Chinese Reminder Theorm

→ To know this need idea of

1. Inverse
2. Congruency

1. Inverse

3 modulo 7 = ?

∴ gcd (3,7) = ?

Find gcd using extendend euli

$7 = 2 \times 3 + 1$

$\therefore 1 = 7 - 2 \times 3$

$\therefore$ 3 modulo 7 = −2

$\frac{1}{7} = \frac{7}{7} - \frac{2 \times 3}{7}$

$\frac{1}{7} = -2 \times \frac{3}{7}$

$\therefore \frac{1}{3} = -2$

$\therefore 3^{-1} = -2$

21 modulo 46

∴ gcd (21, 46) = ?

$46 = 2 \times 21 + 4$ — (I)

$21 = 5 \times 4 + 1$ — (II)

$\Rightarrow 1 = 21 - 5 \times 4$

(I) →

$4 = 46 - 2 \times 21$

$\therefore 1 = 21 - 5 \times (46 - 2 \times 21)$

$= 21 - 5 \times 46 + 10 \times 21$

$= 11 \times 21 - 5 \times 46$

∴ 21 modulo 46 = 11

modular multipicative Inverse

$5 \times 5^{-1} = 1$

$5 \times \frac{1}{5} = 1$

$A \times \frac{1}{A} = 1$

$\frac{1}{5}$ is the multiplicative Inverse of 5

Under mod m

$A \times A^{-1} \equiv 1 \bmod n$

$\therefore (A \times A^{-1})/n = 1$

$\therefore 3 \times \overset{?}{p} \equiv 1 \bmod 5$

Here p is the modulo multipicative Inverse of 3 modulo 5

$\therefore 3 \times 2 \equiv 1 \bmod 5 \quad (3\times2/5 = 1)$

multipicative Inverse

$2 \times ? = 1 \bmod 11 \longrightarrow ? = 6$

a modulo b

$\therefore$ there $\boxed{\text{exist a}}$ multipicative Inverse if a, b relatively prime /coprime or

$\gcd(a,b) = 1$

$\gcd(2, 11) = 1$

$Ax = 1 (\bmod M)$

$x = \frac{1}{A} (\bmod m)$

$x = A^{-1} (\bmod m)$

x is MI

A modulo m

~~3 modulo 5~~

~~$\gcd(3,5) = ?$~~

$\therefore$ ~~$5 = 3\times1 + 2$~~

$3 = 2\times1 + 1$

3 modulo 5

$\gcd(3.5) = ?$

$5 = 3\times1 + 2$

$3 = 2\times1 + 1$

$3 = 1\times(5-3)+1$

$3 = 5-3+1$

$\therefore 1 = -5+6$

$= -5+(2\times3)$

$\therefore MI = 2$

$17x \equiv 1 \pmod{43}$

$\therefore x = \frac{1}{17} \pmod{43}$

$\therefore x = 17^{-1} \bmod 43$

$43 = 17\times2 + 9 \longrightarrow 9 = 43 - 17\times2$

$17 = 9\times1 + 8 \longrightarrow 8 = 17 - 9$

$= 17 - (43 - 17\times2)$

$9 = 8\times1 + 1$

$\Rightarrow 1 = 9 - 8$

~~$= (17-9)$~~

~~$= 9 - 17 + 9$~~

$= 43 - 17\times2 - 17 + \cancel{9}\ 43 - 17\times2$

~~$= 43 - 3\times17 + 9$~~

$= \cancel{43}\ 43\times2 - 5\times17$

$\therefore x = -5$

Congruency

(n)

a, b

$a \bmod m = r_1$

$b \bmod m = r_2$

$\therefore$ if $r_1 = r_2$ $\therefore$ a is congrent of b mod m

$a \equiv b \pmod{m}$

congrent sign

Theorm - 1

Theorm = 2

$$a = b + km$$

$$a - b = km$$

$$k = \frac{a-b}{m}$$

$\therefore$ if $(a-b) \bmod m = 0$ then

$$a \equiv b \pmod{m}$$

CRT (weak)

