

# Filtern

Signal  $s \in \mathbb{R}^n$

Fiter  $a', a$

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & \dots & a_{n-1} \\ a_{n-1} & a_0 & a_1 & a_2 & & a_{n-2} \end{pmatrix}$$

Operation :  $A \cdot s \rightarrow$  Filterung  $a' * s$

$$A = R_n \Delta R_n]$$

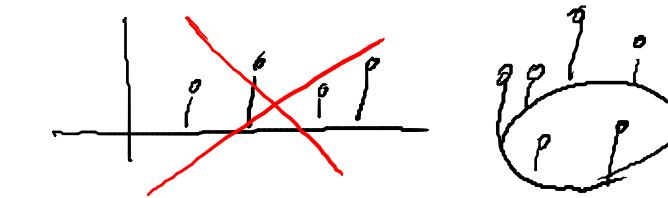
$$\Delta = \text{diag}(-R_n] a')$$

$$As = R_n \Delta R_n] s = R_n \text{diag}(R_n] a') R_n] s$$

$$R_n = \frac{1}{\sqrt{n}} \left( \begin{array}{cccccc} 1 & 1 & 1 & 1 & \dots & \\ 1 & \omega_n & \omega_n^2 & \omega_n^3 & \dots & \\ 1 & \omega_n^3 & \omega_n^4 & \omega_n^5 & \dots & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 1 & \omega_n^{n-1} & \omega_n^{2(n-1)} & \omega_n^{3(n-1)} & \dots & \end{array} \right)$$

$$O(n)$$

$$\omega_n = e^{2\pi i / n}$$



## Rekursive DFT

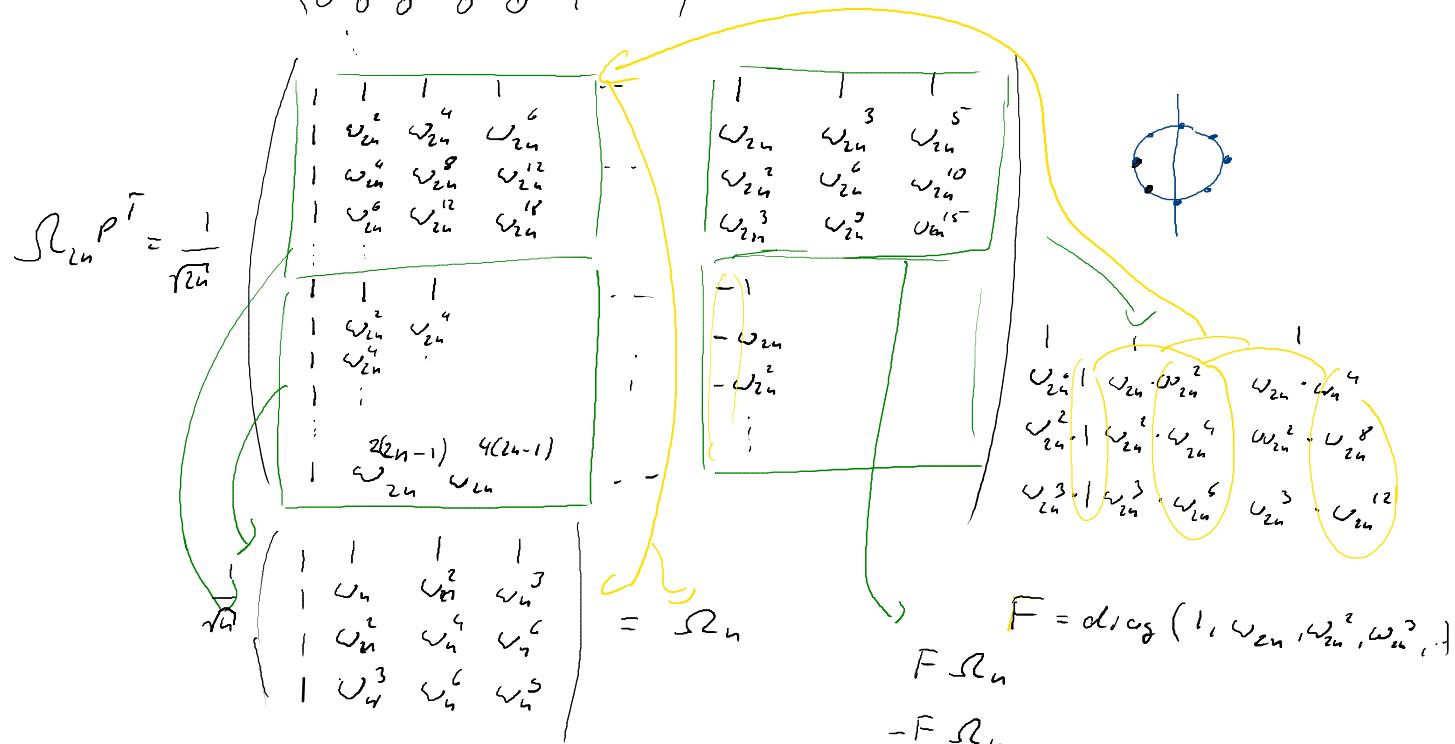
Strategie:  $\mathcal{R}_{2n}$  ausdrücken durch  $\mathcal{R}_n$

$$\omega_n = e^{\frac{2\pi i}{n}} \quad \omega_{2n} = e^{\frac{2\pi i}{2n}}$$

$$\omega_{2n}^{ij} \quad i,j \text{ gerade} \rightarrow \omega_n^{ik} \quad \Rightarrow \quad \omega_{2n}^{2k} = \omega_n^k$$

