```
In [1]: # import datetime
    import os
    import pandas as pd
    import numpy as np
    import matplotlib.pyplot as plt
    from datlib.FRED import *
    from datlib.plots import *
    import pandas_datareader.data as web

%matplotlib inline

# Import Statsmodels

from statsmodels.tsa.api import VAR
    from statsmodels.tsa.stattools import adfuller
    from statsmodels.tsa.stattools import rmse, aic
```

```
In [2]: #FRED.py
        #. . .
        def bil to mil(series):
            return series* 10**3
        #fedProject.py
        # . . .
        data codes = {# Assets
                        "Balance Sheet: Total Assets ($ Mil)": "WALCL",
                        "Balance Sheet Securities, Prem-Disc, Repos, and Loans ($ Mil)":
                        "Balance Sheet: Securities Held Outright ($ Mil)": "WSHOSHO",
                       ### breakdown of securities holdings ###
                       "Balance Sheet: U.S. Treasuries Held Outright ($ Mil)":"WSHOTSL",
                        "Balance Sheet: Federal Agency Debt Securities ($ Mil)" : "WSHOFAD
                       "Balance Sheet: Mortgage-Backed Securities ($ Mil)": "WSHOMCB",
                       # other forms of Lending
                        "Balance Sheet: Repos ($ Mil)": "WORAL",
                       "Balance Sheet: Central Bank Liquidity Swaps ($ Mil)" : "SWPT",
                        "Balance Sheet: Direct Lending ($ Mil)" : "WLCFLL",
                       # unamortized value of securities held (due to changes in interest
                        "Balance Sheet: Unamortized Security Premiums ($ Mil)": "WUPSHO",
                       # Liabilities
                       "Balance Sheet: Total Liabilities ($ Mil)" : "WLTLECL",
                        "Balance Sheet: Federal Reserve Notes Outstanding ($ Mil)" : "WLFN
                        "Balance Sheet: Reverse Repos ($ Mil)": "WLRRAL",
                       ### Major share of deposits
                        "Balance Sheet: Deposits from Dep. Institutions ($ Mil)":"WLODLL"
                        "Balance Sheet: U.S. Treasury General Account ($ Mil)": "WDTGAL",
                        "Balance Sheet: Other Deposits ($ Mil)": "WOTHLB",
                        "Balance Sheet: All Deposits ($ Mil)": "WLDLCL",
                       # Capital
                       "Balance Sheet: Total Capital": "WCTCL",
                       # Interest Rates
                        "Unemployment Rate": "UNRATE",
                        "Nominal GDP ($ Bil)":"GDP",
                        "Real GDP ($ Bil)": "GDPC1",
                        "GDP Deflator": "GDPDEF",
                        "CPI": "CPIAUCSL",
                        "Core PCE": "PCEPILFE",
                        "Private Investment": "GPDI",
                        "Base: Total ($ Mil)": "BOGMBASE",
                        "Base: Currency in Circulation ($ Bil)": "WCURCIR",
                        "1 Month Treasury Rate (%)": "DGS1MO",
                       "3 Month Treasury Rate (%)": "DGS3MO",
                        "1 Year Treasury Rate (%)": "DGS1",
                       "2 Year Treasury Rate (%)": "DGS2",
                        "10 Year Treasury Rate (%)": "DGS10",
                       "30 Year Treasury Rate (%)": "DGS30",
                       "Effective Federal Funds Rate (%)": "DFF",
                        "Federal Funds Target Rate (Pre-crisis)": "DFEDTAR",
                       "Federal Funds Upper Target": "DFEDTARU",
                        "Federal Funds Lower Target": "DFEDTARL",
                        "Interest on Reserves (%)": "IOER",
                        "VIX": "VIXCLS",
                         "5 Year Forward Rate": "T5YIFR"
                       }
```

In [3]: data=pd.read_csv("data up.csv") data

Out[3]:

