

and continue the process until the root is found

Q: Find a root of an equation $f(x) = x^3 + x^2 - 1$ between 0 and 1.

→ Step 1: $f(0) = -1 < 0$

$$f(1) = 1 > 0$$

∴ The root lies between 0 and 1.

$$x_0 = \frac{0+1}{2} = 0.5$$

$$f(x_0) = f(0.5) = -0.625 < 0$$

Step 2: $f(0.5) = -0.625 < 0$

$$f(1) = 1 > 0$$

The root lies between 0.5 and 1.

$$∴ x_1 = \frac{0.5+1}{2} = 0.75$$

$$∴ f(x_1) = f(0.75) = -0.01562 < 0$$

Step 3: $f(0.75) = -0.01562 < 0$

$$f(1) = 1 > 0$$

\therefore The root lies between 0.75 and 1

$$\therefore x_2 = \frac{0.75 + 1}{2} = 0.875$$

$$f(x_2) = f(0.875) = 0.43555 > 0$$

Step 4: $f(0.875) = 0.43555 > 0$

$$f(0.75) = -0.01562 < 0$$

\therefore The root lies between 0.75 and 0.875

$$\therefore x_3 = \frac{0.75 + 0.875}{2} = 0.8125$$

$$\therefore f(x_3) = f(0.8125) = 0.19653 > 0$$

Step 5: $f(0.8125) = 0.19653 > 0$

$$f(0.75) = -0.01562 < 0$$

\therefore The root lies between 0.8125 and 0.75

$$\therefore x_4 = \frac{0.75 + 0.8125}{2} = 0.78125$$

$$\therefore f(x_4) = f(0.78125) = 0.08719 > 0$$

Step 6: $f(0.78125) = 0.08719 > 0$

$$f(0.75) = -0.01562 < 0$$

\therefore The root lies between 0.78125 and 0.75

$$\therefore x_5 = \frac{0.75 + 0.78125}{2} = 0.76562$$

$$f(x_5) = f(0.76562) = 0.03498 > 0$$

Step 7: $f(0.76562) = 0.03498 > 0$

$$f(0.75) = -0.01562 < 0$$

\therefore The root lies between 0.75 and 0.76562

$$\therefore x_6 = \frac{0.75 + 0.76562}{2} = 0.75781$$

$$f(x_6) = f(0.75781) = 0.00948 > 0$$

Step 8: $f(0.75781) = 0.00948 > 0$

$$f(0.75) = -0.01562 < 0$$

$$\therefore x_7 = \frac{0.75 + 0.75781}{2} = 0.75391$$

$$\therefore f(0.75391) = -0.00312 < 0$$

Step 9: $f(0.75391) = -0.00312 < 0$

$$f(0.75781) = 0.00948 > 0$$

$$x_8 = \frac{0.75391 + 0.75781}{2} = 0.75586$$

$$f(0.75586) = 0.00316 < 0$$

Step 10:

$$f(0.75586) = 0.00316 < 0$$

$$f(0.75781) = 0.00948 > 0$$

$$x_9 = \frac{0.75586 + 0.75781}{2} = 0.75488$$

$$\therefore f(0.75488) = 0.00002 > 0$$

\therefore So, the approximate root is $x_9 = 0.755$

Q: Find a root of an equation:

$$f(x) = x^2 + \sin x - 1 = 0$$

Step 1:

Hence, $f(0) = -1 < 0$

$$f(1) = 0.0175 > 0$$

$$\therefore x_0 = \frac{0+1}{2} = 0.5$$

$$f(0.5) = -0.7413 < 0$$

Step 2: $f(0.5) = -0.7413 < 0$

$$f(1) = 0.0175 > 0$$

$$\therefore x_1 = \frac{0.5+1}{2} = 0.75$$

$$f(0.75) = -0.4244 < 0$$

Step 3: $f(0.75) = -0.4244 < 0$

$$f(1) = 0.0175 > 0$$

$$\therefore x_2 = \frac{0.75+1}{2} = 0.875$$

$$\therefore f(0.875) = -0.2191 < 0$$

step 4: $f(0.875) = -0.2191 < 0$

$$f(1) = 0.0175 > 0$$

$$\therefore x_3 = \frac{0.875 + 1}{2} = 0.9375$$

$$f(0.9375) = -0.1047 < 0$$

step 5: $f(0.9375) = -0.1047 < 0$

$$f(1) = 0.0175 > 0$$

$$\therefore x_4 = \frac{0.9375 + 1}{2} = 0.9688$$

$$\therefore f(0.9688) = -0.0446 < 0$$

step 6: $f(0.9688) = -0.0446 < 0$

$$f(1) = 0.0175 > 0$$

$$\therefore x_5 = \frac{0.9688 + 1}{2} = 0.9844$$

$$f(0.9844) = -0.0138 < 0$$

step 7: $f(0.9844) = -0.0138 < 0$

$$f(1) = 0.0175 > 0$$

$$\therefore x_6 = \frac{0.9844 + 1}{2} = 0.9922$$

$$f(0.9922) = 0.0018 > 0$$

Step 8: $f(0.9922) = 0.0018 > 0$

$$f(0.9844) = -0.0138 < 0$$

$$\therefore x_7 = \frac{0.9922 + 0.9844}{2} = 0.9883$$

$$\therefore f(0.9883) = -0.0061 < 0$$

Step 9: $f(0.9883) = -0.0061 < 0$

$$f(0.9922) = 0.0018 > 0$$

$$\therefore x_8 = \frac{0.9883 + 0.9922}{2} = 0.9902$$

$$\therefore f(0.9902) = -0.0022 < 0$$

Step 10: $f(0.9902) = -0.0022 < 0$

$$f(0.9922) = 0.0018 > 0$$

$$\therefore x_9 = \frac{0.9902 + 0.9922}{2} = 0.9912$$

$$\therefore f(0.9912) = -0.0002 < 0$$

\therefore The approximate root is $x_9 = 0.9912$