In [1]:
```
# import datetime
import os
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datlib.FRED import *
from datlib.plots import *
import pandas_datareader.data as web

%matplotlib inline

# Import Statsmodels

from statsmodels.tsa.api import VAR
from statsmodels.tsa.stattools import adfuller
from statsmodels.tools.eval_measures import rmse, aic
```

In [2]:
```
#data cleaning, importing

d_parser = lambda x: pd.datetime.strptime(x, '%m/%d/%Y')
df = pd.read_csv('M4-5.csv', parse_dates=['Date'], date_parser=d_parser)
df
```

```
C:\Users\HP\AppData\Local\Temp/ipykernel_2724/2883410668.py:3: FutureWarning: T
he pandas.datetime class is deprecated and will be removed from pandas in a fut
ure version. Import from datetime module instead.
  d_parser = lambda x: pd.datetime.strptime(x, '%m/%d/%Y')
```

Out[2]:

	Date	M4	Log Total Assets	Effective Federal Funds Rate (%)	Log Currency in Circulation ($ Bil)	Unemployment Rate
0	2010-01-31	0.00	0.000000e+00	0.00	6.83	0.0
1	2010-02-28	-0.01	0.000000e+00	0.00	6.83	0.0
2	2010-03-31	-0.01	2.000000e-02	0.02	6.84	0.1
3	2010-04-30	0.01	-1.000000e-02	-0.01	6.84	-0.1
4	2010-05-31	0.00	-1.000000e-02	-0.03	6.84	-0.3
...	...	...	...	...	...	...
115	2019-08-31	0.01	0.000000e+00	-0.29	7.47	0.1
116	2019-09-30	0.00	2.000000e-02	0.18	7.47	-0.3
117	2019-10-31	0.01	3.000000e-02	-0.12	7.48	0.3
118	2019-11-30	0.01	-2.000000e-02	-0.07	7.49	-0.1
119	2019-12-31	0.00	-1.780000e-15	0.28	7.49	0.0

120 rows × 6 columns

In [3]:
```
df['Date_at_year_month'] = df['Date'].dt.strftime('%Y-%m')
```

In [4]:
```
column_names = {'Date_at_year_month':'DATE',
                'M4':'M4',
                'Log Total Assets': 'TA',
                'Log Currency in Circulation ($ Bil)':'CC',
                'Effective Federal Funds Rate (%)':'FFR',
               'Unemployment Rate': 'U'}

# rename columns
df = df.rename(columns = column_names)

df
```

Out[4]:

	Date	M4	TA	FFR	CC	U	DATE
0	2010-01-31	0.00	0.000000e+00	0.00	6.83	0.0	2010-01
1	2010-02-28	-0.01	0.000000e+00	0.00	6.83	0.0	2010-02
2	2010-03-31	-0.01	2.000000e-02	0.02	6.84	0.1	2010-03
3	2010-04-30	0.01	-1.000000e-02	-0.01	6.84	-0.1	2010-04
4	2010-05-31	0.00	-1.000000e-02	-0.03	6.84	-0.3	2010-05
...	...	...	...	...	...	...	...
115	2019-08-31	0.01	0.000000e+00	-0.29	7.47	0.1	2019-08
116	2019-09-30	0.00	2.000000e-02	0.18	7.47	-0.3	2019-09
117	2019-10-31	0.01	3.000000e-02	-0.12	7.48	0.3	2019-10
118	2019-11-30	0.01	-2.000000e-02	-0.07	7.49	-0.1	2019-11
119	2019-12-31	0.00	-1.780000e-15	0.28	7.49	0.0	2019-12

120 rows × 7 columns

In [5]:
```
df = df.set_index('DATE')
df
```

Out[5]:

DATE	Date	M4	TA	FFR	CC	U
2010-01	2010-01-31	0.00	0.000000e+00	0.00	6.83	0.0
2010-02	2010-02-28	-0.01	0.000000e+00	0.00	6.83	0.0
2010-03	2010-03-31	-0.01	2.000000e-02	0.02	6.84	0.1
2010-04	2010-04-30	0.01	-1.000000e-02	-0.01	6.84	-0.1
2010-05	2010-05-31	0.00	-1.000000e-02	-0.03	6.84	-0.3
...	...	...	...	...	...	...
2019-08	2019-08-31	0.01	0.000000e+00	-0.29	7.47	0.1
2019-09	2019-09-30	0.00	2.000000e-02	0.18	7.47	-0.3
2019-10	2019-10-31	0.01	3.000000e-02	-0.12	7.48	0.3
2019-11	2019-11-30	0.01	-2.000000e-02	-0.07	7.49	-0.1
2019-12	2019-12-31	0.00	-1.780000e-15	0.28	7.49	0.0

120 rows × 6 columns

In [6]:
```
df = df.drop(['Date'], axis = 1)
df
```

Out[6]:

DATE	M4	TA	FFR	CC	U
2010-01	0.00	0.000000e+00	0.00	6.83	0.0
2010-02	-0.01	0.000000e+00	0.00	6.83	0.0
2010-03	-0.01	2.000000e-02	0.02	6.84	0.1
2010-04	0.01	-1.000000e-02	-0.01	6.84	-0.1
2010-05	0.00	-1.000000e-02	-0.03	6.84	-0.3
...	...	...	...	...	...
2019-08	0.01	0.000000e+00	-0.29	7.47	0.1
2019-09	0.00	2.000000e-02	0.18	7.47	-0.3
2019-10	0.01	3.000000e-02	-0.12	7.48	0.3
2019-11	0.01	-2.000000e-02	-0.07	7.49	-0.1
2019-12	0.00	-1.780000e-15	0.28	7.49	0.0

