Markov model

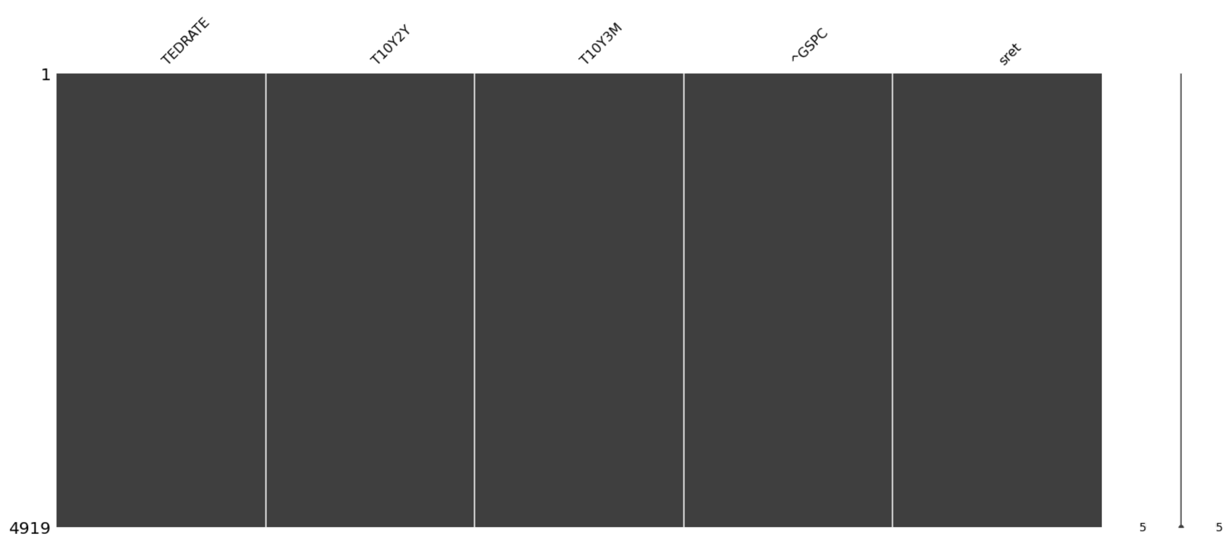
#In this paper, the observable variables I use are: the underlying asset returns, the Ted Spread, the 10 year - 2 year constant maturity spread, and the 10 year - 3 month constant maturity spread.

In [5]:
```
import pandas as pd
import pandas_datareader.data as web
import sklearn.mixture as mix

import numpy as np
import scipy.stats as scs

import matplotlib as mpl
from matplotlib import cm
import matplotlib.pyplot as plt
from matplotlib.dates import YearLocator, MonthLocator
%matplotlib inline

import seaborn as sns
import missingno as msno
from tqdm import tqdm
p=print
```

In [7]:
```
# get fed data

f1 = 'TEDRATE' # ted spread
f2 = 'T10Y2Y' # constant maturity ten yer - 2 year
f3 = 'T10Y3M' # constant maturity 10yr - 3m

start = pd.to_datetime('2002-01-01')
end = pd.datetime.today()

mkt = '^GSPC'
MKT = (web.DataReader([mkt], 'yahoo', start, end)['Adj Close']
       .rename(columns={mkt:mkt})
       .assign(sret=lambda x: np.log(x[mkt]/x[mkt].shift(1)))
       .dropna())

data = (web.DataReader([f1, f2, f3], 'fred', start, end)
        .join(MKT, how='inner')
        .dropna()
       )

p(data.head())

# gives us a quick visual inspection of the data
msno.matrix(data)
```

```
C:\Users\PROMA~1.GUP\AppData\Local\Temp/ipykernel_7812/1834947210.py:8: FutureW
arning: The pandas.datetime class is deprecated and will be removed from pandas
in a future version. Import from datetime module instead.
  end = pd.datetime.today()

            TEDRATE  T10Y2Y  T10Y3M        ^GSPC      sret
2002-01-03     0.18    1.97    3.43  1165.270020  0.009138
2002-01-04     0.18    1.99    3.46  1172.510010  0.006194
2002-01-07     0.21    2.01    3.41  1164.890015 -0.006520
2002-01-08     0.19    2.03    3.42  1160.709961 -0.003595
2002-01-09     0.19    2.07    3.42  1155.140015 -0.004810
```

