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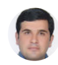

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Kinematics and Control of a 4-DOF Delta Parallel Manipulator

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***Abstract*—In this paper, the design, kinematic problem and control of a 4-DOF Delta parallel manipulator are investigated. Delta is a parallel manipulator which is widely used for pick-and-place applications. A simulation of this manipulator is designed in SimMechanic environment. The verification of the inverse kinematic problem has been done in simulation environment. Afterwards, different controllers like PID and Sliding-Mode based on the inverse kinematic problem are designed in simulation environment and their results are compared with each other. According to value of the root mean square error, PID controller exhibits better performance and tracked the desired path with less error compared to other controllers. Then the Sliding-Mode control is designed and based to the obtained results, leads to less chattering in tracking the desired path.**

***Index Terms*—Delta parallel manipulator, Inverse kinematic problem, PID control, Sliding-Mode control.**

I. Introduction

In recent years, the robotic science has become noticeable for scientists. Due to the high precision in comparison to the mankind and challenges of human resources management, manipulators have been considered as favourite, such as apparatus for different field of industrialist. Manipulators falls into two types, namely, parallels and serial manipulators. The field of parallel mechanism is developed in recent years, but the accuracy limits the industrialization of these manipulators. This mechanism consists of a closed kinematic chain which connects the fixed base to the end-effector. Several parallel manipulators have been proposed in the literature and by industry, such as, Delta [3], Gough-Stewart [4], Novint Falcon [5], Agile Eye [6], Tripteron [7] which Delta is the under study manipulator of the present paper. Delta parallel manipulator is a type of high speed parallel manipulators which based on the performed motion are categorized into two kinds: 3-DOF and 4-DOF. High speed parallel manipulators with the light-weight links and fixed frame actuators have found wide industrial applications in packaging factories where the pick-and-place operations of objects are regularly required [1]. These kinds of manipulators contains four parallel chains $\underline{\mathbf{R}} - (\mathbf{SS})^2$. $(\mathbf{SS})^2$ specifies a four-bar linkage made with four spherical joints and a fourth "telescopic" leg which helps the end-effector to rotate around the vertical axis while $\underline{\mathbf{R}}$ specifies a revolute joint which has been actuated by motors [2]. The manipulator is designed with all actuators fixed on its base for high performances. The advantage of parallel manipulators over serial robots can be regarded as high dynamic capabilities, large moving masses and high precision.

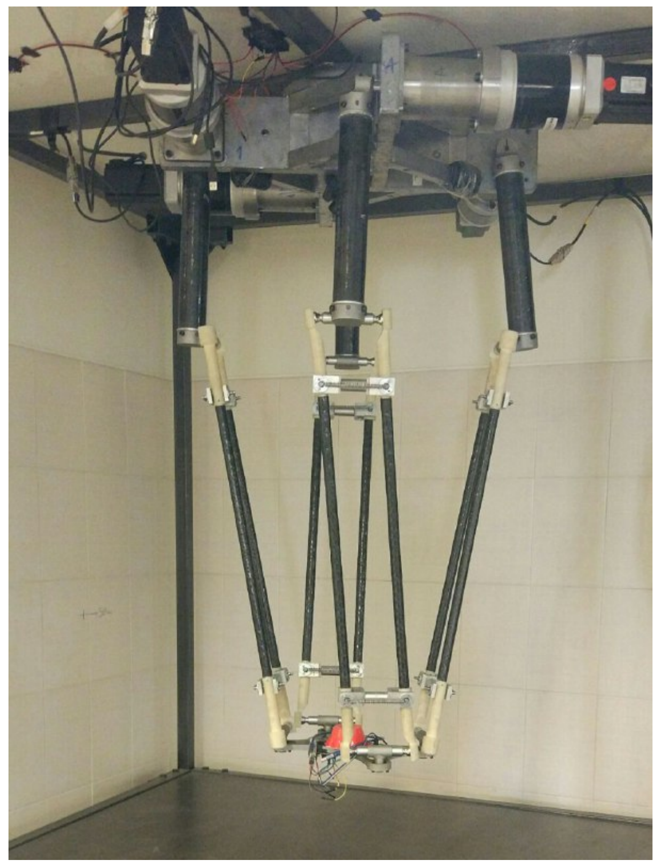

Fig. 1. The 4-DOF Delta manipulator, built at Human and Robot Interaction Laboratory, University of Tehran.

