Magnetohydrodynamics and Electromagnetics in the FUSE Integrated-Modeling Framework

by

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[1] General Atomics

Presented at the

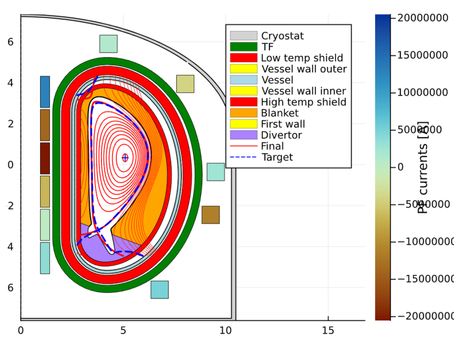
66th Annual Meeting of the APS Division of Plasma Physics
Atlanta, Georgia, USA
October 9th, 2024

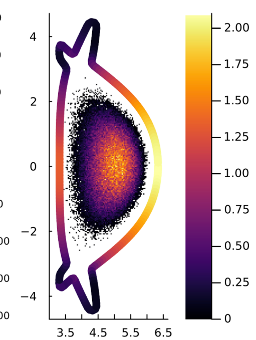
Fusion pilot plant design motivates development of radically new modeling framework

- **FUSE: a flexible, integrated framework for whole-facility FPP modeling**
  - Go from idea to concept design in minutes rather than months
  - Evaluate and optimize wildly different concepts on same footing
  - Provide physics/engineering teams high-quality, integrated design
  - Support modeling needs from design to operations
- **Models span from plasma, through build, to site boundary**
- **Developed from the ground-up based on GA modeling expertise**
  - Built around ITER IMAS ontology for modularity and maximum interoperability
  - All in one language: Julia (high-level like Python but as fast as C/Fortran

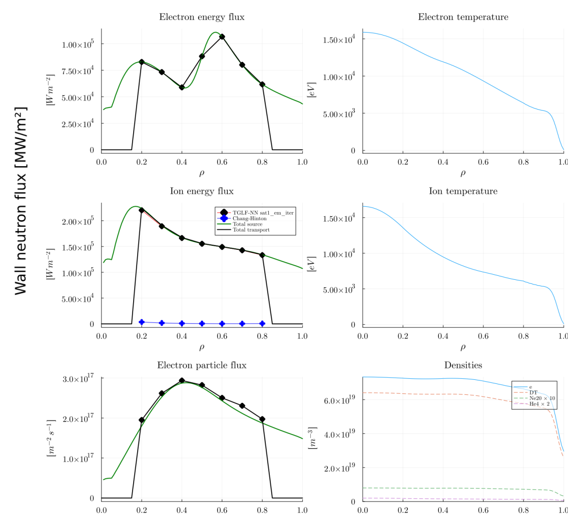
Equilibrium

Parametric Build





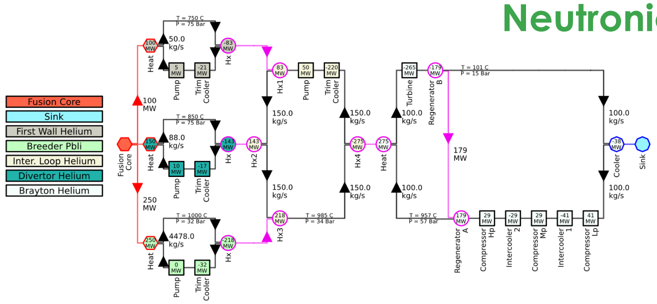
Neutronics

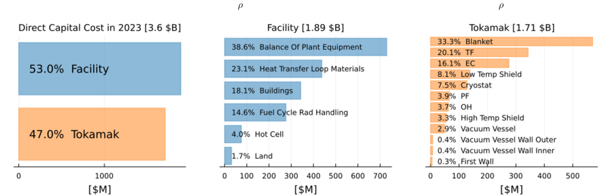


Transport

Heating & Current Drive

Balance of Plant





Costing & Risk

Grad-Shafranov Equilibria

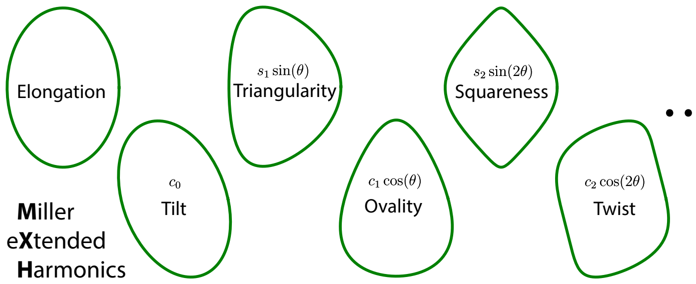
$$\Delta^*\Psi = (2\pi)^2 \mu_0 R J_\varphi$$

