Gaussian Elimination Method:

Gaussian elimination is an algorithm that allows us to find the solutions of a system of simultaneous linear equations by transforming the system into an equivalent system in row echelon form. Elementary row operations are performed on the system until the system is in row echelon form. To find the solution:

1) First solve one of the equations for one variable and then substitute this expression into the remaining equations.

   This results in a new system in which the number of equations and variables is one less than in the original system.

2) This procedure is applied to another variable and the reduction process continued until there remains one equation, in which the only unknown quantity is the last variable.

3) "Back substitute" the value found by solving this equation in an earlier equation that contains this variable and one other unknown to solve for another variable.

This process is continued untill all the original variables have been evaluated.

* Solve the following system using Gaussian Elimination method:

$$x - 2y + z = 0$$
$$2x + y - 3z = 5$$
$$4x - 7y + z = -1$$

Solution: The augmented matrix which represents this system is:

P.T.O

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & 1 & -3 & 5 \\ 4 & -7 & 1 & -1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & 1 & -3 & 5 \\ 0 & -1 & 3 & 1 \end{array}\right] \qquad R_3 \leftarrow 4R_1 - R_3$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -5 & 5 & -5 \\ 0 & -1 & 3 & 1 \end{array}\right] \qquad R_2 \leftarrow 2R_1 - R_2$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -5 & 5 & -5 \\ 0 & 0 & -10 & -10 \end{array}\right] \qquad R_3 \leftarrow R_2 - 5R_3$$

So we find, $x - 2y + z = 0$ ——— (i)

$-5y + 5z = -5$ ——— (ii)

$-10z = -10$ ——— (iii)

$(iii) \rightarrow \quad -10z = -10 \qquad \therefore z = 1$

$(ii) \rightarrow \quad -5y + 5z = -5$

or, $-5y + 5 \cdot 1 = -5$

or, $-5y + 5 = -5$

or, $-5y = -10 \qquad \therefore y = 2$

$(i) \rightarrow \quad x - 2y + z = 0$

or, $x - 2 \cdot 2 + 1 = 0$

or, $x - 4 + 1 = 0 \qquad \therefore x = 3$

$\therefore$ The solution of the system is

$(x, y, z) = (3, 2, 1)$.

(Ans).

Gauss - Jordan Elimination Method:

Gauss Jordan elimination is an algorithm that allows us to find the solutions of a system of simultaneous linear equations by transforming the system into an equivalent system in reduced row echelon form.

In a reduced row echelon form, all the pivots will be equal to 1 and the pivots are the only non-zero entries of the basic columns.

* Solve the following system using Gauss - Jordan Elimination method:

$$x - 2y + z = 0$$

$$2x + y - 3z = 5$$

$$4x - 7y + z = -1$$

Solution:

The augmented matrix which represents this system is:

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 2 & 1 & -3 & 5 \\ 4 & -7 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 1 & 0 \\ 2 & 1 & -3 & 5 \\ 0 & -1 & 3 & -1 \end{bmatrix} \qquad R_3 \leftarrow 4R_1 - R_3$$

$$= \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & -5 & 5 & -5 \\ 0 & -1 & 3 & 1 \end{bmatrix} \qquad R_2 \leftarrow 2R_1 - R_2$$

$$= \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & -5 & 5 & -5 \\ 0 & 0 & -10 & -10 \end{bmatrix} \qquad R_3 \leftarrow R_2 - 5R_3$$

$$= \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \leftarrow \frac{-1}{5} R_2 \\ R_3 \leftarrow \frac{1}{10} R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_1 \leftarrow R_1 + 2R_2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_1 \leftarrow R_1 + R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_2 \leftarrow R_2 + R_3$$

So we find, $x = 3$

$y = 2$

$z = 1$

$\therefore$ The solution of the system is

$(x, y, z) = (3, 2, 1)$ {Ans}