→ is used to solve a set of different congruent equations with one variable but different modulo which are relatively prime (coprime)

$$x \equiv a_1 \pmod{m_1}$$

$$x \equiv a_2 \pmod{m_2}$$

Find x

$$\vdots$$

$$x_1 = \cancel{a} \; a_n \pmod{m_n}$$

→ CRT states that above equations have a unique solution of the moduli are relatively prime

$$x \equiv \overset{a_1}{2} \pmod{\overset{m_1}{3}}$$

$$x \equiv \overset{a_2}{3} \pmod{\overset{m_2}{5}}$$

$$x \equiv \overset{a_3}{2} \pmod{\overset{m_3}{7}}$$

$x = ?$

अर्थ x exist if
$gcd(m_1, m_2) = gcd(m_2, m_3)$
$gcd(m_1, m_3) = 1$
or coprime

Test
#AS

$m = m_1 \cdot m_2 \cdot m_3 = 3 \times 5 \times 7 = 105$

$= m/m_1$

$M_1 = \frac{105}{3} = 35$

$Y_1 = 35 \text{ modulo } 3$

$= M_1 \text{ modulo } m_1$

$3 =$

$35 > 3$
can't do

35 modulo 3

$35 = 3 \times 11 + 2$
$3 = 2 \times 1 + 1$
$3 = 35$
$1 = 3 - 2$
$= 3 - 35 + 3 \times 11$
$= 12 \times 3 - 35$

ans :- 12

$$X = \left(a_1 M_1 M_1^{-1} + a_2 M_2 M_2^{-1} + a_3 M_3 M_3^{-1} \right) \bmod m$$

∴ Here,

$$M = m_1 \times m_2 \times m_3$$
$$= 3 \times 5 \times 7$$
$$= 105$$

$$\therefore M_1 = \frac{M}{m_1} = \frac{105}{3} = 35$$

$$M_2 = \frac{M}{m_2} = \frac{105}{5} = 21$$

$$M_3 = \frac{M}{m_3} = 105/7 = 15$$

$M_1 \times m^{-1} = 1 \bmod m_1$

$35 \times m^{-1} = 1 \bmod 3$

$35 \times 2 = 70 \% 3 = 1$

$\therefore m^{-1} = 35$ modulo 3

$m^{-1} = 2$

$M_2 \times M_2^{-1} = 1 \bmod m_2$

$21 \times M_2^{-1} = 1 \bmod 5$

$21 \times 1 = 21 \% 5 = 1$

$M_2^{-1} = 1$

$M_3 \times M_3^{-1} = 1 \bmod m_3$

$15 \times M_3^{-1} = 1 \bmod 7$

$15 \times 1 = 15 \% 7 = 1$

$M_3^{-1} = 1$

Extended euclid's

ax+by=1

Given $AX + BY = GCD(A, B)$

Find (x, y)

— o —

$$AX + BY = GCD(B, A \% B)$$

$\therefore$ $X = y_1$ → hoon

$\therefore$ $Y = x_1 - \left\lfloor \frac{a}{b} \right\rfloor y_1$

Ans

$(2, -1)$

$(-1, 2)$

$$18x + 30y = 6$$

$$18x + 30y = GCD(18, 30)$$

$$\Rightarrow 30x_1 + 18y_1 = GCD(30, 18\%30)$$
$$(30, 18)$$

$(1, -1)$

$x_2 = 1$

$y_2 = x_3 - \lfloor \frac{a}{b} \rfloor y_3$
$= 0 - \lfloor \frac{18}{12} \rfloor 1$
$= -1$

$$\Rightarrow 18x_2 + 12y_2 = GCD(18, 30\%18)$$
$$(18, 12)$$

$(0, 1)$

$x_3 = y_4 = 0$

$y_3 = x_4 - \lfloor \frac{a}{b} \rfloor y_4$
$= 1 - \frac{12}{6} \times 0$
$= 1$

