Linear Programming Problems in R

Use linear programming tool in R to solve optimization problems.

Asst. Prof. Ashwini Mathur

LPP over

Operations research is a field of study where, we **optimize performance under given constraints**.

Operations research also helps us **develop models for decision making**.

Linear programming problems have applications in various fields some of which include **manufacturing systems, publics systems, business, supply chain management and analytics.**

Now Let’s understand the aspects of formulating a problem into a linear programming problem.

Formulation 1 – Product mix problem

A shop can make two types of sweets (A and B). They use two resources – flour and sugar. To make one packet of A, they need 3 kg of flour and 3 kg of sugar. To make one packet of B, they need 3 kg of flour and 4 kg of sugar. They have 21 kg of flour and 28 kg of sugar. These sweets are sold at Rs 1000 and 900 per packet respectively. Find the best product mix to maximize the revenue.

Let X_1 be the number of packets of sweet A made

Let X_2 be the number of packets of sweet B made

Maximize $1000X_1 + 900X_2$

$3X_1 + 3X_2 \leq 21$

$3X_1 + 4X_2 \leq 28$

$X_1, X_2 \geq 0$

Notations

Let X_1 be the number of packets of sweet A made ← Decision variable
Let X_2 be the number of packets of sweet B made

Maximize $1000X_1 + 900X_2$ ← Objective function

$3X_1 + 3X_2 \leq 21$ ← Constraints
$3X_1 + 4X_2 \leq 28$

$X_1, X_2 \geq 0$ ← Non negativity restriction

Assumptions

1. Proportionality
2. Linearity
3. Deterministic

Descriptive Problem Now Converted into Mathematical Problem

Now we are able to model or **write a formulation** which has an **objective function** which is a linear function of the **decision variables**, **constraints** which are **linear inequalities or equations** and **explicit non-negativity restriction on the decision variables**,

we have formulated a linear programming problem.

Why we study Linear Programming Problem ??

In linear programming, we try to **optimize a linear objective function subject to linear constraints and with non-negativity restrictions** on the variables.

Every activity is carried out with an objective in mind and it is important to **optimize the objective**.

Basic LPP Problem

Example 3: Product Mix Problem

- Maximize your profit
  - Decide how many to produce product types 1,2
  - To be produced, each product requires different amount of 2 resources per unit
  - Each resource is limited

Resource	Resource Usage		Resource Availability
	Product 1	Product 2	
A	1	1	5
B	3	2	12
profit/unit	6	5	

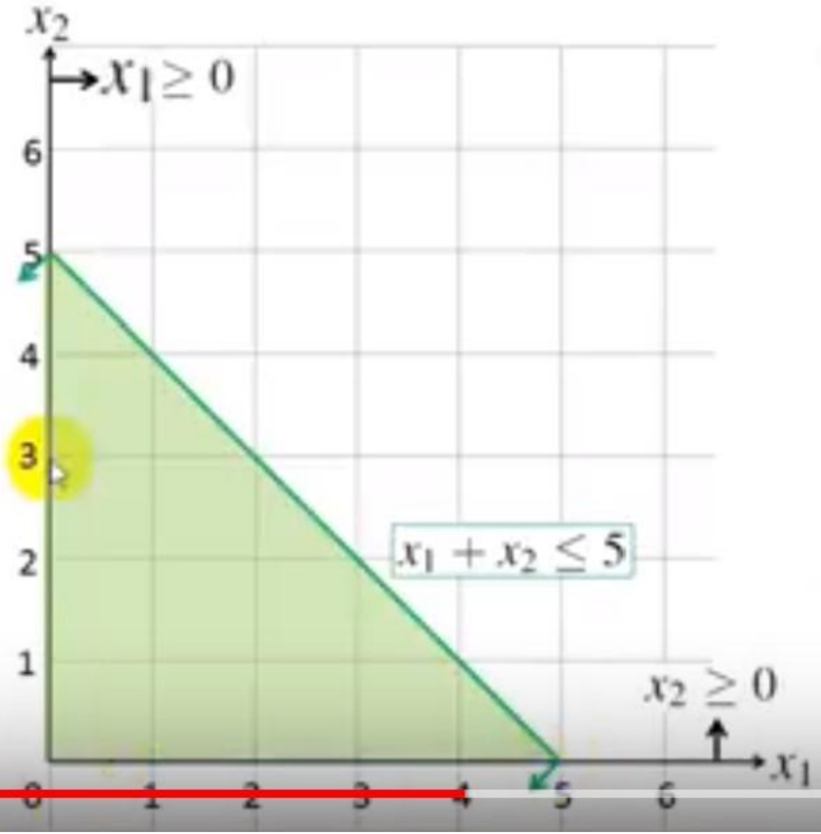
Example 3: Product Mix Problem (cont.)