120 rows × 5 columns

In [7]:
```
data = df
data
```

Out[7]:

DATE	M4	TA	FFR	CC	U
2010-01	0.00	0.000000e+00	0.00	6.83	0.0
2010-02	-0.01	0.000000e+00	0.00	6.83	0.0
2010-03	-0.01	2.000000e-02	0.02	6.84	0.1
2010-04	0.01	-1.000000e-02	-0.01	6.84	-0.1
2010-05	0.00	-1.000000e-02	-0.03	6.84	-0.3
...	...	...	...	...	...
2019-08	0.01	0.000000e+00	-0.29	7.47	0.1
2019-09	0.00	2.000000e-02	0.18	7.47	-0.3
2019-10	0.01	3.000000e-02	-0.12	7.48	0.3
2019-11	0.01	-2.000000e-02	-0.07	7.49	-0.1
2019-12	0.00	-1.780000e-15	0.28	7.49	0.0

120 rows × 5 columns

In [8]:
```
#ADF test

X = data["M4"].values
result = adfuller(X)
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
    print('\t%s: %.3f' % (key, value))

if result[0] < result[4]["5%"]:
    print ("Reject Ho - Time Series is Stationary")
else:
    print ("Failed to Reject Ho - Time Series is Non-Stationary")


X = data["FFR"].values
result = adfuller(X)
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
    print('\t%s: %.3f' % (key, value))

if result[0] < result[4]["5%"]:
    print ("Reject Ho - Time Series is Stationary")
else:
    print ("Failed to Reject Ho - Time Series is Non-Stationary")


X = data["TA"].values
result = adfuller(X)
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
    print('\t%s: %.3f' % (key, value))

if result[0] < result[4]["5%"]:
    print ("Reject Ho - Time Series is Stationary")
else:
    print ("Failed to Reject Ho - Time Series is Non-Stationary")


X = data["CC"].values
result = adfuller(X)
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
    print('\t%s: %.3f' % (key, value))

if result[0] < result[4]["5%"]:
    print ("Reject Ho - Time Series is Stationary")
else:
    print ("Failed to Reject Ho - Time Series is Non-Stationary")
```

```
X = data["U"].values
result = adfuller(X)
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
    print('\t%s: %.3f' % (key, value))

if result[0] < result[4]["5%"]:
    print ("Reject Ho - Time Series is Stationary")
else:
    print ("Failed to Reject Ho - Time Series is Non-Stationary")
```

```
ADF Statistic: -14.773444
p-value: 0.000000
Critical Values:
        1%: -3.487
        5%: -2.886
        10%: -2.580
Reject Ho - Time Series is Stationary
ADF Statistic: -3.996996
p-value: 0.001426
Critical Values:
        1%: -3.491
        5%: -2.888
        10%: -2.581
Reject Ho - Time Series is Stationary
ADF Statistic: -10.281226
p-value: 0.000000
Critical Values:
        1%: -3.487
        5%: -2.886
        10%: -2.580
Reject Ho - Time Series is Stationary
ADF Statistic: -3.653973
p-value: 0.004809
Critical Values:
        1%: -3.492
        5%: -2.889
        10%: -2.581
Reject Ho - Time Series is Stationary
ADF Statistic: -8.219365
p-value: 0.000000
Critical Values:
        1%: -3.491
        5%: -2.888
        10%: -2.581
Reject Ho - Time Series is Stationary
```

In [9]:
```
df = data
```

In [16]:
```
import statsmodels.api as sm

#perform KPSS test
sm.tsa.stattools.kpss(df["M4"], regression='ct')
```

```
C:\Users\HP\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:2022: Inte
rpolationWarning: The test statistic is outside of the range of p-values availa
ble in the
look-up table. The actual p-value is greater than the p-value returned.

  warnings.warn(
```

Out[16]:
```
(0.0547214358889738,
 0.1,
 6,
 {'10%': 0.119, '5%': 0.146, '2.5%': 0.176, '1%': 0.216})
```

In [17]:
```
sm.tsa.stattools.kpss(df["FFR"], regression='ct')
```

```
C:\Users\HP\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:2022: Inte
rpolationWarning: The test statistic is outside of the range of p-values availa
ble in the
look-up table. The actual p-value is greater than the p-value returned.

  warnings.warn(
```

Out[17]:
```
(0.07533063409509935,
 0.1,
 19,
 {'10%': 0.119, '5%': 0.146, '2.5%': 0.176, '1%': 0.216})
```

In [18]:
```

sm.tsa.stattools.kpss(df["CC"], regression='ct')
```

```
C:\Users\HP\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:2018: Inte
rpolationWarning: The test statistic is outside of the range of p-values availa
ble in the
look-up table. The actual p-value is smaller than the p-value returned.

  warnings.warn(
```

Out[18]:
```
(0.39483205140890026,
 0.01,
 6,
 {'10%': 0.119, '5%': 0.146, '2.5%': 0.176, '1%': 0.216})
```