	Date	VIX	Log Total Assets	Log Currency in Circulation (\$ Bil)	Effective Federal Funds Rate (%)	Loss Function
0	12/31/2002	28.210476	13.495030	6.516891	1.238387	-4.053360
1	1/31/2003	27.424286	13.493538	6.521227	1.235161	-3.288373
2	2/28/2003	32.218421	13.488846	6.521846	1.262143	-3.684400
3	3/31/2003	30.634286	13.492065	6.527099	1.252903	-3.663571
4	4/30/2003	23.991905	13.510243	6.532415	1.258000	-4.161742
220	4/30/2021	17.416190	15.866549	7.675462	0.069000	-2.827729
221	5/31/2021	19.760500	15.878174	7.681530	0.058065	-1.116398
222	6/30/2021	16.956818	15.898266	7.686549	0.078000	-1.093716
223	7/31/2021	17.603333	15.918468	7.689882	0.098065	0.611392
224	8/31/2021	17.472727	15.930789	7.690558	0.092258	1.117317

225 rows × 6 columns

```
In [4]: data = data.set_index('Date')
```

In [5]: data

Out[5]:

	VIX	Log Total Assets	Log Currency in Circulation (\$ Bil)	Effective Federal Funds Rate (%)	Loss Function
Date					
12/31/2002	28.210476	13.495030	6.516891	1.238387	-4.053360
1/31/2003	27.424286	13.493538	6.521227	1.235161	-3.288373
2/28/2003	32.218421	13.488846	6.521846	1.262143	-3.684400
3/31/2003	30.634286	13.492065	6.527099	1.252903	-3.663571
4/30/2003	23.991905	13.510243	6.532415	1.258000	-4.161742
4/30/2021	17.416190	15.866549	7.675462	0.069000	-2.827729
5/31/2021	19.760500	15.878174	7.681530	0.058065	-1.116398
6/30/2021	16.956818	15.898266	7.686549	0.078000	-1.093716
7/31/2021	17.603333	15.918468	7.689882	0.098065	0.611392
8/31/2021	17.472727	15.930789	7.690558	0.092258	1.117317

225 rows × 5 columns

In [6]: data_diff = data.diff(year).dropna()
data_diff

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	VIX	Log Total Assets	Log Currency in Circulation (\$ Bil)	Effective Federal Funds Rate (%)	Loss Function
Date					
12/31/2003	-11.384113	0.045793	0.057056	-0.254194	1.032640
1/31/2004	-11.323286	0.047454	0.049309	-0.238065	0.368954
2/29/2004	-16.220000	0.043829	0.047986	-0.254901	1.102860
3/31/2004	-12.946894	0.043121	0.043676	-0.251290	0.404229
4/30/2004	-8.293333	0.033697	0.042352	-0.254000	1.601901
4/30/2021	-24.037619	0.210753	0.132517	0.020000	112.839893
5/31/2021	-11.136500	0.124273	0.116395	0.008065	84.519025
6/30/2021	-14.162727	0.118755	0.108215	0.000333	48.659408
7/31/2021	-9.237121	0.162716	0.099906	0.005484	39.538836
8/31/2021	-5.416797	0.172818	0.087397	-0.002903	20.729400

213 rows × 5 columns

In [7]: data_new = data_diff.diff(year).dropna()

In [8]: data_new

Out[8]:

	VIX	Log Total Assets	Log Currency in Circulation (\$ Bil)	Effective Federal Funds Rate (%)	Loss Function
Date					
12/31/2004	7.017294	0.016904	-0.006413	1.426129	0.032383
1/31/2005	8.660286	0.012750	0.000492	1.520323	0.889740
2/28/2005	11.930526	0.016558	0.002776	1.749446	-0.450845
3/31/2005	8.385867	0.017146	0.008396	1.878710	1.480530
4/30/2005	7.053810	0.019289	0.007823	2.035000	-0.470287
4/30/2021	-52.542381	-0.260106	0.043371	2.394667	228.571523
5/31/2021	-25.311682	-0.460421	0.008424	2.349032	170.165523
6/30/2021	-29.446273	-0.499225	-0.009767	2.300333	98.492688
7/31/2021	-22.771667	-0.441530	-0.024475	2.315806	78.485753
8/31/2021	-9.327229	-0.441747	-0.048900	2.027742	40.409641