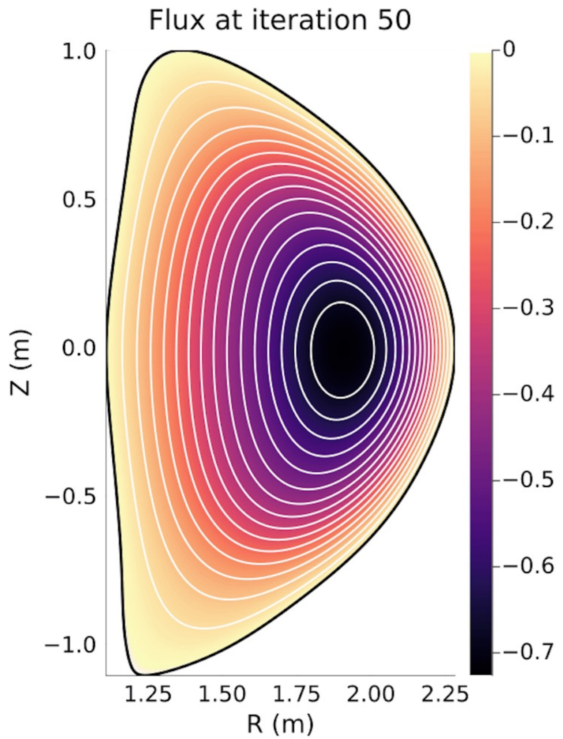
TEQUILA provides fixed-boundary equilibria using efficient spectral representation of flux surface geometry

- **Initialized by what is known in future devices: desired boundary and pressure/current profiles**
- **Solves for Ψ in poloidal coordinates with cubic Hermite finite elements in ρ_{pol} and Fourier modes in θ**
- **Boundary and flux surfaces defined by Miller extended harmonic representation [1]**

$$R(\rho,\theta) = R_0(\rho) + a(\rho)\cos\left[\Theta_R(\rho,\theta)\right]$$
$$Z(\rho,\theta) = Z_0(\rho) + \kappa(\rho)a(\rho)\sin(\theta)$$
$$\Theta_R(\rho,\theta) = \theta + c_0(\rho) + \sum_{m=1}^{M}\left[c_m(\rho)\cos(m\theta) + s_m(\rho)\sin(m\theta)\right]$$

- **G-S solve and poloidal flux coordinate fitting iterated until θ dependence eliminated**

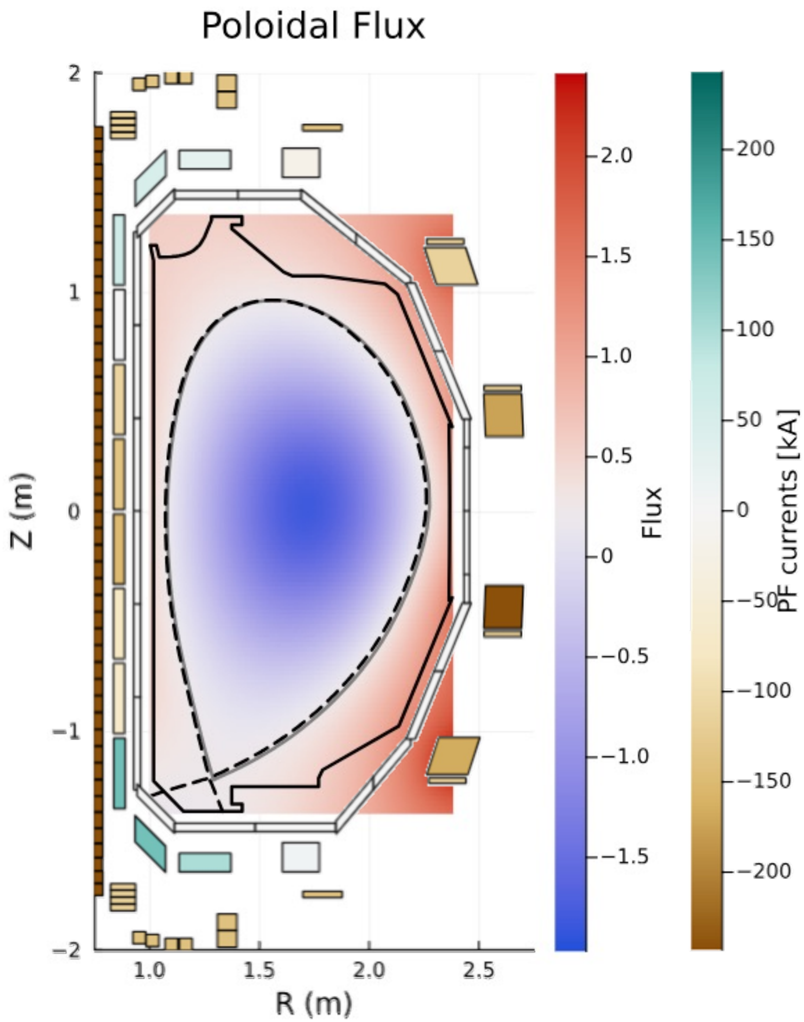




[1] Arbon et al., PPCF 63, 012001 (2021)

FRESCO provides free-boundary equilibria

- **G-S solve by two iterative loops**
- **Inner loop**
  - Invert G-S operator 2D finite differences with Fourier transform in Z coordinate
  - Update plasma boundary and $J_\varphi(R,Z)$
  - Relaxed Picard iterations
- **Outer loop**
  - Update Dirichlet boundary condition on Ψ using von Hagenow [1] boundary integral method
  - <u>Forward solve</u>: Update fictitious control coils for radial/vertical axis feedback
  - <u>Inverse solve</u>: Update physical coil currents to produce desired plasma shape



Forward solve reproduces EFIT equilibrium

[1] Lackner. Comput. Phys. Commun. 12, 33-44 (1976)