Out[7]: <AxesSubplot:>



Next we will use the sklearn's GaussianMixture to fit a model that estimates these regimes. We will explore mixture models in more depth in part 2 of this series. The important takeaway is that mixture models implement a closely related unsupervised form of density estimation. It makes use of the expectation-maximization algorithm to estimate the means and covariances of the hidden states (regimes). For now, it is ok to think of it as a magic button for guessing the transition and emission probabilities, and most likely path.

We have to specify the number of components for the mixture model to fit to the time series. In this example the components can be thought of as regimes. We will arbitrarily classify the regimes as High, Neutral and Low Volatility and set the number of components to three.

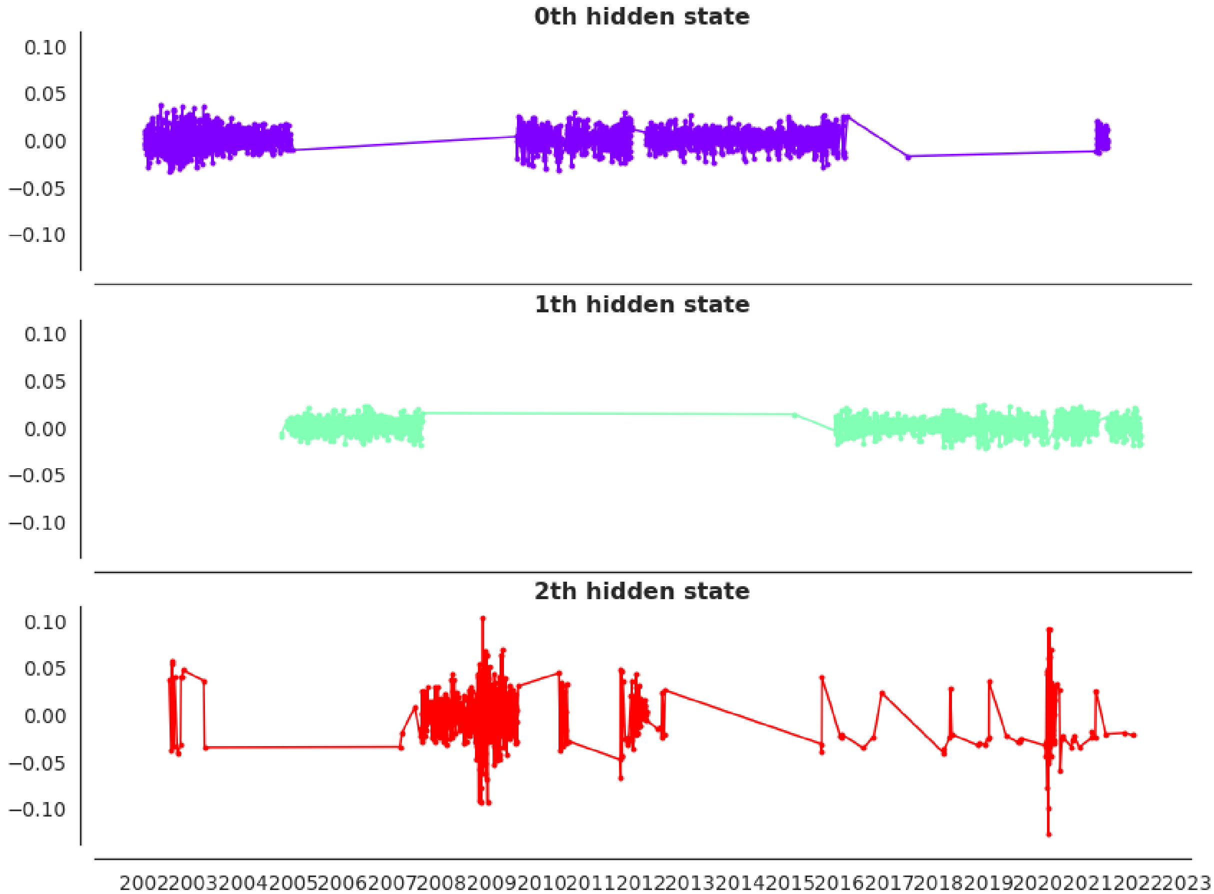
In [8]:

```python
# code adapted from http://hmmlearn.readthedocs.io
# for sklearn 18.1

col = 'sret'
select = data.loc[:].dropna()

ft_cols = [f1, f2, f3, 'sret']
X = select[ft_cols].values

model = mix.GaussianMixture(n_components=3,
                            covariance_type="full",
                            n_init=100,
                            random_state=7).fit(X)

# Predict the optimal sequence of internal hidden state
hidden_states = model.predict(X)

print("Means and vars of each hidden state")
for i in range(model.n_components):
    print("{0}th hidden state".format(i))
    print("mean = ", model.means_[i])
    print("var = ", np.diag(model.covariances_[i]))
    print()

sns.set(font_scale=1.25)
style_kwds = {'xtick.major.size': 3, 'ytick.major.size': 3,
              'font.family':u'courier prime code', 'legend.frameon': True}
sns.set_style('white', style_kwds)

fig, axs = plt.subplots(model.n_components, sharex=True, sharey=True, figsize=(12