201 rows × 5 columns

Υ

Ζ

In [12]: df

Out[12]: P Q X

Date					
12/31/2004	7.017294	0.016904	-0.006413	1.426129	0.032383
1/31/2005	8.660286	0.012750	0.000492	1.520323	0.889740
2/28/2005	11.930526	0.016558	0.002776	1.749446	-0.450845
3/31/2005	8.385867	0.017146	0.008396	1.878710	1.480530
4/30/2005	7.053810	0.019289	0.007823	2.035000	-0.470287
4/30/2021	-52.542381	-0.260106	0.043371	2.394667	228.571523
5/31/2021	-25.311682	-0.460421	0.008424	2.349032	170.165523
6/30/2021	-29.446273	-0.499225	-0.009767	2.300333	98.492688
7/31/2021	-22.771667	-0.441530	-0.024475	2.315806	78.485753
8/31/2021	-9.327229	-0.441747	-0.048900	2.027742	40.409641

201 rows × 5 columns

residuals = pd.DataFrame(residuals)

```
In [14]: ## Partial Correlation

import statsmodels.api as sm

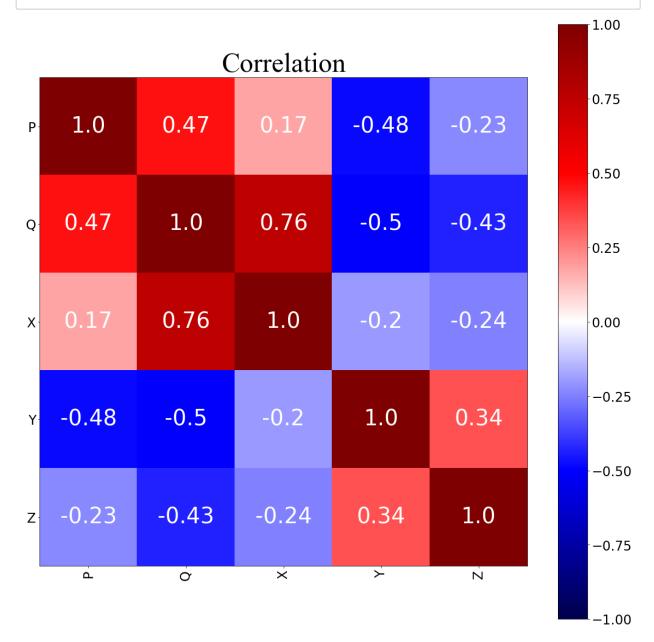
residuals = {}
for y_var in df.keys():
    X_vars = list(df.keys())
    X_vars.remove(y_var)
    X = df[X_vars]
    # Initial estimate should include constant
    # This won't be the case we regress the errors
    X["Constant"] = 1
    # pass y_var as list for consistent structure
    y = df[[y_var]]
    model = sm.OLS(y, X)
    results = model.fit()
    residuals[y_var] = results.resid
```

In [15]: residuals Out[15]: Ρ Q X Υ Ζ **Date** 12/31/2004 9.849021 0.112525 -0.014792 -4.395330 1.754966 1/31/2005 13.077744 0.065692 -0.007423 1.773681 -4.903426 2/28/2005 17.384859 0.051942 -0.005322 2.052677 -7.202058 3/31/2005 15.139816 0.041784 -0.001625 2.030169 -6.030974 4/30/2005 14.196069 0.057093 -0.003338 2.185788 -8.155989 0.230718 4/30/2021 -34.986555 0.019126 -1.142877 208.280595 5/31/2021 -5.785034 0.030449 0.018060 -0.366177 144.187619 6/30/2021 -10.050999 -0.014380 0.008812 0.078302 73.532742 7/31/2021 -7.182427 0.072184 -0.007770 0.681436 57.155418 8/31/2021 0.085653 -0.023287 21.874820 2.183128 1.199790 201 rows × 5 columns In [17]: residuals.corr()[residuals.corr().abs() < 1].mul(-1).fillna(1).round(2)</pre> Out[17]: Р Q X Υ Ζ 1.00 0.36 -0.23 -0.25 0.04 0.36 1.00 0.78 -0.36 -0.27 -0.23 0.78 1.00 0.24 0.10 -0.25 -0.36 0.24 1.00 0.13 0.04 -0.27 0.10 0.13 1.00