In [19]:
```
sm.tsa.stattools.kpss(df["TA"], regression='ct')
```

```
C:\Users\HP\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:2022: Inte
rpolationWarning: The test statistic is outside of the range of p-values availa
ble in the
look-up table. The actual p-value is greater than the p-value returned.

  warnings.warn(
```

Out[19]:
```
(0.07963189413717638,
 0.1,
 13,
 {'10%': 0.119, '5%': 0.146, '2.5%': 0.176, '1%': 0.216})
```

In [20]:
```
sm.tsa.stattools.kpss(df["U"], regression='ct')
```

Out[20]:
```
(0.17877024133327776,
 0.023961159500020836,
 34,
 {'10%': 0.119, '5%': 0.146, '2.5%': 0.176, '1%': 0.216})
```

In [10]:
```
## Johansen test

from statsmodels.tsa.vector_ar.vecm import coint_johansen

def cointegration_test(df, alpha=0.05):
    """Perform Johanson's Cointegration Test and Report Summary"""
    out = coint_johansen(df,-1,5)
    d = {'0.90':0, '0.95':1, '0.99':2}
    traces = out.lr1
    cvts = out.cvt[:, d[str(1-alpha)]]
    def adjust(val, length= 6): return str(val).ljust(length)

    # Summary
    print('Name   ::  Test Stat > C(95%)    =>   Signif  \n', '--'*20)
    for col, trace, cvt in zip(df.columns, traces, cvts):
        print(adjust(col), ':: ', adjust(round(trace,2), 9), ">", adjust(cvt, 8),

cointegration_test(df)
```

```
Name   ::  Test Stat > C(95%)    =>   Signif
 ----------------------------------------
M4     ::  166.56    > 60.0627   =>   True
TA     ::  97.62     > 40.1749   =>   True
FFR    ::  60.85     > 24.2761   =>   True
CC     ::  33.67     > 12.3212   =>   True
U      ::  12.02     > 4.1296    =>   True
```

In [11]:
```
##Testing Causation using Granger's Causality Test

from statsmodels.tsa.stattools import grangercausalitytests
maxlag=12
test = 'ssr_chi2test'
def grangers_causation_matrix(data_new, variables, test='ssr_chi2test', verbose=F
    df = pd.DataFrame(np.zeros((len(variables), len(variables))), columns=variabl
    for c in df.columns:
        for r in df.index:
            test_result = grangercausalitytests(data_new[[r, c]], maxlag=maxlag,
            p_values = [round(test_result[i+1][0][test][1],4) for i in range(maxl
            if verbose: print(f'Y = {r}, X = {c}, P Values = {p_values}')
            min_p_value = np.min(p_values)
            df.loc[r, c] = min_p_value
    df.columns = [var + '_x' for var in variables]
    df.index = [var + '_y' for var in variables]
    return df

grangers_causation_matrix(df, variables = df.columns)
```

Out[11]:

	M4_x	TA_x	FFR_x	CC_x	U_x
M4_y	1.0000	0.0000	0.3924	0.0004	0.0108
TA_y	0.0002	1.0000	0.0077	0.2116	0.1110
FFR_y	0.0625	0.2477	1.0000	0.0156	0.1985
CC_y	0.0019	0.0216	0.7152	1.0000	0.2312
U_y	0.1152	0.0000	0.5653	0.0048	1.0000

In [12]:
```
from statsmodels.tsa.stattools import kpss

def kpss_test(df):
    statistic, p_value, n_lags, critical_values = kpss(df.values)

    print(f'KPSS Statistic: {statistic}')
    print(f'p-value: {p_value}')
    print(f'num lags: {n_lags}')
    print('Critial Values:')
    for key, value in critical_values.items():
        print(f'   {key} : {value}')

print('KPSS Test: M4')
kpss_test(df['M4'])
print('KPSS Test: FFR')
kpss_test(df['FFR'])
print('KPSS Test: TA')
kpss_test(df['TA'])
```

```
KPSS Test: M4
KPSS Statistic: 0.7310020008542962
p-value: 0.010727090831427614
num lags: 3
Critial Values:
   10% : 0.347
   5% : 0.463
   2.5% : 0.574
   1% : 0.739
KPSS Test: FFR
KPSS Statistic: 0.07556965458914269
p-value: 0.1
num lags: 19
Critial Values:
   10% : 0.347
   5% : 0.463
   2.5% : 0.574
   1% : 0.739
KPSS Test: TA
KPSS Statistic: 0.08352134889349987
p-value: 0.1
num lags: 13
Critial Values:
   10% : 0.347
   5% : 0.463
   2.5% : 0.574
   1% : 0.739


C:\Users\HP\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:2022: In
terpolationWarning: The test statistic is outside of the range of p-values av
ailable in the
look-up table. The actual p-value is greater than the p-value returned.

  warnings.warn(
C:\Users\HP\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:2022: In
terpolationWarning: The test statistic is outside of the range of p-values av
ailable in the
```

```
look-up table. The actual p-value is greater than the p-value returned.

  warnings.warn(
```

In [ ]:
```
## Extra
```

In [ ]:
```

```

In [21]:
```
var = VAR(df)
```

```
C:\Users\HP\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:471:
ValueWarning: No frequency information was provided, so inferred frequency MS w
ill be used.
  self._init_dates(dates, freq)
```

In [22]:
```
x= var.select_order()
x.summary()
```

Out[22]:

VAR Order Selection (* highlights the minimums)