colors = cm.rainbow(np.linspace(0, 1, model.n_components))

for i, (ax, color) in enumerate(zip(axs, colors)):
    # Use fancy indexing to plot data in each state.
    mask = hidden_states == i
    ax.plot_date(select.index.values[mask],
                 select[col].values[mask],
                 ".-", c=color)
    ax.set_title("{0}th hidden state".format(i), fontsize=16, fontweight='demi')

    # Format the ticks.
    ax.xaxis.set_major_locator(YearLocator())
    ax.xaxis.set_minor_locator(MonthLocator())
    sns.despine(offset=10)

plt.tight_layout()
fig.savefig('Hidden Markov (Mixture) Model_Regime Subplots.png')
```

```
Means and vars of each hidden state
0th hidden state
mean =  [2.25100703e-01 2.04909870e+00 2.63031637e+00 3.77580523e-04]
var =  [3.91648738e-03 2.18841577e-01 4.01972608e-01 8.99057525e-05]

1th hidden state
mean =  [0.33975318 0.53700146 0.80487591 0.00087417]
var =  [2.27770175e-02 2.07791317e-01 5.13936355e-01 4.86038598e-05]
```

```
2th hidden state
mean =  [ 1.01573604  1.47921652  1.91023814 -0.00198837]
var =  [5.18005341e-01 4.25911451e-01 9.07279539e-01 5.82397332e-04]
```

```
findfont: Font family ['courier prime code'] not found. Falling back to DejaVu Sans.
findfont: Font family ['courier prime code'] not found. Falling back to DejaVu Sans.
```



In the above image, I've highlighted each regime's daily expected mean and variance of SPY returns. It appears the 1th hidden state is our low volatility regime. Note that the 1th hidden state has the largest expected return and the smallest variance.The 0th hidden state is the neutral volatility regime with the second largest return and variance. Lastly the 2th hidden state is high volatility regime. We can see the expected return is negative and the variance is the largest of the group.