- You can formulate the following LP model
  - x_1: *the amount of product type 1 produced (units)*
  - x_2: *the amound of product type 2 produced (units)*

$$
\begin{array}{lrcrcrl}
\texttt{Maximize} & 6x_1 & + & 5x_2 & & & \quad \text{Total profit} \\
s.t. & x_1 & + & x_2 & \leq & 5 & \quad \text{Resource A limit} \\
 & 3x_1 & + & 2x_2 & \leq & 12 & \quad \text{Resource B limit} \\
 & x_1 & & & \geq & 0 & \multirow{2}{*}{} \\
 & & & x_2 & \geq & 0 &
\end{array}
\left.\begin{array}{l} \\ \\ \\ \\ \\ \end{array}\right.
$$

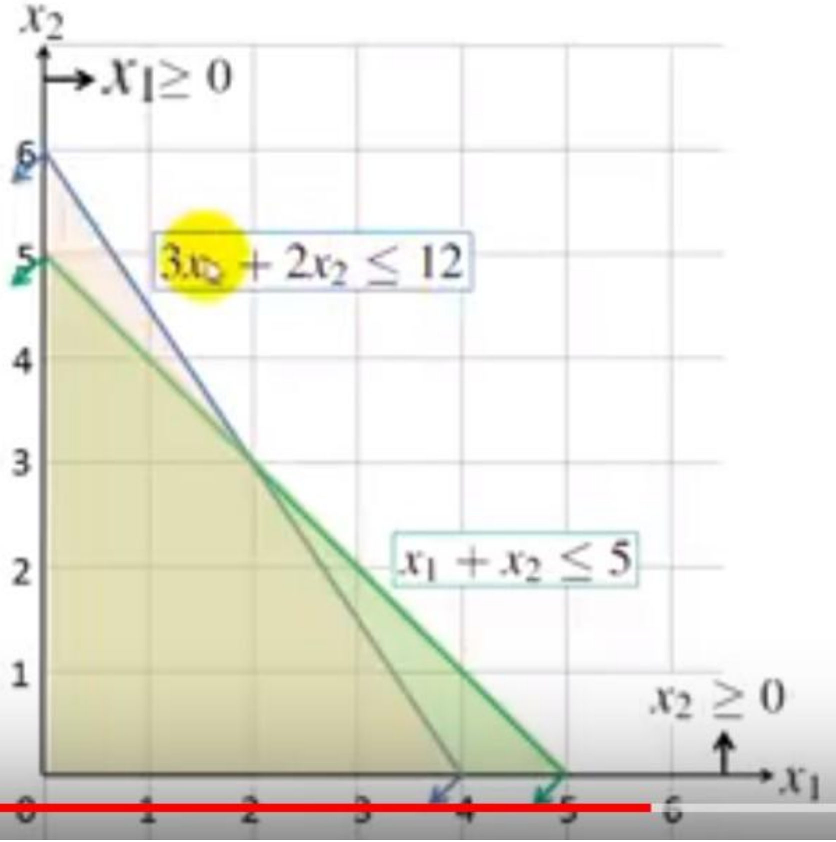
$$
\left.\begin{array}{r} x_1 \geq 0 \\ x_2 \geq 0 \end{array}\right\} \text{Non-negativity}
$$

Graphical Solution to (Example 3: Product Mix Problem) (cont.)