The under study manipulator which has been constructed in

Human and Robot Interaction Laboratory at University of Tehran, is illustrated in Fig. 1. The most important principle in a manipulator's proper operation is control. The purpose of controlling a manipulator is to execute appropriate commands with the goal of achieving a desirable behavior in the end-effector. Manipulators are divided into two categories in terms of control: Non-Servo Control and Servo Control. There are various controllers for controlling manipulators, but the main reason limiting the usage of controllers is their complexity. Some of the controllers that are used to control manipulators are, namely, PID, robust and adaptive control [8], [9], and Sliding-Mode controller(SMC). The purpose of this paper consists in designing a controller with capability of implementing on the 4-DOF Delta parallel manipulator and also high performance and precision.

The main contribution of this paper consists in simulation of Delta parallel manipulator and studying the performance of controllers, SMC and PID, on this plant based on Inverse Kinematic Problem (IKP) in SimMechanics environment of MATLAB software. The desinged SMC which is used for this manipulator in simulation environment can be regarded as a novel approach compared to other approaches.

The remainder of this paper is organized as follows. In Section 2, the IKP of Delta parallel manipulator has been calculated based on a vectorial method. The simulation of the manipulator in SimMechanics environment has been implemented in Section 3. In Section 4, verification of the calculated IKP has been done. In Section 5, the design of the intended controller, i.e., SMC and PID, which are based on the IKP are explained. Finally, the paper concludes with some hints as ongoing works.

II. Inverse Kinematic Problem

Delta parallel manipulator composed of 4 pairs of legs which connect the fixed-base and the end-effector, has been illustrated in Fig. 2. Each leg is composed of a actuated arm which connects the motor of leg to a parallelogram arm. The parallelogram arm composed a pair of links and a pair of spherical joints which connects the actuated arm to the end-effector. The end-effector can move along 4-directions, namely, i.e., three transitional movements and one rotational movement around the z axis. The pendular movment of the actuated arm is passed to the end-effector by the parallelogram arm, and the 4-directions movement of the end-effector is realized [1]. The IKP consists in finding the actuators states θ_1, θ_2, θ_3 and θ_4 by having the end-effector position. If $\mathbf{p}$ is the end-effector's position in the coordinate system of fixed base, it can be represented as follows:

$$\mathbf{p} = (x, y, z)^T \tag{1}$$

and the fourth Degree-of-Freedom (DOF) can be shown by φ.

The coordinate frame of the fixed base is defined with origin O and axes X, Y, Z and the coordinate frame of the end-effector is defined with origin O' and axes X', Y', Z'. According to Fig. 2, $\mathbf{r}_{1i}$ is defined as a vector between O and A_i, $\mathbf{r}_{2i}$ is definded as a vector between O' and C_i, and $\mathbf{l}_{1i}$ is defined as a vector between A_i and B_i. These vectors can be computed readily as follows:

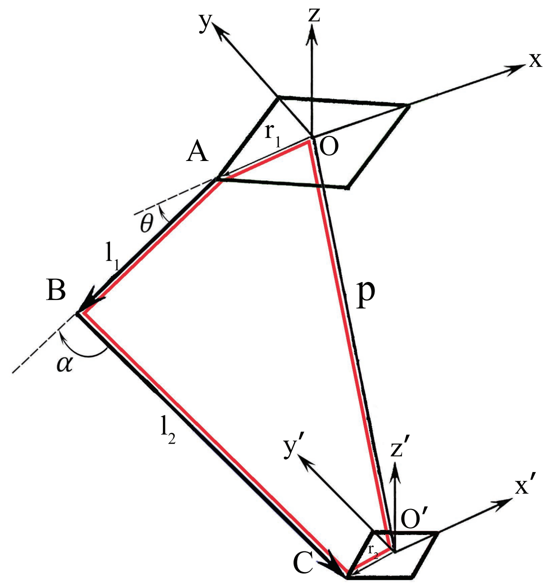

Fig. 2. Kinematic arrangement of one limb of a 4-DOF parallel robot.

$$\mathbf{r}_{1i} = |\mathbf{r}_1|(\cos\beta_i, \sin\beta_i, 0)^T \tag{2}$$

$$\mathbf{r}_{2i} = |\mathbf{r}_2|(\cos(\beta_i + \varphi), \sin(\beta_i + \varphi), 0)^T \tag{3}$$

$$\mathbf{l}_{1i} = |\mathbf{l}_1|(\cos\beta_i\cos\theta_i, \sin\beta_i\cos\theta_i, -\sin\theta_i)^T \tag{4}$$

where the angle between each leg and the positive x axis is defined as β_i for $i = 0, 1, 2, 3$, the angle between actuated link and the negative x axis is defined as θ for $i = 0, 1, 2, 3$. Referring to Fig. 2, the kinematic loop closure of one limb can be written as follows:

$$\mathbf{r}_{1i} + \mathbf{l}_{1i} + \mathbf{l}_{2i} - \mathbf{r}_{2i} = \mathbf{p}$$

and hence,

$$\mathbf{p} - (\mathbf{r}_{1i} - \mathbf{r}_{2i}) - \mathbf{l}_{1i} = \mathbf{l}_{2i} \tag{5}$$

