$u_{i,j} = u(x_i, t_j) = u(i \cdot h, j \cdot k)$ • METODA EULERA

• $h = 0.1$, $k = 0.005$

1^0 • $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ W.P. • $u_{0,j} = 0$, $u_{M-1,j} = 0$, $u_{i,0} = e^{-(x_i - x_{mid})^2}$, $x_i = i \cdot h$, $x_{mid} = \frac{x_{max}}{2}$

• $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{2h}(u_{i+1,j} - u_{i-1,j}) = 0$

• $u_{i,j+1} = u_{i,j} - \frac{k}{2h}(u_{i+1,j} - u_{i-1,j})$

2^0 • $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$ W.P. • $u_{0,j} = 0$, $u_{M-1,j} = 0$, • $u_{i,0} = e^{-(x_i - x_{mid})^2}$

• $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{4h}((u_{i+1,j})^2 - (u_{i-1,j})^2) = 0$

• $u_{i,j+1} = u_{i,j} - \frac{k}{4h}((u_{i+1,j})^2 - u(_{i-1,j})^2)$

3^0 • $\frac{\partial u}{\partial t} - \beta\frac{\partial^2 u}{\partial x^2} = 0$ W.P. = " =

• $\frac{1}{k}(u_{i,j+1} - u_{i,j}) - \beta\frac{1}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$

• $u_{i,j+1} = u_{i,j} + \frac{\beta k}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$ • $\frac{\beta k}{h^2} < \frac{1}{2}$ • $\beta < \frac{h^2}{2k}$

4^0 • $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \beta\frac{\partial^2 u}{\partial x^2} = 0$ Burgers Eq. W.P. jak powyżej

• $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{4h}((u_{i+1,j})^2 - (u_{i-1,j})^2) +$
$- \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$ • $\frac{\beta k}{h^2} < \frac{1}{2}$ • $\beta < \frac{h^2}{2k}$

• $u_{i,j+1} = u_{i,j} - \frac{k}{4h}((u_{i+1,j})^2 - u(_{i-1,j})^2) +$
$+ \frac{\beta k}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$

• METODA NIEJAWNA

4^0 • $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \beta\frac{\partial^2 u}{\partial x^2} = 0$ Burger Eq. W.P. jak pow. • $\frac{\beta k}{h^2} < \frac{1}{2}$ • $\beta < \frac{h^2}{2k}$

• $\frac{1}{k}(u_{i,j} - u_{i,j-1}) + \frac{1}{4h}((u_{i+1,j})^2 - (u_{i-1,j})^2) +$
$- \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$

• $u_{i,j-1} = \left(-\frac{k}{4h} - \frac{k\beta}{h^2}\right)u_{i-1,j} + \left(1 + 2\frac{\beta k}{h^2}\right)u_{i,j} + \left(\frac{k}{4h} - \frac{k\beta}{h^2}\right)u_{i+1,j}$

$S_0 = +\left(\frac{k}{4h} - \frac{k\beta}{h^2}\right)$
$S_1 = \left(1 + \frac{2\beta k}{h^2}\right)$
$S_2 = \left(-\frac{k}{4h} - \frac{k\beta}{h^2}\right)$

$A: \begin{bmatrix} S_1 & S_0 & & 0 \\ S_2 & S_1 & S_0 & \\ & S_2 & S_1 & \ddots \\ 0 & & \ddots & \ddots \end{bmatrix}$

• METODA RK2

1° $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$

W.P. • $u_{0,j} = 0$

• $u_{n-1,j} = 0$

• $u_{i,0} = e^{-(x_i - x_{mid})^2}$

• $u_{i,j+\frac{1}{2}} = u_{i,j} - \frac{k}{4h}(u_{i+1,j} - u_{i-1,j})$

• $u_{i,j+1} = u_{i,j} - \frac{k}{2h}(u_{i+1,j+\frac{1}{2}} - u_{i-1,j+\frac{1}{2}})$

2° $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$

W.P. = " =

• $u_{i,j+\frac{1}{2}} = u_{i,j} - \frac{k}{8h}((u_{i+1,j})^2 - (u_{i-1,j})^2)$

• $u_{i,j+1} = u_{i,j} - \frac{k}{4h}((u_{i+1,j+\frac{1}{2}})^2 - (u_{i-1,j+\frac{1}{2}})^2)$

3° $\frac{\partial u}{\partial t} - \beta\frac{\partial^2 u}{\partial x^2} = 0$

W.P. = " =

• $u_{i,j+\frac{1}{2}} = u_{i,j} + \frac{\beta k}{2h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$

• $u_{i,j+1} = u_{i,j} + \frac{\beta k}{h^2}(u_{i+1,j+\frac{1}{2}} - 2u_{i,j+\frac{1}{2}} + u_{i-1,j+\frac{1}{2}})$

4° $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \beta\frac{\partial^2 u}{\partial x^2} = 0$

W.P. = " =

• $u_{i,j+\frac{1}{2}} = u_{i,j} - \frac{k}{8h}((u_{i+1,j})^2 - (u_{i-1,j})^2) + \frac{\beta k}{2h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$

• $u_{i,j+1} = u_{i,j} - \frac{k}{4h}((u_{i+1,j+\frac{1}{2}})^2 - (u_{i-1,j+\frac{1}{2}})^2) + \frac{\beta k}{h^2}(u_{i+1,j+\frac{1}{2}} - 2u_{i,j+\frac{1}{2}} + u_{i-1,j+\frac{1}{2}})$

• METODA NIEJAWNA

3° $\frac{\partial u}{\partial t} - \beta\frac{\partial^2 u}{\partial x^2} = 0$

W.P. = " =

• $\frac{1}{k}(u_{i,j} - u_{i,j-1}) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$

• $u_{i,j} - u_{i,j-1} = \frac{\beta k}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$

• $u_{i,j-1} = -\frac{\beta k}{h^2}u_{i-1,j} + (1 + 2\underbrace{\frac{\beta k}{h^2}}_{\to s})u_{i,j} - \frac{\beta k}{h^2}u_{i+1,j}$

• $u_{i,j-1} = -s u_{i-1,j} + (1+2s)u_{i,j} - s u_{i+1,j}$

• Mamy $u_{j-1} = A u_j$, gdzie

$$A: \begin{bmatrix} 1+2s & -s & & 0 \\ -s & 1+2s & -s & \\ & -s & \ddots & \ddots \\ 0 & & \ddots & 1+2s \end{bmatrix}$$

• $u_j = A^{-1} u_{j-1}$