$$\Rightarrow 12x_3 + 6y_3 = GCD(12, 6)$$

$(1, 0)$

$$\Rightarrow 6x_4 + 0y_4 = GCD(6, 0)$$

$$6x_4 = 6$$

$\therefore x_4 = 1$

$y_4 = 0$

Base

	Index i	Quotient q_i	Reminder r_i	x_i $x_i = x_{i-2} - q_i x_{i-1}$	y_i $y_i = y_{i-2} - q_i \times y_{i-1}$
	0	~~240~~	240	1 $i-2$	0
	1	~~46~~	46	0 $i-1$	1
240, 46	2	240/46 = 5	240 % 46 = 10 (240 − 5×46)	$x_i = 1 - 5\times0$ $= 1$	$y_i = 0 - 5\times1$ $= -5$
46, 10	3	46/10 = ~~6~~ 4	46 % 10 = 6	$0 - 4\times1$ $= -4$	$1 - 4\times -5$ $= 21$
10, 6	4	10/6 = 1	10 % 6 = 4	1 + 4 = 5	−5 − 21 = 26
6, 4	5	6/4 = 1	6 % 4 = 2	−4 − 5 = −9	21 + 26 = 47
4, 2	6	4/2 = 2	4 % 2 = 0	5 + 18 = ~~13~~ 23	−26 − 2×47 = 120

```
int ext_gcd ( int A, int B, int *x, int *y)
{
    int x1 = 0, y1 = 1 ;
    int x2 = 1, y2 = 0;

    int x, y, r2, r1, q, r ;

    for ( r2 = A , r1 = B ;  r1 != 0 ;  r2 = r1,
                                         r1 = r
                                         x2 = x1
                                         x1 = x
                                         y2 = y1
                                         y1 = y )
    {   q = r2 / r1 ;
        r = r2 % r1 ;
        x = x2 - (q * x1) ;
        y = y2 - (q * y1) ;
    }

    *x = x2 ;
    *y = y2 ;
    return r2 ;
```

$$x_i = \underbrace{x_{i-2}}_{x_2} - q_i \underbrace{x_{i-1}}_{x_1}$$

CamScanner

modular multipicative Inverse using Ext euclid

$$AX \equiv 1 \pmod{m}$$

x modulo Inverse of A modulo m

$$AX + mY = 1$$

mI of 3 mod 5

A	B	Reminder R	Quotient Q	T_1	T_2	T $T = T_1 - T_2 \times Q$
5	3	5 % 3 = 2	5/3 = 1	0	1	-1
3	2	1	1	1	-1	2
2	1	0	2	-1	2	-5
1	0	x	x	2	-5	

Ans

Linear Diophantine Equation

$4x + 10y = 8$

$\therefore \underset{(4,10)}{GCD} = 2 = g$; since 2 divides 8
Solution exists //

$\therefore \frac{a}{g} = \frac{b}{g} = \frac{c}{g}$; $\frac{4}{2} = \frac{10}{2} = \frac{8}{2} \rightarrow 4$

$\therefore 2x + 5y = 4 \rightarrow (1) \rightarrow$ Find solue using ext-gcd

$\therefore$ From ext_gcd $\rightarrow x, y = -2, 1$

$\therefore$ this solution is for $ax + by = 1$

$\therefore$ we need to multiply 4

$\therefore \boxed{(-8, 4)}$.

Ans

```
bool linearDiophantine
        (int A, int B, int c, int *x, int *y)

  int g = gcd (A, B)

    if (c % g != 0) return false

int a = A/g   b = B/g   c = c/g

  ext_gcd (a, b, x, y)

   if (g < 0 )

   { a = a x -1     b = b x -1    c = c x -1 }

*x = x * c

*y = y * c

return true

}
```

Sieve of Eratbsthenes

~~int max 1oes+7;~~
~~bool isPrime[max];~~
~~Vector <int> prime;~~

~~void sieve(){~~
~~isp is~~

Pseducode

Initial all number true or prime

```
Void sieve ( )
{
  int n;
  vectoor<bool> isPrime (n+1, true);

  isPrime[0] = isPrime[1] = false;

  for (int i = 2 ; i*i <= n ; i++) {

      if (isPrime[i] == true)

        { for (int j = i*i ; j<=n ; j += i)

            { isPrime[j] = false }

        }

}
```

$n=16$

→ 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

i=2 Prime and all its গুণিতক not prime

→ 2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ 9 ~~10~~ 11 ~~12~~ 13 ~~14~~ 15 ~~16~~

i=3 Prime

→ 2 ✓ 3 ✓ ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~

i=4

→ isprime[4] = 4 = not prime

= 2nd loop not work

i=5

→ $5 \times 5 = 25 > n$ loops break

2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~

Primes→

Bit sieve

```
int N = 100;
int status [100/32];
```

→ inital all value 0

→ status arr is integer type Here ~~is~~ it has 32bit in 1 index

→ all these 32 bit indicates 32 different numbers whether they prime or not

→ status [0] = 0 — 31 number

→ a ~~number~~ index = i / 32;