	AIC	BIC	FPE	HQIC
0	-31.71	-31.58	1.696e-14	-31.66
1	-39.17	-38.42	9.737e-18	-38.87
2	-39.81	-38.44*	5.139e-18	-39.26*
3	-39.88	-37.88	4.856e-18*	-39.07
4	-39.80	-37.18	5.317e-18	-38.74
5	-39.68	-36.43	6.151e-18	-38.36
6	-39.52	-35.65	7.475e-18	-37.95
7	-39.59	-35.09	7.329e-18	-37.77
8	-39.55	-34.43	8.213e-18	-37.47
9	-39.59	-33.85	8.621e-18	-37.26
10	-39.85	-33.48	7.482e-18	-37.27
11	-39.95	-32.96	7.924e-18	-37.12
12	-39.86	-32.24	1.064e-17	-36.77
13	-40.10*	-31.85	1.082e-17	-36.75

In [24]:
```
model = VAR(df)
for i in [1,2,3,4,5,6,7,8,9]:
    result = model.fit(i)
    print('Lag Order =', i)
    print('AIC : ', result.aic)
    print('BIC : ', result.bic)
    print('FPE : ', result.fpe)
    print('HQIC: ', result.hqic, '\n')
```

```
C:\Users\HP\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:471:
ValueWarning: No frequency information was provided, so inferred frequency MS w
ill be used.
  self._init_dates(dates, freq)

Lag Order = 1
AIC :  -38.86078211668317
BIC :  -38.16016274867186
FPE :  1.3278915406121078e-17
HQIC:  -38.57628270825603

Lag Order = 2
AIC :  -39.486465573246335
BIC :  -38.19504477370725
FPE :  7.119094641989602e-18
HQIC:  -38.96211065467247

Lag Order = 3
AIC :  -39.64941137643481
BIC :  -37.76074543811156
FPE :  6.084484381378661e-18
HQIC:  -38.882636302281455

Lag Order = 4
AIC :  -39.49836574273318
BIC :  -37.005891862852415
FPE :  7.158792165057362e-18
HQIC:  -38.48656372485281

Lag Order = 5
AIC :  -39.36769094399398
BIC :  -36.26472419019204
FPE :  8.319388611277759e-18
HQIC:  -38.10821221950964

Lag Order = 6
AIC :  -39.17946700791118
BIC :  -35.45919718772568
FPE :  1.0352542000694361e-17
HQIC:  -37.66961799359197

Lag Order = 7
AIC :  -39.25706692725263
BIC :  -34.91255535762235
FPE :  1.0017448866866661e-17
HQIC:  -37.49410936060666

Lag Order = 8
```

```
AIC :  -39.43154410718428
BIC :  -34.45572028026022
FPE :  8.962628136202133e-18
HQIC:  -37.412694174675075

Lag Order = 9
AIC :  -39.459317556121846
BIC :  -33.84497569754674
FPE :  9.510178545581809e-18
HQIC:  -37.18174500900278
```

In [30]:
```
x = model.select_order(maxlags=13)
x.summary()
```

Out[30]:

VAR Order Selection (* highlights the minimums)

	AIC	BIC	FPE	HQIC
0	-31.71	-31.58	1.696e-14	-31.66
1	-39.17	-38.42	9.737e-18	-38.87
2	-39.81	-38.44*	5.139e-18	-39.26*
3	-39.88	-37.88	4.856e-18*	-39.07
4	-39.80	-37.18	5.317e-18	-38.74
5	-39.68	-36.43	6.151e-18	-38.36
6	-39.52	-35.65	7.475e-18	-37.95
7	-39.59	-35.09	7.329e-18	-37.77
8	-39.55	-34.43	8.213e-18	-37.47
9	-39.59	-33.85	8.621e-18	-37.26
10	-39.85	-33.48	7.482e-18	-37.27
11	-39.95	-32.96	7.924e-18	-37.12
12	-39.86	-32.24	1.064e-17	-36.77
13	-40.10*	-31.85	1.082e-17	-36.75

In [31]:
```
results = model.fit(maxlags=13, ic='aic')
results.summary()
```

Out[31]:

