3rd approx

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)} = 2.587691337$$

$$f(x_2) = -0.064732741$$

⋮

12th approx. $x_{11} = \dfrac{x_{10} f(b) - b f(x_{10})}{f(b) - f(x_{10})} = 2.594313012$

* Newton Raphson Method

$$F(x_1) = f(x_0 + h)$$

Expanding by Tailor's series.

$$F(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \cdots = 0$$

$\rightarrow F(x_0) + h f'(x_0) = 0$

$\rightarrow h = -\dfrac{f(x_0)}{f'(x_0)}$ , $[f(x_0) \neq 0]$

first approx. $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ $[x_1 = x_0 + h]$

why? $x_{n+1} = x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad [n = 0, 1, 2, 3 \ldots]$$

Ex: $x^3 - 2x^2 - 4 = 0$ $a = 2,\ b = 3$

$x_0 = \frac{a+b}{2} = 2.5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.6$

$x_2 = \cancel{x_1 - f(x_1)}\ x_1 - \frac{f(x_1)}{f'(x_1)} = 2.59433$
$1984$

$\vdots$

$x_5 = 2.59$

Numerical 19.9.22

## Intenpoletion:

* Using Newton's fonmula fon intenpoletion estimate the population fon the yean 1905.

| Yean | 1891 | 1901 | 1911 | 1921 | 1931 |
|---|---|---|---|---|---|
| Population | 98752 | 1,32,285 | 168076 | 195690 | 246050 |

| yean (x) | population (y) | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 1891 | 98752 | 33533 | 2258 | -10435 | 41358 |
| 1901 | 132285 | 35791 | -8177 | 30923 | |
| 1911 | 168076 | 27614 | 22746 | | |
| 1921 | 195690 | 50360 | | | |
| 1931 | 246050 | | | | |

Newton's fonwand intenpoletion fonmula:

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

whene, $u = \frac{x - x_0}{h}$ ; $x = 1905$, $x_0 = 1891$

$h = 10$

$$u = \frac{1905-1891}{10} = 1.4$$

$y_0 = 98752$ , $\Delta y_0 = 33533$

$\Delta^2 y_0 = 2258$ , $\Delta^3 y_0 = -10435$

$\Delta^4 y_0 = 41358$

$$y(1905) = 98752 + (1.4 \times 33533) + \frac{1.4 \times (1.4-1)}{2!} \times 2258$$

$$+ \frac{1.4(1.4-1)(1.4-2)}{3!}(-10435) + \frac{(1.4)(1.4-1)(1.4-2)(1.4-3)}{4!}(41358) + \cdots$$

$= 1,47,841$ (approx.)

Backward:

| Year (x) | population (y) | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ | $\nabla^4 y$ |
|---|---|---|---|---|---|
| 1891 | 98,752 | | | | |
| 1901 | 132285 | 33533 | | | |
| 1911 | 168076 | 35791 | 2258 | | |
| 1921 | 195690 | 27614 | -8177 | -10435 | |
| 1931 | 246050 | 50360 | 22796 | 30923 | 41358 |

Newton's backward interpolation formula:

$$y(x) = y_n + u\nabla y_n + \frac{u(u-1)}{2!}\nabla^2 y_n + \frac{u(u-1)(u-2)}{3!}\nabla^3 y_n$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\nabla^4 y_n + \cdots$$

where, $u = \frac{x - x_n}{h} = \frac{1925-1931}{10} = (-0.6)$

$x = 1925,\ x_n = 1931,\ h = 10,\ y_n = 246050$

$\nabla y = 50360,\ \nabla^2 y = 22746,\ \nabla^3 y = 30923$

$\nabla^4 y = 41358$

$$y(1925) = 246050 + (-0.6)(50360) + \frac{(-0.6)(-0.6-1)}{2!}(22746)$$

$$+ \frac{(-0.6)(-0.6-1)(-0.6-2)}{3!}(30923)$$

$$+ \frac{(-0.6)(-0.6-1)(-0.6-2)(-0.6-3)}{4!}(41358) + \cdots$$

H.W: Find the area of circle of diameter 52. Given that the area 'A' of a circle of diameter d are as follows:

| d | 50 | 55 | 60 | 65 | 70 |
|---|---|---|---|---|---|
| A | 1963 | 2376 | 2827 | 3318 | 3848 |

* Derive Lagranges's interpolation formula.

