Algo 30.10.22

Merge Sort $O(n \log n)$

- Pseudocode

→ merge(arr, left, right) → n

$$T(n) = 2T(n/2) + c.n$$
$$= 2\{2T(n/4) + c.\tfrac{n}{2}\} + c.n$$
$$= 4T(n/4) + 2cn$$
$$= 4\{2T(n/8) + c.\tfrac{n}{4}\} + 2cn$$
$$= 8T(n/8) + 3cn$$

Here, $n = 2^k$

$\rightarrow \log_2 n = k$

$$= 2^k\, T(n/2^k) + kcn$$
$$= n\,T(1) + kcn$$
$$= n + cn\log n$$
$$= n(1 + c\log n)$$
$$= O(n\log n)$$

* Quick Sort -(pseudocode)

~~O(n)~~ O(n log n) - best

if sorted, O(n^2) - worst

1 2 3 4 5 6 $T(n)$

12345 {5} $T(n-1)$

1234 { }

~~n T(n-1) + =~~

$$T(n) = T(n-1) + cn$$
$$= T(n-2) + c(n-1) + cn$$
$$= T(n-2) + 2cn - c$$
$$= T(n-3) + c(n-2) + c(n-1) + cn$$
$$= T(n-3) + 3cn - 2c - c$$
$$\vdots$$
$$= T(n-k) + kcn - (k-1)c \ldots - 2c - c$$
$$= T(n-k) + kcn - c[(k-1) + (k-2) + \ldots 2 + 1]$$

$n = k$ হলে,

$$T(n) = T(0) + n^2 c - c\frac{n(n-1)}{2}$$

$$= n^2 - \frac{n(n-1)}{2} = O(n^2).$$

$$T(n) = cn + T(i) + T(n-i-1)$$

$$= cn + \frac{1}{n}\sum_{i=0}^{n-1}\{T(i) + T(n-i-1)\}$$

$$= cn + \frac{1}{n}\sum_{i=0}^{n-1}\{2T(i)\}$$

$$= cn + \frac{2}{n}\sum_{i=0}^{n-1} T(i)$$

$$= cn + \frac{2}{n}\left(T(0) + T(1) + \cdots T(n-1)\right)$$

$$n.T(n) = cn^2 + 2\left(T(0) + T(1) + \cdots T(n-1)\right) \quad \text{--- (1)}$$

$n = n-1$

$$(n-1)T(n-1) = c(n-1)^2 + 2\left(T(0) + \cdots T(n-2)\right) \quad \text{--- (2)}$$

$(1) - (2) \rightarrow$

$$n(T(n)) - (n-1)T(n-1) = cn^2 - c(n^2 - 2n + 1) + 2T(n-1)$$

$$= 2T(n-1) + 2cn - c$$

$$\Rightarrow nT(n) = (n-1)T(n-1) + 2T(n-1) + 2cn - c$$

$$= (n+1)T(n-1) + 2cn - c$$

$n(n+1)$ দিয়ে ভাগ

$$\Rightarrow \frac{T(n)}{n+1} = \frac{T(n-1)}{n} + \frac{2c}{n+1} - \boxed{\frac{c}{n(n+1)}} \times$$

$$\Rightarrow \frac{T(n)}{n+1} - \frac{T(n-1)}{n} = \frac{2c}{n+1}$$

$$\Rightarrow \frac{T(n)}{n+1} + \frac{T(n-1)}{n} + \frac{T(n-2)}{n-1} + \cdots \frac{T(2)}{3} + \frac{T(1)}{2}$$

$$- \frac{T(n-1)}{n} - \frac{T(n-2)}{n-1} - \cdots \frac{T(2)}{3} - \frac{T(1)}{2} - \frac{T(0)}{1}$$

$$= \frac{2c}{n+1} + \frac{2c}{n} + \frac{2c}{n-1} + \cdots \frac{2c}{2}$$

$$\Rightarrow \frac{T(n)}{n+1} - \frac{T(0)}{1} = 2c\left(\frac{1}{n+1} + \frac{1}{n} + \cdots \frac{1}{3} + \frac{1}{2}\right)$$

$$\Rightarrow \frac{T(n)}{n+1} = \frac{T(0)}{1} + 2c\left(H_{n+1} - 1\right)$$

$$\Rightarrow \frac{T(n)}{n+1} \approx 2c \ln(n+1)$$

$$\Rightarrow T(n) \approx (n+1)\, 2c \ln(n+1)$$

$$\approx (n+1) \log_2(n+1)$$

$$\approx n \log(n)$$