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Wed, 27, Apr, 2022
Time:                     21:05:33
--------------------------------------------------------------------
No. of Equations:         5.00000    BIC:                   -31.8525
Nobs:                     107.000    HQIC:                  -36.7541
Log likelihood:           1716.00    FPE:                1.08164e-17
AIC:                     -40.0958    Det(Omega_mle):     9.78976e-19
--------------------------------------------------------------------
Results for equation M4
==============================================================================
             coefficient       std. error           t-stat            prob
------------------------------------------------------------------------------
const          -0.010154         0.061304           -0.166           0.868
L1.M4          -0.351056         0.150643           -2.330           0.020
L1.TA           0.113330         0.091123            1.244           0.214
L1.FFR         -0.009368         0.015050           -0.622           0.534
L1.CC           0.197373         0.143857            1.372           0.170
L1.U            0.010131         0.006114            1.657           0.097
L2.M4          -0.187186         0.157219           -1.191           0.234
L2.TA           0.099267         0.096044            1.034           0.301
L2.FFR         -0.006114         0.015509           -0.394           0.693
L2.CC          -0.327550         0.168426           -1.945           0.052
L2.U            0.016291         0.010490            1.553           0.120
L3.M4          -0.037137         0.160434           -0.231           0.817
L3.TA          -0.044694         0.107418           -0.416           0.677
L3.FFR         -0.008602         0.017486           -0.492           0.623
L3.CC           0.037761         0.168082            0.225           0.822
L3.U            0.023047         0.013460            1.712           0.087
L4.M4           0.082684         0.154908            0.534           0.594
L4.TA           0.014259         0.104764            0.136           0.892
L4.FFR         -0.003310         0.017686           -0.187           0.852
L4.CC          -0.049561         0.169033           -0.293           0.769
L4.U            0.020344         0.015666            1.299           0.194
L5.M4           0.270228         0.152783            1.769           0.077
L5.TA          -0.034992         0.109536           -0.319           0.749
L5.FFR          0.014465         0.019084            0.758           0.448
L5.CC           0.118714         0.177426            0.669           0.503
L5.U            0.010933         0.017062            0.641           0.522
L6.M4          -0.071392         0.154466           -0.462           0.644
L6.TA          -0.021534         0.097905           -0.220           0.826
L6.FFR          0.003603         0.019565            0.184           0.854
L6.CC          -0.126382         0.168435           -0.750           0.453
L6.U            0.009874         0.017536            0.563           0.573
L7.M4          -0.114941         0.172471           -0.666           0.505
L7.TA          -0.020286         0.087500           -0.232           0.817
L7.FFR          0.025035         0.020220            1.238           0.216
L7.CC           0.055011         0.160517            0.343           0.732
L7.U            0.004001         0.017335            0.231           0.817
L8.M4           0.184361         0.201125            0.917           0.359
L8.TA           0.000217         0.092117            0.002           0.998
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L8.FFR          0.016512         0.020806           0.794           0.427
L8.CC          -0.014507         0.157415          -0.092           0.927
L8.U            0.004426         0.016440           0.269           0.788
L9.M4           0.123313         0.212771           0.580           0.562
L9.TA          -0.103341         0.090980          -1.136           0.256
L9.FFR          0.015365         0.020623           0.745           0.456
L9.CC          -0.070997         0.148447          -0.478           0.632
L9.U           -0.001642         0.015255          -0.108           0.914
L10.M4          0.121578         0.218787           0.556           0.578
L10.TA         -0.220970         0.092512          -2.389           0.017
L10.FFR         0.002255         0.022287           0.101           0.919
L10.CC          0.472067         0.147486           3.201           0.001
L10.U           0.000622         0.013579           0.046           0.963
L11.M4          0.028059         0.201746           0.139           0.889
L11.TA          0.009058         0.097979           0.092           0.926
L11.FFR        -0.020890         0.021344          -0.979           0.328
L11.CC         -0.303771         0.151094          -2.010           0.044
L11.U          -0.009221         0.011378          -0.810           0.418
L12.M4         -0.061586         0.169627          -0.363           0.717
L12.TA          0.065874         0.103625           0.636           0.525
L12.FFR        -0.004264         0.019033          -0.224           0.823
L12.CC          0.170396         0.162574           1.048           0.295
L12.U          -0.007225         0.008353          -0.865           0.387
L13.M4         -0.100686         0.141660          -0.711           0.477
L13.TA          0.075054         0.087439           0.858           0.391
L13.FFR        -0.027063         0.018268          -1.481           0.138
L13.CC         -0.156395         0.143627          -1.089           0.276
L13.U          -0.006134         0.005480          -1.119           0.263
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Results for equation TA
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             coefficient       std. error           t-stat            prob
------------------------------------------------------------------------------
const           0.150506         0.102053           1.475           0.140
L1.M4           0.161852         0.250777           0.645           0.519
L1.TA          -0.470895         0.151693          -3.104           0.002
L1.FFR         -0.004708         0.025054          -0.188           0.851
L1.CC          -0.276814         0.239481          -1.156           0.248
L1.U            0.010635         0.010178           1.045           0.296
L2.M4           0.338347         0.261724           1.293           0.196
L2.TA          -0.508959         0.159885          -3.183           0.001
L2.FFR         -0.041902         0.025817          -1.623           0.105
L2.CC          -0.187784         0.280380          -0.670           0.503
L2.U            0.011361         0.017463           0.651           0.515
L3.M4           0.678268         0.267076           2.540           0.011
L3.TA          -0.342083         0.178819          -1.913           0.056
L3.FFR         -0.050968         0.029109          -1.751           0.080
L3.CC           0.614209         0.279807           2.195           0.028
L3.U           -0.001354         0.022407          -0.060           0.952
L4.M4           0.358210         0.257878           1.389           0.165
L4.TA          -0.403491         0.174401          -2.314           0.021
L4.FFR         -0.019485         0.029442          -0.662           0.508
L4.CC          -0.465900         0.281390          -1.656           0.098
L4.U            0.001011         0.026079           0.039           0.969
L5.M4           0.238761         0.254340           0.939           0.348
L5.TA          -0.151409         0.182346          -0.830           0.406
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L5.FFR	-0.027764	0.031770	-0.874	0.382
L5.CC	0.252494	0.295362	0.855	0.393
L5.U	-0.021056	0.028403	-0.741	0.458
L6.M4	0.736816	0.257141	2.865	0.004
L6.TA	-0.023016	0.162984	-0.141	0.888
L6.FFR	-0.020907	0.032570	-0.642	0.521
L6.CC	-0.106153	0.280396	-0.379	0.705
L6.U	-0.030803	0.029192	-1.055	0.291
L7.M4	1.078341	0.287115	3.756	0.000
L7.TA	-0.179133	0.145663	-1.230	0.219
L7.FFR	-0.002316	0.033661	-0.069	0.945
L7.CC	-0.370334	0.267215	-1.386	0.166
L7.U	-0.036825	0.028858	-1.276	0.202
L8.M4	0.705000	0.334815	2.106	0.035
L8.TA	-0.291335	0.153349	-1.900	0.057
L8.FFR	-0.017741	0.034637	-0.512	0.609
L8.CC	0.227138	0.262050	0.867	0.386
L8.U	-0.034570	0.027368	-1.263	0.207
L9.M4	0.634004	0.354203	1.790	0.073
L9.TA	-0.169010	0.151456	-1.116	0.264
L9.FFR	-0.034441	0.034332	-1.003	0.316
L9.CC	0.037223	0.247121	0.151	0.880
L9.U	-0.030481	0.025395	-1.200	0.230
L10.M4	0.437344	0.364217	1.201	0.230
L10.TA	-0.102331	0.154006	-0.664	0.506
L10.FFR	-0.040022	0.037102	-1.079	0.281
L10.CC	0.455499	0.245521	1.855	0.064
L10.U	-0.021001	0.022605	-0.929	0.353
L11.M4	-0.455661	0.335848	-1.357	0.175
L11.TA	-0.058989	0.163106	-0.362	0.718
L11.FFR	-0.067470	0.035531	-1.899	0.058
L11.CC	-0.044937	0.251528	-0.179	0.858
L11.U	-0.014359	0.018940	-0.758	0.448
L12.M4	-0.309032	0.282380	-1.094	0.274
L12.TA	0.140581	0.172505	0.815	0.415
L12.FFR	0.010702	0.031685	0.338	0.736
L12.CC	-0.154686	0.270638	-0.572	0.568
L12.U	-0.009560	0.013905	-0.688	0.492
L13.M4	-0.399750	0.235824	-1.695	0.090
L13.TA	0.276067	0.145561	1.897	0.058
L13.FFR	-0.045785	0.030411	-1.506	0.132
L13.CC	-0.001298	0.239098	-0.005	0.996
L13.U	0.004053	0.009122	0.444	0.657