In [18]: # !pip install pingouin import pingouin df.pcorr().round(2)

Out[18]:

	Р	Q	X	Y	Z
Р	1.00	0.36	-0.23	-0.25	0.04
Q	0.36	1.00	0.78	-0.36	-0.27
X	-0.23	0.78	1.00	0.24	0.10
Υ	-0.25	-0.36	0.24	1.00	0.13
z	0.04	-0.27	0.10	0.13	1.00



TypeError: corr_matrix_heatmap() got an unexpected keyword argument 'title'

```
In [21]: residuals
Out[21]:
                              Ρ
                                        Q
                                                  Χ
                                                            Υ
                                                                        Ζ
                 Date
            12/31/2004
                        9.849021
                                  0.112525 -0.014792
                                                     1.754966
                                                                 -4.395330
             1/31/2005
                       13.077744
                                  0.065692 -0.007423
                                                      1.773681
                                                                 -4.903426
             2/28/2005
                       17.384859
                                  0.051942 -0.005322
                                                      2.052677
                                                                 -7.202058
             3/31/2005
                                  0.041784 -0.001625
                       15.139816
                                                      2.030169
                                                                 -6.030974
             4/30/2005
                                  0.057093 -0.003338
                      14.196069
                                                      2.185788
                                                                 -8.155989
             4/30/2021 -34.986555
                                 0.230718
                                            0.019126 -1.142877
                                                               208.280595
             5/31/2021
                       -5.785034
                                  0.030449
                                            0.018060
                                                     -0.366177
                                                               144.187619
             6/30/2021 -10.050999
                                 -0.014380
                                                      0.078302
                                            0.008812
                                                                73.532742
             7/31/2021
                       -7.182427
                                  0.072184 -0.007770
                                                      0.681436
                                                                57.155418
             8/31/2021
                                  0.085653 -0.023287
                                                      1.199790
                        2.183128
                                                                21.874820
          201 rows × 5 columns
In [22]: |pcorr_pvalues = {}
          for y, Y in residuals.items():
               pcorr_pvalues[y] = {}
               for x, X in residuals.items():
                    if x != y:
                        pcorr_pvalues[y][x] = sm.OLS(Y,X).fit().pvalues[x]
                    else:
                        pcorr_pvalues[y][x] = np.NaN
          pd.DataFrame(pcorr pvalues).round(2)
Out[22]:
                 Ρ
                                  Υ
                                       Ζ
                      Q
                            X
                     0.0 0.00 0.00 0.56
              NaN
           Q 0.00
                    NaN 0.00 0.00 0.00
              0.00
                     0.0 NaN 0.00 0.15
               0.00
                     0.0 0.00 NaN 0.07
```