Solⁿ: Let $y = f(x)$ be a polynomial of $n^{th}$ degree, which takes the value $f(x), f(x_1), f(x_2), \dots f(x_n)$ for any values $x_0, x_1, x_2 \dots x_n$ of the argument $x$.

This polynomial may be written as:

$$f(x) = a_0 (x - x_1)(x - x_2) \dots (x - x_n) + a_1 (x - x_2)(x - x_3) \dots (x - x_n) + \dots a_n (x - x_1) \dots (x - x_{n-1}) \quad \text{--- (1)}$$

[a's are constant]

To find the values of a's we put $x = x_0, x_1, x_2 \dots x_n$ in (1)

$$f(x_0) = a_0 (x_0 - x_1)(x_0 - x_2) \dots (x_0 - x_n)$$

$$\therefore a_0 = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2) \dots (x_0 - x_n)}$$

$$a_1 = \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}$$

$\vdots$

$$a_n = \frac{f(x_n)}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

$$(1) \rightarrow f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_1) + \cdots\cdots$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \quad \text{———} (2)$$

Numerical 17.10.22

## ODE / NI / ND / SLE

ND : We have Newton's forward formula:

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

where $u = \frac{x - x_0}{h} = \frac{1}{h}(x - x_0)$

$$\therefore \frac{du}{dx} = \frac{1}{h}(1-0) = \frac{1}{h}$$

$$\frac{dy}{dx} = 0 + \frac{1}{h}\Delta y_0 + \frac{(2u-1)}{h \cdot 2!}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{h \cdot 3!}\Delta^3 y_0 + \cdots$$

$$= \frac{1}{h}\left(\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0 + \cdots\right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \cdots\right)$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left(\Delta^3 y_0 + \cdots\right)$$

* Find the $1^{st}$, $2^{nd}$ and $3^{rd}$ derivatives of the function tabulated below.

| $x$ | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|
| $y$ | 3.375 | 7.0 | 13.625 | 24.0 | 38.875 | 59.0 |

Form the forward difference table :

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 1.5 | 3.375 | 3.625 | 3.0 | 0.75 |
| 2.0 | 7.0 | 6.625 | 3.75 | 0.75 |
| 2.5 | 13.625 | 10.375 | 4.5 | 0.75 |
| 3.0 | 24.0 | 14.875 | 5.25 | |
| 3.5 | 38.875 | 20.125 | | |
| 4.0 | 59.0 | | | |

at the point $x = 1.5$,

$x_0 = 1.5$, $x = 1.5$, $u = \frac{x - x_0}{h} = 0$, $h = 0.5$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0\right]$$

$$= \frac{1}{0.5}\left[3.625 + \frac{1}{2}(-1)(3.0) + \frac{2}{6}(0.75)\right]$$

$$= 4.750$$

CamScanner

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\Delta^2 y_0 + (u-1)\Delta^3 y_0\right)$$

$$= \frac{1}{0.25}\left[3.0 + (-1)(0.75)\right] = 9.0$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\Delta^3 y_0 + \cdots\right] = \frac{1}{0.125}\left[0.75\right] = 6$$

Thus at $x = 1.5$

$$\frac{dy}{dx} = 4.75$$

$$\frac{d^2y}{dx^2} = 9.0 \; ; \quad \frac{d^2y}{dx^2} = \cancel{6}\ 6.0$$

* Trapezoidal rule for numerical integration.

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \cdots y_{n-1}) + y_n\right]$$

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*** Simpson's $\frac{1}{3}$ rule :**

US

$$\int_a^b f(x)\,dx = \frac{h}{3}\Big[y_0 + 4(y_1 + y_3 + \cdots y_{n-1}) + 2(y_2 + y_4 + \cdots y_{n-2}) + y_n\Big]$$

3 এর গুণিতক
n ~~বিজোড়~~ হলে

*** Simpson's $\frac{3}{8}$ rule:**

$$\int_a^b f(x)\,dx = \frac{3h}{8}\Big[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + y_9 + \cdots y_{n-3}) + y_n\Big]$$

* Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using ⓐ trapezoidal rule ⓑ simpson's $\frac{1}{3}$ rule ⓒ simpson's $\frac{3}{8}$ rule.