Since the product of a vector and its transpose equals to square of vector size, Eq. (5) can be expressed as follows:

$$(\mathbf{p} - (\mathbf{r}_{1i} - \mathbf{r}_{2i}) - \mathbf{l}_{1i})^T(\mathbf{p} - (\mathbf{r}_{1i} - \mathbf{r}_{2i}) - \mathbf{l}_{1i}) = |\mathbf{l}_2|^2 \tag{6}$$

Notice that the above equations require that all involved vectors should be expressed in the same coordinate frame.

The expansion of the Eq. (6) yields

$$A_i\sin\theta_i + B_i\cos\theta_i + C_i = 0 \tag{7}$$

where

$$A_i = 2|\mathbf{l}_1|(\mathbf{p} - (\mathbf{r}_{1i} - \mathbf{r}_{2i}))^T \mathbf{e}_3 = 2|\mathbf{l}_1|(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} |\mathbf{r}_1|\cos\beta_i \\ |\mathbf{r}_1|\sin\beta_i \\ 0 \end{bmatrix} + \begin{bmatrix} |\mathbf{r}_2|\cos(\beta_i+\varphi) \\ |\mathbf{r}_2|\sin(\beta_i+\varphi) \\ 0 \end{bmatrix})^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 2|\mathbf{l}_1|z \tag{8}$$

$$\begin{aligned} B_i &= -2\,|\mathbf{l}_1|\,(\mathbf{p} - (\mathbf{r}_{i1} - \mathbf{r}_{2i}))^T (\mathbf{e}_1 \cos\beta_i + \mathbf{e}_2 \sin\beta_i) \\ &= -2\,|\mathbf{l}_1| \begin{bmatrix} x - |\mathbf{r}_1|\cos\beta_i - |\mathbf{r}_2|\cos(\beta_i+\varphi) \\ y - |\mathbf{r}_1|\sin\beta_i - |\mathbf{r}_2|\sin(\beta_i+\varphi) \\ z \end{bmatrix}^T \begin{bmatrix} \cos\beta_i \\ \sin\beta \\ 0 \end{bmatrix} \\ &= -2\,|\mathbf{l}_1|\,((x - |\mathbf{r}_1|\cos\beta_i + |\mathbf{r}_2|\cos(\beta_i+\varphi))\cos\beta_i \\ &\quad +(y - |\mathbf{r}_1|\sin\beta_i + |\mathbf{r}_2|\sin(\beta_i+\varphi))\sin\beta_i) \\ &= -2\,|\mathbf{l}_1|\,(x\cos\beta_i + y\sin\beta_i - |\mathbf{r}_1| + |\mathbf{r}_2|\cos\varphi) \end{aligned} \tag{9}$$

$$\begin{aligned} C_i &= (\mathbf{p} - (\mathbf{r}_{i1} - \mathbf{r}_{2i}))(\mathbf{p} - (\mathbf{r}_{i1} - \mathbf{r}_{2i}))^T + |\mathbf{l}_1|^2 - |\mathbf{l}_2|^2 \\ &= (x - |\mathbf{r}_1|\cos\beta_i + |\mathbf{r}_2|\cos(\beta_i+\varphi))^2 + (y - |\mathbf{r}_1|\sin\beta_i \\ &\quad + |B_2|\sin(\beta_i+\varphi))^2 + |\mathbf{l}_1|^2 - |\mathbf{l}_2|^2 \\ &= x^2 + y^2 + |\mathbf{r}_1|^2 + |\mathbf{r}_2|^2 + |\mathbf{l}_1|^2 - |\mathbf{l}_2|^2 - 2\,|\mathbf{r}_1|\,(x\cos\beta_i \\ &\quad + y\sin\beta_i) + 2\,|\mathbf{r}_2|\,(x\cos(\beta_i+\varphi) + y\sin(\beta_i+\varphi)) \\ &\quad -2\,|\mathbf{r}_1|\,|\mathbf{r}_2|\cos\varphi \end{aligned} \tag{10}$$

Finally, by substituting A_i, B_i and C_i into Eq. (7), θ_i obtained as follows:

$$\theta_i = 2\arctan[\frac{(-A_i - \sqrt{A_i^2 - C_i^2 + B_i^2})}{(C_i - B_i)}] \tag{11}$$

III. SIMULATION

In this section, a model of the 4-DOF Delta parallel manipulator is simulated in MATLAB software and in the SimMechanics environment [10]. An overview of simulated blocks of Delta manipulator, including solver configuration, world frame, mechanism configuration and fixed base, manipulator arms and end-effector is depicted in Fig. 3.