Poloidal Vacuum Fields

Flux from poloidal field coils and vacuum vessel computed from Green's functions

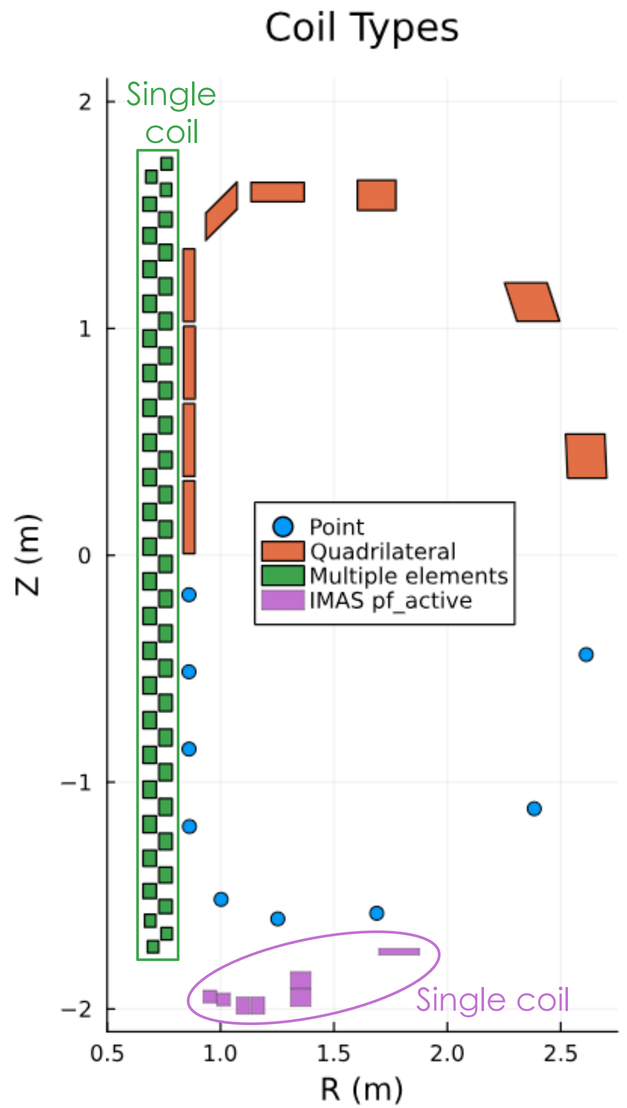
- **Various coil types have been implemented, or can work directly from IMAS coils**
- **Use toroidal Green's function (including integration over coil area) to compute poloidal flux at any point**

$$\Psi_c = 2\pi\mu_0 G\left(R, Z; R_c, Z_c\right)\, I_c$$

$$G\left(R, Z; R', Z'\right) = \frac{\sqrt{RR'}}{2\pi k}\left[2E(k) - (2-k^2)K(k)\right] \qquad k^2 = \frac{4RR'}{\left(R+R'\right)^2 + \left(Z-Z'\right)^2}$$

- **Self/mutual inductance computed by double integral over coils**

$$M_{12} = -\frac{2\pi\mu_0 N_1 N_2}{A_1 A_2}\int dA_1 \int dA_2\, G\left(R_1, Z_1; R_2, Z_2\right)$$