In [9]:
```
sns.set(font_scale=1.5)
states = (pd.DataFrame(hidden_states, columns=['states'], index=select.index)
          .join(select, how='inner')
          .assign(mkt_cret=select.sret.cumsum())
          .reset_index(drop=False)
          .rename(columns={'index':'Date'}))
p(states.head())

sns.set_style('white', style_kwds)
order = [0, 1, 2]
fg = sns.FacetGrid(data=states, hue='states', hue_order=order, aspect=1.31, size=
fg.map(plt.scatter, 'Date', mkt, alpha=0.8).add_legend()
sns.despine(offset=10)
fg.fig.suptitle('Historical S&P 500 Regimes', fontsize=24, fontweight='demi')
fg.savefig('Hidden Markov (Mixture) Model_SP Regimes.png')
```

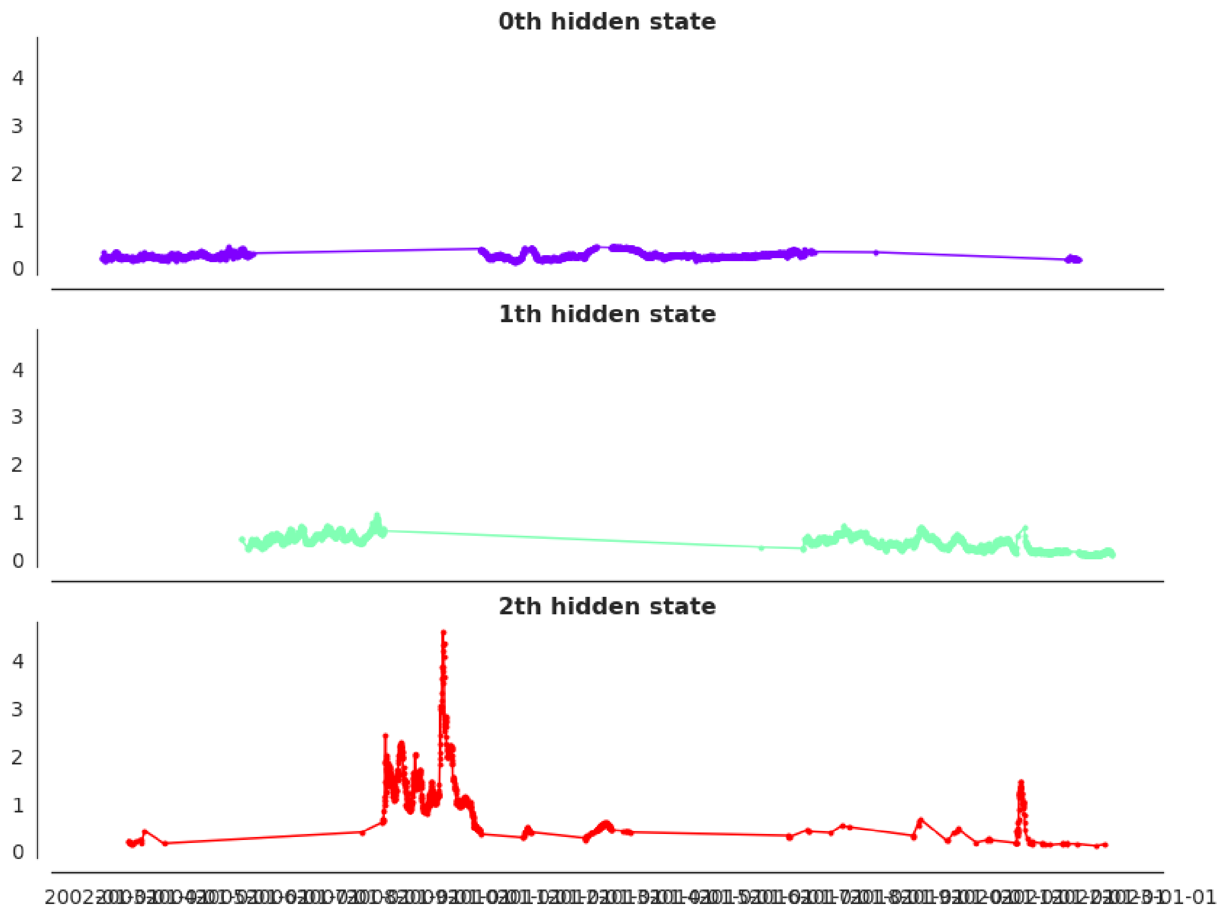
```
C:\Users\proma.gupta\Anaconda3\lib\site-packages\seaborn\axisgrid.py:337: UserW
arning: The `size` parameter has been renamed to `height`; please update your c
ode.
  warnings.warn(msg, UserWarning)
findfont: Font family ['courier prime code'] not found. Falling back to DejaVu
Sans.

        Date  states  TEDRATE  T10Y2Y  T10Y3M        ^GSPC      sret  mkt_cret
0 2002-01-03       0     0.18    1.97    3.43  1165.270020  0.009138  0.009138
1 2002-01-04       0     0.18    1.99    3.46  1172.510010  0.006194  0.015332
2 2002-01-07       0     0.21    2.01    3.41  1164.890015 -0.006520  0.008812
3 2002-01-08       0     0.19    2.03    3.42  1160.709961 -0.003595  0.005217
4 2002-01-09       0     0.19    2.07    3.42  1155.140015 -0.004810  0.000407

findfont: Font family ['courier prime code'] not found. Falling back to DejaVu
Sans.
findfont: Font family ['courier prime code'] not found. Falling back to DejaVu
Sans.
```