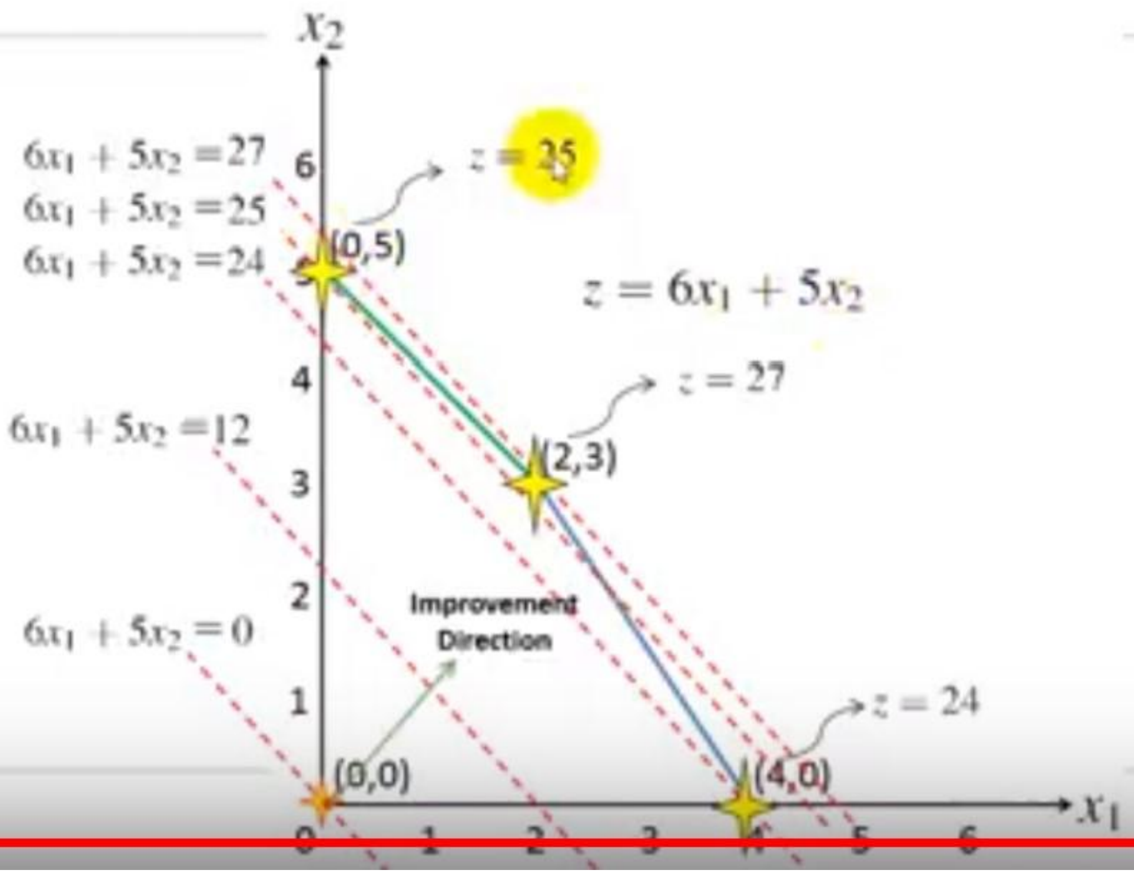
- Resource A limit constraint
  - Draw $x_1 + x_2 = 5$
  - Find the region where $x_1 + x_2 \leq 5$
  - Hint: take a point in each of one of the regions and see if this point satisfies the constraint. The region including the point satisfying your constraint is the region defined by your constraint (you can take the origin as your point)

Graphical Solution to (Example 3: Product Mix Problem) (cont.)