→ number status of prime is at status [i/32] in i% 32 bit

```
bool check (int index,
            int position)
{
    return
    (bool)(index & (1<<position));
}

~~bool~~ int set (int index, int position)
{ return index = index | (1 << position)
}
```

```
void sieve
   { * we only hunt for odd number
       even num by deafault not prime ]

   for (int i = 3 ; i <= ~~sqrt~~ sqrt(N) ; i += 2 )
```

```
{
if (check(status[i/32], i%32) == 0)
for (int j = i*i ; j <= N ; j += (2i) )
  {
    status[j/32] = set (status[j/32], j%32)
  }

prime.pushback(2);
for( int i=3 ; i <= N; i += 2)
{
   if (check(status[i/32], i%32) == 0 )
        prime.push back[i];

}
```

CamScanner

How works

suppose $\boxed{i=3}$

now $i/32 = 3/32 = 0$ status[0] index 0

$i\%32 = 3\%32 = 3$ pos : 3 → 3rd bit

1

* check (status[$\underbrace{\cancel{3\%32}\ 3/32}_{0}$], $\underbrace{3\%32}_{3}$)

→ $\boxed{\text{index}}$ value = 0
∴ in binary index value = 0 (binary)

→ $\boxed{1 << \text{position}}$ = $1 * 2^{pos}$
$= 1 * 2^3$
$= 8$
$= 1000$ (in binary)

now $\boxed{\text{index} \ \& \ (1 << \text{position})}$

→ 0 & 1000

→ 0

and
1000
& 0000
0000

∴ ans == 0 ; 3 is prime number

now mark all its multpies

$J = i \times i \rightarrow J = 3 \times 3 = 9$

$J = 9$

$status[9/32] = 0 \leftarrow$ ~~index~~ something

$9 \% 32 = 9 \rightarrow$ position

status[9/32] =

$\boxed{set(status[9/32], 9 \% 32)}$

$\rightarrow$ Index $= 0$

$= 0$ (binary)

$\rightarrow 1 << position = 1 * 2^9$

$= 512$

$= 100\ 000\ 000$ (binary)

$\rightarrow$ index | (1 << position) $\rightarrow 0 \mid 100000000$

$\rightarrow 100000000$

$\therefore$ index $= 512$

$\therefore$ status[9/32] $= 512$ (setted) i = ~~72~~ 71

~~71~~/32 = 2 → status[2] → index 2

~~72~~ 71 % 32 = 7 → pos → 7

check (2 , 7)

∴ index 2 = 10 (binary)

∴ 1 << position = 2^7 = 128

= 10000000 (binary)

∴ Index & 1 << position = 10 & 10000000

= 0

∴ $\boxed{\text{71 is prime}}$

Segmented Sieve

L → R

21 – 50

```
vector<int> prime;
sieve();

void segSieve( int L , int R )
{
    vector<bool> isprime(R-L+1, true)   ↗ Intial all true

    if (L==1) isprime[0] = false   → extreme case

    for (int i=0 ; prime[i]* prime[i] <=R
        ; i++)
    {
    ② int current prime = prime[i]
       int currbase = (L/currprime)* curr prime
            20          21      2        2

       if (currbase <L) currbase += curr prime
            20     21
```

```
for (int j = currbase ; j <= r ; j += currprime)
{
    int curr index = j - L        * index

    isprime [currIndex] = false

}

if (currbase == currprime)
            isprime [currbase - L] = true

}

for (int i = 0 ; i < (r-l+1) ; i++)
{
    if (isprime [i])
        cout << i+L << endl.
}
```

CS CamScanner

Prime ~~start~~ 2nd 3rd

1 2 3 ~~4~~ 5

6 7 8 9 10

11 12 13 14 15

16 17 18 19 20

21 22 $\boxed{23}$ 24 25

26 27 28 $\boxed{29}$ 30

prime
in Range
21 – 30

Bellman Ford

→ complexity O(VE)

basic pseudocode

① Initial all distance [vertice] = INT_max (∞)

