2021

Time Series Modelling on Shanghai Stock Exchange (SSE) Composite Index Return

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Contents

Section 1. Background Introduction

Foreign Companies are allowed to invest in the banking, healthcare insurance, logistics, human resources and tourism sectors through subsidiaries or joint ventures within the Shanghai Pilot Free Trade Zone (SPFTZ) since September, 2013.

The company that intends to invest in SPFTZ plans to create a portfolio resembling Shanghai Stock Exchange (SSE) Composite Index. Accordingly, it is interested in the behavior of the SSE Index return series and would like to model the return series and the volatility.

Section 2. Data Preparation

The dataset contains daily SSE index series from 04/Jan/2005 to 14/Oct/2021, a total of 4072 observations. Taking $R_t = 100 * \log\left(\frac{Price_t}{Price_{t-1}}\right)$ as index return variable, I further split the data with 70% as training data (from 04/Jan/2005 to 27/Sept/2016) and 30% (from 28/Sept/2016 to 14/Oct/2021) as validation data.

Section 3. ARMA Model Selection

Section 3.1 Unit Root Test

ARMA models require the time series to be stationary. I first perform unit root tests(ADF and KPSS) to check whether the data is stationary.

Null Hypothesis: R has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=27)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-52.16950	0.0001
Test critical values:	1% level	-3.432455	
	5% level	-2.862356	
	10% level	-2.567249	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R)
Method: Least Squares
Date: 12/08/21 Time: 15:16
Sample (adjusted): 2 2849
Included observations: 2848 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R(-1)	-0.977657	0.018740	-52.16950	0.0000
C	0.029978	0.032933	0.910256	0.3628

R-squared	0.488833	Mean dependent var	-4.95E-05
Adjusted R-squared	0.488654	S.D. dependent var	2.457433
S.E. of regression	1.757273	Akaike info criterion	3.966105
Sum squared resid	8788.470	Schwarz criterion	3.970287
Log likelihood	-5645.734	Hannan-Quinn criter.	3.967613
F-statistic	2721.657	Durbin-Watson stat	1.998457
Prob(F-statistic)	0.000000		

Figure 1: ACF Test

Null Hypothesis: R is stationary
Exogenous: Constant
Bandwidth: 11 (Newey-West automatic) using Bartlett kernel

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.193322
Asymptotic critical values*:	1% level	0.739000
	5% level	0.463000
	10% level	0.347000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Residual variance (no correction)	3.086471
HAC corrected variance (Bartlett kernel)	3.450965

KPSS Test Equation
Dependent Variable: R
Method: Least Squares
Date: 12/08/21 Time: 15:18
Sample (adjusted): 1 2849
Included observations: 2849 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030911	0.032920	0.938973	0.3478

R-squared	0.000000	Mean dependent var	0.030911
Adjusted R-squared	0.000000	S.D. dependent var	1.757144
S.E. of regression	1.757144	Akaike info criterion	3.965607
Sum squared resid	8793.355	Schwarz criterion	3.967697
Log likelihood	-5648.008	Hannan-Quinn criter.	3.966361
Durbin-Watson stat	1.955223		

Figure 2: KPSS Test

From the test result, we see that we can reject the null hypothesis of ADF test and cannot reject that of KPSS test. Therefore, I conclude that the return data is stationary.

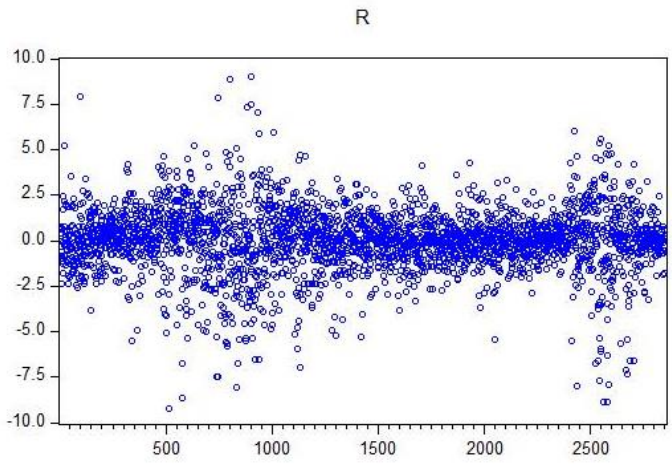


Figure 3: Time Series Distribution

Section 3.2 ARMA Models Selection

To find out the possible p and q order of ARMA models, I run a combination of p, q taking p from 0 to 5 and q from 0 to 5. The results are attached in appendix 1.

Based on information criteria AIC, ARMA(5,5) is recommended while ARMA(1,0) is recommended based on SBIC.

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:22
Sample: 1 2849
Included observations: 2849
Convergence achieved after 51 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030930	0.037642	0.821692	0.4113
AR(1)	-0.835131	0.148256	-5.633041	0.0000
AR(2)	-0.940416	0.196563	-4.784291	0.0000
AR(3)	-0.149268	0.263032	-0.567491	0.5704
AR(4)	-0.235308	0.186397	-1.262399	0.2069
AR(5)	0.315260	0.133159	2.367540	0.0180
MA(1)	0.860398	0.151750	5.669856	0.0000
MA(2)	0.941682	0.200371	4.699694	0.0000
MA(3)	0.183546	0.265112	0.692332	0.4888
MA(4)	0.313270	0.187088	1.674453	0.0942
MA(5)	-0.226823	0.135695	-1.671563	0.0947
SIGMASQ	3.039078	0.050579	60.08533	0.0000

R-squared	0.015355	Mean dependent var	0.030911
Adjusted R-squared	0.011537	S.D. dependent var	1.757144
S.E. of regression	1.746978	Akaike info criterion	3.957921
Sum squared resid	8658.332	Schwarz criterion	3.983002
Log likelihood	-5626.059	Hannan-Quinn criter.	3.966967
F-statistic	4.021975	Durbin-Watson stat	1.999822
Prob(F-statistic)	0.000007		

Inverted AR Roots	.48 -.69+.69i	.03-.83i	.03+.83i	-.69-.69i
Inverted MA Roots	.38 -.68+.71i	.06+.78i	.06-.78i	-.68-.71i

Figure 4: ARMA(5,5) Model Result

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:57
Sample: 1 2849
Included observations: 2849
Convergence achieved after 16 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030922	0.034622	0.893126	0.3719
MA(1)	0.023650	0.013514	1.750027	0.0802
SIGMASQ	3.084840	0.049591	62.20564	0.0000

R-squared	0.000528	Mean dependent var	0.030911
Adjusted R-squared	-0.000174	S.D. dependent var	1.757144
S.E. of regression	1.757297	Akaike info criterion	3.966483
Sum squared resid	8788.710	Schwarz criterion	3.972753
Log likelihood	-5647.255	Hannan-Quinn criter.	3.968744
F-statistic	0.752077	Durbin-Watson stat	2.001205
Prob(F-statistic)	0.471480		

Inverted MA Roots	-.02

Figure 5: ARMA(1,0) Model Result

Section 3.3 ARCH Effect Detection

In this stage, let's check whether the models recommended are good enough, which means they are supposed to have no ARCH effect.

Heteroskedasticity Test: ARCH

F-statistic	34.88615	Prob. F(7,2834)	0.0000
Obs*R-squared	225.4644	Prob. Chi-Square(7)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 12/19/21 Time: 20:47
Sample (adjusted): 8 2849
Included observations: 2842 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.437104	0.167198	8.595236	0.0000
RESID^2(-1)	0.100734	0.018745	5.373879	0.0000
RESID^2(-2)	0.071788	0.018818	3.814821	0.0001
RESID^2(-3)	0.103847	0.018843	5.511121	0.0000
RESID^2(-4)	0.088596	0.018871	4.694934	0.0000
RESID^2(-5)	0.049773	0.018843	2.641450	0.0083
RESID^2(-6)	0.048890	0.018818	2.597996	0.0094
RESID^2(-7)	0.064721	0.018745	3.452637	0.0006

R-squared	0.079333	Mean dependent var	3.045802
Adjusted R-squared	0.077059	S.D. dependent var	7.232214
S.E. of regression	6.947975	Akaike info criterion	6.717588
Sum squared resid	136809.5	Schwarz criterion	6.734344
Log likelihood	-9537.693	Hannan-Quinn criter.	6.723632
F-statistic	34.88615	Durbin-Watson stat	2.005278
Prob(F-statistic)	0.000000		

Figure 6: Heteroskedasticity Test for ARMA(5,5) Model Result

Heteroskedasticity Test: ARCH

F-statistic	36.93999	Prob. F(7,2834)	0.0000
Obs*R-squared	237.6282	Prob. Chi-Square(7)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 12/19/21 Time: 20:49
Sample (adjusted): 8 2849
Included observations: 2842 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.427532	0.169922	8.401115	0.0000
RESID^2(-1)	0.098115	0.018745	5.234113	0.0000
RESID^2(-2)	0.070517	0.018814	3.748021	0.0002
RESID^2(-3)	0.109989	0.018833	5.840174	0.0000
RESID^2(-4)	0.093077	0.018865	4.933739	0.0000
RESID^2(-5)	0.054586	0.018833	2.898374	0.0038
RESID^2(-6)	0.047507	0.018815	2.524997	0.0116
RESID^2(-7)	0.064637	0.018745	3.448133	0.0006

R-squared	0.083613	Mean dependent var	3.091647
Adjusted R-squared	0.081350	S.D. dependent var	7.415988
S.E. of regression	7.107947	Akaike info criterion	6.763115
Sum squared resid	143181.9	Schwarz criterion	6.779870
Log likelihood	-9602.386	Hannan-Quinn criter.	6.769158
F-statistic	36.93999	Durbin-Watson stat	2.005378
Prob(F-statistic)	0.000000		

Figure 7: Heteroskedasticity Test for ARMA(1,0) Model Result

However, both these two models don't pass the Heteroskedasticity Test. They all have ARCH effects. We need to further model the volatility of the time series data.

