

* Analytical vs numerical solution:

- Analytical

1) obtained exactly with pencil and paper

2) give exact value/solutions

Numerical

1) can't be obtained exactly in finite time and can't be solved typically with pencil and paper.

2) approximate value.

Analytical approach example:

Find the root of $f(x) = x - 5$

Analytical solution:

$$f(x) = x - 5 = 0$$

add +5 to both sides to get the answer.

$$\dots x = 5$$

Numerical solution:

Suppose $x = 1$

$$\therefore f(1) = 1 - 5 = -4 < 0$$

Again $x = 6$

$$\therefore f(6) = 6 - 5 = 1 > 0$$

So, the answer is between 1 and 6.

$$\text{Now, } x = \frac{6+1}{2}$$

$$\therefore f\left(\frac{7}{2}\right) < 0$$

So, the root is between $\frac{7}{2}$ and 6.

continues like this. This is called bisection method

* Gauss Seidal vs Gauss Jacobi:

→ Gauss Seidal method is more efficient than Jacobi method as Gauss-Seidal method requires less iterations to converge to the actual solution with a certain degree of accuracy.

(Gauss Seidal and Gauss Jacobi method both are used iterative methods for determining the solutions for the SLE, which are diagonally dominant). But in Jacobi method, even after the modified value of a variable is evaluated in the present iteration, it is not used until the next iteration. So, it increases the number of iterations to reach the exact solution.

Gauss-Seidel method always applies the latest updated values during the iterative procedures.

Gauss Jacobi

Gauss Seidel

Each step general form:

$$x_i = g_1(y_{i-1}, z_{i-1})$$

$$y_i = g_2(x_{i-1}, z_{i-1})$$

$$z_i = g_3(x_{i-1}, y_{i-1})$$

$$x_i = g_1(y_{i-1}, z_{i-1})$$

$$y_i = g_2(x_i, z_{i-1})$$

$$z_i = g_3(x_i, y_i)$$

until the approximate values are found.

* Diagonally Dominant Matrix: (Example) ^{with}

A square matrix A is called diagonally dominant if, for every row of the matrix, the magnitude of the diagonal entry in a row is larger than the sum of the magnitudes of the non-diagonal entries in that row.

That's to say,

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}| \text{ for all } i$$

where $a_{ij} \rightarrow$ entry in the i^{th} row and j^{th} column

* Simpson vs trapezoidal:

→ both are used for finding approximate value of integrals

Simpson

— uses quadratic approximation

— gives accurate value

— this rule can be applied if the area is divided in even numbers.

— number of ordinates → odd

— works better in case of the area under a parabola

— formula:

Trapezoidal

— uses linear approx.

— doesn't give accurate value (gives approx. value)

— this rule can be applied for any number of ordinates

— under a straight line.

— formula:

* Significant figure/digit:

The significant digits of a number are those that can be used with confidence. They correspond to the number of certain digits plus one estimated digit.

1-9 → always significant

* 0 → significant

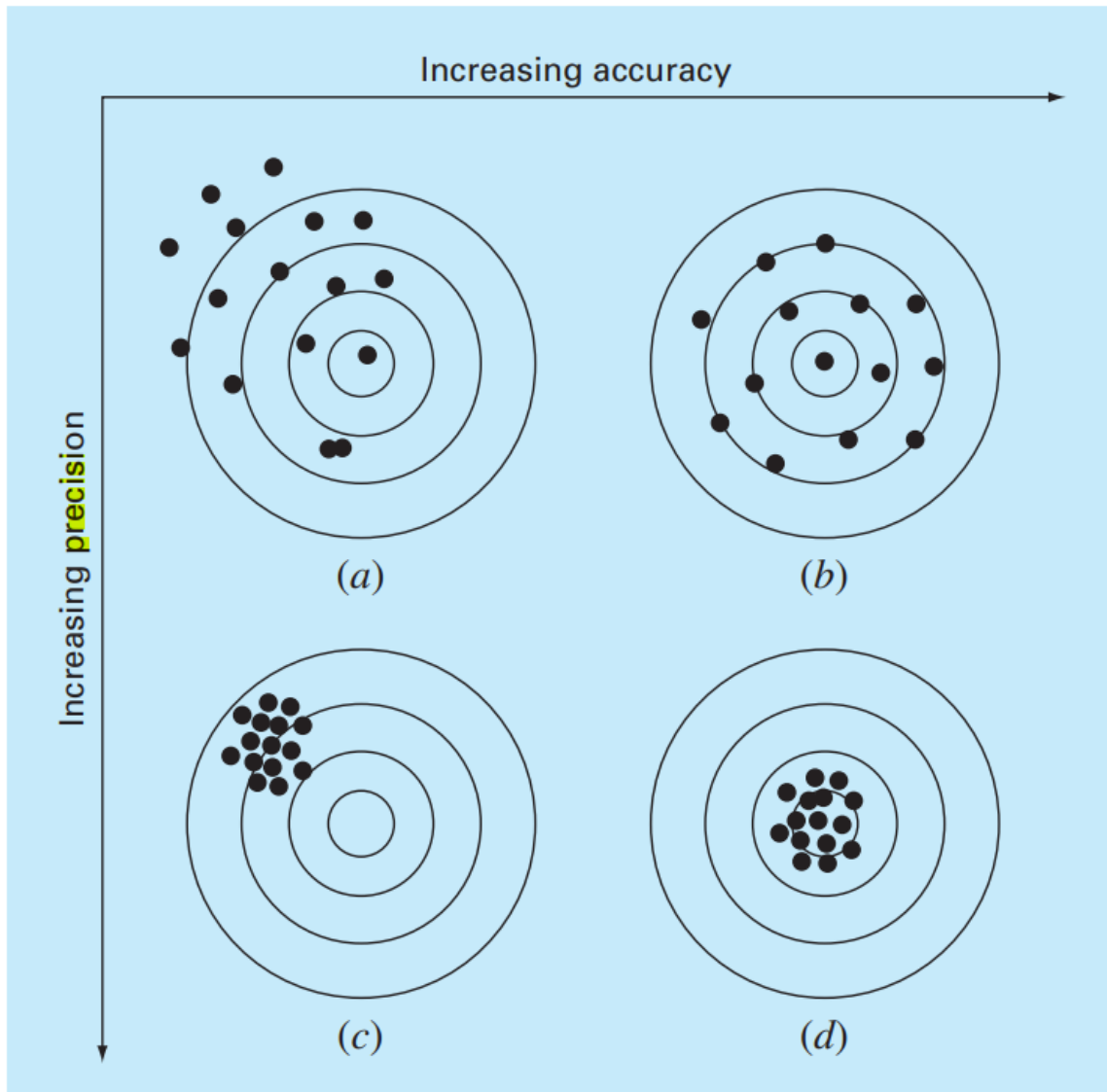
if trailing zeroes: 120

if middle zeroes: 109

if decimal point: 12.05

* Accuracy: accuracy refers to how closely a computed value agrees with the true value.

* Precision: refers to how closely individual computed values agree with each other.



True value = approximation + error

$$\text{True fractional relative error} = \frac{\text{true error}}{\text{true value}}$$