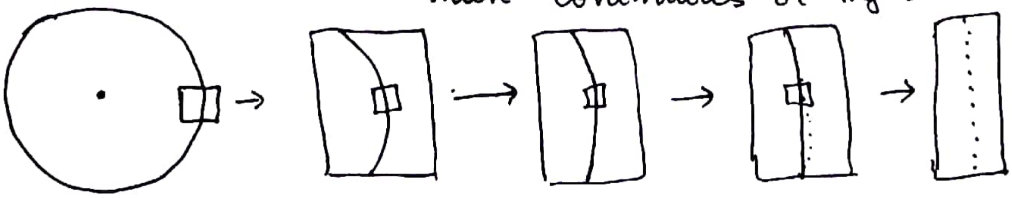
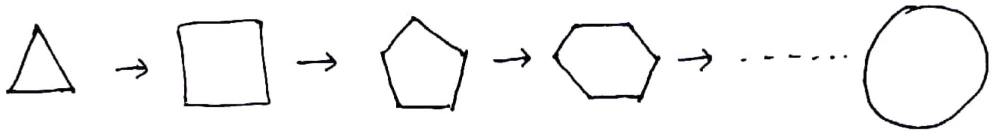
Maths for Programmers

Discrete Mathematics → Deals with finite set of elements rather than continuous or infinit set of elements





increasing vertices to regular polygon approximates circle.

Approximate continuous sets → discrete sets.

Set → collection of distinct objects called elements.

↳ denoted by capital letters

Roster Notation

$A = \{5\}$ $B = \{2,3,4\}$ $C = \{d, f, g\}$ $E = \{\ \}$

$F = \{A, B, C\} = \{\{5\}, \{2,3,4\}, \{d, f, g\}\}$

Interval Notation

$x \in (0, 1)$ $0 < x < 1$

$x \in [0, 1)$ $0 \leq x < 1$

$x \in [0, 1]$ $0 \leq x \leq 1$

Common sets

$\phi = \{\ \}$ $N = \{1, 2, 3, \cdots\}$

$N_0 = \{0, 1, 2, 3, \cdots\}$ $Z = \{\cdots, -1, 0, 1, \cdots\}$

Rational Numbers

↳ Ratio of two integers

↳ Non-terminating repeating decimals + integers + terminating decimals

$Q = \{a/b : a, b \in Z, b \neq 0\}$

Set builder Notation

Non-Rational Number

if $\sqrt{2}$ rational,

$\sqrt{2} = \frac{a}{b}$ → irreducable fraction

$\Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \Rightarrow a$ is even   let $a = 2K$

$\Rightarrow a^2 = 2(2K^2)$

$\Rightarrow \cancel{2}b^2 = \cancel{2}(2K^2)$

$\Rightarrow b^2 = 2K^2 \Rightarrow b$ is even   let $b = 2l$

$\sqrt{2} = a/b \Rightarrow \sqrt{2} = \frac{2K}{2l} = \frac{K}{l}$

a/b irreducable   reduced version

so $\sqrt{2} = a/b$ is false.

we to define these rational, non-rational more specifically

Set Operators

$A = \{x, y, z\}$ $B = \{c, x, y\}$

Union $A \cup B = \{P : P \in A \text{ or } P \in B\}$ $A \cup B = \{c, x, y, z\}$

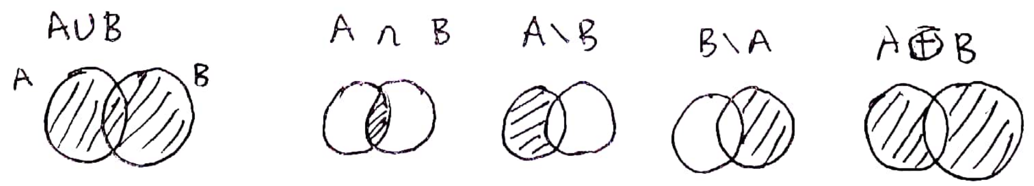
Intersection $A \cap B = \{P : P \in A \text{ and } P \in B\}$ $A \cap B = \{x, y\}$

Set Diff $A \setminus B = \{P : P \in A \text{ and } P \notin B\}$ $A \setminus B = \{z\}$