The block diagram of a Delta manipulator arm is shown in Fig. 4. It should be noticed that CAD files with ".Step" extension was exported from SolidWorks and then imported in SimMechanics for simulation purpose. The constituting blocks of an arm are illustrated in full details in Fig. 4.

The first block, represents an electrical motor which is simulated with a revolute joint. Furthermore, as it is shown, the actuated leg is related to an interface link, hereinafter "connector", by a revolute joint and the connector is connected to two limbs that are parallel to each other. Finally,similar to the same approach, the parallelogram limbs are connected by a connector to the end-effector.
The assembled manipulator in SimMechanics environment is depicted in Fig. 5. Available sensors in simulation has been modelled with transform sensor block, in real case, the MPU sensor with Arduino is used which has been illustrated in Fig. 5.
since other arms of manipulator are similar, they have not been shown in separate figures.

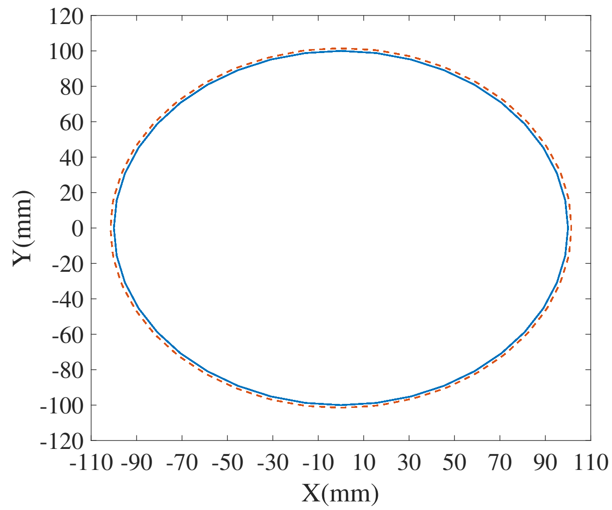

Fig. 6. The circular path given to the robot.

IV. VERIFICATION OF INVERSE KINEMATIC PROBLEM IN SIMULATION

In the present section, kinematic equations are verified, in other words, the validity of these equations is investigated by using simulation in MATLAB software. It is observed that without a controller, the desired path is tracked with high precision.
For the verification, a circular path is given to the kinematic equations of the Delta manipulator and the angles of each four motors are calculated. These angles are given to the revolute joints in the simulation environment which are simulated instead of the motors. The position and orientation of the end-effector can be read using the available sensors in the simulation and these values are compared with the desired position and orientation. According to Fig. 6 and Fig. 7, the circular path and snail path are tracked with low error. Thus, it is concluded that IKP are calculated correctly.

V. DESIGNING CONTROLLER BASED ON INVERSE KINEMATIC EQUATIONS IN SIMULATION ENVIRONMENT

Designing an appropriate and robust controller is one of the most important topics in any mechanical-electrical system.
In this section, control of the 4-DOF Delta manipulator