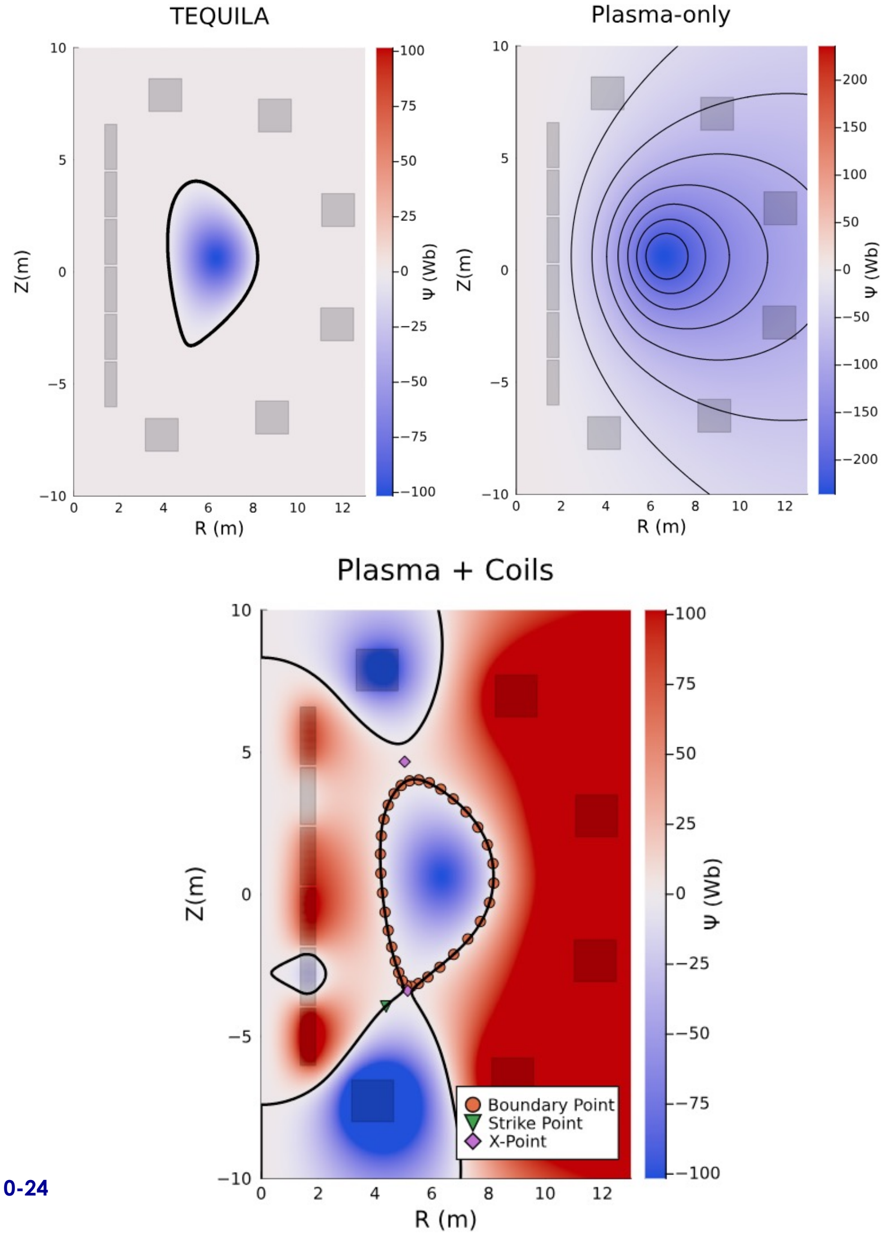


Coil currents can be set to match plasma boundary with X-points and strike points

- **Plasma flux in vacuum computed efficiently from virtual-casing method**
  - Fixed-boundary simulations assume Ψ=0 on boundary, which requires image current on boundary
  - Compute flux from image current

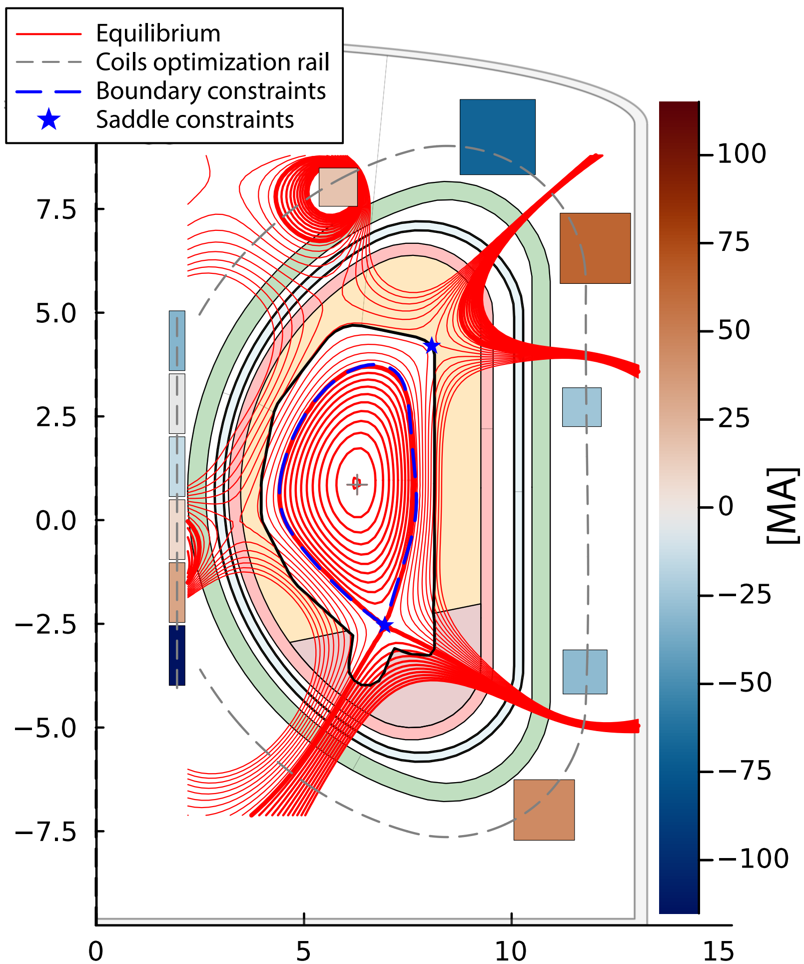
$$\Psi_{im}(R,Z) = \oint_{\Omega_p} \frac{d\ell'}{R'} G\left(R,Z;R',Z'\right) \frac{\partial \Psi}{\partial n}$$

  - Then on & outside boundary, plasma flux is opposite image flux
- **Coil currents found from least-squares linear optimization to match control points when summed with plasma flux**
- **Various control points implemented**
  - Flux value
  - Isoflux ($\Psi_1 = \Psi_2 = \Psi_3 \ldots$)
  - X-point ($d\Psi/dR = d\Psi/dZ = 0$)