② Except source node Distance [source] = 0

③ Relax all edges (node−1) times

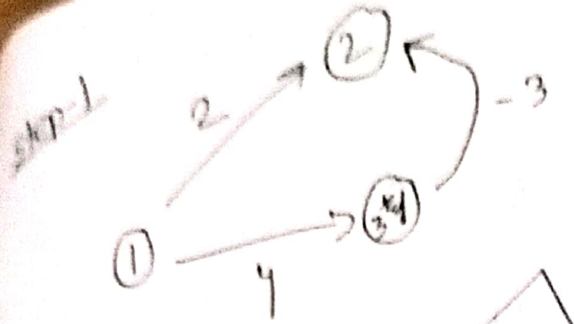
```
for( node-1)
{
    for (edges)
    {
        if dis[v] > dis[u] + weight
          ∴ dis[v] = dis[u] + weight
    }
}
```

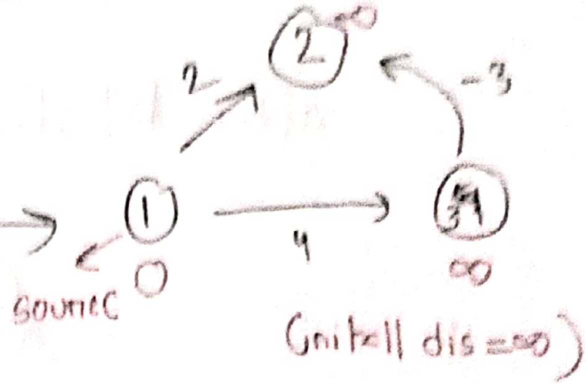
Relaxing

$u \xrightarrow{wt} v$

④ Negative weight cycle Detection

step-1





list of edge

3 2 ✓
1 3 ✓
1 2

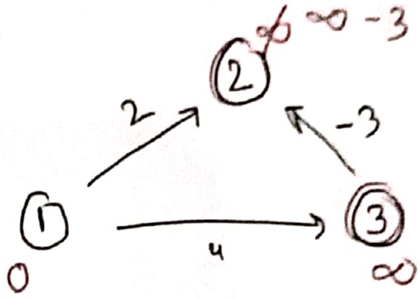
any order

adj List

nodes = 3
(3-1)|2 times iteration

(3,2)
(u,v)

First iteration



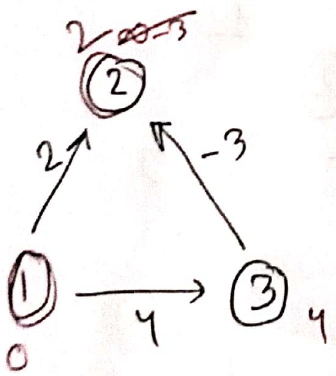
$dis[v] > dist[u] + wt$

$\therefore dis[v] = dis[u] + wt$

$dis[2] = dist[3] + wt$

$= \infty - 3$

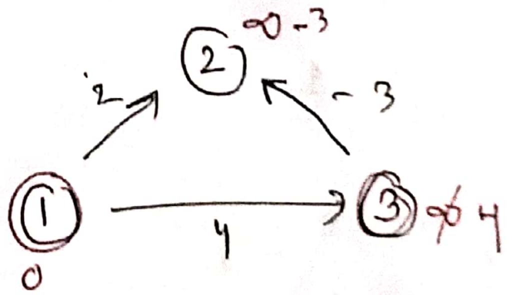
(1,2)



$dis[2] > dis[1] + 2$

$\therefore dis[2] = 2$

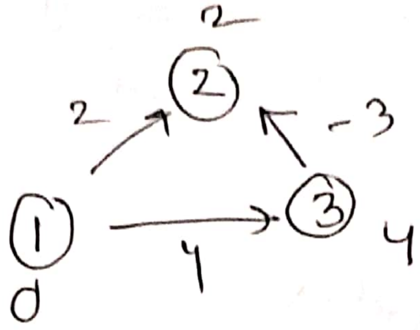
(1,3)