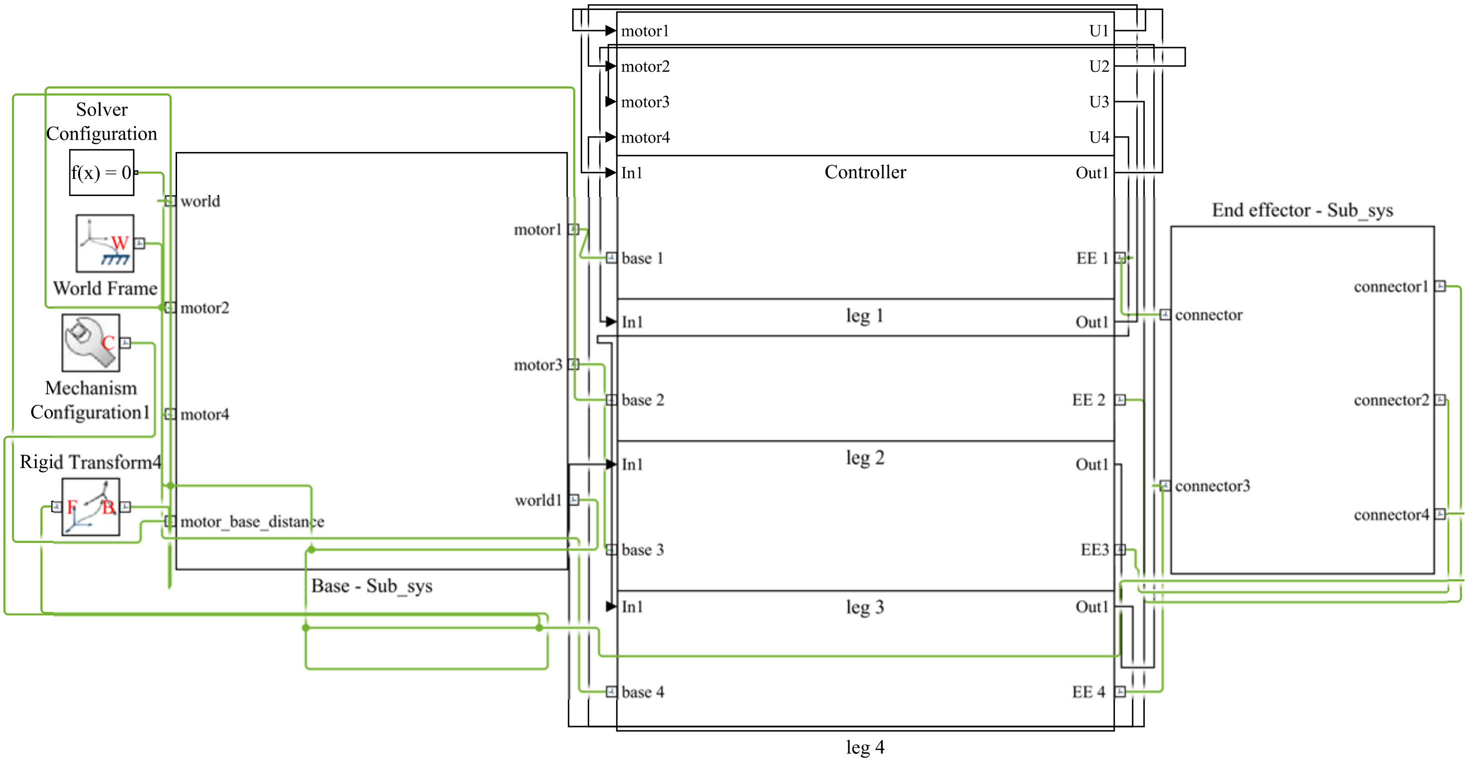

Fig. 3. An overview of simulated blocks of Delta manipulator.

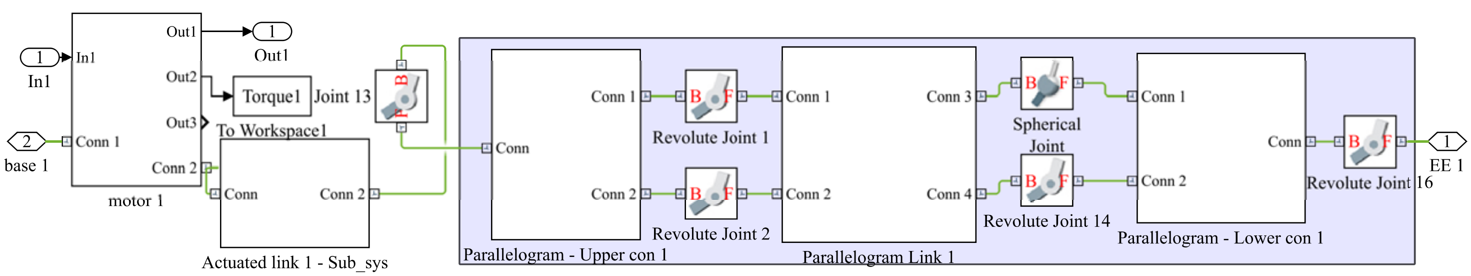

Fig. 4. The block diagram of a Delta manipulator arm.

is expressed in a joint space and is implemented in SimMechanics environment.

A. PID Controller

First, the P-controller is designed, according to Fig. 8, the steady state error is not reduced by increasing the value of proportional gain and if this value is increased continuously, the Delta manipulator becomes unstable [3].