Section 4. Hybrid Models Selection

Section 4.1 Models Attempt

As ARMA model selected by criteria information has ARCH effect, we need to consider building ARMA- GARCH model. I first anchor mean equation part of the model to ARMA(3,3)-GARCH(1,1) after several attempt. Below are ARMA-GARCH models of four types:

Remarks:

1.For the following pictures, from left to right first, then first row to second row to are results of ARMA-GARCH, ARMA-TGARCH, ARMA-EGARCH, ARMA-GARCH-in-mean respectively

2.All models use Bollerslev-Wooldridge as covariance method as non-normality are tested for all four models

Dependent Variable: R
Method: ML ARCH - Normal distribution (OPG - BHHH / Marquardt steps)
Date: 12/19/21 Time: 19:43
Sample (adjusted): 4 2849
Included observations: 2846 after adjustments
Failure to improve likelihood (non-zero gradients) after 44 iterations
Coefficient covariance computed using Bollerslev-Wooldridge QML sandwich with expected Hessian
MA Backcast: 1 3
Presample variance: backcast (parameter = 0.7)
GARCH = C(8) + C(9)*RESID(-1)^2 + C(10)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.042215	0.033489	1.260564	0.2075
AR(1)	0.559781	0.357486	1.565881	0.1174
AR(2)	-0.761017	0.148396	-5.128276	0.0000
AR(3)	0.355635	0.323026	1.100946	0.2709
MA(1)	-0.528245	0.362411	-1.457584	0.1450
MA(2)	0.708908	0.157106	4.512278	0.0000
MA(3)	-0.278996	0.321084	-0.868920	0.3849
Variance Equation				
C	1.929226	0.894297	2.157254	0.0310
RESID(-1)^2	0.071300	0.030548	2.334034	0.0196
GARCH(-1)	0.521300	0.217145	2.400702	0.0164

R-squared	0.009706	Mean dependent var	0.030888
Adjusted R-squared	0.007613	S.D. dependent var	1.757898
S.E. of regression	1.751193	Akaike info criterion	3.983836
Sum squared resid	8706.296	Schwarz criterion	4.004755
Log likelihood	-5658.998	Hannan-Quinn criter.	3.991381
Durbin-Watson stat	2.019579		

Inverted AR Roots	.49	.04+.85i	.04-.85i
Inverted MA Roots	.42	.05+.81i	.05-.81i

Dependent Variable: R
Method: ML ARCH - Normal distribution (OPG - BHHH / Marquardt steps)
Date: 12/19/21 Time: 19:38
Sample (adjusted): 4 2849
Included observations: 2846 after adjustments
Convergence achieved after 72 iterations
Coefficient covariance computed using Bollerslev-Wooldridge QML sandwich with expected Hessian
MA Backcast: 1 3
Presample variance: backcast (parameter = 0.7)
GARCH = C(8) + C(9)*RESID(-1)^2 + C(10)*RESID(-1)^2*(RESID(-1)<0) + C(11)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.039136	0.030568	1.280278	0.2004
AR(1)	0.456506	0.096945	4.708913	0.0000
AR(2)	-0.480583	0.055207	-8.705187	0.0000
AR(3)	0.895852	0.088734	10.09591	0.0000
MA(1)	-0.434036	0.102440	-4.236963	0.0000
MA(2)	0.463973	0.060146	7.714079	0.0000
MA(3)	-0.881922	0.094311	-9.351238	0.0000
Variance Equation				
C	0.019688	0.007597	2.591522	0.0096
RESID(-1)^2	0.055004	0.015550	3.537212	0.0004
RESID(-1)^2*(RESID(-1)<0)	0.003597	0.018967	0.189644	0.8496
GARCH(-1)	0.937519	0.011009	85.15924	0.0000

R-squared	0.008685	Mean dependent var	0.030888
Adjusted R-squared	0.006590	S.D. dependent var	1.757898
S.E. of regression	1.752096	Akaike info criterion	3.704437
Sum squared resid	8715.277	Schwarz criterion	3.727449
Log likelihood	-5260.414	Hannan-Quinn criter.	3.712737
Durbin-Watson stat	1.999832		

Inverted AR Roots	.95	-.25-.94i	-.25+.94i
Inverted MA Roots	.94	-.25+.94i	-.25-.94i

Dependent Variable: R
Method: ML ARCH - Normal distribution (OPG - BHHH / Marquardt steps)
Date: 12/19/21 Time: 19:58
Sample (adjusted): 4 2849
Included observations: 2846 after adjustments
Convergence achieved after 40 iterations
Coefficient covariance computed using Bollerslev-Wooldridge QML sandwich with expected Hessian
MA Backcast: 1 3
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(8) + C(9)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(10) *LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.048683	0.027522	1.768890	0.0769
AR(1)	0.464879	0.063180	7.358000	0.0000
AR(2)	0.282373	0.086332	3.270794	0.0011
AR(3)	-0.899132	0.060688	-14.81569	0.0000
MA(1)	-0.458704	0.056010	-8.189724	0.0000
MA(2)	-0.303097	0.077524	-3.909720	0.0001
MA(3)	0.926124	0.054888	16.87312	0.0000
Variance Equation				
C(8)	-0.091269	0.015483	-5.894840	0.0000
C(9)	0.133888	0.022659	5.908800	0.0000
C(10)	0.991325	0.003356	295.4002	0.0000

R-squared	0.005107	Mean dependent var	0.030888
Adjusted R-squared	0.003004	S.D. dependent var	1.757898
S.E. of regression	1.755255	Akaike info criterion	3.700628
Sum squared resid	8746.736	Schwarz criterion	3.721547
Log likelihood	-5255.994	Hannan-Quinn criter.	3.708173
Durbin-Watson stat	1.966676		

Inverted AR Roots	.69+.71i	.69-.71i	-.92
Inverted MA Roots	.70+.71i	.70-.71i	-.93

Dependent Variable: R
Method: ML ARCH - Normal distribution (OPG - BHHH / Marquardt steps)
Date: 12/19/21 Time: 20:03
Sample (adjusted): 4 2849
Included observations: 2846 after adjustments
Convergence achieved after 68 iterations
Coefficient covariance computed using Bollerslev-Wooldridge QML sandwich with expected Hessian
MA Backcast: 1 3
Presample variance: backcast (parameter = 0.7)
GARCH = C(9) + C(10)*RESID(-1)^2 + C(11)*RESID(-1)^2*(RESID(-1)<0) + C(12)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.002049	0.065318	0.031374	0.9750
C	0.036552	0.086304	0.423522	0.6719
AR(1)	0.455753	0.097491	4.674803	0.0000
AR(2)	-0.480961	0.055550	-8.658213	0.0000
AR(3)	0.895124	0.089200	10.03508	0.0000
MA(1)	-0.433208	0.102875	-4.211015	0.0000
MA(2)	0.464395	0.060509	7.674868	0.0000
MA(3)	-0.881125	0.094767	-9.297765	0.0000
Variance Equation				
C	0.019692	0.007599	2.591552	0.0096
RESID(-1)^2	0.055022	0.015564	3.535167	0.0004
RESID(-1)^2*(RESID(-1)<0)	0.003575	0.018962	0.188535	0.8505
GARCH(-1)	0.937511	0.011020	85.07241	0.0000

R-squared	0.008651	Mean dependent var	0.030888
Adjusted R-squared	0.006205	S.D. dependent var	1.757898
S.E. of regression	1.752435	Akaike info criterion	3.705140
Sum squared resid	8715.578	Schwarz criterion	3.730243
Log likelihood	-5260.414	Hannan-Quinn criter.	3.714194
Durbin-Watson stat	1.999844		

Inverted AR Roots	.95	-.25-.94i	-.25+.94i
Inverted MA Roots	.94	-.25-.94i	-.25+.94i

Figure 8: Model Result for Four GARCH-Type Models

In terms of the significant of independent variables, models involving extension GARCH are recommended, while regular GARCH model has 4 insignificant indicators. Accordingly, we can abandon regular GARCH model in this step.

Section 4.2 Residual Analysis

Section 4.2.1 White Noise Checking

Date: 12/19/21 Time: 19:46
Sample: 1 2850
Included observations: 2846

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.066	0.066	12.405	0.000
		2	0.094	0.090	37.728	0.000
		3	0.124	0.114	81.586	0.000
		4	0.115	0.096	119.20	0.000
		5	0.094	0.066	144.35	0.000
		6	0.102	0.068	174.33	0.000
		7	0.109	0.070	208.01	0.000
		8	0.089	0.046	230.55	0.000
		9	0.058	0.010	240.03	0.000
		10	0.133	0.088	290.65	0.000
		11	0.079	0.032	308.65	0.000
		12	0.072	0.023	323.57	0.000
		13	0.127	0.077	369.99	0.000
		14	0.126	0.075	415.33	0.000
		15	0.059	0.003	425.19	0.000
		16	0.131	0.073	474.22	0.000
		17	0.084	0.019	494.45	0.000
		18	0.095	0.033	520.32	0.000
		19	0.073	0.011	535.51	0.000
		20	0.113	0.046	572.15	0.000
		21	0.129	0.069	620.19	0.000
		22	0.053	-0.009	628.26	0.000
		23	0.098	0.028	655.66	0.000
		24	0.078	0.003	672.92	0.000
		25	0.082	0.023	692.14	0.000
		26	0.097	0.028	719.28	0.000
		27	0.065	-0.006	731.54	0.000
		28	0.075	0.008	747.77	0.000
		29	0.055	-0.011	756.37	0.000
		30	0.128	0.064	803.38	0.000
		31	0.076	0.010	820.00	0.000
		32	0.055	-0.009	828.59	0.000
		33	0.069	-0.001	842.26	0.000
		34	0.099	0.028	870.53	0.000
		35	0.075	0.015	886.72	0.000
		36	0.069	0.004	900.42	0.000

*Probabilities may not be valid for this equation specification.

Date: 12/19/21 Time: 19:45
Sample: 1 2850
Included observations: 2846
Q-statistic probabilities adjusted for 6 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.003	-0.003	0.0346	
		2	0.016	0.016	0.7262	
		3	0.004	0.004	0.7773	
		4	0.009	0.009	1.0107	
		5	0.008	0.008	1.2149	
		6	-0.028	-0.028	3.3980	
		7	0.023	0.022	4.9085	0.027
		8	-0.014	-0.013	5.4455	0.066
		9	0.007	0.006	5.5810	0.134
		10	0.018	0.018	6.4611	0.167
		11	0.026	0.026	8.3555	0.138
		12	0.000	-0.001	8.3555	0.213
		13	0.065	0.065	20.326	0.005
		14	-0.026	-0.028	22.244	0.004
		15	0.064	0.063	33.991	0.000
		16	-0.002	-0.002	34.003	0.000
		17	-0.002	-0.003	34.013	0.000
		18	0.033	0.031	37.044	0.000
		19	-0.013	-0.010	37.552	0.000
		20	0.006	-0.000	37.673	0.001
		21	-0.001	0.005	37.675	0.001
		22	0.010	0.004	37.953	0.002
		23	-0.028	-0.029	40.267	0.001
		24	0.002	0.001	40.276	0.002
		25	0.011	0.008	40.606	0.003
		26	-0.009	-0.015	40.828	0.004
		27	0.000	0.003	40.828	0.006
		28	0.045	0.036	46.532	0.002
		29	0.009	0.010	46.781	0.002
		30	-0.031	-0.034	49.632	0.002
		31	-0.006	-0.011	49.745	0.002
		32	-0.028	-0.026	52.030	0.002
		33	-0.014	-0.018	52.617	0.002
		34	0.023	0.029	54.076	0.002
		35	0.028	0.027	56.390	0.002
		36	-0.010	-0.009	56.693	0.002

*Probabilities may not be valid for this equation specification.