Sol$^n$: $h = \frac{6-0}{6} = 1$ ; $y = \frac{1}{1+x^2}$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y=\frac{1}{1+x^2}$ | 1 | 0.5 | 0.2 | 0.1 | 0.058 | 0.03846 | 0.0270270 |
| | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ |

(a) Trapezoidal:

$$\int_0^6 y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5) + y_6\right]$$

$$= 1.4107985$$

(b) Simpson's $\frac{1}{3}$ rule:

$$\int_0^6 y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6\right]$$

$$= 1.3661734$$

(c) Simpson's $\frac{3}{8}$ rule:

$$\int_0^6 y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3) + y_6\right]$$

$$= 1.3570808$$

Eulen Method :

$\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0, h$

$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$

$\frac{dy}{dx} = x + 1 \qquad y(0) = 0$

$\rightarrow dy = (x+1)\,dx$

$\rightarrow \int dy = \int (x+1)\,dx$

$\rightarrow y = \frac{1}{2}x^2 + x + c$

at $x = 0$, $y(0) = 0 + c \quad \therefore c = 0$

$\therefore y = \frac{1}{2}x^2 + x$

* Solve $\frac{dy}{dx} = x + y$, $y(0) = 1$, fon $0 \le x \le 1$

with $h = 0.1$

$\rightarrow y_1 = y_0 + hf(x_{n-1}, y_{n-1})$

$= 1 + 0.1(0 + 1) = 1.1$

$y_2 = 1.1 + 0.1(0.1 + 1.1) = 1.22$

* Run-ge-kuttaq Method:

| $x$ | $K_1$ | $K_2$ | $y$ |
|---|---|---|---|

$K_{1,n} = hf(x_{n-1}, y_{n-1})$ $\qquad y_n = y_{n-1} + \frac{1}{2}(K_1 + K_2)$

$K_{2,n} = hf(x_{n-1} + h, y_{n-1} + K_{1,n})$

* Use Runge-kutta method, solve $\frac{dy}{dx} = \frac{x^2 + y^2}{10}$

$y(0) = 1$, for $0 \le x \le 0.4$; $h = 0.1$.

$\rightarrow \frac{dy}{dx} = \frac{1}{10}(x^2 + y^2) = f(x, y)$; $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$K_{1,1} = 0.1\left(\frac{0+1}{10}\right) = 0.01$

$K_{2,1} = 0.1\left(\frac{0.1^2 + 1.01}{10}\right) = 0.10301$

$$y_1 = y_0 + \frac{1}{2}(K_1 + K_2)$$

$$= 1 + \frac{1}{2}(0.01 + 0.10301)$$

$$= 1.01015$$

Now, $x_1 = 0.1$, $y_1 = 1.01015$, $h = 0.1$

$k_1 =$

$K_2 =$

$y_2 = y_1 + \frac{1}{2}(K_1 + K_2)$

* Solution of Linear equation.

Assignment.

$$\left.\begin{array}{l} x + 2y = 1 \\ 2x - 3y = 0 \end{array}\right\}$$

1) Gaussian Elimination Method

2) Gauss Jordan " "

diagonally dominated

# * Gauss Jacobi Iterative method:

$$10x - 2y + z = 2$$
$$-3x + 11y + 2z = 5$$
$$x - y + 5z = 1$$

$$x = (2 + 2y - z)/10$$
$$y = (5 + 3x - 2z)/11$$
$$z = (1 - x + y)/5$$

$$x = g_1(y, z)$$
$$y = g_2(x, z)$$
$$z = g_3(x, y)$$

General form: $x_i = g_1(y_{i-1}, z_{i-1})$

$$y_i = g_2(x_{i-1}, z_{i-1})$$
$$z_i = g_3(x_{i-1}, y_{i-1})$$

# * Gauss Seidal:

$$x_i = g_1(y_{i-1}, z_{i-1})$$
$$y_i = g_2(x_i, z_{i-1})$$
$$z_i = g_3(x_i, y_i)$$

| x | y | z |
|---|---|---|
| 0 | 0 | 0 |
| 0.2 | 0.4545 | 0.2 |
| 0.2709 | 0.477 | 0.2509 |
| ⋮ | ⋮ | ⋮ |