In the next step, the PD-controller is illustrated. In the initial design of the PD-controller, the difference between the instant error and the previous moment error is utilized. In other words, the PD-controller is designed without any low-pass filters. According to Fig. 9, the result of this controller is not acceptable and does not have good performance. For this reason, the PD-controller is designed with low-pass filter, according to Fig. 10, the result is better than before, but there is still a steady state error. Finally, the PID controller is designed. In this case, the steady state error is less than the previous controllers. In Fig. 11, as depicted the snail path is tracked with high precision [11]. In order to compare between the controllers, the root mean square error are calculated. This comparison can be shown in Table I.

It should be noticed that in all types of controllers, end-effector is at a fixed height.

B. Sliding Mode Control

One type of the robust controllers is Sliding-Mode controller which has been designed in this section. This type of controllers can undertake remarkable amounts of uncerainties [12]. In order to design SMC, the following sliding surface is

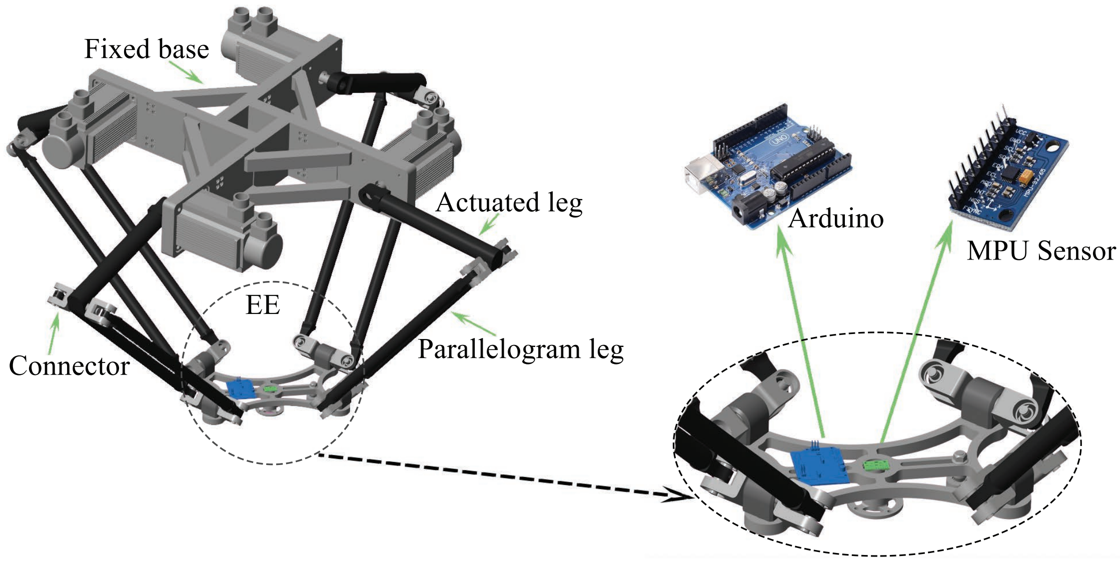

Fig. 5. Assembled manipulator in SimMechanics environment.

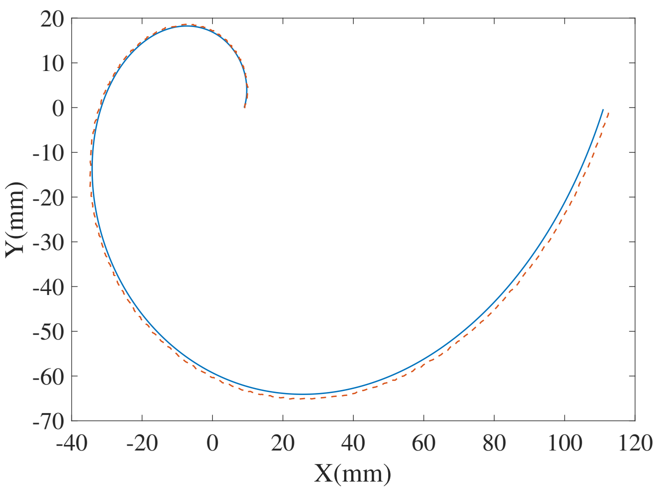

Fig. 7. The snail path given to the robot.

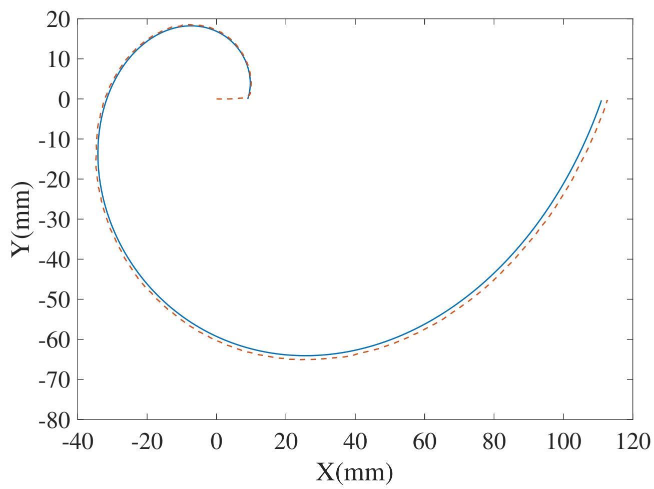

Fig. 8. The result of P-controller.