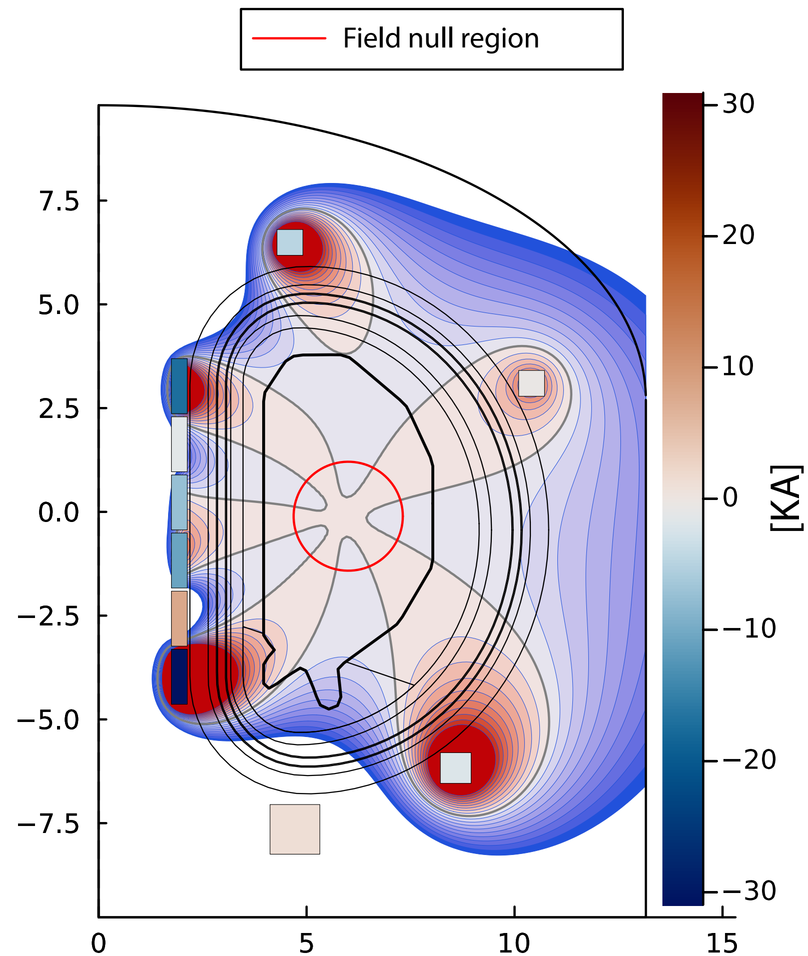


Coil locations can be optimized to produce desired plasma boundary and/or initial field null

Plasma Boundary



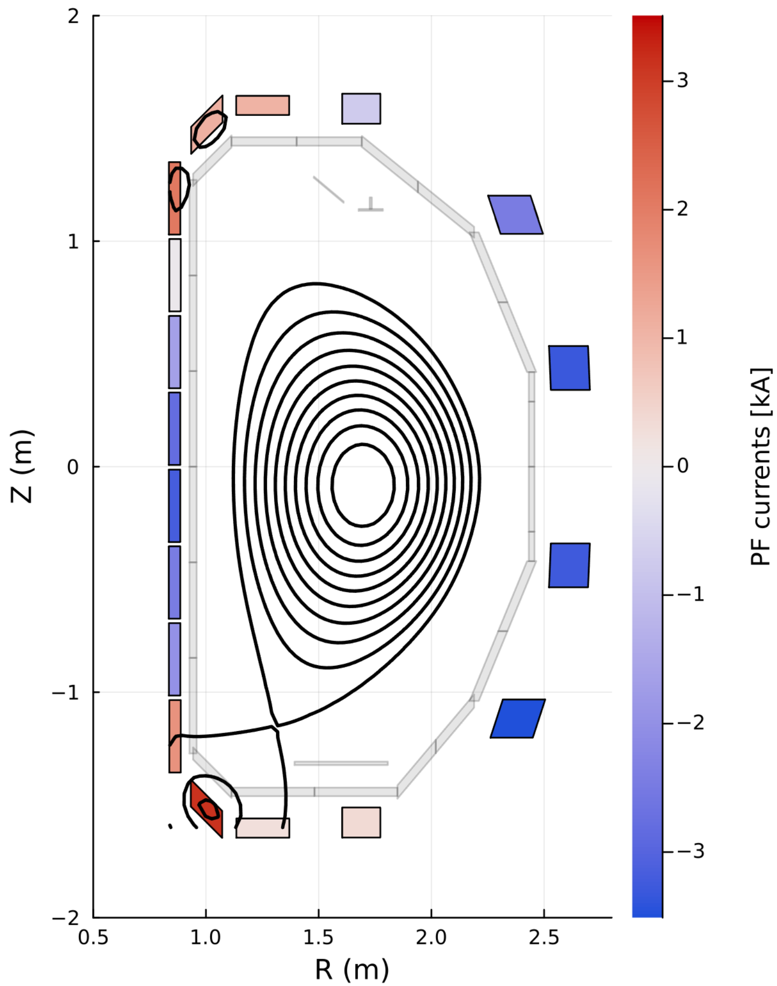
Initial Field Null



Coupled plasma-coil system provides metrics for vertical stability

- **Vacuum vessel and other conducting structures treated as coils**
- **Two metrics for vertical stability**
  - Inductive stability margin m_s
    - Ideal instability for $m_s < 0$
    - $m_s > 0.15$ for practical requirement
    - Depends on
      - Coil/vessel currents and mutuals
      - Plasma current
      - First & second vertical derivative of plasma-coil/vessel mutuals
  - Normalized, massless growth rate $\gamma\tau$
    - Requires coil/vessel resistances as well
    - $\gamma\tau < 10$ for controllability
- **Benchmarked to TokSys**
- **Future work: constrained optimization of passive plate locations to maximize vertical stability**

