d] = [a,b] + [d,c]$

Repräsentation von $\mathbb{Z}$

$a + x = b$

$x = b - a$

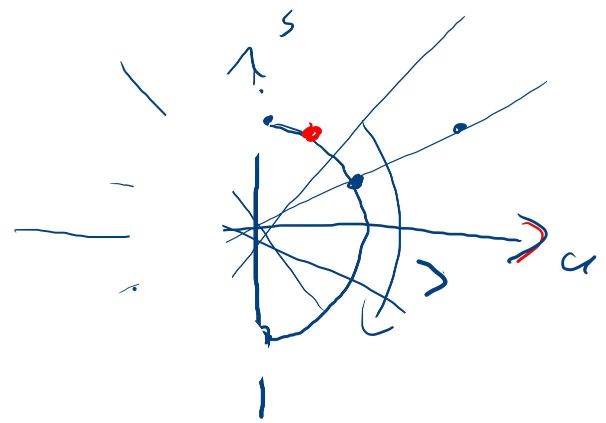
$x \ge 0$	$y \le 0$	
$(0, x)$	$(\lvert y \rvert, 0)$	Vorzeichendarstellung
$(0, x)$	$(2^{n-1} - 1, y)$	(1er Komplement)
$(0, x)$	$(2^{n-1}, y)$	(2er Komplement)
$(2^{n-1}, x)$		(Exzess Darstellung)

$$\begin{array}{l} (2^{n-1}, x) \\ (2^{n-1}, y) \\ \hline (2^n, x+y) \end{array} \to (2^{n-1}, x+y-2^{n-1})$$

Rationale Zahlen $\mathbb{Q}$

$ax = b$

$x = \frac{b}{a}$



$(a,b) \sim (c,d) \qquad ad = bc$

$\mathbb{Q} = \{ [a,b] \;,\; a \in \mathbb{Z}\setminus\{0\},\; b \in \mathbb{N} \}$

$[a,b] + [c,d] := [ac,\; ad + bc]$

$[a,b] + [c,d] := [ac,\; bd]$

$[a,b]^{-1} := [b,a]$

$[a,b] \le [c,d] \quad (ac \le 0 \wedge ad \le bc) \vee (0 \le ac \wedge bc \le ad)$

Repräsentation von $\mathbb{Q}$

fest $(b^{e}, x) \;\hat{=}\; \frac{x}{b^{e}} \longrightarrow$ Festkommadarstellung

$x_{n-1}\, x_{n-2} \cdots x_e, x_{e-1} \cdots x_2\, x_1\, x_0 = \sum_{i=0}^{n-1} x_i\, b^{i-e}$

variabel $(b^{e}, m) \;\hat{=}\; m \cdot b^{e}$

Mantisse

$\pm \underbrace{x_{m+n_e} \cdots x_{m+1}}\; \underbrace{x_m \cdots x_0}_{m} =$

$1 \le m \le b$

Gleitkommazahlen

$\boxed{1{,}23} = 123 \cdot 10^{-2}$
$= 1230 \cdot 10^{-3}$
$= 12300 \cdot 10^{-4}$