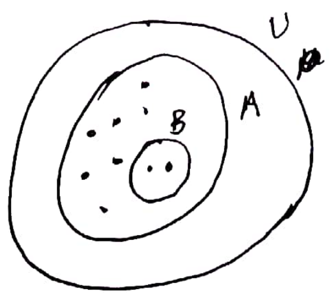
Symmetric difference $A \oplus B = (A \setminus B) \cup (B \setminus A)$ $A \oplus B = \{z, c\}$

$R \setminus Q = I$

Real   Rational   IrRational



Subsets & Supersets



B is subset (proper subset) of A

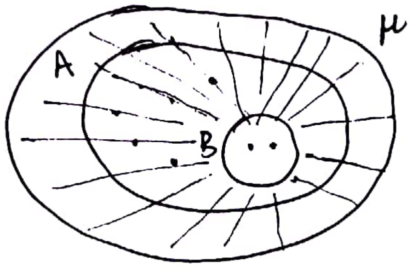
A is superset (proper superset) of A

U and A have same elements

So A is subset, superset of U and vise-versa

$A = U$

Sets: The Universe & Compliments



Universal Set
↳ max boundaries, it contains everything

compliment

$B^c = \{x \in \mu : x \notin B\}$

$A = \{1, x, 3\}$ $B = \{3, \beta, \gamma\}$ $C = \{1, 2, 3\}$

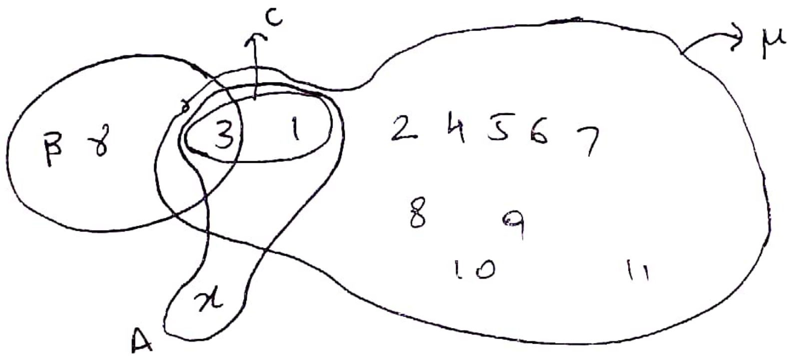
$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$C \subset \mu$

$\mu \supset C$

$B \not\subset \mu$

$\mu \not\supseteq A$



$A^c = \{x : x \in \mu \text{ and } x \notin A\}$ $A^c = \{2, 4, 5, 6, 7, 8, 9, 10, 11\}$

$B^c = \{x : x \in \mu \text{ and } x \notin B\}$ $B^c = \{1, 2, 4, 5, 6, 7, 8, 9, 10, 11\}$

$C^c = \{x : x \in \mu \text{ and } x \notin C\}$ $C^c = \{2, 4, 5, 6, 7, 8, 9, 10, 11\}$

Set Idempotence Law

$A \cup A = A$ $A \cap A = A$

Set Identity Laws

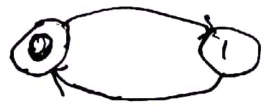
$A \cup \phi = A$ $\mu \cup A = \mu$ $A \cap \mu = A$ $\phi \cap A = \phi$

Set Complements laws

$A \cup A^c = \mu$ $\phi^c = \mu$ $A \cap A^c = \phi$ $\mu^c = \phi$

Set Involution laws

$(A^c)^c = A$



$A^c = \{x : x \notin A\}$

Set Associative Laws

$A \cap (B \cap C) = (A \cap B) \cap C \qquad A \cup (B \cup C) = (A \cup B) \cup C$