DIII-D model imported from TokSys



Plasma and Coil Current Evolution

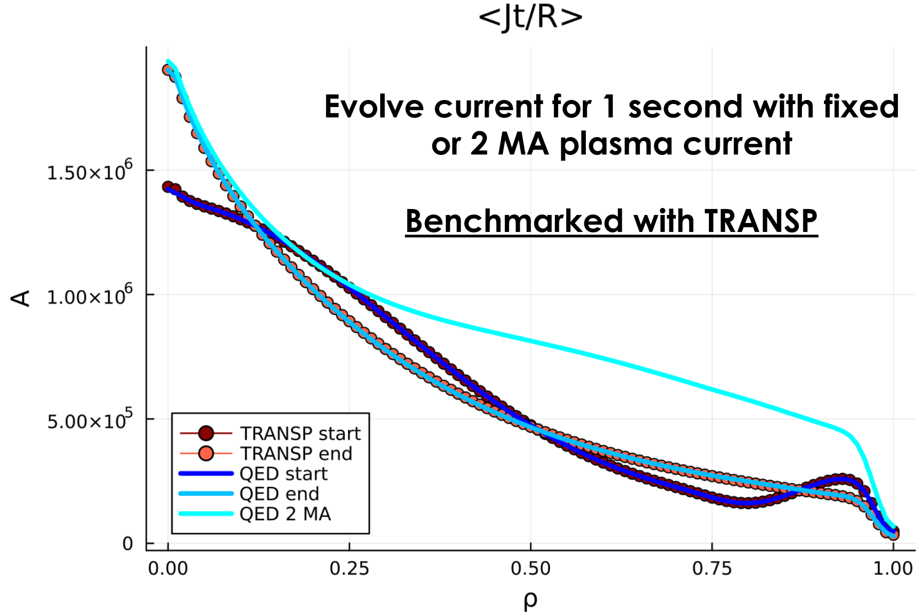
QED solves diffusion equation for rotational transform

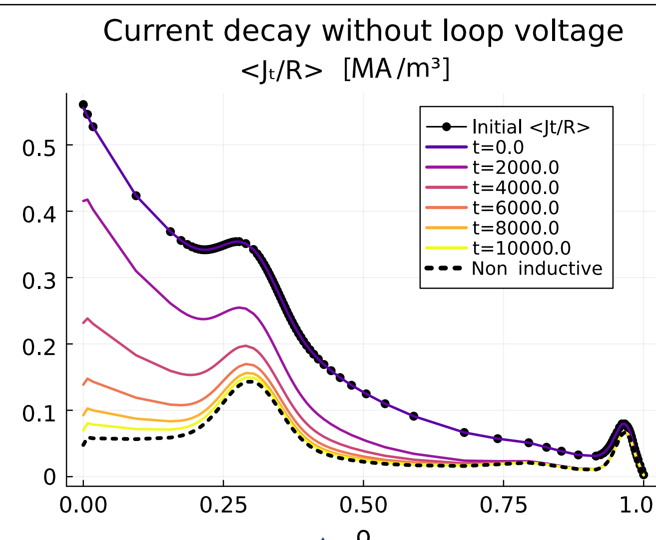
- **Lightweight, computationally efficient, stand-alone current-diffusion solver**
- **θ-implicit time advance of diffusion equation on rotational transform with non-inductive source**

$$\frac{\partial \iota}{\partial t} = \frac{1}{\Phi'}\frac{\partial V_L}{\partial \rho}$$

$$V_L = \frac{\eta F}{\mu_0 V' \langle R^{-2}\rangle}\frac{\partial}{\partial \rho}\left[\frac{\Phi' V'}{2\pi F}\left\langle \frac{|\nabla \rho|^2}{R^2}\right\rangle \iota\right] - \frac{\eta}{F\langle R^{-2}\rangle}\langle \mathbf{B}\cdot\mathbf{J}_{NI}\rangle$$

- **Rotational transform and all geometric quantities represented by cubic Hermite finite elements**
- **Boundary conditions**
  - Constant I_p – Dirichlet on ι
  - Constant edge V_L – Robin on ι





Coupling QED to time-dependent coil equations models inductive current drive and vessel response

- **All coils obey coupled circuit equations**

$$L_i \frac{dI_i}{dt} + \sum_{j \neq i} M_{ij} \frac{dI_j}{dt} + M_{pi} \frac{dI_p}{dt} + \frac{dM_{pi}}{dt} I_p + R_i I_i = V_i$$