Results for equation FFR

	coefficient	std. error	t-stat	prob
const	-0.118475	0.711350	-0.167	0.868
L1.M4	1.329502	1.748012	0.761	0.447
L1.TA	0.668148	1.057356	0.632	0.527
L1.FFR	-0.298245	0.174638	-1.708	0.088
L1.CC	2.491483	1.669270	1.493	0.136
L1.U	0.009623	0.070943	0.136	0.892
L2.M4	-2.271899	1.824312	-1.245	0.213
L2.TA	-0.080470	1.114461	-0.072	0.942

L2.FFR	-0.391628	0.179956	-2.176	0.030
L2.CC	1.579825	1.954356	0.808	0.419
L2.U	0.018999	0.121720	0.156	0.876
L3.M4	-1.936567	1.861622	-1.040	0.298
L3.TA	0.916087	1.246438	0.735	0.462
L3.FFR	-0.040797	0.202901	-0.201	0.841
L3.CC	-3.363305	1.950362	-1.724	0.085
L3.U	0.060027	0.156183	0.384	0.701
L4.M4	-1.736900	1.797504	-0.966	0.334
L4.TA	-0.018017	1.215644	-0.015	0.988
L4.FFR	-0.115780	0.205220	-0.564	0.573
L4.CC	-1.061880	1.961396	-0.541	0.588
L4.U	0.081067	0.181783	0.446	0.656
L5.M4	0.548013	1.772846	0.309	0.757
L5.TA	0.131858	1.271022	0.104	0.917
L5.FFR	-0.412799	0.221448	-1.864	0.062
L5.CC	0.537375	2.058787	0.261	0.794
L5.U	0.114187	0.197981	0.577	0.564
L6.M4	0.513473	1.792366	0.286	0.775
L6.TA	-0.561721	1.136060	-0.494	0.621
L6.FFR	0.135196	0.227026	0.596	0.552
L6.CC	-1.346024	1.954468	-0.689	0.491
L6.U	0.124737	0.203480	0.613	0.540
L7.M4	0.539000	2.001297	0.269	0.788
L7.TA	-0.731469	1.015326	-0.720	0.471
L7.FFR	-0.171525	0.234629	-0.731	0.465
L7.CC	3.953172	1.862587	2.122	0.034
L7.U	0.070852	0.201150	0.352	0.725
L8.M4	-1.202137	2.333788	-0.515	0.606
L8.TA	-1.459001	1.068899	-1.365	0.172
L8.FFR	-0.247906	0.241430	-1.027	0.305
L8.CC	0.361417	1.826587	0.198	0.843
L8.U	0.068974	0.190762	0.362	0.718
L9.M4	0.373815	2.468924	0.151	0.880
L9.TA	0.785121	1.055703	0.744	0.457
L9.FFR	0.035280	0.239304	0.147	0.883
L9.CC	-1.561051	1.722526	-0.906	0.365
L9.U	0.058659	0.177014	0.331	0.740
L10.M4	-0.119938	2.538730	-0.047	0.962
L10.TA	0.309601	1.073477	0.288	0.773
L10.FFR	-0.399196	0.258614	-1.544	0.123
L10.CC	-2.302247	1.711374	-1.345	0.179
L10.U	-0.002833	0.157565	-0.018	0.986
L11.M4	3.745470	2.340988	1.600	0.110
L11.TA	-0.056995	1.136913	-0.050	0.960
L11.FFR	0.158154	0.247668	0.639	0.523
L11.CC	-1.044679	1.753243	-0.596	0.551
L11.U	-0.011467	0.132021	-0.087	0.931
L12.M4	1.074226	1.968294	0.546	0.585
L12.TA	-0.632154	1.202426	-0.526	0.599
L12.FFR	-0.005756	0.220854	-0.026	0.979
L12.CC	3.352744	1.886448	1.777	0.076
L12.U	0.009942	0.096921	0.103	0.918
L13.M4	1.173703	1.643779	0.714	0.475
L13.TA	-1.148842	1.014617	-1.132	0.258
L13.FFR	-0.163840	0.211973	-0.773	0.440
L13.CC	-1.591184	1.666603	-0.955	0.340