Historical S&P 500 Regimes
^GSPC
4000
3000
2000
states
0
1
2

```
In [16]: #TEDRATE  T10Y2Y  T10Y3M
col = 'TEDRATE'
select = data.loc[:].dropna()

ft_cols = [f1, f2, f3, 'sret']
X = select[ft_cols].values

model = mix.GaussianMixture(n_components=3,
                            covariance_type="full",
                            n_init=100,
                            random_state=7).fit(X)

# Predict the optimal sequence of internal hidden state
hidden_states = model.predict(X)

print("Means and vars of each hidden state")
for i in range(model.n_components):
    print("{0}th hidden state".format(i))
    print("mean = ", model.means_[i])
    print("var = ", np.diag(model.covariances_[i]))
    print()

sns.set(font_scale=1.25)
style_kwds = {'xtick.major.size': 3, 'ytick.major.size': 3,
              'font.family':u'courier prime code', 'legend.frameon': True}
sns.set_style('white', style_kwds)

fig, axs = plt.subplots(model.n_components, sharex=True, sharey=True, figsize=(12
colors = cm.rainbow(np.linspace(0, 1, model.n_components))

for i, (ax, color) in enumerate(zip(axs, colors)):
    # Use fancy indexing to plot data in each state.
    mask = hidden_states == i
    ax.plot_date(select.index.values[mask],
                 select[col].values[mask],
                 ".-", c=color)
    ax.set_title("{0}th hidden state".format(i), fontsize=16, fontweight='demi')

    # Format the ticks.
    ax.xaxis.set_major_locator(YearLocator())
    ax.xaxis.set_minor_locator(MonthLocator())
    sns.despine(offset=10)

plt.tight_layout()
fig.savefig('Hidden Markov (Mixture) Model_Regime Subplots.png')
```