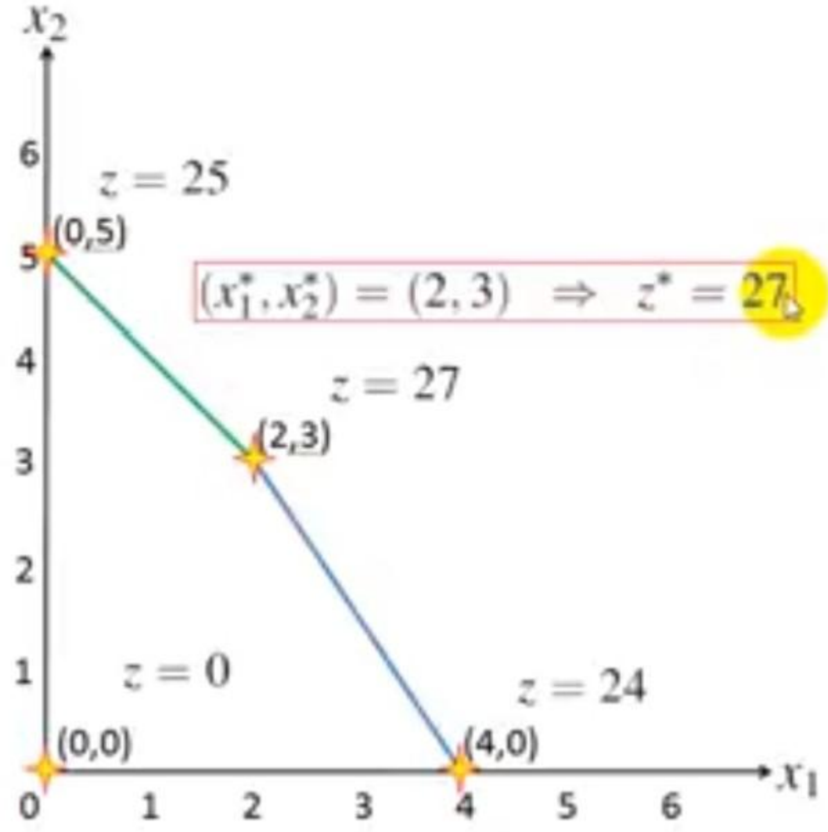


- Resource B limit constraint
  - Draw $3x_1 + 2x_2 = 12$
  - Find the region where $3x_1 + 2x_2 \leq 12$

Graphical Solution to (Example 3: Product Mix Problem) (cont.)



Graphical Solution to (Example 3: Product Mix Problem) (cont.)



- **Optimum solution will be one of the corner solutions!!**
  1. Graph the constraints
  2. Define your feasible region
  3. Evaluate the corner points (or draw iso-lines until you leave feasible region)
  4. Choose the best corner point

Let's Demonstrate this Problem in R

Problem:

objective fun.

$$Z = 12x_1 + 16x_2$$

constraints $\begin{cases} 10x_1 + 20x_2 \leq 120 \\ 8x_1 + 8x_2 \leq 80 \end{cases}$

$$x_1 + x_2 \geq 0$$

lpSolve (Library in R)

Description: Lp_solve is freely available software for solving linear, integer and mixed integer programs.

License: LGPL-2

URL: https://github.com/gaborcsardi/lpSolve

Repository CRAN: Date/Publication 2020-01-24 22:20:02 UTC

Linear Programming using R

Description: Interface to lp_solve linear/integer programming system

Usage: lp (direction = "min", objective.in, const.mat, const.dir, const.rhs, transpose.constraints = TRUE, int.vec, presolve=0, compute.sens=0, binary.vec, all.int=FALSE, all.bin=FALSE, scale = 196, dense.const, num.bin.solns=1, use.rw=FALSE)

Arguments

Direction: Character string giving direction of optimization: "min" (default) or "max."

Objective.in: Numeric vector of coefficients of objective function const.mat Matrix of numeric constraint coefficients, one row per constraint, one column per variable (unless transpose.constraints = FALSE; see below).

Const.dir: Vector of character strings giving the direction of the constraint: each value should be one of "<," "<=," "=," "==," ">," or ">=". (In each pair the two values are identical.)

Const.rhs: Vector of numeric values for the right-hand sides of the constraints.