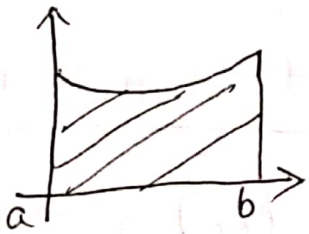
Numerical Integration:

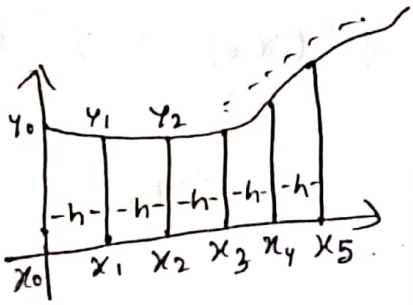
The area bounded by the curve $f(x)$ and x axis between limit a and b is denoted by

$$I = \int_a^b f(x)\,dx \quad \text{---(1)}$$



divide the interval (a,b) into n equal ~~in~~ part with h interval.

where, $h = \frac{b-a}{n}$



$a = x_0$

$x_1 = x_0 + h$

$x_2 = x_1 + h$

$\vdots$

$x_n = x_{n-1} + h$

3 rules:

1. Trapezoidal Rule
2. Simpson's $1/3$ rule
3. Simpson's $3/8$ rule

Trapezoidal's rule:

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \cdots y_{n-1}) + y_n\right]$$

Simpson's 1/3 rule: (n-even)

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + \cdots y_{n-1}) + 2(y_2 + y_4 + \cdots y_{n-2}) + y_n\right]$$

Simpson's 3/8 rule: (n-divisible by 3)

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + \cdots y_{n-3}) + y_n\right]$$

Ex: Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using

a) trapezoidal formula

b) simpson's 1/3 "

c) " 3/8 "

Given

Soln: $h = \frac{6-0}{6} = 1$; $y = \frac{1}{1+x^2}$

x	$y = \frac{1}{1+x^2}$
$x_0 = 0$	$y_0 = 1$
$x_1 = 0+1 = 1$ ($x_1 = x_0 + h$)	$y_1 = \frac{1}{1+1} = 0.5$
$x_2 = 1+1 = 2$	$y_2 = \frac{1}{1+4} = 0.2$
$x_3 = 3$	$y_3 = \frac{1}{1+9} = 0.1$
$x_4 = 4$	$y_4 = \frac{1}{1+16} = 0.058$
$x_5 = 5$	$y_5 = 0.03846$
$x_6 = 6$	$y_6 = 0.027027$

a) Trapezoidal Rule:

$$\int_0^6 y\,dx = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+y_4+y_5) + y_6\right]$$

$$= \frac{1}{2}\left[1 + 2(0.5+0.2+0.1+0.058 + 0.03846) + 0.027027\right]$$

$= 1.4099735$

b) Simpson's 1/3 rule:

$$\int_0^6 f(x)\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6\right]$$

$$= \frac{1}{3}\left[1 + 4(0.5 + 0.1 + 0.038476) + 2(0.2 + 0.058) + 0.027027\right]$$

$$= 1.365622$$

c) Simpson's 3/8 rule:

$$\int_0^6 y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3) + y_6\right]$$

$$= \frac{3\times 1}{8}\left[1 + 3(0.5 + 0.2 + 0.058 + 0.038476) + 2(0.1) + 0.027027\right]$$

$$= 1.356153$$

Interpolation: method of estimating unknown values from given set of observation.

3 formula's:

① Newton's forward formula
② Newton's Backward formula
③ Lagrange's interpolation formula

equal interval

x	f(x)
1971	1000
1981	1025
1991	1080
2001	1120

Forward and Backward used

unequal

x	f(x)
75	670
80	685
87	750
90	800
99	812

Lagrange's formula used

Find 1983 - you've to see the progness of forward

1995 - back