Root Finding method

① Bisection Method : (Bolzano method)

x, x^2, x^4

(solutions for Algebric and Transcedental equation)

log, $\sin x$, $\cos x$

→ based on true repeated application of Intermediate property

Theory

Let $f(x)$ be continous between a & b

Let. $f(a)$ be -ive
$f(b)$ be +ive

or $f(a)f(b) < 0$

Then the first approximation of the root is

in $x_1 = \frac{a+b}{2}$

Ⓨ then if $f(x_1) = 0$; x_1 is root of $f(x) = 0$

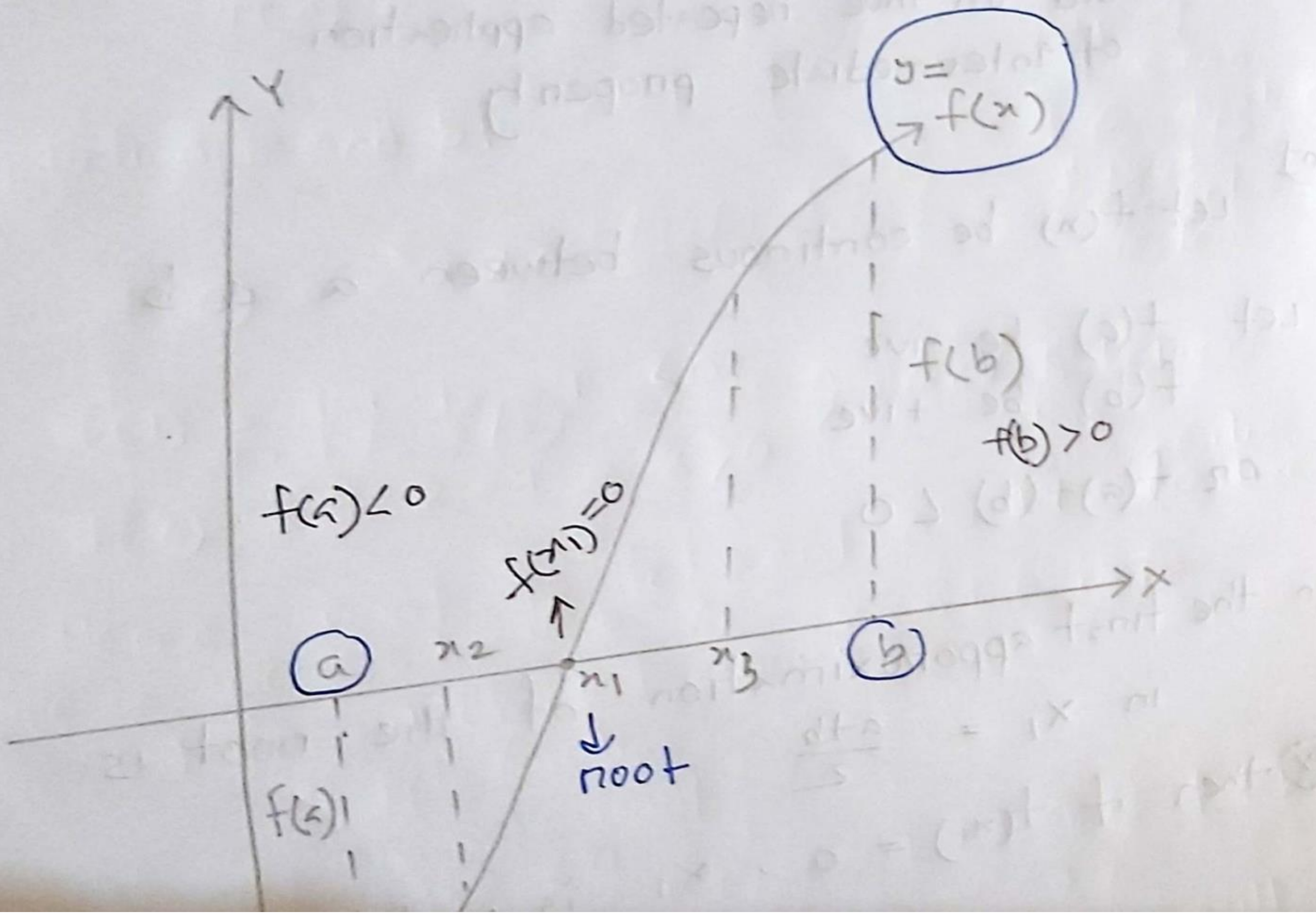
otherwise;