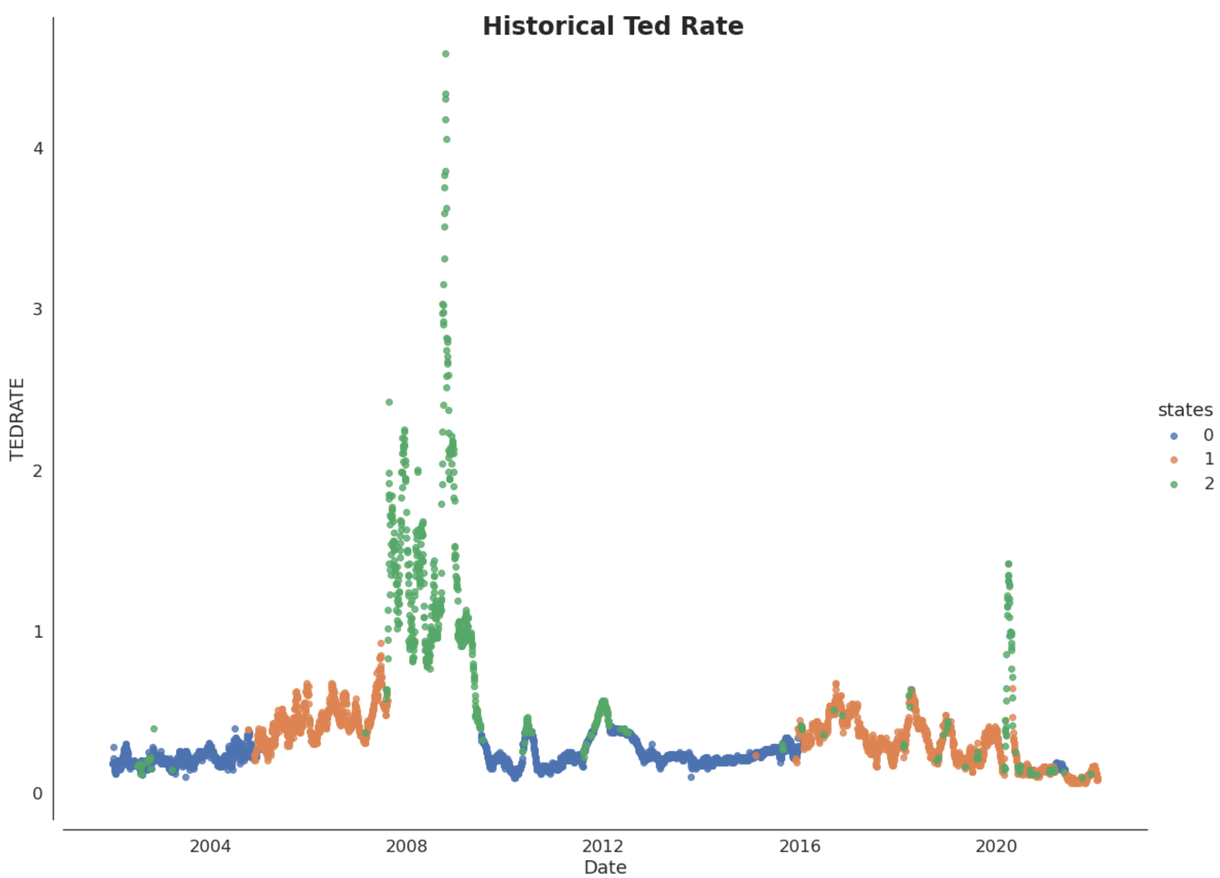
```
Means and vars of each hidden state
0th hidden state
mean =  [2.25086543e-01 2.04890896e+00 2.63007936e+00 4.37994419e-04]
var =  [3.91438511e-03 2.19060148e-01 4.02046533e-01 8.92091549e-05]

1th hidden state
mean =  [0.33979542 0.53638135 0.80381946 0.00095597]
var =  [2.27922160e-02 2.07413442e-01 5.13214844e-01 4.86792853e-05]

2th hidden state
```

```
mean =  [ 1.01493636  1.48071078  1.91291297 -0.0019215 ]
var =  [5.18046381e-01 4.24401712e-01 9.04472869e-01 5.58988846e-04]
```



```
In [17]: sns.set(font_scale=1.5)
states = (pd.DataFrame(hidden_states, columns=['states'], index=select.index)
          .join(select, how='inner')
          .assign(mkt_cret=select.TEDRATE.cumsum())
          .reset_index(drop=False)
          .rename(columns={'index':'Date'}))
p(states.head())

sns.set_style('white', style_kwds)
order = [0, 1, 2]
fg = sns.FacetGrid(data=states, hue='states', hue_order=order, aspect=1.31, size=
fg.map(plt.scatter, 'Date','TEDRATE', alpha=0.8).add_legend()
sns.despine(offset=10)
fg.fig.suptitle('Historical Ted Rate', fontsize=24, fontweight='demi')
fg.savefig('Ted Rate.png')
```

```
        Date  states  TEDRATE  T10Y2Y  T10Y3M        SPY      sret  mkt_cret
0 2002-01-03       0     0.18    1.97    3.43  79.387589  0.011275      0.18
1 2002-01-04       0     0.18    1.99    3.46  79.917587  0.006654      0.36
2 2002-01-07       0     0.21    2.01    3.41  79.353622 -0.007082      0.57
3 2002-01-08       0     0.19    2.03    3.42  79.170174 -0.002314      0.76
4 2002-01-09       0     0.19    2.07    3.42  78.524681 -0.008187      0.95

C:\Users\HP\anaconda3\lib\site-packages\seaborn\axisgrid.py:337: UserWarning: T
he `size` parameter has been renamed to `height`; please update your code.
  warnings.warn(msg, UserWarning)
```