Z 0.56

0.0 0.15 0.07 NaN

```
In [23]: undirected_graph = {key:[] for key in df.keys()}
for x in undirected_graph:
    remaining_vars = [y for y in df.keys() if y != x]
    for y in remaining_vars:
        undirected_graph[x].append(y)

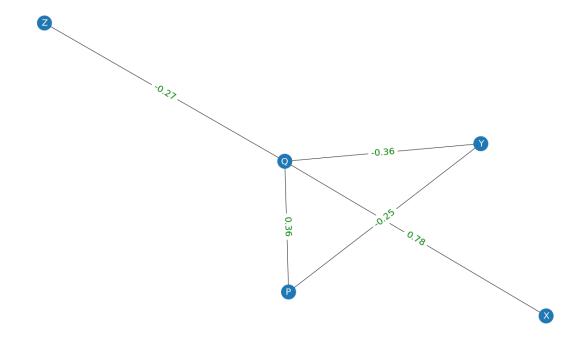
undirected_graph

Out[23]: {'P': ['Q', 'X', 'Y', 'Z'],
        'Q': ['P', 'X', 'Y', 'Z'],
        'X': ['P', 'Q', 'Y', 'Z'],
        'Y': ['P', 'Q', 'X', 'Z'],
        'Z': ['P', 'Q', 'X', 'Y']}
```

```
In [24]: import copy
         p_val = .01
         def build skeleton(df, undirected graph):
             def check remaining controls(control vars, undirected graph, x, y, controls (
                 for c var in control vars:
                     # set c_used every time use cycle through a new control
                     # the program will then iterate through remaining controls
                     # until statistical significance is broken
                      c used = copy.copy(controls used)
                      if y in undirected_graph[x]:
                         c used.append(c var)
                         test = df.partial_corr(x = x, y = y, covar=c_used,
                                                method = "pearson")
                         if test["p-val"].values[0] > p_val:
                              undirected graph[x].remove(y)
                              #breakout of the for
                             break
                         else:
                              remaining controls = copy.copy(control vars)
                             remaining_controls.remove(c_var)
                             # recursive function that iterates through remaining variable
                             # uses them as controls statistical significance holds with
                             # otherwise break
                              check remaining controls(remaining controls, undirected grap∤
             for x in df.keys():
                 ys = undirected graph[x]
                 for y in df.keys():
                     if x != y:
                      # first check for correlation with no controls
                         test = df.partial corr(x = x,
                                                 y = y,
                                                 covar = None,
                                                 method = "pearson")
                         if test["p-val"].values[0] > p val:
                             undirected_graph[x].remove(y)
                      # if correlated check for deseparation controlling for other variable
                         else:
                              control_vars = [z for z in df.keys() if z != y and z != x]
                              check remaining controls(control vars, undirected graph, x, )
             return undirected graph
         undirected graph = build skeleton(df, undirected graph)
         undirected graph
Out[24]: {'P': ['Q', 'Y'],
           'Q': ['P', 'X', 'Y', 'Z'],
           'X': ['Q'],
           'Y': ['P', 'Q'],
           'Z': ['Q']}
```

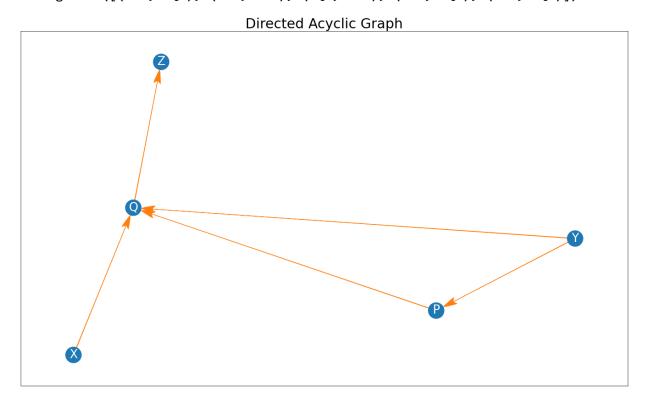
```
In [25]: import matplotlib.pyplot as plt
         import networkx as nx
         def graph_DAG(undirected_graph, df, title = "DAG Structure"):
             # generate partial correlation matrix to draw values from
             # for graph edges
             pcorr_matrix = df.pcorr()
             graph = nx.Graph()
             edges = []
             edge_labels = {}
             for key in undirected graph:
                 for key2 in undirected_graph[key]:
                     if (key2, key) not in edges:
                         edge = (key.replace(" ","\n"), key2[0].replace(" ","\n"))
                         edges.append(edge)
                         # edge label is partial correlation between
                         # key and key2
                         edge_labels[edge] = str(round(pcorr_matrix.loc[key][key2],2))
             # edge format: ("i", "j") --> from node i to node j
             graph.add edges from(edges)
             color_map = ["CO" for g in graph]
             fig, ax = plt.subplots(figsize = (20,12))
             graph.nodes()
             plt.tight layout()
             pos = nx.spring layout(graph)#, k = 5/(len(sig\ corr.keys())**.5))
             plt.title(title, fontsize = 30)
             nx.draw_networkx(graph, pos, node_color=color_map,
                              node_size = 1000,
                              with_labels=True, arrows=False,
                              font size = 20, alpha = 1,
                              font_color = "white",
                              ax = ax)
             nx.draw_networkx_edge_labels(graph,pos,
                                           edge_labels=edge_labels,
                                           font_color='green',
                                           font size=20)
             plt.axis("off")
             plt.savefig("g1.png", format="PNG")
             plt.show()
```