Date: 12/19/21 Time: 19:59
Sample: 1 2850
Included observations: 2846
Q-statistic probabilities adjusted for 6 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.023	0.023	1.4716	
		2	0.008	0.008	1.6647	
		3	0.035	0.034	5.0608	
		4	0.047	0.045	11.306	
		5	0.000	-0.002	11.306	
		6	-0.033	-0.035	14.495	
		7	0.039	0.037	18.823	0.000
		8	0.026	0.023	20.692	0.000
		9	0.015	0.016	21.325	0.000
		10	0.046	0.046	27.395	0.000
		11	0.020	0.013	28.503	0.000
		12	0.006	0.000	28.602	0.000
		13	0.026	0.024	30.568	0.000
		14	-0.001	-0.007	30.572	0.000
		15	0.048	0.046	37.034	0.000
		16	0.012	0.011	37.476	0.000
		17	0.004	-0.002	37.518	0.000
		18	0.033	0.027	40.581	0.000
		19	-0.003	-0.009	40.600	0.000
		20	0.005	-0.000	40.680	0.000
		21	-0.015	-0.015	41.291	0.000
		22	-0.004	-0.010	41.347	0.000
		23	-0.022	-0.026	42.703	0.001
		24	0.013	0.015	43.175	0.001
		25	0.020	0.014	44.284	0.001
		26	0.004	0.001	44.324	0.001
		27	0.003	0.002	44.347	0.002
		28	0.030	0.024	46.990	0.001
		29	0.008	0.004	47.169	0.002
		30	-0.023	-0.023	48.687	0.002
		31	0.007	0.007	48.838	0.003
		32	-0.012	-0.014	49.251	0.004
		33	-0.002	-0.002	49.263	0.006
		34	0.012	0.016	49.693	0.007
		35	0.031	0.027	52.400	0.005
		36	-0.017	-0.020	53.244	0.006

*Probabilities may not be valid for this equation specification.

Date: 12/19/21 Time: 20:06
Sample: 1 2850
Included observations: 2846
Q-statistic probabilities adjusted for 6 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.015	0.015	0.6140	
		2	0.002	0.001	0.6214	
		3	0.039	0.039	4.9835	
		4	0.014	0.013	5.5353	
		5	0.016	0.016	6.2847	
		6	-0.037	-0.039	10.194	
		7	0.024	0.024	11.800	0.001
		8	0.003	0.001	11.829	0.003
		9	0.026	0.028	13.738	0.003
		10	0.033	0.031	16.757	0.002
		11	0.004	0.003	16.800	0.005
		12	0.009	0.005	17.048	0.009
		13	0.040	0.038	21.531	0.003
		14	-0.025	-0.028	23.291	0.003
		15	0.043	0.044	28.495	0.001
		16	0.015	0.011	29.122	0.001
		17	-0.008	-0.009	29.302	0.002
		18	0.016	0.012	30.003	0.003
		19	0.004	0.004	30.056	0.005
		20	0.007	0.001	30.205	0.007
		21	-0.023	-0.021	31.742	0.007
		22	-0.008	-0.011	31.913	0.010
		23	-0.017	-0.020	32.765	0.012
		24	-0.003	-0.000	32.786	0.018
		25	0.008	0.006	32.987	0.024
		26	0.005	0.005	33.045	0.033
		27	0.008	0.008	33.250	0.044
		28	0.021	0.017	34.573	0.043
		29	0.011	0.010	34.928	0.053
		30	-0.023	-0.023	36.412	0.050
		31	-0.002	-0.003	36.423	0.065
		32	-0.025	-0.025	38.286	0.057
		33	0.000	0.003	38.287	0.073
		34	0.008	0.011	38.483	0.090
		35	0.023	0.024	39.968	0.084
		36	-0.015	-0.016	40.639	0.093

*Probabilities may not be valid for this equation specification.

Figure 9: Correlogram of Residuals for Four GARCH-Type Models

In terms of white noise of the residual, TGARCH and GARCH-in-mean are recommended as they have relatively white noise signal.

Section 4.2.2. ARCH Effects Checking

Date: 12/19/21 Time: 19:51
Sample: 1 2850
Included observations: 2846

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.066	0.066	12.405	0.000
		2	0.094	0.090	37.728	0.000
		3	0.124	0.114	81.586	0.000
		4	0.115	0.096	119.20	0.000
		5	0.094	0.066	144.35	0.000
		6	0.102	0.068	174.33	0.000
		7	0.109	0.070	208.01	0.000
		8	0.089	0.046	230.55	0.000
		9	0.058	0.010	240.03	0.000
		10	0.133	0.088	290.65	0.000
		11	0.079	0.032	308.65	0.000
		12	0.072	0.023	323.57	0.000
		13	0.127	0.077	369.99	0.000
		14	0.126	0.075	415.33	0.000
		15	0.059	0.003	425.19	0.000
		16	0.131	0.073	474.22	0.000
		17	0.084	0.019	494.45	0.000
		18	0.095	0.033	520.32	0.000
		19	0.073	0.011	535.51	0.000
		20	0.113	0.046	572.15	0.000
		21	0.129	0.069	620.19	0.000
		22	0.053	-0.009	628.26	0.000
		23	0.098	0.028	655.66	0.000
		24	0.078	0.003	672.92	0.000
		25	0.082	0.023	692.14	0.000
		26	0.097	0.028	719.28	0.000
		27	0.065	-0.006	731.54	0.000
		28	0.075	0.008	747.77	0.000
		29	0.055	-0.011	756.37	0.000
		30	0.128	0.064	803.38	0.000
		31	0.076	0.010	820.00	0.000
		32	0.055	-0.009	828.59	0.000
		33	0.069	-0.001	842.26	0.000
		34	0.099	0.028	870.53	0.000
		35	0.075	0.015	886.72	0.000
		36	0.069	0.004	900.42	0.000

*Probabilities may not be valid for this equation specification.

Date: 12/19/21 Time: 19:54
Sample: 1 2850
Included observations: 2846

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.021	-0.021	1.2646	0.261
		2	0.018	0.018	2.2217	0.329
		3	0.011	0.012	2.5867	0.460
		4	0.013	0.013	3.0340	0.552
		5	0.004	0.005	3.0905	0.686
		6	-0.012	-0.012	3.4976	0.744
		7	0.012	0.011	3.9222	0.789
		8	-0.017	-0.017	4.7943	0.779
		9	-0.023	-0.024	6.3218	0.707
		10	0.027	0.027	8.4476	0.585
		11	-0.013	-0.011	8.9556	0.626
		12	0.010	0.009	9.2437	0.682
		13	0.015	0.016	9.8749	0.704
		14	0.009	0.009	10.108	0.754
		15	-0.029	-0.030	12.516	0.640
		16	0.013	0.012	13.037	0.670
		17	0.007	0.007	13.195	0.723
		18	-0.003	-0.002	13.217	0.779
		19	0.002	0.003	13.231	0.827
		20	0.005	0.004	13.292	0.864
		21	-0.003	-0.002	13.311	0.897
		22	-0.005	-0.004	13.370	0.922
		23	-0.001	-0.003	13.374	0.943
		24	0.007	0.005	13.506	0.957
		25	0.012	0.015	13.938	0.963
		26	0.007	0.006	14.093	0.972
		27	-0.003	-0.003	14.123	0.980
		28	-0.015	-0.015	14.793	0.981
		29	-0.019	-0.020	15.808	0.978
		30	0.022	0.020	17.147	0.971
		31	-0.009	-0.006	17.369	0.977
		32	-0.028	-0.028	19.620	0.958
		33	0.000	-0.000	19.620	0.968
		34	0.010	0.012	19.918	0.974
		35	0.020	0.021	21.099	0.969
		36	-0.012	-0.011	21.540	0.973

*Probabilities may not be valid for this equation specification.

Date: 12/19/21 Time: 20:00
Sample: 1 2850
Included observations: 2846

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.017	-0.017	0.7806	0.377
		2	0.024	0.024	2.4859	0.289
		3	0.020	0.021	3.6574	0.301
		4	0.006	0.006	3.7668	0.438
		5	0.014	0.013	4.3091	0.506
		6	-0.016	-0.016	5.0105	0.542
		7	0.020	0.019	6.1829	0.519
		8	-0.018	-0.017	7.0871	0.527
		9	-0.009	-0.010	7.3292	0.603
		10	0.029	0.028	9.6610	0.471
		11	-0.012	-0.009	10.046	0.526
		12	0.016	0.015	10.821	0.544
		13	0.013	0.014	11.299	0.586
		14	0.006	0.005	11.394	0.655
		15	-0.034	-0.035	14.652	0.477
		16	0.013	0.013	15.169	0.512
		17	0.006	0.006	15.287	0.575
		18	-0.002	0.000	15.299	0.641
		19	-0.004	-0.004	15.334	0.701
		20	-0.001	-0.002	15.338	0.757
		21	-0.003	-0.003	15.360	0.804
		22	-0.008	-0.007	15.535	0.838
		23	-0.005	-0.006	15.595	0.872
		24	-0.000	-0.000	15.595	0.902
		25	0.022	0.025	17.004	0.882
		26	0.005	0.004	17.067	0.907
		27	-0.009	-0.010	17.322	0.923
		28	-0.007	-0.008	17.466	0.939
		29	-0.023	-0.023	18.954	0.922
		30	0.026	0.024	20.925	0.890
		31	-0.011	-0.008	21.305	0.904
		32	-0.027	-0.028	23.447	0.864
		33	0.006	0.005	23.545	0.888
		34	0.012	0.015	23.974	0.899
		35	0.024	0.023	25.579	0.878
		36	-0.016	-0.014	26.326	0.881

*Probabilities may not be valid for this equation specification.