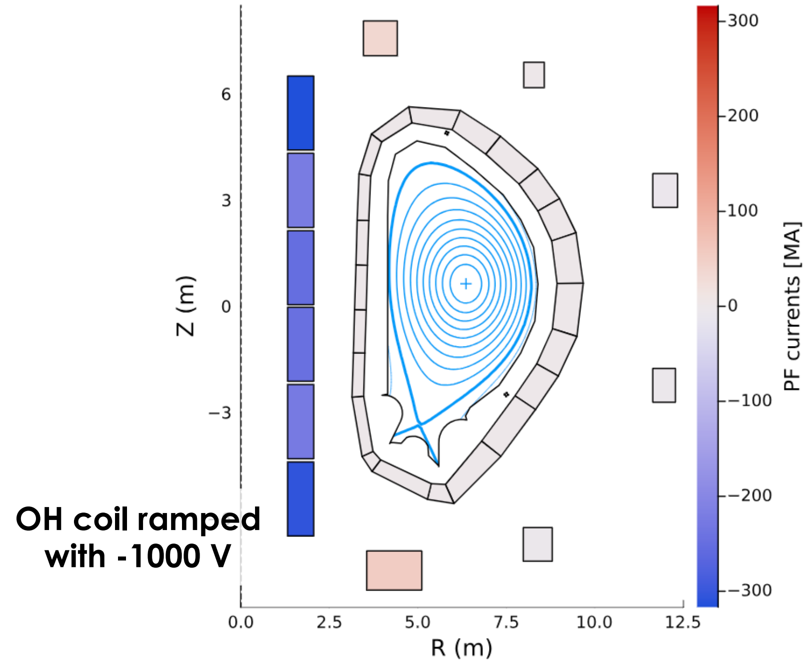
- **Coupled to plasma through additional circuit equation with scalar resistance, inductance, and non-inductive voltage**

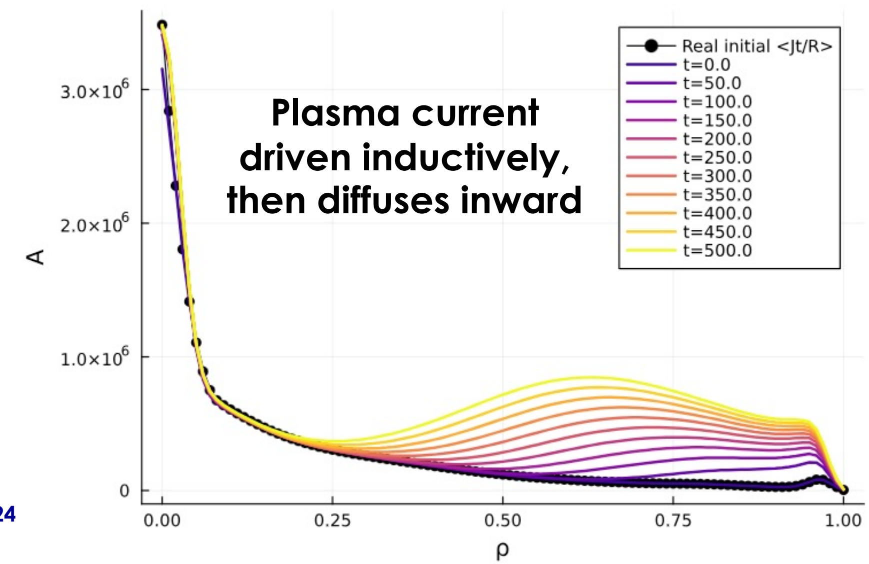
$$L_p \frac{dI_p}{dt} + \sum_j M_{pj} \frac{dI_j}{dt} + R_p I_p = V_{NI}$$

- **Couples to internal plasma current diffusion equation through boundary condition**

$$I_p = \frac{1}{(2\pi)^2 \mu_0} \frac{d\Phi}{d\rho} \frac{dV}{d\rho} \left\langle \frac{|\nabla \rho|^2}{R^2} \right\rangle \iota \Bigg|_{\rho=1}$$

- **Whole linear system can be advanced implicitly (e.g., Crank-Nicholson)**
- **Accuracy can be improved through iterative solving of G-S equilibrium and/or predictor-corrector on scalar plasma quantities**





Towards a Time-Dependent, Whole-Device Model

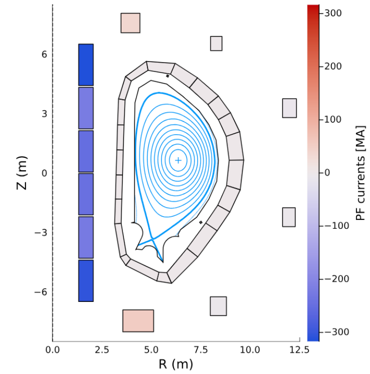
Plasma-build evolution coupled to implicit flux matching will create Grad-Hogan-like solver

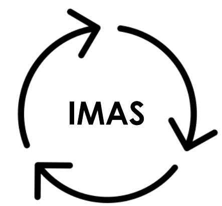
- **Profiles found from turbulent & neoclassical flux matching (cf. TGYRO), with implicit time-derivative “sources”**

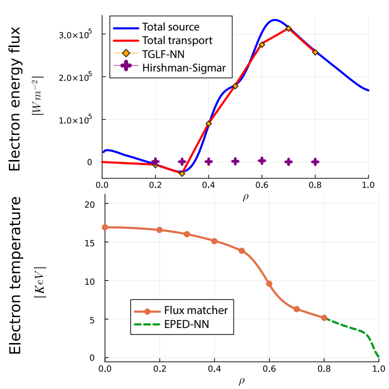
$$\langle \nabla \cdot \Gamma \rangle = S - \frac{\partial f}{\partial t}$$

