

i. For the `balanceTreeTwo()` function, it invoke the `buildBST` function, it's a recursion function, in each recursion , it add the middle element to the tree, and then recursion again but the parameter is the two side of the array. It's two parameter `begin` and `end`, represent the range that is going to build. for the first step , it put the middle value in the top of the tree, with the recursion, it will be a balance tree.

```
public void buildBST(BST bst, int[] v, int begin, int end) {
    if(begin > end)
        return;
    if(begin == end)
        bst.put(v[begin]);
    else if(begin + 1 == end)
    {
        bst.put(v[begin]);
        bst.put(v[end]);
    }else
    {
        int middle = (begin+end)/2;
        bst.put(v[middle]);
        buildBST(bst, v, begin, middle - 1);
        buildBST(bst, v, middle+1, end);
    }
}
```

ii. the total time complexity of the `balanceTreeOne()` is  $O(n)$

$O(\text{BalanceTreeOne}) = O(\text{sortedTree}) + O(\text{put}(\text{int}[a]))$

for `sortedTree`, it does a traverse in order, it's time complexity is  $O(n)$ , for `put(int[a])`, which is called in `buildBST`, because each index is viewed once, it's time complexity is also  $O(n)$ .

So the total time complexity of the `balanceTreeOne()` is  $O(n)$

iii. the space complexity of the `balanceTreeOne` is  $O(2n)$ . because it needs an extra array to store the data which is sorted.

the total time complexity of the `balanceTreeTwo()` is  $O(\log n)$

iv.

For the first left rotations loop, consider the worst situation, it's time complexity is  $O(n - \log n)$ , for the second left rotations loop, it's time complexity is  $O(\log n)$ , so the finally time complexity is  $O(\log n)$

v.

the space complexity of `balanceTreeTwo()` is  $O(n)$ , because it need't extra space to complete the operation, so it's depend on the quantity of number.