Date: 12/19/21 Time: 20:07
Sample: 1 2850
Included observations: 2846

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.021	-0.021	1.2675	0.260
		2	0.018	0.018	2.2274	0.328
		3	0.011	0.012	2.5946	0.458
		4	0.013	0.013	3.0423	0.551
		5	0.004	0.005	3.0990	0.685
		6	-0.012	-0.012	3.5072	0.743
		7	0.012	0.011	3.9324	0.788
		8	-0.018	-0.017	4.8107	0.778
		9	-0.023	-0.024	6.3348	0.706
		10	0.027	0.027	8.4591	0.584
		11	-0.013	-0.011	8.9671	0.625
		12	0.010	0.009	9.2560	0.681
		13	0.015	0.016	9.8886	0.703
		14	0.009	0.009	10.122	0.753
		15	-0.029	-0.030	12.530	0.639
		16	0.014	0.012	13.052	0.669
		17	0.007	0.007	13.210	0.722
		18	-0.003	-0.002	13.232	0.778
		19	0.002	0.003	13.246	0.826
		20	0.005	0.004	13.307	0.864
		21	-0.003	-0.002	13.326	0.897
		22	-0.005	-0.004	13.386	0.922
		23	-0.001	-0.003	13.390	0.943
		24	0.007	0.005	13.522	0.957
		25	0.012	0.015	13.957	0.962
		26	0.007	0.006	14.113	0.971
		27	-0.003	-0.003	14.143	0.980
		28	-0.015	-0.015	14.812	0.980
		29	-0.019	-0.020	15.825	0.977
		30	0.022	0.020	17.161	0.971
		31	-0.009	-0.006	17.385	0.977
		32	-0.028	-0.028	19.639	0.957
		33	0.000	-0.000	19.639	0.968
		34	0.010	0.012	19.936	0.974
		35	0.020	0.021	21.112	0.969
		36	-0.012	-0.011	21.555	0.973

*Probabilities may not be valid for this equation specification.

Figure 10: Correlogram of Residuals Squared for Four GARCH-Type Models

Heteroskedasticity Test: ARCH

F-statistic	20.21475	Prob. F(7,2831)	0.0000
Obs*R-squared	135.1479	Prob. Chi-Square(7)	0.0000

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 12/19/21 Time: 19:53
Sample (adjusted): 11 2849
Included observations: 2839 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.357121	0.037084	9.630103	0.0000
WGT_RESID^2(-1)	0.023174	0.018748	1.236103	0.2165
WGT_RESID^2(-2)	0.057435	0.018710	3.069787	0.0022
WGT_RESID^2(-3)	0.090810	0.018708	4.854014	0.0000
WGT_RESID^2(-4)	0.081785	0.018723	4.368163	0.0000
WGT_RESID^2(-5)	0.059290	0.018708	3.169183	0.0015
WGT_RESID^2(-6)	0.066228	0.018710	3.539643	0.0004
WGT_RESID^2(-7)	0.070388	0.018746	3.754780	0.0002

R-squared	0.047604	Mean dependent var	0.648538
Adjusted R-squared	0.045249	S.D. dependent var	1.487141
S.E. of regression	1.453106	Akaike info criterion	3.588098
Sum squared resid	5977.704	Schwarz criterion	3.604867
Log likelihood	-5085.304	Hannan-Quinn criter.	3.594147
F-statistic	20.21475	Durbin-Watson stat	2.006552
Prob(F-statistic)	0.000000		

Heteroskedasticity Test: ARCH

F-statistic	0.542404	Prob. F(7,2831)	0.8028
Obs*R-squared	3.802459	Prob. Chi-Square(7)	0.8022

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 12/19/21 Time: 19:56
Sample (adjusted): 11 2849
Included observations: 2839 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.975901	0.062245	15.67842	0.0000
WGT_RESID^2(-1)	-0.020390	0.018793	-1.085006	0.2780
WGT_RESID^2(-2)	0.017629	0.018804	0.937533	0.3486
WGT_RESID^2(-3)	0.012568	0.018806	0.668279	0.5040
WGT_RESID^2(-4)	0.012570	0.018806	0.668419	0.5039
WGT_RESID^2(-5)	0.004310	0.018807	0.229187	0.8187
WGT_RESID^2(-6)	-0.012224	0.018803	-0.650109	0.5157
WGT_RESID^2(-7)	0.011248	0.018800	0.598293	0.5497

R-squared	0.001339	Mean dependent var	1.001621
Adjusted R-squared	-0.001130	S.D. dependent var	2.014416
S.E. of regression	2.015554	Akaike info criterion	4.242479
Sum squared resid	11500.82	Schwarz criterion	4.259249
Log likelihood	-6014.199	Hannan-Quinn criter.	4.248528
F-statistic	0.542404	Durbin-Watson stat	1.999579
Prob(F-statistic)	0.802769		

Heteroskedasticity Test: ARCH

F-statistic	0.893909	Prob. F(7,2831)	0.5102
Obs*R-squared	6.261204	Prob. Chi-Square(7)	0.5096

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 12/19/21 Time: 20:01
Sample (adjusted): 11 2849
Included observations: 2839 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.949094	0.061592	15.40925	0.0000
WGT_RESID^2(-1)	-0.016927	0.018789	-0.900916	0.3677
WGT_RESID^2(-2)	0.024402	0.018773	1.299820	0.1938
WGT_RESID^2(-3)	0.021700	0.018777	1.155656	0.2479
WGT_RESID^2(-4)	0.006767	0.018781	0.360317	0.7186
WGT_RESID^2(-5)	0.012656	0.018777	0.673998	0.5004
WGT_RESID^2(-6)	-0.015668	0.018774	-0.834594	0.4040
WGT_RESID^2(-7)	0.018863	0.018773	1.004787	0.3151

R-squared	0.002205	Mean dependent var	1.001014
Adjusted R-squared	-0.000262	S.D. dependent var	2.003887
S.E. of regression	2.004149	Akaike info criterion	4.231130
Sum squared resid	11371.03	Schwarz criterion	4.247900
Log likelihood	-5998.089	Hannan-Quinn criter.	4.237179
F-statistic	0.893909	Durbin-Watson stat	1.999327
Prob(F-statistic)	0.510199		

Heteroskedasticity Test: ARCH

F-statistic	0.543850	Prob. F(7,2831)	0.8016
Obs*R-squared	3.812580	Prob. Chi-Square(7)	0.8011

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 12/19/21 Time: 20:08
Sample (adjusted): 11 2849
Included observations: 2839 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.975846	0.062245	15.67751	0.0000
WGT_RESID^2(-1)	-0.020415	0.018793	-1.086326	0.2774
WGT_RESID^2(-2)	0.017653	0.018804	0.938788	0.3479
WGT_RESID^2(-3)	0.012607	0.018806	0.670365	0.5027
WGT_RESID^2(-4)	0.012579	0.018806	0.668846	0.5036
WGT_RESID^2(-5)	0.004312	0.018806	0.229258	0.8187
WGT_RESID^2(-6)	-0.012240	0.018803	-0.650976	0.5151
WGT_RESID^2(-7)	0.011255	0.018800	0.598693	0.5494

R-squared	0.001343	Mean dependent var	1.001605
Adjusted R-squared	-0.001126	S.D. dependent var	2.014539
S.E. of regression	2.015673	Akaike info criterion	4.242597
Sum squared resid	11502.17	Schwarz criterion	4.259367
Log likelihood	-6014.367	Hannan-Quinn criter.	4.248646
F-statistic	0.543850	Durbin-Watson stat	1.999577
Prob(F-statistic)	0.801635		

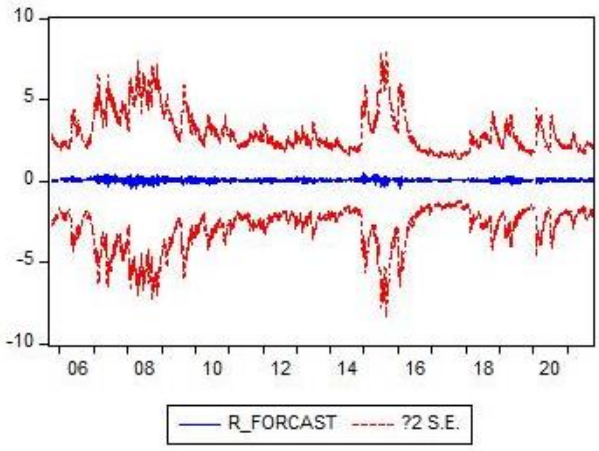
Figure 11: Heteroskedasticity Test for Four GARCH-Type Models

In terms of squared residual and ARCH effects, all three models involving extension GARCH are recommended as they don't have ARCH effects.

Therefore, at this step, I finally recommend the model to be ARMA(3,3)-TGARCH (1,1) since its white noise performance is better than ARMA(3,3)-GARCH-in-mean (1,1)

Section 5. Model Prediction

Finally, let's see the performance of the model recommended.



Forecast: R_FORCAST	
Actual: R	
Forecast sample: 9/28/2005 10/14/2021	
Included observations: 3893	
Root Mean Squared Error	1.581005
Mean Absolute Error	1.068237
Mean Abs. Percent Error	121.3359
Theil Inequality Coefficient	0.935342
Bias Proportion	0.000028
Variance Proportion	0.885226
Covariance Proportion	0.114746
Theil U2 Coefficient	1.045436
Symmetric MAPE	174.4949

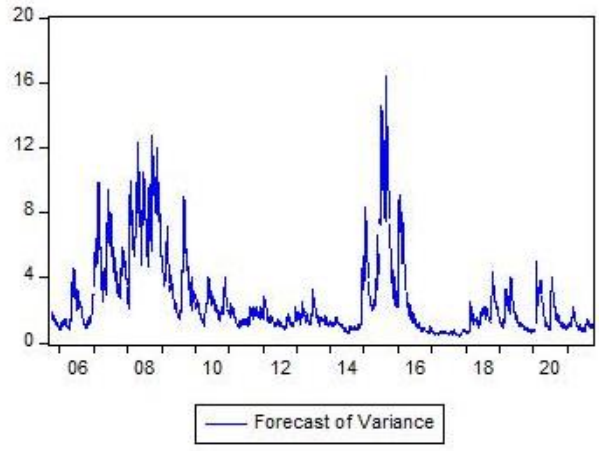


Figure 12: Forecast Result 1

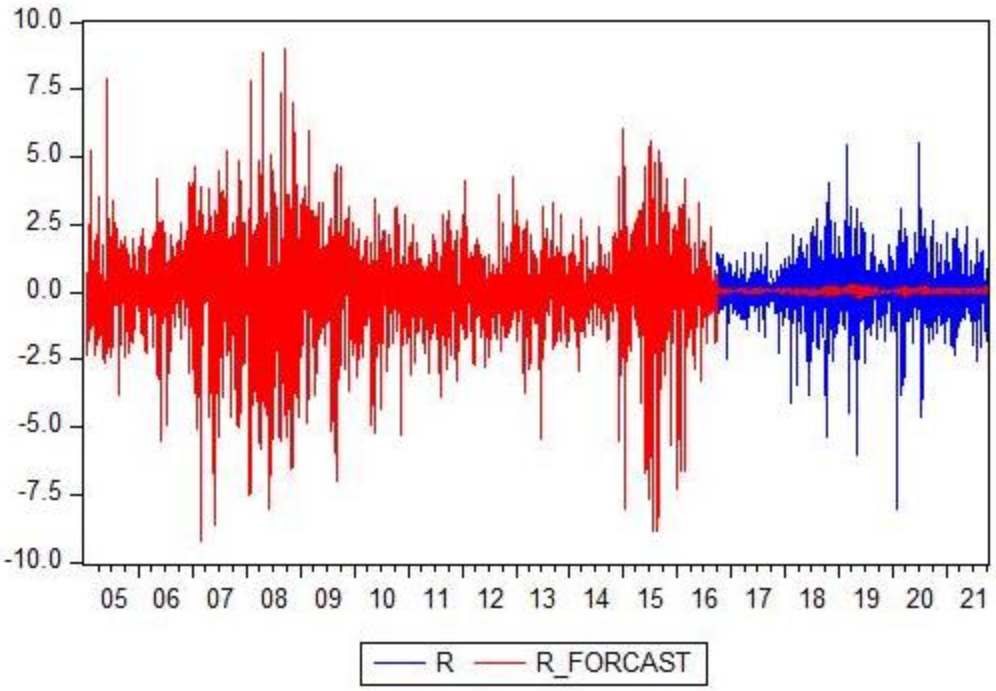


Figure 13: Forecast Result 2

Section 6. Interpreting and Limitations

From above discussion, I finally proposed an ARMA(3,3)-TGARCH(1,1) model for the Shanghai Stock Exchange (SSE) Composite Index return series. However, the model has limitations: white noise result is not perfectly ideal although all the indicators in the model are significant and the model is of no ARCH effects. The process can be further improved if some more complex hybrid model, like ARMA-EGARCH(1,1)-M model are further attempted.