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & 1 & 0 & \dots \\ \vdots & & & & & \dots \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{R}_{2n} = \mathcal{R}_{2n} \underbrace{P^\top}_{I} P_S$$



$$\mathcal{R}_{2n} \cdot P^\top = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathcal{R}_n + F \cdot \mathcal{R}_n \\ \mathcal{R}_n - F \cdot \mathcal{R}_n \end{pmatrix}$$

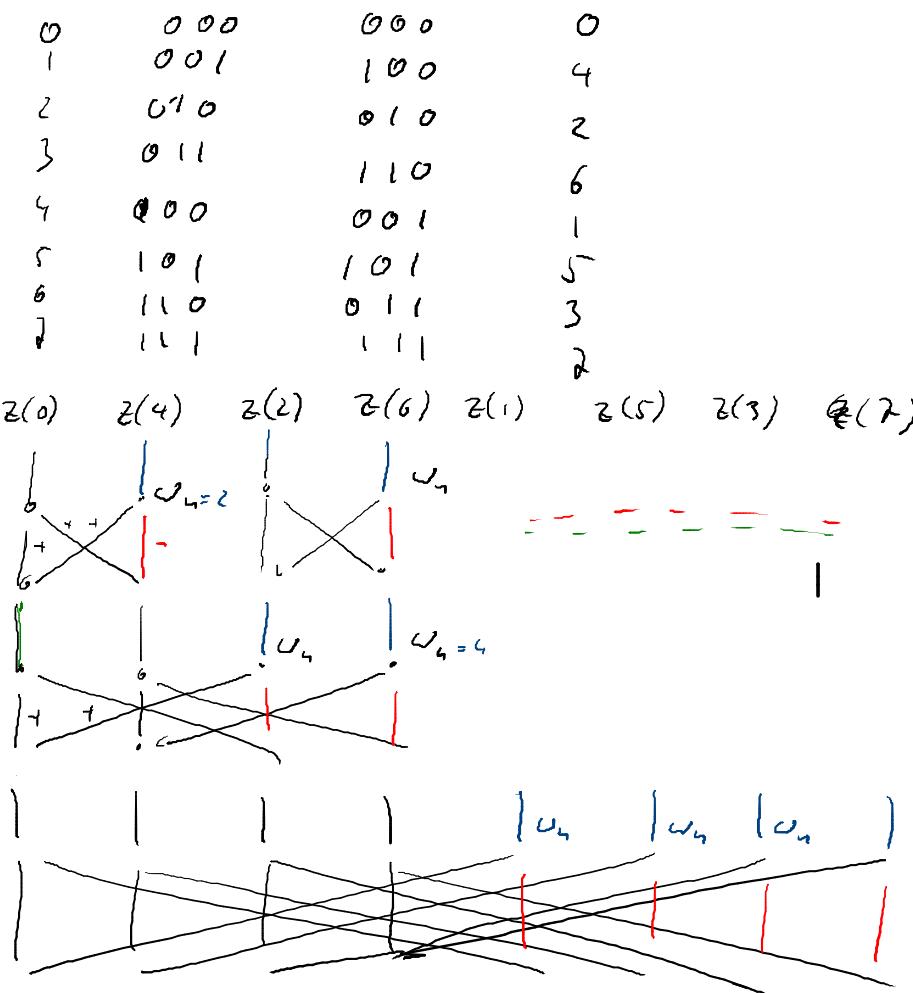
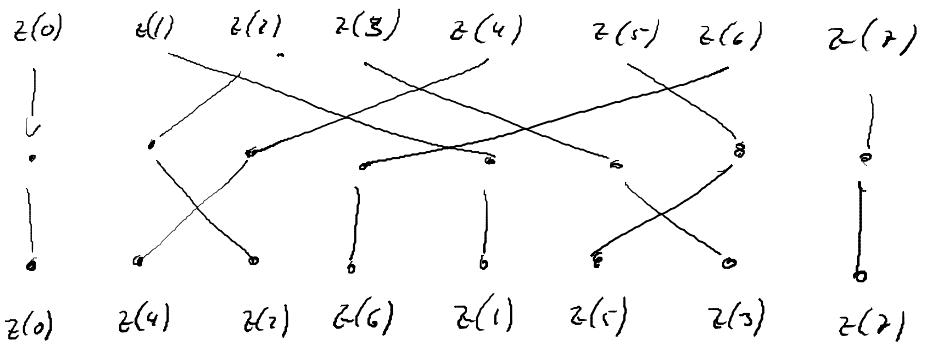
Aufwand

$F(n)$  Kosten für FT der Länge  $n$

$$\begin{aligned} F(n) &= C \cdot n + 2 F\left(\frac{n}{2}\right) \\ &= C \cdot n + 2 \left(C \cdot \frac{n}{2} + 2 F\left(\frac{n}{4}\right)\right) \\ &= 2C \cdot n + 4 F\left(\frac{n}{4}\right) \\ &= 3C \cdot n + 8 F\left(\frac{n}{8}\right) \end{aligned}$$

$$\sum_{k=0}^{2^m} C_k = C \cdot \log_2(n) \cdot n \rightarrow O(C \cdot \log_2(n))$$

# Schnelle DFT $\rightarrow$ FFT



Butterfly - Algorithmus f.d. FFT

Was macht man für  $n \neq 2^m$

1. zero-padding - Auffüllen mit Nullen
2. Sprenkeln
- 3.zyklisch Fortsetzen