$dis[3] > dis[1] + 4$

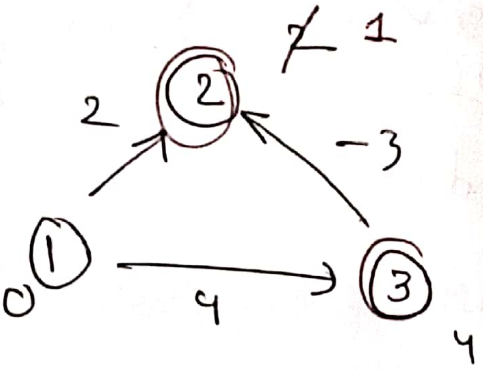
$\therefore dis[3] = 4$

After 1st iteration



↓ (2nd iteration)

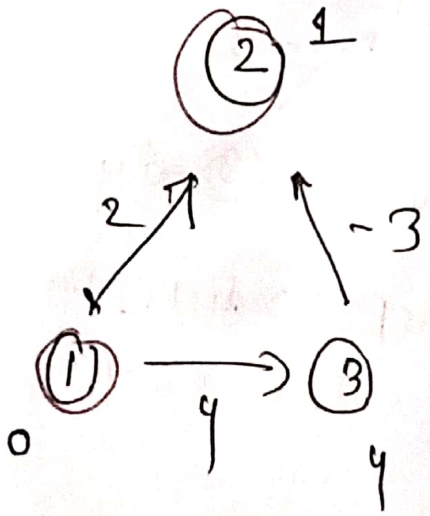
(2,3)



dis[2] > dis[3]-3

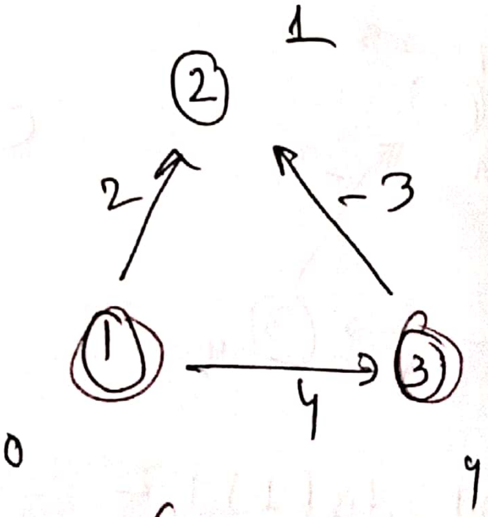
dis[2] = 1

(1,2)



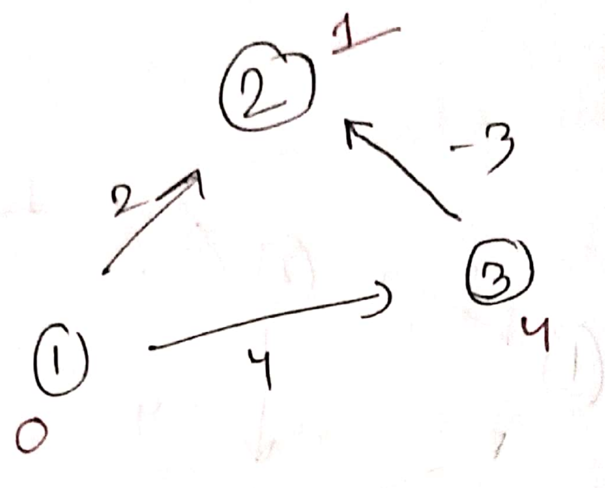
same

(1,3)