Appendix

A1. Reference

Introductory Econometrics for Finance 3rd (third) Edition by Chris Brooks

A2. ARMA Models Result

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 16:55
Sample: 1 2849
Included observations: 2849
Convergence achieved after 6 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030921	0.034617	0.893231	0.3718
AR(1)	0.022337	0.013535	1.650324	0.0990
SIGMASQ	3.084930	0.049555	62.25259	0.0000

R-squared	0.000499	Mean dependent var	0.030911
Adjusted R-squared	-0.000203	S.D. dependent var	1.757144
S.E. of regression	1.757322	Akaike info criterion	3.966512
Sum squared resid	8788.965	Schwarz criterion	3.972782
Log likelihood	-5647.296	Hannan-Quinn criter.	3.968773
F-statistic	0.710760	Durbin-Watson stat	1.998682
Prob(F-statistic)	0.491358		

Inverted AR Roots	.02

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 16:58
Sample: 1 2849
Included observations: 2849
Convergence achieved after 13 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030936	0.034314	0.901556	0.3674
AR(1)	0.022946	0.013516	1.697672	0.0897
AR(2)	-0.027022	0.014111	-1.914919	0.0556
SIGMASQ	3.082675	0.050174	61.44017	0.0000

R-squared	0.001230	Mean dependent var	0.030911
Adjusted R-squared	0.000176	S.D. dependent var	1.757144
S.E. of regression	1.756989	Akaike info criterion	3.966484
Sum squared resid	8782.542	Schwarz criterion	3.974844
Log likelihood	-5646.256	Hannan-Quinn criter.	3.969499
F-statistic	1.167535	Durbin-Watson stat	1.998190
Prob(F-statistic)	0.320624		

Inverted AR Roots	.01+.16i	.01-.16i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:01
Sample: 1 2849
Included observations: 2849
Convergence achieved after 12 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030921	0.035616	0.868171	0.3854
AR(1)	0.023799	0.013534	1.758500	0.0788
AR(2)	-0.027752	0.014042	-1.976328	0.0482
AR(3)	0.031522	0.013264	2.376506	0.0175
SIGMASQ	3.079608	0.050077	61.49789	0.0000

R-squared	0.002224	Mean dependent var	0.030911
Adjusted R-squared	0.000820	S.D. dependent var	1.757144
S.E. of regression	1.756423	Akaike info criterion	3.966191
Sum squared resid	8773.803	Schwarz criterion	3.976642
Log likelihood	-5644.839	Hannan-Quinn criter.	3.969960
F-statistic	1.584435	Durbin-Watson stat	2.004315
Prob(F-statistic)	0.175627		

Inverted AR Roots	.29	-.14-.30i	-.14+.30i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:03
Sample: 1 2849
Included observations: 2849
Convergence achieved after 13 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030910	0.038525	0.802331	0.4224
AR(1)	0.021594	0.013429	1.607987	0.1079
AR(2)	-0.025817	0.014065	-1.835569	0.0665
AR(3)	0.029842	0.013267	2.249411	0.0246
AR(4)	0.070009	0.013628	5.137202	0.0000
SIGMASQ	3.064482	0.049726	61.62694	0.0000

R-squared	0.007124	Mean dependent var	0.030911
Adjusted R-squared	0.005378	S.D. dependent var	1.757144
S.E. of regression	1.752412	Akaike info criterion	3.961976
Sum squared resid	8730.708	Schwarz criterion	3.974517
Log likelihood	-5637.835	Hannan-Quinn criter.	3.966499
F-statistic	4.079919	Durbin-Watson stat	1.999897
Prob(F-statistic)	0.001080		

Inverted AR Roots	.53	-.02-.53i	-.02+.53i	-.47

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:05
Sample: 1 2849
Included observations: 2849
Convergence achieved after 12 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030910	0.038569	0.801402	0.4230
AR(1)	0.021599	0.013542	1.595004	0.1108
AR(2)	-0.025815	0.014227	-1.814463	0.0697
AR(3)	0.029840	0.013275	2.247928	0.0247
AR(4)	0.070010	0.013739	5.095680	0.0000
AR(5)	-6.93E-05	0.014546	-0.004764	0.9962
SIGMASQ	3.064482	0.049797	61.53968	0.0000

R-squared	0.007124	Mean dependent var	0.030911
Adjusted R-squared	0.005028	S.D. dependent var	1.757144
S.E. of regression	1.752721	Akaike info criterion	3.962678
Sum squared resid	8730.708	Schwarz criterion	3.977309
Log likelihood	-5637.835	Hannan-Quinn criter.	3.967955
F-statistic	3.398739	Durbin-Watson stat	1.999915
Prob(F-statistic)	0.002415		

Inverted AR Roots	.53	.00	-.02+.53i	-.02-.53i
	-.47			

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:13
Sample: 1 2849
Included observations: 2849
Convergence achieved after 29 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030923	0.039025	0.792400	0.4282
AR(1)	-0.880913	0.051856	-16.98756	0.0000
AR(2)	-0.007148	0.018833	-0.379528	0.7043
AR(3)	0.007249	0.018176	0.398852	0.6900
AR(4)	0.096332	0.017646	5.459104	0.0000
AR(5)	0.088785	0.014204	6.250814	0.0000
MA(1)	0.903946	0.050069	18.05401	0.0000
SIGMASQ	3.053507	0.049990	61.08262	0.0000

R-squared	0.010680	Mean dependent var	0.030911
Adjusted R-squared	0.008242	S.D. dependent var	1.757144
S.E. of regression	1.749887	Akaike info criterion	3.959804
Sum squared resid	8699.441	Schwarz criterion	3.976525
Log likelihood	-5632.741	Hannan-Quinn criter.	3.965834
F-statistic	4.381388	Durbin-Watson stat	1.994815
Prob(F-statistic)	0.000076		

Inverted AR Roots	.56	-.00+.56i	-.00-.56i	-.59
	-.85			
Inverted MA Roots	-.90			

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:13
Sample: 1 2849
Included observations: 2849
Convergence achieved after 29 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030923	0.039025	0.792400	0.4282
AR(1)	-0.880913	0.051856	-16.98756	0.0000
AR(2)	-0.007148	0.018833	-0.379528	0.7043
AR(3)	0.007249	0.018176	0.398852	0.6900
AR(4)	0.096332	0.017646	5.459104	0.0000
AR(5)	0.088785	0.014204	6.250814	0.0000
MA(1)	0.903946	0.050069	18.05401	0.0000
SIGMASQ	3.053507	0.049990	61.08262	0.0000

R-squared	0.010680	Mean dependent var	0.030911
Adjusted R-squared	0.008242	S.D. dependent var	1.757144
S.E. of regression	1.749887	Akaike info criterion	3.959804
Sum squared resid	8699.441	Schwarz criterion	3.976525
Log likelihood	-5632.741	Hannan-Quinn criter.	3.965834
F-statistic	4.381388	Durbin-Watson stat	1.994815
Prob(F-statistic)	0.000076		

Inverted AR Roots	.56	-.00+.56i	-.00-.56i	-.59
	-.85			
Inverted MA Roots	-.90			

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:16
Sample: 1 2849
Included observations: 2849
Convergence achieved after 55 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030907	0.037567	0.822716	0.4107
AR(1)	0.006611	0.194935	0.033914	0.9729
AR(2)	-0.583075	0.103412	-5.638373	0.0000
AR(3)	0.042396	0.016096	2.633998	0.0085
AR(4)	0.060044	0.017619	3.407898	0.0007
AR(5)	0.010689	0.026624	0.401496	0.6881
MA(1)	0.015239	0.195912	0.077784	0.9380
MA(2)	0.562630	0.104136	5.402853	0.0000
SIGMASQ	3.054097	0.050076	60.98943	0.0000

R-squared	0.010489	Mean dependent var	0.030911
Adjusted R-squared	0.007702	S.D. dependent var	1.757144
S.E. of regression	1.750364	Akaike info criterion	3.960695
Sum squared resid	8701.122	Schwarz criterion	3.979506
Log likelihood	-5633.010	Hannan-Quinn criter.	3.967479
F-statistic	3.763024	Durbin-Watson stat	1.999708
Prob(F-statistic)	0.000213		

Inverted AR Roots	.38	-.01-.82i	-.01+.82i	-.17+.11i
	-.17-.11i			
Inverted MA Roots	-.01+.75i	-.01-.75i		

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:19
Sample: 1 2849
Included observations: 2849
Convergence achieved after 27 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030924	0.037848	0.817067	0.4140
AR(1)	-0.990050	0.114824	-8.622298	0.0000
AR(2)	-0.548266	0.137214	-3.995700	0.0001
AR(3)	-0.396548	0.104892	-3.780530	0.0002
AR(4)	0.097680	0.020608	4.739797	0.0000
AR(5)	0.085797	0.016328	5.254705	0.0000
MA(1)	1.014262	0.116716	8.689985	0.0000
MA(2)	0.549685	0.141678	3.879823	0.0001
MA(3)	0.420933	0.107669	3.909512	0.0001
SIGMASQ	3.045791	0.050447	60.37555	0.0000

R-squared	0.013180	Mean dependent var	0.030911
Adjusted R-squared	0.010052	S.D. dependent var	1.757144
S.E. of regression	1.748290	Akaike info criterion	3.958683
Sum squared resid	8677.459	Schwarz criterion	3.979584
Log likelihood	-5629.144	Hannan-Quinn criter.	3.966221
F-statistic	4.213069	Durbin-Watson stat	1.998866
Prob(F-statistic)	0.000020		