Undirected Graph with Partial Correlations from Full Set of Controls



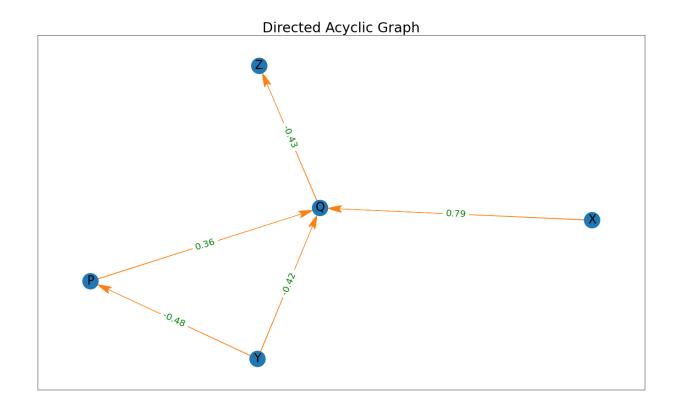
```
In [28]: from matplotlib.patches import ArrowStyle
         def graph_DAG(edges, df, title = ""):
             graph = nx.DiGraph()
             graph.add_edges_from(edges)
             color_map = ["CO" for g in graph]
             fig, ax = plt.subplots(figsize = (20,12))
             graph.nodes()
             plt.tight_layout()
             pos = nx.spring_layout(graph)#, k = 5/(Len(sig_corr.keys())**.5))
             plt.title(title, fontsize = 30)
             nx.draw_networkx(graph, pos, node_color=color_map, node_size = 1200,
                              with labels=True, arrows=True,
                              font_color = "white",
                              font_size = 26, alpha = 1,
                              width = 1, edge_color = "C1",
                              arrowstyle=ArrowStyle("Fancy, head_length=3, head_width=1.5,
         graph_DAG(edges, df, title = "Directed Acyclic Graph")
         edges
```

Out[28]: OutEdgeView([('Y', 'Q'), ('Y', 'P'), ('Q', 'Z'), ('X', 'Q'), ('P', 'Q')])



```
In [29]: ## D-separation
         def graph DAG(edges, df, title = ""):
             graph = nx.DiGraph()
             edge labels = {}
             ########## Add ###########
             for edge in edges:
                 controls = [key for key in df.keys() if key not in edge]
                 controls = list(set(controls))
                 keep controls = []
                 for control in controls:
                     control_edges = [ctrl_edge for ctrl_edge in edges if control == ctrl]
                     if (control, edge[1]) in control_edges:
                         print("keep control:", control)
                         keep controls.append(control)
                 print(edge, keep_controls)
                 pcorr = df[[edge[0], edge[1]]+keep_controls].pcorr()
                   corr_matrix_heatmap(pcorr, save_fig = False, pp = None, title = "Partic
                 edge_labels[edge] = str(round(pcorr[edge[0]].loc[edge[1]],2))
             graph.add edges from(edges)
             color map = ["C0" for g in graph]
             fig, ax = plt.subplots(figsize = (20,12))
             graph.nodes()
             plt.tight_layout()
             pos = nx.spring layout(graph)#, k = 5/(len(siq\ corr.keys())**.5))
             plt.title(title, fontsize = 30)
             nx.draw_networkx(graph, pos, node_color=color_map, node_size = 1200,
                              with labels=True, arrows=True,
                              # turn text black for larger variable names in homework
                              font color = "k",
                              font size = 26, alpha = 1,
                              width = 1, edge color = "C1",
                              arrowstyle=ArrowStyle("Fancy, head_length=3, head_width=1.5]
             ########## Add ##########
             nx.draw_networkx_edge_labels(graph,pos,
                                          edge_labels=edge_labels,
                                          font color='green',
                                          font size=20)
         graph_DAG(edges, df, title = "Directed Acyclic Graph")
         keep control: X
         keep control: P
         ('Y', 'Q') ['X', 'P']
         ('Y', 'P') []
         ('Q', 'Z') []
         keep control: Y
         keep control: P
         ('X', 'Q') ['Y', 'P']
         keep control: Y
         keep control: X
```