- **Long time stepping, limited only by**
  - Current-diffusion time
  - Timescale of interest, e.g.,
    - Rapid shape change
    - LH transition
    - Burn initiation
- **Multi-objective optimization can be used for actuator trajectory optimization**

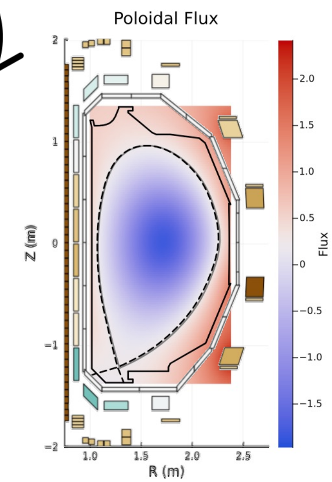
QED: Advance coil & plasma currents w/ control







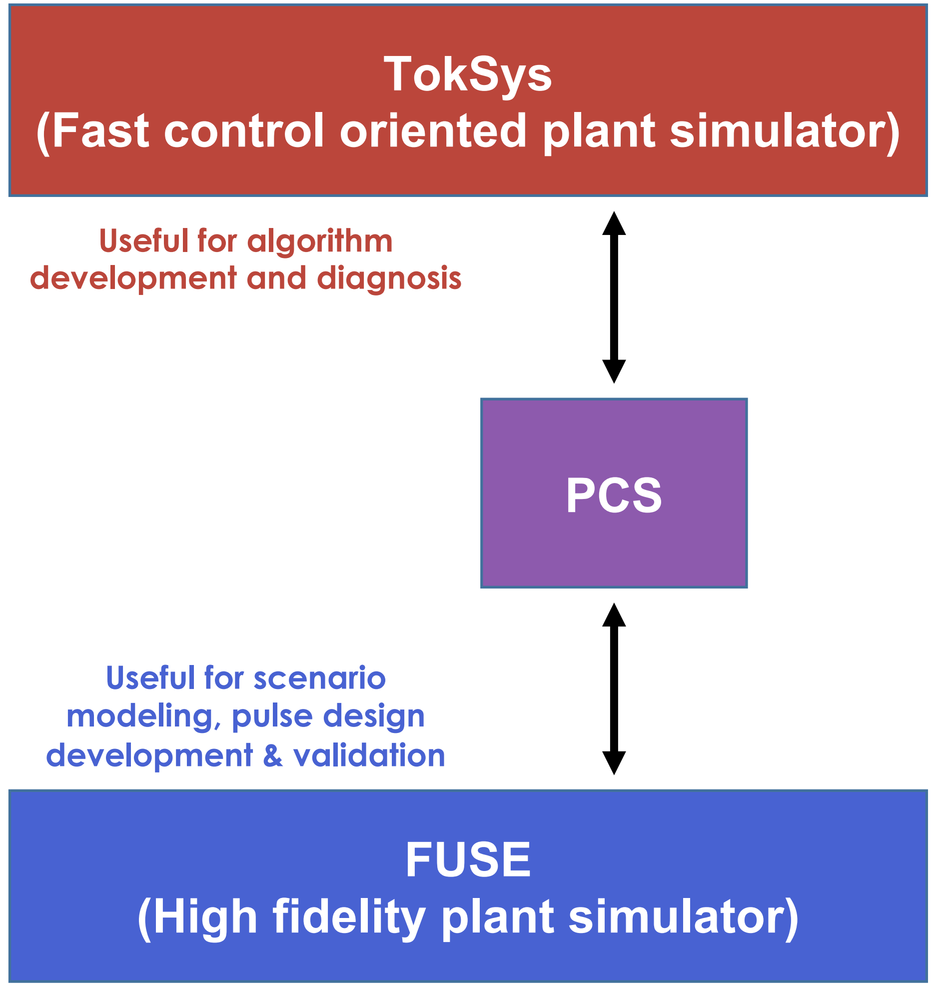
FluxMatcher: update kinetic profiles



FRESCO: update equilibrium

Coupling FUSE and TokSys will create high-fidelity plasma, build, & control model for scenario development

- Integrated simulations essential for testing control and safety, from commissioning through operation
- Co-simulation between TokSys/FUSE
  - FUSE provides high-fidelity plasma model and parametric builds
  - TokSys provides realistic control algorithms via plasma control system (PCS)
- Allows evaluation of all phases of tokamak operations
  - Pulse schedule development/validation
  - Control problem diagnosis
  - Control system development



Conclusions

- **FUSE has an extensive suite of MHD and EM capabilities**
  - Fixed and free-boundary plasma equilibria
  - PF coil current and position optimization
  - Vertical stability assessment
  - Coupled plasma current profile and coil current evolution
- **Ongoing work**
  - Couple all of the above plus transport solver to make Grad-Hogan solver
  - Couple to TokSys for realistic control simulations ("flight simulator")
- **Long-term goals**
  - Time-dependent scenario development of existing & prospective devices
  - Digital twins of experiments and reactors
- **FUSE and all packages are available as open-source Julia packages**
  - Contact lyonsbc@fusion.gat.com or meneghini@fusion.gat.com
  - For more information, visit the fuse.help website:

Acknowledgments

- This material is based upon work supported by General Atomics corporate funding and the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences under Award DE-FG02-95ER54309.