Inverted AR Roots	.39 -.88	-.04+.78i	-.04-.78i	-.41
Inverted MA Roots	-.05+.68i	-.05-.68i	-.92	

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:21
Sample: 1 2849
Included observations: 2849
Convergence achieved after 85 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030916	0.036014	0.858443	0.3907
AR(1)	-1.543634	0.106212	-14.53352	0.0000
AR(2)	-1.098393	0.188145	-5.838003	0.0000
AR(3)	-0.974535	0.180142	-5.409804	0.0000
AR(4)	-0.448880	0.102846	-4.364573	0.0000
AR(5)	0.087984	0.016965	5.186327	0.0000
MA(1)	1.570434	0.106463	14.75105	0.0000
MA(2)	1.109702	0.188381	5.890742	0.0000
MA(3)	0.985434	0.177751	5.543913	0.0000
MA(4)	0.543546	0.095486	5.692431	0.0000
SIGMASQ	3.044074	0.050213	60.62339	0.0000

R-squared	0.013736	Mean dependent var	0.030911
Adjusted R-squared	0.010261	S.D. dependent var	1.757144
S.E. of regression	1.748105	Akaike info criterion	3.958829
Sum squared resid	8672.567	Schwarz criterion	3.981820
Log likelihood	-5628.352	Hannan-Quinn criter.	3.967120
F-statistic	3.952645	Durbin-Watson stat	2.002048
Prob(F-statistic)	0.000022		

Inverted AR Roots	.14 -.91-.22i	.07+.84i	.07-.84i	-.91+.22i
Inverted MA Roots	.11-.80i	.11+.80i	-.89-.20i	-.89+.20i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:22
Sample: 1 2849
Included observations: 2849
Convergence achieved after 51 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030930	0.037642	0.821692	0.4113
AR(1)	-0.835131	0.148256	-5.633041	0.0000
AR(2)	-0.940416	0.196563	-4.784291	0.0000
AR(3)	-0.149268	0.263032	-0.567491	0.5704
AR(4)	-0.235308	0.186397	-1.262399	0.2069
AR(5)	0.315260	0.133159	2.367540	0.0180
MA(1)	0.860398	0.151750	5.669856	0.0000
MA(2)	0.941682	0.200371	4.699694	0.0000
MA(3)	0.183546	0.265112	0.692332	0.4888
MA(4)	0.313270	0.187088	1.674453	0.0942
MA(5)	-0.226823	0.135695	-1.671563	0.0947
SIGMASQ	3.039078	0.050579	60.08533	0.0000

R-squared	0.015355	Mean dependent var	0.030911
Adjusted R-squared	0.011537	S.D. dependent var	1.757144
S.E. of regression	1.746978	Akaike info criterion	3.957921
Sum squared resid	8658.332	Schwarz criterion	3.983002
Log likelihood	-5626.059	Hannan-Quinn criter.	3.966967
F-statistic	4.021975	Durbin-Watson stat	1.999822
Prob(F-statistic)	0.000007		

Inverted AR Roots	.48 -.69+.69i	.03-.83i	.03+.83i	-.69-.69i
Inverted MA Roots	.38 -.68+.71i	.06+.78i	.06-.78i	-.68-.71i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:25
Sample: 1 2849
Included observations: 2849
Convergence achieved after 22 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030910	0.038571	0.801368	0.4230
AR(1)	0.021228	0.206470	0.102816	0.9181
AR(2)	-0.025809	0.015550	-1.659719	0.0971
AR(3)	0.029832	0.014488	2.059124	0.0396
AR(4)	0.070020	0.015773	4.439388	0.0000
MA(1)	0.000368	0.207775	0.001769	0.9986
SIGMASQ	3.064482	0.049796	61.54021	0.0000

R-squared	0.007124	Mean dependent var	0.030911
Adjusted R-squared	0.005028	S.D. dependent var	1.757144
S.E. of regression	1.752721	Akaike info criterion	3.962678
Sum squared resid	8730.708	Schwarz criterion	3.977309
Log likelihood	-5637.835	Hannan-Quinn criter.	3.967955
F-statistic	3.398738	Durbin-Watson stat	1.999903
Prob(F-statistic)	0.002415		

Inverted AR Roots	.53	-.02-.53i	-.02+.53i	-.47
Inverted MA Roots	-.00			

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:25
Sample: 1 2849
Included observations: 2849
Convergence achieved after 22 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030910	0.038571	0.801368	0.4230
AR(1)	0.021228	0.206470	0.102816	0.9181
AR(2)	-0.025809	0.015550	-1.659719	0.0971
AR(3)	0.029832	0.014488	2.059124	0.0396
AR(4)	0.070020	0.015773	4.439388	0.0000
MA(1)	0.000368	0.207775	0.001769	0.9986
SIGMASQ	3.064482	0.049796	61.54021	0.0000

R-squared	0.007124	Mean dependent var	0.030911
Adjusted R-squared	0.005028	S.D. dependent var	1.757144
S.E. of regression	1.752721	Akaike info criterion	3.962678
Sum squared resid	8730.708	Schwarz criterion	3.977309
Log likelihood	-5637.835	Hannan-Quinn criter.	3.967955
F-statistic	3.398738	Durbin-Watson stat	1.999903
Prob(F-statistic)	0.002415		

Inverted AR Roots	.53	-.02-.53i	-.02+.53i	-.47
Inverted MA Roots	-.00			

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:26
Sample: 1 2849
Included observations: 2849
Convergence achieved after 23 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030908	0.037209	0.830649	0.4062
AR(1)	0.078994	0.108456	0.728356	0.4665
AR(2)	-0.589666	0.100487	-5.868078	0.0000
AR(3)	0.042544	0.016004	2.658291	0.0079
AR(4)	0.057046	0.017683	3.226100	0.0013
MA(1)	-0.056957	0.110461	-0.515628	0.6062
MA(2)	0.567883	0.100578	5.646173	0.0000
SIGMASQ	3.054188	0.049958	61.13557	0.0000

R-squared	0.010460	Mean dependent var	0.030911
Adjusted R-squared	0.008021	S.D. dependent var	1.757144
S.E. of regression	1.750082	Akaike info criterion	3.960023
Sum squared resid	8701.380	Schwarz criterion	3.976744
Log likelihood	-5633.053	Hannan-Quinn criter.	3.966053
F-statistic	4.289945	Durbin-Watson stat	2.000240
Prob(F-statistic)	0.000099		

Inverted AR Roots	.33	.01+.82i	.01-.82i	-.26
Inverted MA Roots	.03-.75i	.03+.75i		

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:29
Sample: 1 2849
Included observations: 2849
Convergence achieved after 22 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030907	0.037494	0.824322	0.4098
AR(1)	0.141888	0.311866	0.454965	0.6492
AR(2)	-0.595399	0.120079	-4.958390	0.0000
AR(3)	0.100178	0.256249	0.390938	0.6959
AR(4)	0.054261	0.025795	2.103545	0.0355
MA(1)	-0.119965	0.312849	-0.383461	0.7014
MA(2)	0.572129	0.116117	4.927194	0.0000
MA(3)	-0.055298	0.244788	-0.225902	0.8213
SIGMASQ	3.054136	0.050171	60.87474	0.0000

R-squared	0.010476	Mean dependent var	0.030911
Adjusted R-squared	0.007689	S.D. dependent var	1.757144
S.E. of regression	1.750376	Akaike info criterion	3.960708
Sum squared resid	8701.235	Schwarz criterion	3.979519
Log likelihood	-5633.029	Hannan-Quinn criter.	3.967492
F-statistic	3.758376	Durbin-Watson stat	1.999950
Prob(F-statistic)	0.000217		

Inverted AR Roots	.37	-.00+.82i	-.00-.82i	-.22
Inverted MA Roots	.10	.01-.75i	.01+.75i	

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:30
Sample: 1 2849
Included observations: 2849
Convergence achieved after 38 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030909	0.037280	0.829119	0.4071
AR(1)	0.165032	0.440927	0.374284	0.7082
AR(2)	-0.674315	0.325540	-2.071378	0.0384
AR(3)	0.127102	0.351501	0.361598	0.7177
AR(4)	-0.030635	0.264802	-0.115692	0.9079
MA(1)	-0.143104	0.441063	-0.324452	0.7456
MA(2)	0.651107	0.317559	2.050349	0.0404
MA(3)	-0.081804	0.332247	-0.246213	0.8055
MA(4)	0.082584	0.239457	0.344879	0.7302
SIGMASQ	3.054002	0.050323	60.68778	0.0000

R-squared	0.010520	Mean dependent var	0.030911
Adjusted R-squared	0.007383	S.D. dependent var	1.757144
S.E. of regression	1.750645	Akaike info criterion	3.961366
Sum squared resid	8700.851	Schwarz criterion	3.982267
Log likelihood	-5632.966	Hannan-Quinn criter.	3.968904
F-statistic	3.353668	Durbin-Watson stat	1.999799
Prob(F-statistic)	0.000428		

Inverted AR Roots	.10-.19i	.10+.19i	-.02-.80i	-.02+.80i
Inverted MA Roots	.09-.40i	.09+.40i	-.02+.70i	-.02-.70i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:33
Sample: 1 2849
Included observations: 2849
Convergence achieved after 34 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030927	0.036909	0.837918	0.4021
AR(1)	-1.152172	0.206883	-5.569194	0.0000
AR(2)	-0.812129	0.221420	-3.667826	0.0002
AR(3)	-0.676912	0.207636	-3.260095	0.0011
AR(4)	-0.159357	0.179608	-0.887246	0.3750
MA(1)	1.176869	0.206960	5.686458	0.0000
MA(2)	0.817748	0.225315	3.629347	0.0003
MA(3)	0.700305	0.204060	3.431865	0.0006
MA(4)	0.254388	0.179743	1.415292	0.1571
MA(5)	0.086028	0.016602	5.181902	0.0000
SIGMASQ	3.045402	0.050566	60.22630	0.0000

R-squared	0.013306	Mean dependent var	0.030911
Adjusted R-squared	0.009829	S.D. dependent var	1.757144
S.E. of regression	1.748487	Akaike info criterion	3.959258
Sum squared resid	8676.350	Schwarz criterion	3.982249
Log likelihood	-5628.963	Hannan-Quinn criter.	3.967550
F-statistic	3.827175	Durbin-Watson stat	1.999719
Prob(F-statistic)	0.000037		