('P', 'Q') ['Y', 'X']



```
In [30]: def graph_DAG(edges, df, title = "", fig = False, ax = False):
             graph = nx.DiGraph()
             edge labels = {}
             for edge in edges:
                 controls = [key for key in df.keys() if key not in edge]
                 controls = list(set(controls))
                 keep controls = []
                 for control in controls:
                      control_edges = [ctrl_edge for ctrl_edge in edges if control == ctrl]
                      if (control, edge[1]) in control_edges:
                          keep controls.append(control)
                 pcorr = df[[edge[0], edge[1]]+keep_controls].pcorr()
                 edge_labels[edge] = str(round(pcorr[edge[0]].loc[edge[1]],2))
             graph.add edges from(edges)
             color_map = ["C0" for g in graph]
             # add fig and ax if none passed to function
             if not fig and not ax:
                 fig, ax = plt.subplots(figsize = (20,12))
             graph.nodes()
             plt.tight layout()
             pos = nx.spring_layout(graph)
             # use ax.set_title to access subplot when setting title
             ax.set_title(title, fontsize = 30)
             nx.draw_networkx(graph, pos, node_color=color_map, node_size = 1200,
                               with labels=True, arrows=True,
                               font_color = "k",
                               font size = 26, alpha = 1,
                               width = 1, edge color = "C1",
                               arrowstyle = ArrowStyle("Fancy, head_length=3, head_width=1.
                               ax = ax) # reference ax for specific subplot since that is p
                                      # just using plt.title will only add title to very ld
             nx.draw_networkx_edge_labels(graph,pos,
                                          edge_labels=edge_labels,
                                          font color='green',
                                          font size=20,
                                          ax = ax)
         algorithms = ["orig", "stable", "parallel"]
         p_{vals} = [0.1, 0.2, 0.3]
         i,j = 0,0
         fig, ax = plt.subplots(len(algorithms), len(p_vals), figsize = (30,30))
         # use i in range(len(algorithm)) instead of algorithm in algorithms for ax refere
         for i in range(len(algorithms)):
             for j in range(len(p_vals)):
                 a = ax[i]
                 algorithm = algorithms[i]
                 p_val = p_vals[j]
                 c = PC(df)
                 max_cond_vars = len(df.keys()) - 2
                 model = c.estimate(return_type = "dag",
                                     variant = algorithm,
                                     significance level = p val,
                                     max_cond_vars = max_cond_vars,
```

```
ci_test = "pearsonr")
edges = model.edges()
a = ax[i][j]
graph_DAG(edges, df, fig = fig, ax = a)

if j == 0:
    a.set_ylabel(algorithm, fontsize = 20)
if i == len(algorithms) - 1:
    a.set_xlabel("$p \leq$ "+ str(p_val), fontsize = 20)
if i == 3:
    break

import pingouin as pg
import matplotlib.pyplot as plt
import networkx as nx
from pgmpy.estimators import PC
from matplotlib.patches import ArrowStyle
```

```
0% | 0/3 [00:00<?, ?it/s]
```

C:\Users\HP\anaconda3\lib\site-packages\pgmpy\base\DAG.py:1187: UserWarning: PD
AG has no faithful extension (= no oriented DAG with the same v-structures as P
DAG). Remaining undirected PDAG edges oriented arbitrarily.
warn(

```
0%|
             | 0/3 [00:00<?, ?it/s]
```