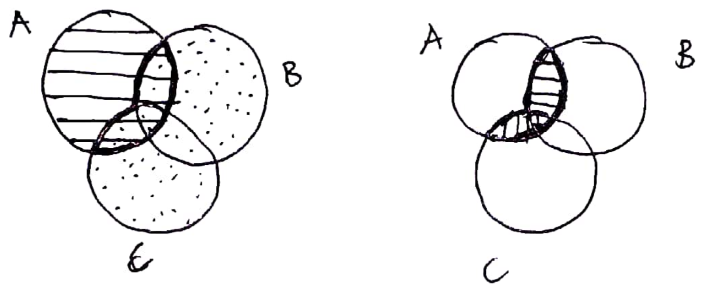
Set commutativity law

$A \cap B = B \cap A \qquad A \cup B = B \cup A$

$\{x : x \in A \text{ and } x \in B\} = \{x : x \in B \text{ and } x \in A\}$

Set Distributive law

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



$A \cap (B \cup C) =$ = $(A \cap B) \cup (A \cap C) =$ +

Distributivity

Case 1: $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C)$

$x \in A$ and $x \in (B \cup C)$

$x \in A$ and $((x \in B)$ or $(x \in C))$

$(x \in A$ and $x \in C)$ or $(x \in A$ or $x \in C)$

$x \in (A \cap B)$ or $x \in (A \cap C)$

$x \in (A \cap B) \cup (A \cap C)$

case 2:- $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

$x \in (A \cap B) \cup (A \cap C)$

$(x \in A \cap B)$ or $(x \in A \cap C)$

$(x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$x \in A$ and $(x \in B$ or $x \in C)$

$x \in A$ and $x \in (B \cup C)$

$x \in (A \cap (B \cup C))$

* $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$

So case 1 + case 2 $\Rightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A = \{1, x, 3\}$ $B = \{3, \beta, \gamma\}$ $C = \{1, 3\}$

$A \cap B = \{3\}$ $A \cap C = \{1, 3\}$ $B \cup C = \{1, 3, \beta, \gamma\}$

$(A \cap B) \cup (A \cap C) = \{1, 3\}$ $A \cap (B \cup C) = \{1, 3\}$

$\Rightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$B \cap C = \{3\}$ $A \cup B = \{1, x, 3, \beta, \gamma\}$ $A \cup C = \{1, 3, x\}$

$A \cup (B \cap C) = \{1, x, 3\}$ $(A \cup B) \cap (A \cup C) = \{1, x, 3\}$

$\Rightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan Law

$(A \cup B)^c \subseteq A^c \cap B^c$

$x \in (A \cup B)^c$

$x \notin (A \cup B)$

$x \notin A$ and $x \notin B$

$x \in A^c$ and $x \in B^c$

$x \in A^c \cap B^c$

$A^c \cap B^c \subseteq (A \cup B)^c$

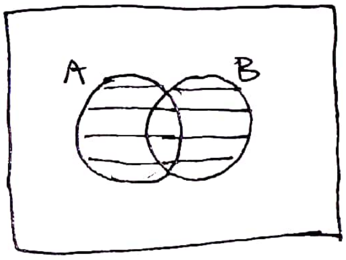
$x \in A^c \cap B^c$

$x \in A^c$ and $x \in B^c$

$x \notin A$ and $x \notin B$

$x \notin A \cup B$

$x \in (A \cup B)^c$



$A = \{1, x, 3\}$ $B = \{3, \beta, \gamma\}$ $C = \{1, 3\}$ $\mu = \{x \in \mathbb{Z} : 0 \le x \le 5\}$

$A \cup B$

$(A \cup B)^c$

$A^c = \{0, 2, 4, 5\}$ $B^c = \{0, 1, 2, 4, 5\}$ $(A \cap B) = \{3\}$

$A^c \cup B^c = \{0, 1, 2, 4, 5\}$ $(A \cap B)^c = \{0, 1, 2, 4, 5\}$

$\Rightarrow (A \cap B)^c = A^c \cup B^c$

$A^c \cap B^c = \{0, 2, 4, 5\}$ $(A \cup B) = \{1, 3, x, \beta, \gamma\}$

$(A \cup B)^c = \{0, 2, 4, 5\}$

$\Rightarrow (A \cup B)^c = A^c \cap B^c$

Logic $\rightarrow$ systematic way of thinking that helps us deduce new info from old info and to parse meaning of sentences.