Inverted AR Roots	.02-.76i	.02+.76i	-.32	-.87
Inverted MA Roots	.10-.67i -.91	.10+.67i	-.23+.39i	-.23-.39i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:35
Sample: 1 2849
Included observations: 2849
Convergence achieved after 29 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030906	0.038399	0.804869	0.4210
AR(1)	0.556168	0.185455	2.998931	0.0027
AR(2)	-0.040276	0.016767	-2.402094	0.0164
AR(3)	0.059423	0.013554	4.384231	0.0000
MA(1)	-0.533515	0.185058	-2.882959	0.0040
SIGMASQ	3.073926	0.050038	61.43160	0.0000

R-squared	0.004065	Mean dependent var	0.030911
Adjusted R-squared	0.002313	S.D. dependent var	1.757144
S.E. of regression	1.755111	Akaike info criterion	3.965049
Sum squared resid	8757.614	Schwarz criterion	3.977589
Log likelihood	-5642.212	Hannan-Quinn criter.	3.969571
F-statistic	2.320515	Durbin-Watson stat	2.002926
Prob(F-statistic)	0.040938		

Inverted AR Roots	.64	-.04+.30i	-.04-.30i
Inverted MA Roots	.53		

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:36
Sample: 1 2849
Included observations: 2849
Convergence achieved after 50 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030921	0.034545	0.895095	0.3708
AR(1)	0.208605	0.073258	2.847534	0.0044
AR(2)	-0.816560	0.053917	-15.14464	0.0000
AR(3)	0.056612	0.015951	3.549172	0.0004
MA(1)	-0.185026	0.073066	-2.532309	0.0114
MA(2)	0.770797	0.059425	12.97081	0.0000
SIGMASQ	3.059036	0.049983	61.20151	0.0000

R-squared	0.008889	Mean dependent var	0.030911
Adjusted R-squared	0.006796	S.D. dependent var	1.757144
S.E. of regression	1.751163	Akaike info criterion	3.960907
Sum squared resid	8715.194	Schwarz criterion	3.975537
Log likelihood	-5635.312	Hannan-Quinn criter.	3.966183
F-statistic	4.247998	Durbin-Watson stat	2.002885
Prob(F-statistic)	0.000289		

Inverted AR Roots	.07	.07-.90i	.07+.90i
Inverted MA Roots	.09+.87i	.09-.87i	

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:38
Sample: 1 2849
Included observations: 2849
Convergence achieved after 36 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030900	0.038018	0.812796	0.4164
AR(1)	0.705658	0.132632	5.320417	0.0000
AR(2)	-0.820286	0.074976	-10.94071	0.0000
AR(3)	0.485936	0.128661	3.776877	0.0002
MA(1)	-0.682464	0.138214	-4.937717	0.0000
MA(2)	0.776425	0.082834	9.373240	0.0000
MA(3)	-0.408172	0.132883	-3.071664	0.0021
SIGMASQ	3.055458	0.050344	60.69194	0.0000

R-squared	0.010048	Mean dependent var	0.030911
Adjusted R-squared	0.007609	S.D. dependent var	1.757144
S.E. of regression	1.750446	Akaike info criterion	3.960439
Sum squared resid	8705.001	Schwarz criterion	3.977160
Log likelihood	-5633.645	Hannan-Quinn criter.	3.966469
F-statistic	4.119338	Durbin-Watson stat	2.002977
Prob(F-statistic)	0.000163		

Inverted AR Roots	.63	.04+.88i	.04-.88i
Inverted MA Roots	.57	.06-.84i	.06+.84i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:39
Sample: 1 2849
Included observations: 2849
Convergence achieved after 26 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030908	0.037319	0.828221	0.4076
AR(1)	0.152029	0.279967	0.543026	0.5872
AR(2)	-0.644027	0.095785	-6.723702	0.0000
AR(3)	0.114944	0.232170	0.495087	0.6206
MA(1)	-0.130106	0.280402	-0.463998	0.6427
MA(2)	0.621105	0.094728	6.556695	0.0000
MA(3)	-0.070107	0.221576	-0.316403	0.7517
MA(4)	0.053483	0.022590	2.367520	0.0180
SIGMASQ	3.054020	0.050223	60.80943	0.0000

R-squared	0.010514	Mean dependent var	0.030911
Adjusted R-squared	0.007727	S.D. dependent var	1.757144
S.E. of regression	1.750342	Akaike info criterion	3.960670
Sum squared resid	8700.903	Schwarz criterion	3.979481
Log likelihood	-5632.974	Hannan-Quinn criter.	3.967454
F-statistic	3.772068	Durbin-Watson stat	1.999851
Prob(F-statistic)	0.000207		

Inverted AR Roots	.18	-.01+.81i	-.01-.81i	
Inverted MA Roots	.07-.31i	.07+.31i	-.00+.72i	-.00-.72i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:41
Sample: 1 2849
Included observations: 2849
Convergence achieved after 30 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030925	0.037425	0.826337	0.4087
AR(1)	-0.983534	0.108748	-9.044180	0.0000
AR(2)	-0.626395	0.129114	-4.851474	0.0000
AR(3)	-0.484701	0.099620	-4.865480	0.0000
MA(1)	1.008743	0.110332	9.142783	0.0000
MA(2)	0.628544	0.133089	4.722739	0.0000
MA(3)	0.512416	0.103288	4.961037	0.0000
MA(4)	0.097016	0.021158	4.585376	0.0000
MA(5)	0.083442	0.016091	5.185624	0.0000
SIGMASQ	3.045948	0.050462	60.36150	0.0000

R-squared	0.013129	Mean dependent var	0.030911
Adjusted R-squared	0.010001	S.D. dependent var	1.757144
S.E. of regression	1.748335	Akaike info criterion	3.958734
Sum squared resid	8677.906	Schwarz criterion	3.979635
Log likelihood	-5629.217	Hannan-Quinn criter.	3.966272
F-statistic	4.196595	Durbin-Watson stat	2.000967
Prob(F-statistic)	0.000021		

Inverted AR Roots	-.05-.74i	-.05+.74i	-.89	
Inverted MA Roots	.10-.50i	.10+.50i	-.14+.57i	-.14-.57i
	-.92			

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:42
Sample: 1 2849
Included observations: 2849
Convergence achieved after 19 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030935	0.034432	0.898427	0.3690
AR(1)	-0.171430	0.433724	-0.395251	0.6927
AR(2)	-0.025094	0.018799	-1.334840	0.1820
MA(1)	0.194606	0.433258	0.449169	0.6533
SIGMASQ	3.082149	0.050229	61.36201	0.0000

R-squared	0.001400	Mean dependent var	0.030911
Adjusted R-squared	-0.000004	S.D. dependent var	1.757144
S.E. of regression	1.757148	Akaike info criterion	3.967015
Sum squared resid	8781.043	Schwarz criterion	3.977466
Log likelihood	-5646.013	Hannan-Quinn criter.	3.970784
F-statistic	0.996896	Durbin-Watson stat	1.999272
Prob(F-statistic)	0.407878		

Inverted AR Roots	-.09+.13i	-.09-.13i	
Inverted MA Roots	-.19		

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:43
Sample: 1 2849
Included observations: 2849
Convergence achieved after 43 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030938	0.033272	0.929833	0.3525
AR(1)	-0.122075	0.078972	-1.545796	0.1223
AR(2)	-0.776769	0.070391	-11.03508	0.0000
MA(1)	0.131260	0.087071	1.507502	0.1318
MA(2)	0.726279	0.078185	9.289235	0.0000
SIGMASQ	3.065469	0.050325	60.91375	0.0000

R-squared	0.006804	Mean dependent var	0.030911
Adjusted R-squared	0.005058	S.D. dependent var	1.757144
S.E. of regression	1.752695	Akaike info criterion	3.962300
Sum squared resid	8733.521	Schwarz criterion	3.974840
Log likelihood	-5638.296	Hannan-Quinn criter.	3.966822
F-statistic	3.895493	Durbin-Watson stat	1.969851
Prob(F-statistic)	0.001604		

Inverted AR Roots	-.06+.88i	-.06-.88i	
Inverted MA Roots	-.07-.85i	-.07+.85i	

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:44
Sample: 1 2849
Included observations: 2849
Convergence achieved after 49 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030921	0.034218	0.903634	0.3663
AR(1)	0.136451	0.057442	2.375468	0.0176
AR(2)	-0.833865	0.048090	-17.33962	0.0000
MA(1)	-0.115445	0.059524	-1.939463	0.0525
MA(2)	0.789770	0.053647	14.72173	0.0000
MA(3)	0.048559	0.015370	3.159282	0.0016
SIGMASQ	3.059589	0.050003	61.18751	0.0000

R-squared	0.008709	Mean dependent var	0.030911
Adjusted R-squared	0.006617	S.D. dependent var	1.757144
S.E. of regression	1.751321	Akaike info criterion	3.961088
Sum squared resid	8716.770	Schwarz criterion	3.975719
Log likelihood	-5635.570	Hannan-Quinn criter.	3.966365
F-statistic	4.161605	Durbin-Watson stat	1.997309
Prob(F-statistic)	0.000360		

Inverted AR Roots	.07+.91i	.07-.91i	
Inverted MA Roots	.09+.89i	.09-.89i	-.06

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:46
Sample: 1 2849
Included observations: 2849
Convergence achieved after 21 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030911	0.036707	0.842085	0.3998
AR(1)	0.022021	0.095610	0.230320	0.8179
AR(2)	-0.641001	0.088192	-7.268228	0.0000
MA(1)	-0.000442	0.098355	-0.004495	0.9964
MA(2)	0.620882	0.089087	6.969425	0.0000
MA(3)	0.038801	0.015249	2.544531	0.0110
MA(4)	0.058385	0.016872	3.460393	0.0005
SIGMASQ	3.054204	0.050000	61.08394	0.0000

R-squared	0.010454	Mean dependent var	0.030911
Adjusted R-squared	0.008016	S.D. dependent var	1.757144
S.E. of regression	1.750087	Akaike info criterion	3.960028
Sum squared resid	8701.428	Schwarz criterion	3.976749
Log likelihood	-5633.061	Hannan-Quinn criter.	3.966059
F-statistic	4.287703	Durbin-Watson stat	1.999216
Prob(F-statistic)	0.000100		

Inverted AR Roots	.01+.80i	.01-.80i		
Inverted MA Roots	.05+.72i	.05-.72i	-.05+.33i	-.05-.33i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:47
Sample: 1 2849
Included observations: 2849
Convergence achieved after 30 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030925	0.037425	0.826337	0.4087
AR(1)	-0.983534	0.108748	-9.044180	0.0000
AR(2)	-0.626395	0.129114	-4.851474	0.0000
AR(3)	-0.484701	0.099620	-4.865480	0.0000
MA(1)	1.008743	0.110332	9.142783	0.0000
MA(2)	0.628544	0.133089	4.722739	0.0000
MA(3)	0.512416	0.103288	4.961037	0.0000
MA(4)	0.097016	0.021158	4.585376	0.0000
MA(5)	0.083442	0.016091	5.185624	0.0000
SIGMASQ	3.045948	0.050462	60.36150	0.0000

