**Thermodynamic Power Cycle Analysis: A Crash Course**

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## DEFINITION OF VARIABLES

### Operational Parameters

$P$ : Pressure
$T$ : Temperature
$u$ : Internal Energy
$s$ : Entropy$ke$ : Kinetic energy
$pe$ : Potential
$V$ : Velocity
$g$ : Gravitational Acceleration

### Fluid Properties

$c_p, c_v$ : Specific heat at constant pressure ($c_p$) and constant volume ($c_v$)
$k$ : Ratio of specific heats, $k = c_p/c_v$
$R$ : Gas Constant for substance, $R = c_p - c_v = R_u/M$
$\rho$ : Density
$v$ : Specific Volume
$M, R_u$ : Molar mass ($M$) and universal gas constant ($R_u$)

1. PRELIMINARIES
   The fundamental equations associated with thermodynamics are known and well documented. The purpose of this paper is not to rederive these relationships, therefore, a certain level of familiarity with thermo is assumed for the reader. Nonetheless, this section presents several useful and neccesary relationships used throughout the report.

   (a) NOTATION
   In the majority of this report energy will be conveyed in **Unit Mass Form**, also known as "specific" form. The Total Energy, $E$ [kJ], and specific energy, $e$ [kJ/kg], are related by: $e = E/m$, where m is the mass [kg]. Similairly, often energy is expressed in time-rate form ($d/dt$) which is denoted by: $\frac{d}{dt}E = \dot{E}$ [kW]. $\dot{E}$ and $e$ are related by $\dot{E} = \dot{m}e$, where $\dot{m}$ is the mass flow rate in [kg/s].

   (b) THERMODYNAMIC PROPERTIES
   This section provides a brief review of thermodynamic properties and their implications.

   **Intensive Properties** are the mass-independent properties of a system ($T, P, \rho$). These properties cannot be put into into "specific" form as the relationship is meaningless.

   **Extensive Properties** are characterstic properties that depend on the system configuration. Meaning they are not innate to the fluid, rather a product of the conditions. Some examples of these proprties are $E$ and volume $\mathbb{V}$. These properties can be put into into "specific" form as they are a function of mass, and therefor of the form $X = mx$, wher $X$ is the total properties and $x$ is the specific property. It should also be noted that the specific volume $\nu = \frac{\mathbb{V}}{m} = \frac{1}{\rho}$

   ***In the remaining sections of the paper, any reference to $V$ is velocity while $\mathbb{V}$ is volume*

(c) KEY RELATIONSHIPS

First we review the definition of the **Total Differential,** $df$, of some function $f(x, y)$

$$df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy \tag{1}$$

We should also introduce the definition of enthalpy

$$h = u + Pdv \tag{2}$$

the gibbs relationships

$$\begin{aligned} Tds &= du + Pdv \\ Tds &= dh - vdP \end{aligned} \tag{3}$$

We now take the time to further evaluate two special cases relevant to power cycles.

i. Incompressible Substances

By nature of incompressible substances

$$\begin{aligned} \nu = \text{constant } &\rightarrow dv = 0 \\ c_p(T) = c_v(T) &= c(T) \\ du &= c(T)dT \end{aligned}$$

From the definition of enthalpy in Eq 2, we can say:

$$dh = c(T)dT + \nu dP \tag{4}$$

ii. Ideal Gas

Intuitively, the first equaton nececarry to evaluate an ideal gas is the ideal gas law:

$$P\nu = RT \tag{5}$$

In addition, we show the specific relationships for an ideal gas:

$$\begin{aligned} c_p &= c_v + R \\ k &= c_p/c_v \\ du &= c_v dT \\ dh &= c_p dT \end{aligned} \tag{6}$$

Finally, when an ideal gas with *constant specific heats* undergoes an **isentropic** process, we can say:

$$\left(\frac{T_2}{T_1}\right)_{s=const} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{v_1}{v_2}\right)^{k-1} \tag{7}$$

2. ENERGY BALANCE

(a) Energy Transport by Mass

$$\theta = h + ke + pe = h + \frac{V^2}{2} + gz \tag{8}$$

(b) Total Energy of Flowing Fluid

$$\begin{aligned} E_{mass} &= m\theta \ \textit{(Amount)} \\ \dot{E}_{mass} &= \dot{m}\theta \ \textit{(Rate)} \end{aligned} \tag{9}$$

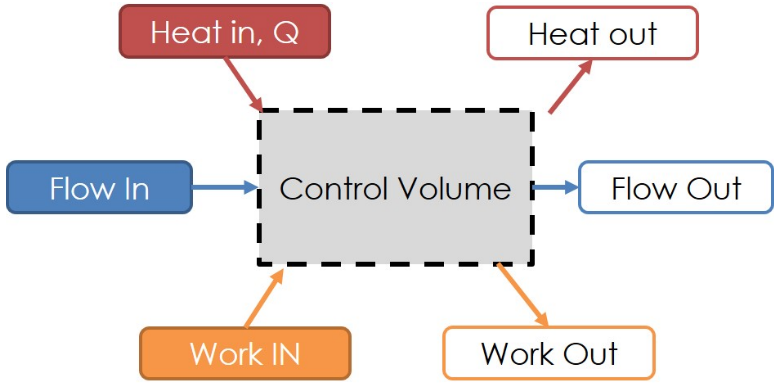


Figure 1: Control Volume General Diagram

(c) **ENERGY BALANCE**

$$\begin{aligned}
\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}\theta &= \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}\theta \\
\dot{Q}_{net} - \dot{W}_{net} &= \sum_{out} \dot{m}\theta - \sum_{in} \dot{m}\theta \\
Where\ \dot{Q}_{net} &= \dot{Q}_{in} - \dot{Q}_{out} \\
\dot{W}_{net} &= \dot{W}_{out} - \dot{W}_{in}
\end{aligned} \tag{10}$$

3. **WORK RELATED COMPONENTS**

(a) Pumps and Compressors
Pumps and compressors are components which convert external work into flow work by increasing the pressure and of the working fluid. It is standard to assume $\dot{Q} = \dot{q} = \Delta ke = \Delta pe = 0$ for said components. Furthermore, there is no output component for work, so $\dot{W}_{net} = -\dot{W}_{in}$. Equation 10 then reduces to:

$$\begin{aligned}
\dot{W}_{in} &= \sum_{out} \dot{m}\theta - \sum_{in} \dot{m}\theta \\
&= \dot{m}(h_{out} - h_{in})
\end{aligned} \tag{11}$$

mirror components, the only difference being the working fluid of the circuit. Both components require external work input in order to increase the pressure of the working fluid.

(b) Turbines

4. **HEAT TRANSFER COMPONENTS**

(a) Heat Exchangers