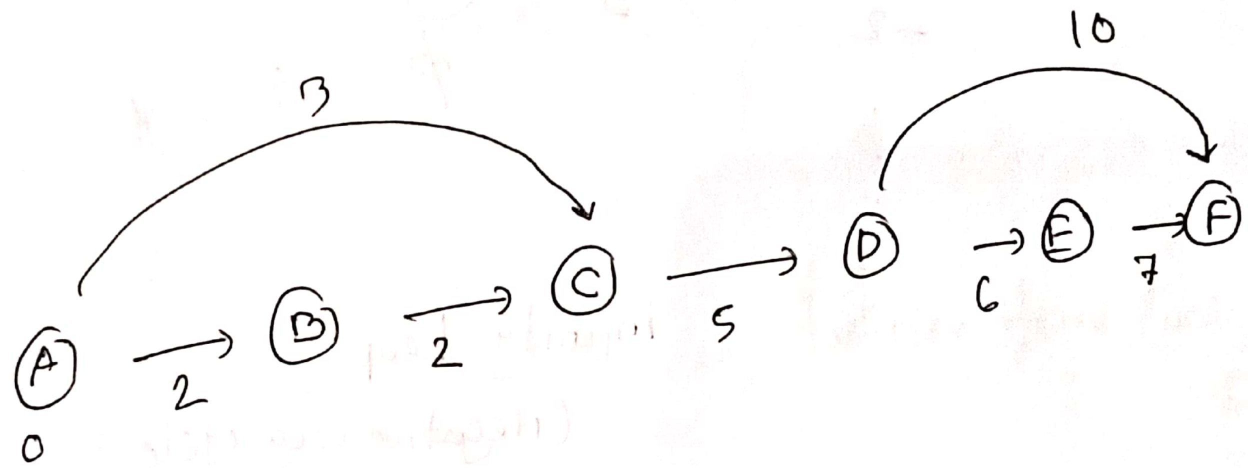


(same)

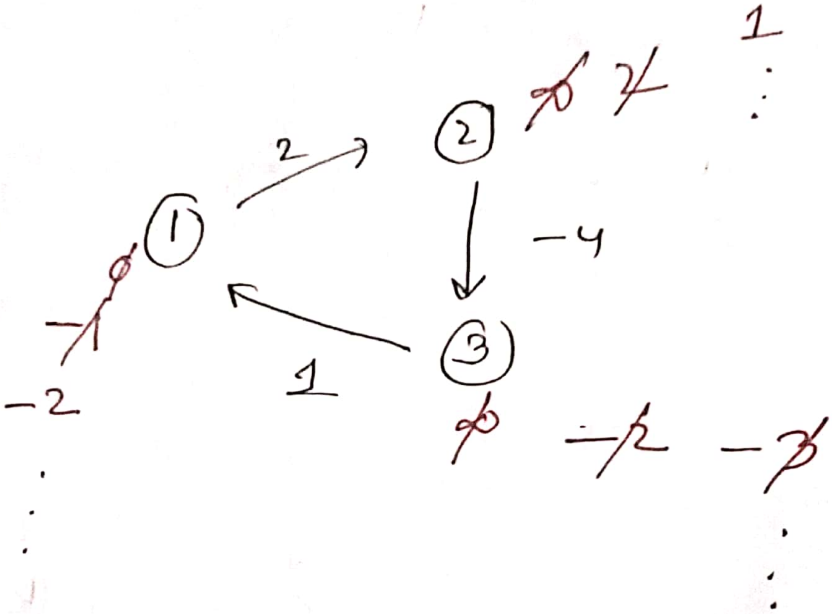
After all iteration graph is now



1	2	3
0	1	4



Negative wt cycle



infinity Loop
(Negative weg cycle)

```
int main( )
{     node, edge ;
    vector <vector <int>> adj ;

   for(int i=0   ;  i < edge  ;  i++)
  {        node1, node2, weight
        adj. pushback({node1, node2, weight})

  }


vector <int>  dis    =   bellman ford (node, 1,
                                         adj )
```

```
Vector <int> bellmanford (int node, int sounec,
                          vector<vector<int>> adj)
{
   # inital all ∞ distance
   vecton <int> dis (node+1, Int_max)
   vector <int> parent ( node, 0)   ∞
        dis[sounec] = 0 ;
# Relaxing nodes/edges
  for( i = 0 to node - 1 )
   {
       for (auto edge : adj)
       {  node1 = edge[0]     wt = edge[2]
          node2 = edge[1]

          if (distance [node1] + wt < dis [node2]

          dis [node 2 = dis [node 1] + wt
                 parent [node2] = node 1
       }
   }
}
```