the root lies between a & x_1 or x_1 & b

according to $f(x_1)$ is negative or positive

Then we bisect the interval as before & continue

the process untill the root is found



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Understanding the method

$$f(x) = x^2 - 2x - 7 = 0$$

suppose $a = 3$; $f(a) = f(3) = 9 - 6 - 7 = -4$

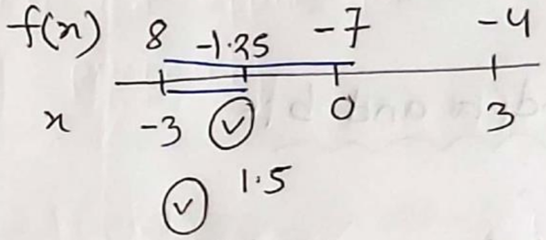
$b = -3$; $f(-3) = 8$

$\therefore f(a)f(+b) < 0$

$\therefore x_1 = \frac{a+b}{2}$

$= \frac{3-3}{2}$

$= 0$



$x_1 = a+b/2$

② if $f(x_1)\ \boxed{< 0} \Rightarrow$ root lies between x_1 and b

$\therefore$ 2nd approximate root is $x_2 = \frac{x_1 + b}{2}$

③ if $f(x_1)\ \boxed{> 0} \Rightarrow$ root lies between x_1 and a

$\therefore$ 2nd approximate $x_2 = \frac{x_1 + a}{2}$ ($a + x_1$)

math-1

Find the root of equation

$x^3 - x - 4 = 0$ using bisection Method

(method correct upto 3 decimal places)

Sol:- Let

$f(x) = x^3 - x - 4 = 0$ — ①

Tips:-
if we can take a, b closer to root we can get ans faster

To find a and b:-

$f(0) = -4 < 0$

$f(1) = 1 - 1 - 4 = -4 < 0$

$f(2) = 8 - 2 - 4 = 2 > 0$

→ root is between this cause positive negative

we can use $a = 1$ or $b = 2$

but we will check for more to apply our tips

$f(1.5) = -2.125 < 0$

$f(1.6) = -1.504 < 0$

$f(1.7) = -0.787 \boxed{< 0}$ ✓

$f(1.8) = 0.032 \boxed{> 0}$ ✓

Instead of taking 1, 2 Now we will take 1.7 and 1.8

choosing $a = 1.7$ $b = 1.8$

First approximate root using bisection method

$$x_1 = \frac{a+b}{2} = \frac{1.7+1.8}{2} = 1.75$$

$$f(1.75) = (1.75)^3 - 1.75 - 4 = \boxed{-0.39 < 0}$$

Hence, root lies between $x_1 = 1.75$ and $b = 1.8$

2nd approximate root using bisection method

$$x_2 = \frac{a+b}{2} = \frac{1.75+1.8}{2} = 1.775$$

$$f(1.775) = (1.775)^3 - (1.775) - 4 = \boxed{-0.182 < 0}$$

Hence, root lies between $x_2 = 1.775$ and $b = 1.8$

3rd approximate root

$$x_3 = \frac{1.775 + 1.8}{2} = 1.7875$$

$$f(1.7875) = (1.7875)^3 - (1.7875) - 4 = \boxed{-0.076 < 0}$$

Hence root lies between $x_3 = 1.7875$ & $b = 1.8$

4th approximate root

$$x_4 = \frac{1.7875 + 1.8}{2} = 1.79375$$

$$f(1.79375) = -0.0022 \quad \boxed{<0}$$

Hence, root is between 1.79375 to 1.8

5th approximate root

$$x_5 = \frac{1.79375 + 1.8}{2} = \cancel{1.79683} \; 1.796875$$

$$f(1.796875) = 0.0048 \quad \boxed{>0}$$

Hence root is between $a = 1.79375$

$x_5 = \boxed{1.796875}$

6th apponximate root

$$x_6 = \frac{1.79375 + 1.796875}{2}$$

$$= 1.795312$$

$$\therefore f(1.795312) = \boxed{-0.0087 < 0}$$

Hence, root is between

$b = 1.796875$

$x_6 = 1.795312$

$\boxed{7^{th} \text{ approximate root}}$

$$x_7 = \frac{1.795312 + 1.796875}{2}$$

$$= \boxed{1.796}093$$

x_7

∴ so hence the approximate root is correct upto 3-decimal places is

$x = 1.796$

Ans

(*) Sample Math

① $f(x) = x^3 - 2x^2 - 4$ $(a = 2 \,;\, b = 3)$

② $x^3 + x^2 - 1 = 0$ $(a = 0 . b = 1$

③ $\sin x = 1 - x^2$