TABLE I
THE ROOT MEANT SQUARE ERROR VALUES

Control method	Root mean square error value
P	0.009
PD without low-pass filter	0.0091
PD with low-pass filter	0.028
PID	0.0068
Sliding	0.0463

suggested:

$$S = k_1\tilde{\boldsymbol{\theta}}_i + k_2\dot{\tilde{\boldsymbol{\theta}}}_i = k_1x_1 + k_2x_2 \tag{12}$$

If the multiplication of k_1 and k_1 is positive, it can be concluded that the defined sliding surface can stabilize the controlled system [13]. In order to add disturbance and noise in the mathematical model, the system steady state is as follows,

$$\begin{cases} x_1 = \tilde{\boldsymbol{\theta}}_i \\ x_2 = \dot{\tilde{\boldsymbol{\theta}}}_i \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = x_2 + \alpha_1 \\ \dot{x}_2 = u + \alpha_2 \end{cases} \tag{13}$$

It should be noticed $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_{\text{real}} - \boldsymbol{\theta}_{\text{desired}}$ and α_1 and α_2 are bounded to $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$.
By differentiating the sliding surface with respect to time, the control effort is obtained in the following Formula:

$$\begin{aligned} &\dot{S} = k_1\dot{x}_1 + k_2\dot{x}_2 = k_1(x_2+\alpha_1) + k_2(u+\alpha_2) \\ &\dot{S} = k_1x_2 + k_2u + (k_1\alpha_1 + k_2\alpha_2) \\ &\dot{S} = k_2(u + \tfrac{k_1}{k_2}x_2) + (k_1\alpha_1 + k_2\alpha_2) \rightarrow \text{for}\dot{S} = 0 \\ &k_1x_2 + k_2u = k_2U \Rightarrow u = U - \tfrac{k_1}{k_2}x_2 \end{aligned} \tag{14}$$

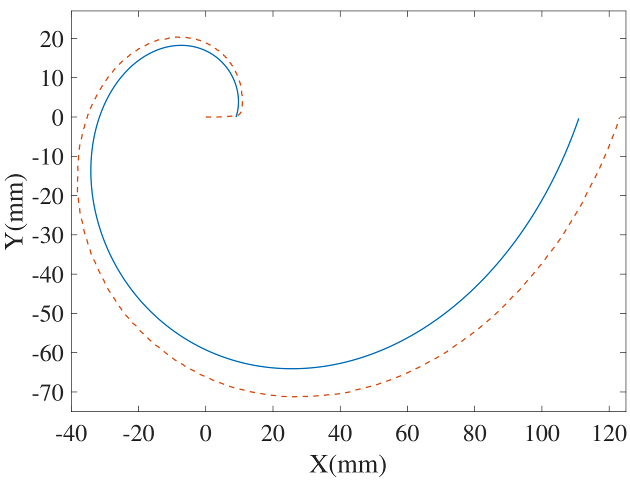

Fig. 9. The result of PD-controller without low-pass filter.

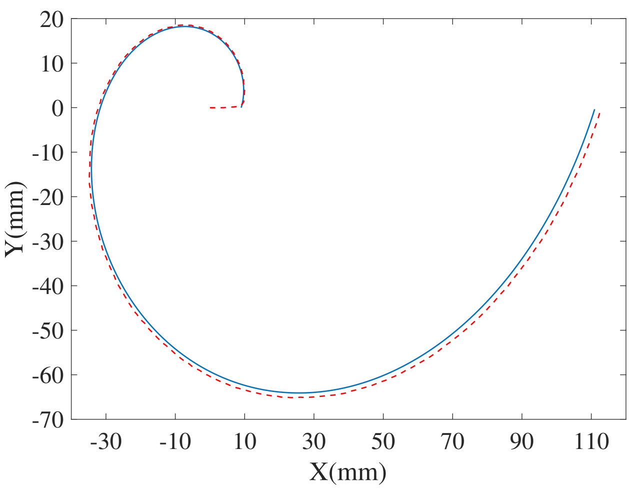

Fig. 10. The result of PD-controller with low-pass filter.