After doing 1 more relaxtion if any distance get change that means graph → negative wt cycle

```
for (auto edge : adj)
{
    node1, node2, a edge wt
    if (dis[node2] > dis[node1] + wt)
    {  "Negative wt cycle"
       exit(0);
    }
}
```

parent printing / path

∵ int presentnode = node

∴ vector <int> path

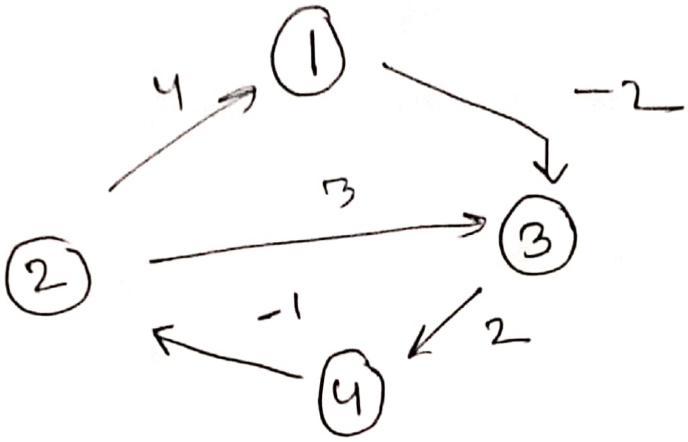
```
while ( parent [presentnode] != 0 )
{
    path_ pushback(prsentnode)
    presentnode = parent[presentnode]
}

reverse (path.begin(), path.end())

for (auto node : path )
    cout<<node
```

Floyd warshall

$\rightarrow O(N^3)$ time



1st

adj List

dis_mat

	1	2	3	4
1	0	∞ noedge	-2	∞
2	4	0	3	∞
3	∞	∞	0	2
4	∞	-1	∞	0

$1 \xrightarrow{-2} 3$

$2 \xrightarrow{4} 1$

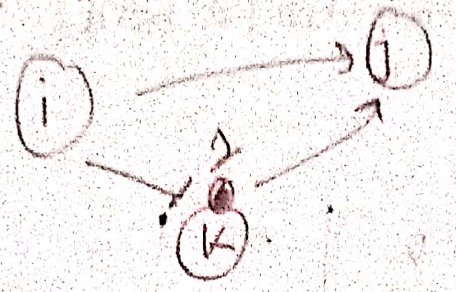
$2 \xrightarrow{3} 3$

$3 \xrightarrow{2} 4$

$4 \xrightarrow{-1} 2$

2nd

1 as intermediate vertex



update path

1

as 1

1st row and col not chang

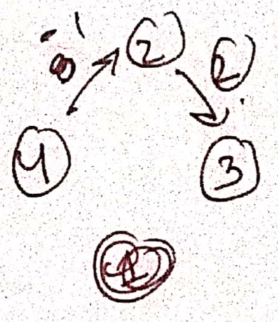
	1	2	3	4
1	0	∞	-2	∞
2	4	0	~~3~~ 2	∞
3	∞	∞	0	∞
4	∞	-1	∞	0

$d[i][j] = \min(d[i][j],\ d[i][k] + d[k][j])$

3rd

2 as intermediate

check dis from table not graph

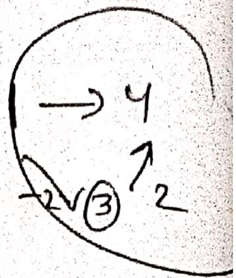


$4 \xrightarrow[1]{} 3$

	1	2	3	4
1	0	∞	-2	∞
2	4	0	2 (2→3)	∞
3	∞	∞	0	2
4	~~∞~~ 3	-1	~~∞~~ 2x 1	0

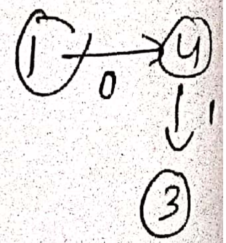
(4) 3rd as intermediate node

	1	2	3	4
1	0	~~∞~~	-2	~~∞~~ 0
2	4	0	2	~~∞~~ 2+2=4
3	∞	∞	0	2
4	3	-1	1	0



(5) 4 as intermediate node

	1	2	3	4
1	0	~~∞~~ 0+-1=-1	-2	0
2	4	0	2	4
3	~~∞~~ 2+3=5	~~∞~~ 1 2+3=5 2+-1=1	0	2
4	3	-1	1	0



	1	2	3	4
1	0	-1	-2	0
2	4	0	2	4
3	5	1	0	2
4	3	-1	1	0

→ 1st check graph then table if possible

1 → 4 → 3 ✗ not work

3 ← 2 ✓

overlap doesn't work