Finding Equillibrium Matrix

In [24]:
```python
start = pd.to_datetime('2002-01-01')
end = pd.datetime.today()

df = web.DataReader("SPY", 'yahoo', start, end)
```

```
C:\Users\HP\AppData\Local\Temp/ipykernel_6272/2789651500.py:2: FutureWarning: T
he pandas.datetime class is deprecated and will be removed from pandas in a fut
ure version. Import from datetime module instead.
  end = pd.datetime.today()
```

In [25]:
```python
df
```

Out[25]:

Date	High	Low	Open	Close	Volume	Adj Close
2002-01-02	115.750000	113.809998	115.110001	115.529999	18651900.0	78.497528
2002-01-03	116.949997	115.540001	115.650002	116.839996	15743000.0	79.387604
2002-01-04	117.980003	116.550003	117.169998	117.620003	20140700.0	79.917580
2002-01-07	117.989998	116.559998	117.699997	116.790001	13106500.0	79.353622
2002-01-08	117.059998	115.970001	116.790001	116.519997	12683700.0	79.170151
...	...	...	...	...	...	...
2022-04-21	450.010010	437.100006	448.540009	438.059998	85417300.0	438.059998
2022-04-22	438.079987	425.440002	436.910004	426.040009	132354400.0	426.040009
2022-04-25	428.690002	418.839996	423.670013	428.510010	119647700.0	428.510010
2022-04-26	426.040009	416.070007	425.829987	416.100006	103996300.0	416.100006
2022-04-27	422.920013	415.010010	417.239990	417.269989	121820100.0	417.269989

5116 rows × 6 columns

In [26]:
```python
df["state"]=df["Close"].astype(float).pct_change()
df['state']=df['state'].apply(lambda x: 'Upside' if (x > 0.001) else ('Downside'

df['priorstate']=df['state'].shift(1)

states = df [['priorstate','state']].dropna()
states_matrix = states.groupby(['priorstate','state']).size().unstack().fillna(0)

transition_matrix= states_matrix.apply(lambda x: x/float(x.sum()),axis=1)
print(transition_matrix)
```

```
state           Downside    Upside
priorstate
Consolidation   0.000000  1.000000
Downside        0.497570  0.502430
Upside          0.551046  0.448954
```

Forecasting Futures Probabilities of States using Python

Let's consider time as t. Now, We will see observe other probability matrices."Transition Matrix" is the probability matrix at t=0. It shows the probability at t=0. However, we will build the Markov Chain by multiplying this transition matrix by itself to obtain the probability matrix in t=1 which would allow us to make one-day forecasts