R-squared	0.013129	Mean dependent var	0.030911
Adjusted R-squared	0.010001	S.D. dependent var	1.757144
S.E. of regression	1.748335	Akaike info criterion	3.958734
Sum squared resid	8677.906	Schwarz criterion	3.979635
Log likelihood	-5629.217	Hannan-Quinn criter.	3.966272
F-statistic	4.196595	Durbin-Watson stat	2.000967
Prob(F-statistic)	0.000021		

Inverted AR Roots	-.05-.74i	-.05+.74i	-.89	
Inverted MA Roots	.10-.50i -.92	.10+.50i	-.14+.57i	-.14-.57i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:49
Sample: 1 2849
Included observations: 2849
Convergence achieved after 40 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030933	0.034142	0.906001	0.3650
AR(1)	-0.911133	0.048254	-18.88200	0.0000
MA(1)	0.931897	0.043524	21.41100	0.0000
SIGMASQ	3.078638	0.049722	61.91658	0.0000

R-squared	0.002538	Mean dependent var	0.030911
Adjusted R-squared	0.001486	S.D. dependent var	1.757144
S.E. of regression	1.755838	Akaike info criterion	3.965179
Sum squared resid	8771.041	Schwarz criterion	3.973540
Log likelihood	-5644.398	Hannan-Quinn criter.	3.968194
F-statistic	2.412593	Durbin-Watson stat	1.992163
Prob(F-statistic)	0.064921		

Inverted AR Roots	-.91
Inverted MA Roots	-.93

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:51
Sample: 1 2849
Included observations: 2849
Convergence achieved after 19 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030934	0.034569	0.894839	0.3709
AR(1)	-0.182572	0.487159	-0.374768	0.7079
MA(1)	0.206229	0.486469	0.423930	0.6716
MA(2)	-0.021967	0.020169	-1.089098	0.2762
SIGMASQ	3.082360	0.050203	61.39773	0.0000

R-squared	0.001332	Mean dependent var	0.030911
Adjusted R-squared	-0.000073	S.D. dependent var	1.757144
S.E. of regression	1.757208	Akaike info criterion	3.967083
Sum squared resid	8781.645	Schwarz criterion	3.977534
Log likelihood	-5646.110	Hannan-Quinn criter.	3.970852
F-statistic	0.948059	Durbin-Watson stat	2.000379
Prob(F-statistic)	0.435023		

Inverted AR Roots	-.18	
Inverted MA Roots	.08	-.28

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:52
Sample: 1 2849
Included observations: 2849
Convergence achieved after 29 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030906	0.038070	0.811811	0.4170
AR(1)	0.510142	0.193868	2.631395	0.0085
MA(1)	-0.490492	0.193163	-2.539262	0.0112
MA(2)	-0.032049	0.016410	-1.953008	0.0509
MA(3)	0.056501	0.013752	4.108524	0.0000
SIGMASQ	3.074707	0.049842	61.68898	0.0000

R-squared	0.003811	Mean dependent var	0.030911
Adjusted R-squared	0.002059	S.D. dependent var	1.757144
S.E. of regression	1.755334	Akaike info criterion	3.965302
Sum squared resid	8759.840	Schwarz criterion	3.977843
Log likelihood	-5642.573	Hannan-Quinn criter.	3.969825
F-statistic	2.175444	Durbin-Watson stat	1.997048
Prob(F-statistic)	0.054182		

Inverted AR Roots	.51			
Inverted MA Roots	.39+.21i	.39-.21i	-.29	

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:54
Sample: 1 2849
Included observations: 2849
Convergence achieved after 28 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030923	0.036939	0.837127	0.4026
AR(1)	-0.005740	0.211309	-0.027165	0.9783
MA(1)	0.027122	0.211414	0.128289	0.8979
MA(2)	-0.018000	0.015646	-1.150421	0.2501
MA(4)	0.067009	0.013989	4.790105	0.0000
SIGMASQ	3.068625	0.049633	61.82662	0.0000

R-squared	0.005782	Mean dependent var	0.030911
Adjusted R-squared	0.004033	S.D. dependent var	1.757144
S.E. of regression	1.753597	Akaike info criterion	3.963325
Sum squared resid	8742.512	Schwarz criterion	3.975866
Log likelihood	-5639.757	Hannan-Quinn criter.	3.967848
F-statistic	3.306741	Durbin-Watson stat	1.999943
Prob(F-statistic)	0.005565		

Inverted AR Roots	-.01			
Inverted MA Roots	.36-.35i	.36+.35i	-.37-.35i	-.37+.35i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:55
Sample: 1 2849
Included observations: 2849
Convergence achieved after 25 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030924	0.038748	0.798069	0.4249
AR(1)	-0.865744	0.060916	-14.21210	0.0000
MA(1)	0.893253	0.061490	14.52687	0.0000
MA(2)	0.003306	0.019319	0.171145	0.8641
MA(3)	0.012557	0.018952	0.662577	0.5077
MA(4)	0.090751	0.018417	4.927587	0.0000
MA(5)	0.085818	0.014623	5.868647	0.0000
SIGMASQ	3.055070	0.050028	61.06675	0.0000

R-squared	0.010174	Mean dependent var	0.030911
Adjusted R-squared	0.007735	S.D. dependent var	1.757144
S.E. of regression	1.750335	Akaike info criterion	3.960315
Sum squared resid	8703.895	Schwarz criterion	3.977035
Log likelihood	-5633.468	Hannan-Quinn criter.	3.966345
F-statistic	4.171457	Durbin-Watson stat	2.004167
Prob(F-statistic)	0.000140		

Inverted AR Roots	-.87			
Inverted MA Roots	.39+.39i	.39-.39i	-.38+.40i	-.38-.40i
	-.91			

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:57
Sample: 1 2849
Included observations: 2849
Convergence achieved after 16 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030922	0.034622	0.893126	0.3719
MA(1)	0.023650	0.013514	1.750027	0.0802
SIGMASQ	3.084840	0.049591	62.20564	0.0000

R-squared	0.000528	Mean dependent var	0.030911
Adjusted R-squared	-0.000174	S.D. dependent var	1.757144
S.E. of regression	1.757297	Akaike info criterion	3.966483
Sum squared resid	8788.710	Schwarz criterion	3.972753
Log likelihood	-5647.255	Hannan-Quinn criter.	3.968744
F-statistic	0.752077	Durbin-Watson stat	2.001205
Prob(F-statistic)	0.471480		

Inverted MA Roots	-.02

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 17:59
Sample: 1 2849
Included observations: 2849
Convergence achieved after 15 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030935	0.034467	0.897532	0.3695
MA(1)	0.024341	0.013489	1.804486	0.0713
MA(2)	-0.024487	0.014115	-1.734837	0.0829
SIGMASQ	3.082739	0.050123	61.50305	0.0000

R-squared	0.001209	Mean dependent var	0.030911
Adjusted R-squared	0.000156	S.D. dependent var	1.757144
S.E. of regression	1.757007	Akaike info criterion	3.966504
Sum squared resid	8782.723	Schwarz criterion	3.974865
Log likelihood	-5646.285	Hannan-Quinn criter.	3.969519
F-statistic	1.148007	Durbin-Watson stat	2.001223
Prob(F-statistic)	0.328291		

Inverted MA Roots	.14	-.17

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 18:00
Sample: 1 2849
Included observations: 2849
Convergence achieved after 23 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030917	0.035545	0.869800	0.3845
MA(1)	0.019378	0.013538	1.431388	0.1524
MA(2)	-0.024153	0.014161	-1.705604	0.0882
MA(3)	0.031810	0.013316	2.388822	0.0170
SIGMASQ	3.080072	0.050012	61.58630	0.0000

R-squared	0.002073	Mean dependent var	0.030911
Adjusted R-squared	0.000670	S.D. dependent var	1.757144
S.E. of regression	1.756555	Akaike info criterion	3.966342
Sum squared resid	8775.125	Schwarz criterion	3.976792
Log likelihood	-5645.054	Hannan-Quinn criter.	3.970111
F-statistic	1.477075	Durbin-Watson stat	1.995623
Prob(F-statistic)	0.206414		

Inverted MA Roots	.16+.25i	.16-.25i	-.35	

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 18:01
Sample: 1 2849
Included observations: 2849
Convergence achieved after 20 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030905	0.038207	0.808882	0.4187
MA(1)	0.021098	0.013539	1.558310	0.1193
MA(2)	-0.017858	0.014232	-1.254828	0.2096
MA(3)	0.029249	0.013664	2.140602	0.0324
MA(4)	0.066353	0.013949	4.756888	0.0000
SIGMASQ	3.066311	0.049590	61.83332	0.0000

R-squared	0.006531	Mean dependent var	0.030911
Adjusted R-squared	0.004784	S.D. dependent var	1.757144
S.E. of regression	1.752935	Akaike info criterion	3.962572
Sum squared resid	8735.921	Schwarz criterion	3.975112
Log likelihood	-5638.683	Hannan-Quinn criter.	3.967094
F-statistic	3.738213	Durbin-Watson stat	1.999325
Prob(F-statistic)	0.002243		

Inverted MA Roots	.36+.38i	.36-.38i	-.37-.32i	-.37+.32i

Dependent Variable: R
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/08/21 Time: 18:02
Sample: 1 2849
Included observations: 2849
Convergence achieved after 48 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.030930	0.037586	0.822911	0.4106
AR(1)	-0.986669	0.087771	-11.24142	0.0000
AR(2)	-0.616629	0.091794	-6.717504	0.0000
MA(1)	1.009962	0.086924	11.61890	0.0000
MA(2)	0.619463	0.094849	6.531071	0.0000
MA(3)	0.023808	0.021958	1.084226	0.2784
MA(4)	0.082134	0.020372	4.031639	0.0001
MA(5)	0.091363	0.015064	6.064946	0.0000
SIGMASQ	3.051730	0.050551	60.36899	0.0000

R-squared	0.011256	Mean dependent var	0.030911
Adjusted R-squared	0.008471	S.D. dependent var	1.757144
S.E. of regression	1.749686	Akaike info criterion	3.959926
Sum squared resid	8694.378	Schwarz criterion	3.978737
Log likelihood	-5631.915	Hannan-Quinn criter.	3.966710
F-statistic	4.041322	Durbin-Watson stat	1.998795
Prob(F-statistic)	0.000086		

Inverted AR Roots	-.49-.61i	-.49+.61i		
Inverted MA Roots	.29+.39i	.29-.39i	-.53+.67i	-.53-.67i
	-.53			