Sprawozdanie programu Burgers Equation 2024

$u_{i,j} = u(x_i, t_j) = u(i \cdot h, j \cdot k)$
$h = 0.1, k = 0.005$
Warunek początkowy: $u_{0,j} = u_{M-1,j} = 0$
$u_{i,0} = e^{-(x_i - x_{mid})^2}$

Równania

1. $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$
2. $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$
3. $\frac{\partial u}{\partial t} - \beta\frac{\partial^2 u}{\partial x^2} = 0$
4. $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \beta\frac{\partial^2 u}{\partial x^2} = 0$ (Równanie Burgers'a)

Metoda Eulera

1. $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{2h}(u_{i+1,j} - u_{i-1,j}) = 0$

   $u_{i,j+1} = u_{i,j} - \frac{k}{2h}(u_{i+1,j} - u_{i-1,j})$
2. $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{4h}(u^2_{i+1,j} - u^2_{i-1,j}) = 0$

   $u_{i,j+1} = u_{i,j} - \frac{k}{4h}(u^2_{i+1,j} - u^2_{i-1,j})$
3. $\frac{1}{k}(u_{i,j+1} - u_{i,j}) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$

   $u_{i,j+1} = u_{i,j} + \frac{\beta k}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$
4. $\frac{1}{k}(u_{i,j+1} - u_{i,j}) + \frac{1}{4h}(u^2_{i+1,j} - u^2_{i-1,j}) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$

   $u_{i,j+1} = u_{i,j} - \frac{k}{4h}(u^2_{i+1,j} - u^2_{i-1,j}) + \frac{\beta k}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$

Metoda Rungego-Kutty 2-go rzędu

1. $v_{i,j+1} = u_{i,j} - \frac{k}{4h}(u_{i+1,j} - u_{i-1,j})$

   $u_{i,j+1} = u_{i,j} - \frac{k}{2h}(v_{i+1,j+1} - v_{i-1,j+1})$
2. $v_{i,j+1} = u_{i,j} - \frac{k}{8h}(u^2_{i+1,j} - u^2_{i-1,j})$

   $u_{i,j+1} = u_{i,j} - \frac{k}{4h}(v^2_{i+1,j+1} - v^2_{i-1,j+1})$
3. $v_{i,j+1} = u_{i,j} + \frac{\beta k}{2h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$

   $u_{i,j+1} = u_{i,j} + \frac{\beta k}{h^2}(v_{i+1,j+1} - 2v_{i,j+1} + v_{i-1,j+1})$
4. $v_{i,j+1} = u_{i,j} - \frac{k}{8h}(u^2_{i+1,j} - u^2_{i-1,j}) + \frac{\beta k}{2h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$

   $u_{i,j+1} = u_{i,j} - \frac{k}{4h}(v^2_{i+1,j+1} - v^2_{i-1,j+1}) + \frac{\beta k}{h^2}(v_{i+1,j+1} - 2v_{i,j+1} + v_{i-1,j+1})$

Metoda Niejawna

3. $\frac{1}{k}(u_{i,j} - u_{i,j-1}) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$

   $u_{i,j-1} = -\frac{\beta k}{h^2}u_{i-1,j} + (1 + 2\frac{\beta k}{h^2})u_{i,j} - \frac{\beta k}{h^2}u_{i+1,j}$

   $$A = \begin{bmatrix} 1+2s & -s & 0 & \cdots & 0 \\ -s & 1+2s & -s & \cdots & 0 \\ 0 & -s & 1+2s & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1+2s \end{bmatrix}, s = \frac{\beta k}{h^2}$$

   $u_{i,j} = A^{-1}u_{j-1}$

4. $\frac{1}{k}(u_{i,j} - u_{i,j-1}) + \frac{1}{4h}(u^2_{i+1,j-1} - u^2_{i-1,j-1}) - \frac{\beta}{h^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) = 0$

   $u_{i,j-1} - \frac{k}{4h}(u^2_{i+1,j-1} - u^2_{i-1,j-1}) = -\frac{\beta k}{h^2}u_{i-1,j} + (1 + 2\frac{\beta k}{h^2})u_{i,j} - \frac{\beta k}{h^2}u_{i+1,j}$

   $$A = \begin{bmatrix} 1+2s & -s & 0 & \cdots & 0 \\ -s & 1+2s & -s & \cdots & 0 \\ 0 & -s & 1+2s & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1+2s \end{bmatrix}, s = \frac{\beta k}{h^2}$$

   $u_{i,j} = A^{-1}(u_{i,j-1} - \frac{k}{4h}(u^2_{i+1,j-1} - u^2_{i-1,j-1}))$