In [27]:
```
df["state"]=df["Close"].astype(float).pct_change()
df['state']=df['state'].apply(lambda x: 'Upside' if (x > 0) else 'Downside' )

df['priorstate']=df['state'].shift(1)

states = df [['priorstate','state']].dropna()
states_matrix = states.groupby(['priorstate','state']).size().unstack().fillna(0)

transition_matrix= states_matrix.apply(lambda x: x/float(x.sum()),axis=1)
print(transition_matrix)
```

```
state       Downside    Upside
priorstate
Downside    0.436910  0.563090
Upside      0.470736  0.529264
```

In [28]:
```
t_0 = transition_matrix.copy()
t_1 =t_0.dot(t_0)
t_1
```

Out[28]:

state	Downside	Upside
priorstate		
Downside	0.455957	0.544043
Upside	0.454813	0.545187

Now, We need to extend the Markov Chain similarly.If we continue multiplying the transition matrix that we have obtained in t=1 by the original transition matrix in t0, we obtain the probabilities in time t=2. Let's find the transition matrix at t=2 and t=3 and so on in a similar manner.

In [29]:
```
t_2 = t_0.dot(t_1)
t_2
```

Out[29]:

state	Downside	Upside
priorstate		
Downside	0.455313	0.544687
Upside	0.455351	0.544649

Equilibrium Matrix using Python

In [30]:
```python
## Equilibrium Matrix using Python

t_0 = transition_matrix.copy()

t_m = t_0.copy()
t_n = t_0.dot(t_0)

i = 1
while(not(t_m.equals(t_n))):
    i += 1
    t_m = t_n.copy()
    t_n = t_n.dot(t_0)

print("Equilibrium Matrix Number: " + str(i))
print(t_n)
```

```
Equilibrium Matrix Number: 12
state       Downside    Upside
priorstate
Downside    0.455334  0.544666
Upside      0.455334  0.544666
```

The equilibrium Matrix is a stationary state. So, As per the theory of the Markov Chain, This figure will stay the same for foreseeable data points

In [32]:
```python
import datetime
```

In [34]:

```
##Random Walk

symbol = "SPY"
days = 9209
end_date = datetime.datetime.now().strftime("%d-%b-%Y")
end_date = str(end_date)

start_date = (datetime.datetime.now()- datetime.timedelta(days=days)).strftime("%
start_date = str(start_date)

#df=index_history("SPY",start_date,end_date)
df = web.DataReader("SPY", 'yahoo', start_date, end_date)

df["state"]=df["Close"].astype(float).pct_change()
df['state']=df['state'].apply(lambda x: 'Upside' if (x > 0) else 'Downside' )

df['priorstate']=df['state'].shift(1)

states = df [['priorstate','state']].dropna()
states_matrix = states.groupby(['priorstate','state']).size().unstack().fillna(0)

transition_matrix= states_matrix.apply(lambda x: x/float(x.sum()),axis=1)
t_0 = transition_matrix.copy()

t_m = t_0.copy()
t_n = t_0.dot(t_0)

i = 1
while(not(t_m.equals(t_n))):
    i += 1
    t_m = t_n.copy()
    t_n = t_n.dot(t_0)

print("Equilibrium Matrix Number: " + str(i))
print(t_n)
```

```
Equilibrium Matrix Number: 11
state        Downside    Upside
priorstate
Downside     0.462416  0.537584
Upside       0.462416  0.537584
```

So, t = 11, We get our equilibrium matrix.

Anyways, the number of the matrix where we get the equilibrium does not matter much. What matters is the values. We can see the values are really close!

Interpreting,

P(priorstate="Downside"/state="Downside) i.e. If We had a Downside day today, Tomorrow there is 53.7584% probability of having a Downside day.

Similarly,

If We had a Downside day today, Tomorrow there is 46% probability of having an Upside day.