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L13.U          -0.084166         0.063584           -1.324           0.186
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Results for equation CC
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             coefficient       std. error           t-stat            prob
--------------------------------------------------------------------------------
const           0.095499         0.068292            1.398           0.162
L1.M4           0.206950         0.167815            1.233           0.217
L1.TA           0.038115         0.101510            0.375           0.707
L1.FFR          0.000866         0.016766            0.052           0.959
L1.CC           0.524289         0.160255            3.272           0.001
L1.U           -0.005099         0.006811           -0.749           0.454
L2.M4           0.249504         0.175140            1.425           0.154
L2.TA          -0.014090         0.106992           -0.132           0.895
L2.FFR          0.021740         0.017276            1.258           0.208
L2.CC           0.114708         0.187624            0.611           0.541
L2.U           -0.015001         0.011686           -1.284           0.199
L3.M4           0.102788         0.178722            0.575           0.565
L3.TA          -0.068645         0.119662           -0.574           0.566
L3.FFR          0.021171         0.019479            1.087           0.277
L3.CC           0.089952         0.187241            0.480           0.631
L3.U           -0.022035         0.014994           -1.470           0.142
L4.M4          -0.033127         0.172566           -0.192           0.848
L4.TA          -0.039271         0.116706           -0.336           0.736
L4.FFR          0.002141         0.019702            0.109           0.913
L4.CC           0.329191         0.188300            1.748           0.080
L4.U           -0.028823         0.017452           -1.652           0.099
L5.M4          -0.001752         0.170199           -0.010           0.992
L5.TA           0.020585         0.122022            0.169           0.866
L5.FFR          0.022413         0.021260            1.054           0.292
L5.CC          -0.246330         0.197650           -1.246           0.213
L5.U           -0.025587         0.019007           -1.346           0.178
L6.M4          -0.235150         0.172073           -1.367           0.172
L6.TA          -0.011825         0.109065           -0.108           0.914
L6.FFR         -0.005189         0.021795           -0.238           0.812
L6.CC           0.111994         0.187635            0.597           0.551
L6.U           -0.018926         0.019535           -0.969           0.333
L7.M4          -0.192321         0.192131           -1.001           0.317
L7.TA          -0.030560         0.097475           -0.314           0.754
L7.FFR         -0.004860         0.022525           -0.216           0.829
L7.CC           0.020541         0.178814            0.115           0.909
L7.U           -0.012268         0.019311           -0.635           0.525
L8.M4           0.134115         0.224051            0.599           0.549
L8.TA          -0.042134         0.102618           -0.411           0.681
L8.FFR         -0.022075         0.023178           -0.952           0.341
L8.CC           0.023969         0.175358            0.137           0.891
L8.U           -0.007936         0.018314           -0.433           0.665
L9.M4          -0.255484         0.237025           -1.078           0.281
L9.TA           0.012429         0.101351            0.123           0.902
L9.FFR         -0.019628         0.022974           -0.854           0.393
L9.CC          -0.051065         0.165368           -0.309           0.757
L9.U           -0.004612         0.016994           -0.271           0.786
L10.M4          0.011762         0.243726            0.048           0.962
L10.TA         -0.084096         0.103057           -0.816           0.414
L10.FFR        -0.010445         0.024828           -0.421           0.674
L10.CC         -0.100649         0.164297           -0.613           0.540
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L10.U	-0.003238	0.015127	-0.214	0.831
L11.M4	0.194145	0.224742	0.864	0.388
L11.TA	0.060995	0.109147	0.559	0.576
L11.FFR	-0.005560	0.023777	-0.234	0.815
L11.CC	0.009666	0.168317	0.057	0.954
L11.U	-0.003054	0.012674	-0.241	0.810
L12.M4	-0.010244	0.188963	-0.054	0.957
L12.TA	-0.089226	0.115437	-0.773	0.440
L12.FFR	0.017028	0.021203	0.803	0.422
L12.CC	0.265206	0.181105	1.464	0.143
L12.U	-0.000127	0.009305	-0.014	0.989
L13.M4	-0.011809	0.157808	-0.075	0.940
L13.TA	-0.011719	0.097406	-0.120	0.904
L13.FFR	0.009309	0.020350	0.457	0.647
L13.CC	-0.102742	0.159999	-0.642	0.521
L13.U	-0.002171	0.006104	-0.356	0.722

