

## Sprawozdanie programu Burgers Equation 2024

$$u_{i,j} = u(x_i, t_j) = u(i \cdot h, j \cdot k)$$

$$h = 0.1, k = 0.005$$

Warunek początkowy:  $u_{0,j} = u_{M-1,j} = 0$

$$u_{i,0} = e^{-(x_i - x_{mid})^2}$$

### Równania

1.  $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$
2.  $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0$
3.  $\frac{\partial u}{\partial t} - \beta \frac{\partial^2 u}{\partial x^2} = 0$
4.  $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - \beta \frac{\partial^2 u}{\partial x^2} = 0$  (Równanie Burgers'a)

### Metoda Eulera

1.  $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{2h}(u_{i+1,j} - u_{i-1,j}) = 0$   
$$u_{i,j+1} = u_{i,j} - \frac{k}{2h}(u_{i+1,j} - u_{i-1,j})$$
2.  $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{4h}(u_{i+1,j}^2 - u_{i-1,j}^2) = 0$   
$$u_{i,j+1} = u_{i,j} - \frac{k}{4h}(u_{i+1,j}^2 - u_{i-1,j}^2)$$
3.  $\frac{1}{k}(u_{i,j+1} - u_{i,j}) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$   
$$u_{i,j+1} = u_{i,j} + \frac{\beta k}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$
4.  $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{4h}(u_{i+1,j}^2 - u_{i-1,j}^2) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$   
$$u_{i,j+1} = u_{i,j} - \frac{k}{4h}(u_{i+1,j}^2 - u_{i-1,j}^2) + \frac{\beta k}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

### Metoda Rungego-Kutty 2-go rzędu

1.  $v_{i,j+1} = u_{i,j} - \frac{k}{4h}(u_{i+1,j} - u_{i-1,j})$   
$$u_{i,j+1} = u_{i,j} - \frac{k}{2h}(v_{i+1,j+1} - v_{i-1,j+1})$$
2.  $v_{i,j+1} = u_{i,j} - \frac{k}{8h}(u_{i+1,j}^2 - u_{i-1,j}^2)$   
$$u_{i,j+1} = u_{i,j} - \frac{k}{4h}(v_{i+1,j+1}^2 - v_{i-1,j+1}^2)$$
3.  $v_{i,j+1} = u_{i,j} + \frac{\beta k}{2h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$   
$$u_{i,j+1} = u_{i,j} + \frac{\beta k}{h^2}(v_{i+1,j+1} - 2v_{i,j+1} + v_{i-1,j+1})$$
4.  $v_{i,j+1} = u_{i,j} - \frac{k}{8h}(u_{i+1,j}^2 - u_{i-1,j}^2) + \frac{\beta k}{2h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$   
$$u_{i,j+1} = u_{i,j} - \frac{k}{4h}(v_{i+1,j+1}^2 - v_{i-1,j+1}^2) + \frac{\beta k}{h^2}(v_{i+1,j+1} - 2v_{i,j+1} + v_{i-1,j+1})$$

## Metoda Niejawna

$$3. \frac{1}{k}(u_{i,j} - u_{i,j-1}) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$$

$$u_{i,j-1} = -\frac{\beta k}{h^2}u_{i-1,j} + (1 + 2\frac{\beta k}{h^2})u_{i,j} - \frac{\beta k}{h^2}u_{i+1,j}$$

$$A = \begin{bmatrix} 1+2s & -s & 0 & \cdots & 0 \\ -s & 1+2s & -s & \cdots & 0 \\ 0 & -s & 1+2s & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1+2s \end{bmatrix}, s = \frac{\beta k}{h^2}$$

$$u_{i,j} = A^{-1}u_{j-1}$$

$$4. \frac{1}{k}(u_{i,j} - u_{i,j-1}) + \frac{1}{4h}(u_{i+1,j-1}^2 - u_{i-1,j-1}^2) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$$

$$u_{i,j-1} - \frac{k}{4h}(u_{i+1,j-1}^2 - u_{i-1,j-1}^2) = -\frac{\beta k}{h^2}u_{i-1,j} + (1 + 2\frac{\beta k}{h^2})u_{i,j} - \frac{\beta k}{h^2}u_{i+1,j}$$

$$A = \begin{bmatrix} 1+2s & -s & 0 & \cdots & 0 \\ -s & 1+2s & -s & \cdots & 0 \\ 0 & -s & 1+2s & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1+2s \end{bmatrix}, s = \frac{\beta k}{h^2}$$

$$u_{i,j} = A^{-1}(u_{i,j-1} - \frac{k}{4h}(u_{i+1,j-1}^2 - u_{i-1,j-1}^2))$$