Logic $\rightarrow$ math $\rightarrow$ algorithms $\rightarrow$ code

Logic : Propositions → a declaration statement with a verifiable truth value.

p = "Rain falls from the sky" true

q = "Ghara is a country in asia." false

r = "What are you doing?" → not declarative

s = "Wash the laundry" → not declarative

$5 = 4 + 89$ false

$7 = \alpha$ → can't verify truth value

$3 = \frac{5}{0}$ false

$99 \cdot \frac{1}{3} = 33$ true

Logic: Composite propositions: group of propositions making a new proposition.

p = "Rains falls from the sky." q = "Ghana is a country in Asia."

Conjunction

$p \wedge q$ = "Rain falls from the sky AND Ghana is a country in Asia."

$\boxed{\text{false}}$

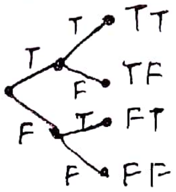
Disjunction

$p \vee q$ = "Rain falls from the sky OR Ghana is a country in Asia."

$\boxed{\text{true}}$

Truth Tables

p	q
T	T
F	F



p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

Idempotence

p	$p \vee p$	$p \wedge p$
T	T	T
F	F	F

$p \equiv p \vee p \equiv p \wedge p$

Identities

T	F	$p \vee F$	$T \vee p$	$p \wedge T$	$p \wedge F$
T	F	T	F	T	F
T	F	F	T	F	F

$p \equiv p \vee F \equiv p \wedge T$

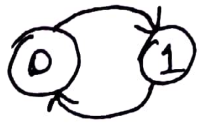
$T \equiv p \vee T$ $\quad F \equiv p \wedge F$

Complements

$P \vee \neg P \equiv T \quad T \equiv \neg F \quad P \wedge \neg P \equiv F \quad F \equiv \neg T$

Involution

$P \equiv \neg\neg P$



P	¬P	¬¬P	P∨¬P	P∧¬P	T	¬F	F	¬T
T	F	T	T	F	T	T	F	F
F	T	F	T	F	T	T	F	F

Always T → Tautology Always F → ~~Fatay~~ Fallacy

Commutative laws

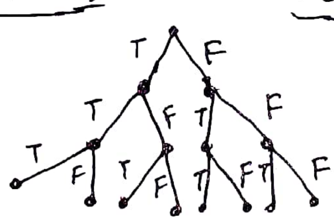
$p \vee q \equiv q \vee p \qquad p \wedge q \equiv q \wedge p$

p	q	p∨q	q∨p	p∧q	q∧p
T	T	T	T	T	T
T	F	T	T	F	F
F	T	T	T	F	F
F	F	F	F	F	F

Associative Laws

$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

$p \vee (q \vee r) \equiv (p \vee q) \vee r$



Distributive Laws

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

De Morgan's Law

$\neg(p \wedge q) \equiv \neg p \vee \neg q \qquad \neg(p \vee q) \equiv \neg p \wedge \neg q$

p	¬p	q	¬q	p∧q	¬(p∧q)	¬p∨¬q	p∨q	¬(p∨q)	¬p∧¬q
T	F	T	F	T	F	F	T	F	F
T	F	F	T	F	T	T	T	F	F
F	T	T	F	F	T	T	T	F	F
F	T	F	T	F	T	T	F	T	T

Conditionals

p	q	¬p	¬q	conditional $p \Rightarrow q$	converse $q \Rightarrow p$	inverse $\neg p \Rightarrow \neg q$	contrapositive $\neg q \Rightarrow \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Logical Quantifiers

$(\forall x \in \mu)\, p(x) \rightarrow \forall x\, P(x) \rightarrow$ Propositional function

↳ takes true or false

Universal : $\forall$ (for all) Existential: $\exists$ (there exists)

$(\forall x \in N)(x+3>4)$ False $(\forall x \in R)(x+3>4)$ False

$(\exists x \in N)(x+3>4)$ False $[\exists x \in R)(x+3>4)$ True

Tautologies → always true

p	q	$\neg p$	$\neg q$	$p \vee \neg p$	$\neg(p \wedge \neg p)$		$p \Rightarrow q$	$(p \Rightarrow q) \wedge \neg q$	$[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$
T	T	F	F	T	T	F	T	F	T
T	F	F	T	T	T	F	F	F	T
F	T	T	F	T	T	F	T	F	T
F	F	T	T	T	T	F	T	T	T

Law of excluded middle

Law of contridiction

Modus Tollens
(or)
Denying the consequent