Results for equation U

	coefficient	std. error	t-stat	prob
const	-2.871000	1.479136	-1.941	0.052
L1.M4	4.121291	3.634705	1.134	0.257
L1.TA	-0.464041	2.198600	-0.211	0.833
L1.FFR	0.408497	0.363130	1.125	0.261
L1.CC	4.511599	3.470974	1.300	0.194
L1.U	-1.376923	0.147514	-9.334	0.000
L2.M4	-1.699078	3.793359	-0.448	0.654
L2.TA	2.233701	2.317339	0.964	0.335
L2.FFR	0.417901	0.374189	1.117	0.264
L2.CC	-5.976411	4.063765	-1.471	0.141
L2.U	-1.330214	0.253098	-5.256	0.000
L3.M4	2.556066	3.870939	0.660	0.509
L3.TA	3.751247	2.591764	1.447	0.148
L3.FFR	0.340517	0.421898	0.807	0.420
L3.CC	5.380960	4.055459	1.327	0.185
L3.U	-1.323527	0.324757	-4.075	0.000
L4.M4	4.195899	3.737616	1.123	0.262
L4.TA	2.652969	2.527732	1.050	0.294
L4.FFR	-0.032525	0.426721	-0.076	0.939
L4.CC	0.405499	4.078403	0.099	0.921
L4.U	-1.247878	0.377987	-3.301	0.001
L5.M4	3.373987	3.686344	0.915	0.360
L5.TA	3.826524	2.642882	1.448	0.148
L5.FFR	0.202375	0.460464	0.440	0.660
L5.CC	4.745521	4.280911	1.109	0.268
L5.U	-1.122643	0.411670	-2.727	0.006
L6.M4	-8.847740	3.726933	-2.374	0.018
L6.TA	3.722344	2.362251	1.576	0.115
L6.FFR	-0.655745	0.472063	-1.389	0.165
L6.CC	-3.728888	4.063997	-0.918	0.359
L6.U	-0.870236	0.423103	-2.057	0.040
L7.M4	-6.493525	4.161371	-1.560	0.119
L7.TA	5.316672	2.111204	2.518	0.012
L7.FFR	-0.149244	0.487873	-0.306	0.760
L7.CC	-2.079581	3.872946	-0.537	0.591

L7.U	-0.661573	0.418259	-1.582	0.114
L8.M4	-5.109789	4.852732	-1.053	0.292
L8.TA	3.433722	2.222601	1.545	0.122
L8.FFR	-0.180430	0.502014	-0.359	0.719
L8.CC	0.300267	3.798090	0.079	0.937
L8.U	-0.482083	0.396659	-1.215	0.224
L9.M4	-3.788690	5.133725	-0.738	0.461
L9.TA	-1.058897	2.195162	-0.482	0.630
L9.FFR	0.096648	0.497593	0.194	0.846
L9.CC	-3.907601	3.581711	-1.091	0.275
L9.U	-0.447613	0.368072	-1.216	0.224
L10.M4	-3.749175	5.278875	-0.710	0.478
L10.TA	-2.768511	2.232120	-1.240	0.215
L10.FFR	0.310715	0.537746	0.578	0.563
L10.CC	8.770319	3.558524	2.465	0.014
L10.U	-0.151913	0.327632	-0.464	0.643
L11.M4	-5.604326	4.867703	-1.151	0.250
L11.TA	-1.725422	2.364025	-0.730	0.465
L11.FFR	0.311223	0.514985	0.604	0.546
L11.CC	-5.687085	3.645583	-1.560	0.119
L11.U	-0.069420	0.274516	-0.253	0.800
L12.M4	-5.434843	4.092745	-1.328	0.184
L12.TA	2.136109	2.500248	0.854	0.393
L12.FFR	0.605472	0.459230	1.318	0.187
L12.CC	-0.527999	3.922560	-0.135	0.893
L12.U	-0.166408	0.201531	-0.826	0.409
L13.M4	-3.371887	3.417969	-0.987	0.324
L13.TA	4.666779	2.109730	2.212	0.027
L13.FFR	0.191958	0.440763	0.436	0.663
L13.CC	-1.828102	3.465428	-0.528	0.598
L13.U	-0.232108	0.132212	-1.756	0.079

==========

Correlation matrix of residuals

	M4	TA	FFR	CC	U
M4	1.000000	-0.086158	-0.235988	-0.082634	0.000437
TA	-0.086158	1.000000	-0.224708	0.153547	-0.128980
FFR	-0.235988	-0.224708	1.000000	-0.104136	-0.045316
CC	-0.082634	0.153547	-0.104136	1.000000	0.181906
U	0.000437	-0.128980	-0.045316	0.181906	1.000000

In [34]:
```
from arch.unitroot import engle_granger

eg_test = engle_granger(df.M4, df.TA, trend="n")
eg_test
```

Out[34]:

Engle-Granger Cointegration Test

Test Statistic	-1.643
P-value	0.392
ADF Lag length	5
Estimated Root ρ (γ+1)	0.747

Trend: Constant
Critical Values: -2.50 (10%), -2.82 (5%), -3.43 (1%)
Null Hypothesis: No Cointegration
Alternative Hypothesis: Cointegration
Distribution Order: 1

In [35]:
```
eg_test.cointegrating_vector
```

Out[35]:
```
M4    1.000000
TA    0.086207
dtype: float64
```

In [ ]:

In [ ]:

In [ ]: