Numerical

19.9.22

Interpolation:

* Using Newton's formula for interpolation estimate the population for the year 1905.

Year	1891	1901	1911	1921	1931
Population	98752	1,32,285	168076	195690	246050

year (x)	population (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	98752	33533	2258	-10435	41358
1901	132285	35791	-8177	30923	
1911	168076	27614	22746		
1921	195690	50360			
1931	246050				

Newton's forward interpolation formula:

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

where, $u = \frac{x - x_0}{h}$; $x = 1905$, $x_0 = 1891$

$h = 10$

$$u = \frac{1905-1891}{10} = 1.4$$

$y_0 = 98752$, $\Delta y_0 = 33533$

$\Delta^2 y_0 = 2258$, $\Delta^3 y_0 = -10435$

$\Delta^4 y_0 = 41358$

$$y(1905) = 98752 + (1.4 \times 33533) + \frac{1.4 \times (1.4-1)}{2!} \times 2258$$

$$+ \frac{1.4(1.4-1)(1.4-2)}{3!}(-10435) + \frac{(1.4)(1.4-1)(1.4-2)(1.4-3)}{4!}(41358) + \cdots$$

$$= 1,47,841 \text{ (approx.)}$$

Backward:

Year (x)	population (y)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	98,752				
1901	132285	33533			
1911	168076	35791	2258		
1921	195690	27614	-8177	-10435	
1931	246050	50360	22746	30923	41358

Newton's backward interpolation formula:

$$y(x) = y_n + u\nabla y_n + \frac{u(u-1)}{2!}\nabla^2 y_n + \frac{u(u-1)(u-2)}{3!}\nabla^3 y_n$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\nabla^4 y_n + \cdots$$

where, $u = \frac{x-x_n}{h} = \frac{1925-1931}{10} = (-0.6)$

$x = 1925$, $x_n = 1931$, $h = 10$, $y_n = 246050$

$\nabla y = 50360$, $\nabla^2 y = 22746$, $\nabla^3 y = 30923$

$\nabla^4 y = 41358$

$$y(1925) = 246050 + (-0.6)(50360) + \frac{(-0.6)(-0.6-1)}{2!}(22746)$$

$$+ \frac{(-0.6)(-0.6-1)(-0.6-2)}{3!}(30923)$$

$$+ \frac{(-0.6)(-0.6-1)(-0.6-2)(-0.6-3)}{4!}(41358) + \cdots$$

H.W: Find the area of circle of diameter 52. Given that the area 'A' of a circle of diameter d are as follows:

d	50	55	60	65	70
A	1963	2376	2827	3318	3848

6/16

* Derive Lagrange's interpolation formula.

Soln: Let $y = f(x)$ be a polynomial of n^{th} degree, which takes the value $f(x_0), f(x_1), f(x_2), \dots f(x_n)$ for any values $x_0, x_1, x_2 \dots x_n$ of the argument x.

This polynomial may be written as:

$$f(x) = a_0 (x - x_1)(x - x_2) \dots (x - x_n) + a_1 (x - x_0)(x - x_2) \dots (x - x_n) + \dots a_n (x - x_1) \dots (x - x_{n-1}) \quad \text{—— (1)}$$

[a's are constant]

To find the values of a's we put $x = x_0, x_1, x_2 \dots x_n$ in (1)

$$f(x_0) = a_0 (x_0 - x_1)(x_0 - x_2) \dots (x_0 - x_n)$$

$$\therefore a_0 = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2) \dots (x_0 - x_n)}$$

$$a_1 = \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}$$

7/16

$$\vdots$$

$$a_n = \frac{f(x_n)}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

$$(1) \rightarrow f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_1) + \cdots$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \quad \text{——— (2)}$$

Numerical 17.10.22

ODE / NI / ND / SLE

ND : We have Newton's forward formula:

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

where $u = \frac{x - x_0}{h} = \frac{1}{h}(x - x_0)$

$\therefore \frac{du}{dx} = \frac{1}{h}(1-0) = \frac{1}{h}$

$$\frac{dy}{dx} = 0 + \frac{1}{h}\Delta y_0 + \frac{(2u-1)}{h \cdot 2!}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{h \cdot 3!}\Delta^3 y_0 + \cdots$$

$$= \frac{1}{h}\left(\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{3!}\Delta^3 y_0 + \cdots\right)$$

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2}\left(\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \cdots\right)$$

$$\frac{d^3 y}{dx^3} = \frac{1}{h^3}\left(\Delta^3 y_0 + \cdots\right)$$

[redacted]

* Find the 1^{st}, 2^{nd} and 3^{rd} derivatives of the function tabulated below.

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.0	13.625	24.0	38.875	59.0

Form the forward difference table :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1.5	3.375	3.625	3.0	0.75
2.0	7.0	6.625	3.75	0.75
2.5	13.625	10.375	4.5	0.75
3.0	24.0	14.875	5.25	
3.5	38.875	20.125		
4.0	59.0			

at the point $x = 1.5$,

$x_0 = 1.5$, $x = 1.5$, $u = \frac{x - x_0}{h} = 0$, $h = 0.5$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0\right]$$

$$= \frac{1}{0.5}\left[3.625 + \frac{1}{2}(-1)(3.0) + \frac{2}{6}(0.75)\right]$$

$$= 4.750$$

9/16

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\Delta^2 y_0 + (u-1)\Delta^3 y_0\right)$$

$$= \frac{1}{0.25}\left[3.0 + (-1)(0.75)\right] = 9.0$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\Delta^3 y_0 + \ldots\right] = \frac{1}{0.125}\left[0.75\right] = 6$$

thus at $x = 1.5$

$$\frac{dy}{dx} = 4.75$$

$$\frac{d^2y}{dx^2} = 9.0 \; ; \; \frac{d^2y}{dx^2} = \cancel{6}\ 6.0$$

... rule for numerical

10/16