In order to remain on sliding surface, by considering the bounds of α_1 and α_2, the following relation can be concluded :

$$\left|\frac{k_1\alpha_1 + k_2\alpha_2}{k_2}\right| \leq \frac{k_1}{k_2}\tilde{\alpha}_1 + \tilde{\alpha}_2 = \alpha^* \Rightarrow U = -\alpha^* \text{sign}(S) \tag{15}$$

Thus, the control effort is as follows:

$$u = -\frac{k_1}{k_2}x_2 - \alpha^* \text{sign}(S) \tag{16}$$

The control effort is the input of the Delta parallel manipulator motors. In order to proof the stability of SMC, Lyapanov condidate can be taken as follows:

$$v = \frac{1}{2}S^2 \tag{17}$$

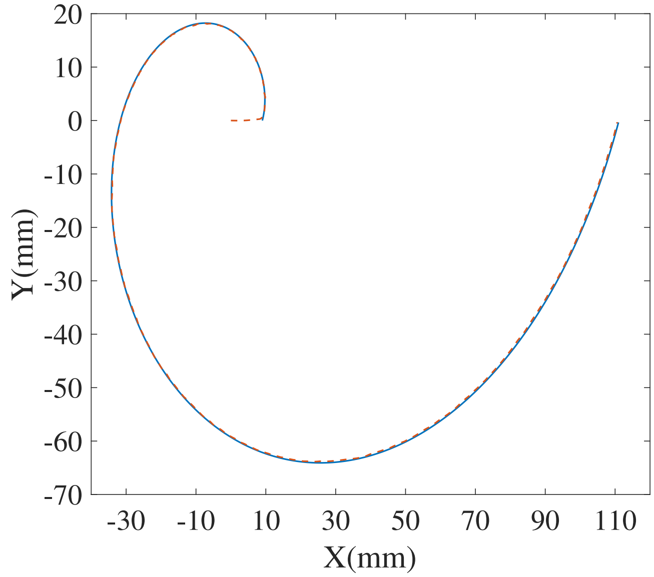

Fig. 11. The result of PID-controller.

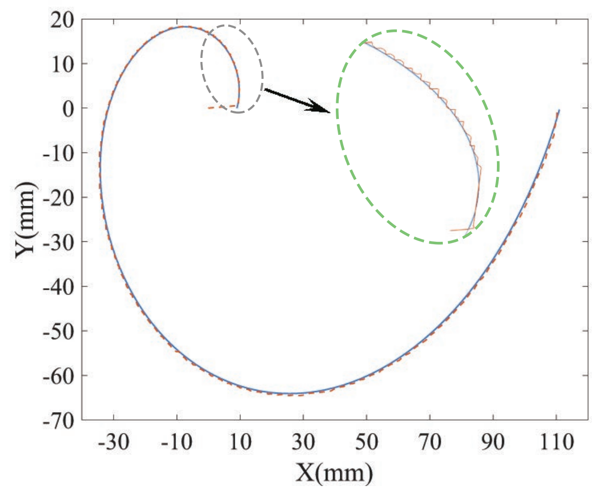

Fig. 12. The result of SMC.

Now, time derivative of Lyapanov function is obtained as in the following relation:

$$\begin{aligned} \dot{v} &= S\dot{S} = -S[k_1(\tilde{\alpha}_1 \text{sign}(S) - \alpha_1) + k_2(\tilde{\alpha}_2 \text{sign}(S) - \alpha_2)] \\ &\leq -S[k_1\tilde{\alpha}_1 \text{sign}(S) + k_2\tilde{\alpha}_2 \text{sign}(S)] \leq 0 \end{aligned} \tag{18}$$

According to the Lyapanove stability criteria, the designed SMC algorithm would stabilize the system of Eq. (13), in the existence of disturbance and noise.
The result of SMC is depicted in Fig. 12.

VI. CONCLUSION

In this paper a simulation study of a 4-DOF parallel manipulator was introduced. In the following, the IKP was calculated from vectorial method and a PID and SMC based on the IKP was designed. In order to verify the equations, a sample path given as the input of IKP, eventually the tracked path was compared with the desired path. Due to the results of the controllers PID has less error campared with other control approaches and the root mean square error was 0.0023. According to the designed SMC algorithm, the under study manipulator was stabilized in the existence of disturbance and noise. It should be noticed that the chattering was less in tracking path and the steady state error was acceptable. As an ongoing work, beside the presented controllers in this paper, an adaptive controller will be implemented in experimental mode.

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