Interpoletion

Equal Interval

∴ Newton's $\boxed{\text{forward}}$ Interpoletion formula

$$\therefore y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

∴ y_0 = Table's most Left side value

$$\therefore u = \frac{x - x_0}{h}$$

∴ Newton's $\boxed{\text{backward}}$ Interpoletion formula

$$\therefore y(x) = y_n + u\Delta y_n + \frac{u(u-1)}{2!}\Delta^2 y_n + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_n +$$

$$\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_n + \cdots$$

$$\therefore u = \frac{x - x_n}{h}$$

y_n = Table's most ~~le~~ Right side

(7) Using Newton formula for interpoletion

(1) estimate the population for year 1905

(2) the population for year 1925

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Year	1891	1901	1911	1921	1931
Population	98,752	1,32,285	1,68,076	1,95,690	246,050

Ans (1) For 1905 = forward Interpoletion

Year x	Population Y	Δy $x_2 - x_1$	Δy^2	$\Delta^3 y$	$\Delta^4 y$
1891 (x_1)	98,752	33,533	2,258	−10,435	41,358
8 1901 (x_2)	1,32,287	35,791	−8,177	30,923	
1911	1,68,076	27,614	22,746		
1921	1,95690	50,360			
1931	2,46,050				

Here,

$x = 1905$

$x_0 = 1891$

$\therefore h = 10$

$\therefore u = \frac{x - x_0}{h}$

$= 1.4$

$\therefore y_0 = 98.752$

$\therefore \Delta y_0 = 33.533$

$\therefore \Delta^2 y_0 = 2258$

$\Delta^3 y_0 = -10.435$

$\Delta^4 y_0 = 41.358$

$$\therefore y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

$y(1905) = 1,47,841$ (Approximately)

Ans

② for 1925 → backward Interpoletion

Year x	Population y	Δy $y_9 - y_4$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	98,752				
1901	1,32,285	33.533			
1911	1,68,076	35,791	2258		
1921	1,95,690	27,614	-8,177	-10,435	
1931	2,46,050	50,0360	22,746	30,923	41,358

Here ∴ $x = 1925$

$x_n = 1931$

$h = 10$

$$\therefore u = \frac{x - x_n}{h}$$

$$= -.6$$

$$y(1925) = y_n + u\Delta y_n + \frac{u(u-1)}{2!}\Delta^2 y_n + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_n$$
$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_n + \cdots$$

$$= 2,25,982 \text{ (approxi.)}$$

Ans

— o —

Find the area of circle diameter 52 Given that the area 'A' of a circle of diameter d are as follows :-

d	50	55	60	65	70
A	1963	2376	2827	3318	3848

Ans:- d=52 ; forward Interpoletion

Diameter (x)	Area (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
50	1963	413	38	2	-3
55	2376	451	40	-1	
60	2827	491	39		
65	3318	530			
70	3848				

$$\therefore y(52) = y_0 + u\Delta y_0 + \frac{(u-1)u}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$\therefore u = \frac{x - x_0}{h} = \frac{52-50}{5} = 0.4$$

$x = 52$

$x_0 = 50$

$h = 5$

$$\therefore \; y_{(52)} = 1963 + 0.4 \times 413 + \frac{0.4(.4-1)}{2!} \times 38$$

$$+ \frac{0.4(.4-1)(.4-2)}{3!} \times 2$$

$$+ \frac{.4(.4-1)(.4-2)(.4-3)}{4!}(-3)$$

$$= 1963 + 165.2 + (-4.56) + .128 + .1248$$

$$y(52) = 2123.8928 \text{ (approx.)}$$

Ans

$\Delta^2 y_0$

$y_0 + \cdots$

$\frac{u^2-6u+2}{3!} \Delta^3 y_0$

$+$

$+24$

u^2-100u

$\Delta^5 y_0$

x_0	y_0
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4

Lagrange's Interpolation formula

(for unequal Interval)

formula for any x, $y(x)$ is given by

$$\therefore y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}\, y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times y_4$$

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Q. Given valus

	x_0	x_1	x_2	x_3	x_4
x :	5	7	11	13	17
$f(x)$:	150	392	1452	2366	5262
	y_0	y_1	y_2	y_3	y_4

$\therefore$ find $f(9)$ using Lagrange's formula

$y(9)$ /

$$\therefore f(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)} \times 150$$

$$+ \frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)} \times 392$$

$$+ \frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)} \times 1452$$

$$+ \frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)} \times 2366$$

$$+ \frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)} \times 5202$$

$y(9) =$

$f(9) = 810$

Ans

Q. Using Lagrange's Interpolation formula compute the value of $\log_{10} 321.5$

x	X	x_0 321.0	x_1 322.8	x_2 324.2	x_3 325.0
$y(x)$	$\log_{10} x$	2.50651 y_0	2.50893 y_1	2.51081 y_2	2.51188 y_3

Ans:- find $y(\log_{10} 321.5)$ using lagrange formula

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$\log_{10} 321.5 = \frac{-12.28500}{-23.0400} \times 2.50651$$

$$+ \frac{5.54400}{} \times 2.50893$$

$$\boxed{\log_{10} 321.5 = 2.50718}$$

$$= 1.33148 + 2.13829 + (-1.59378) + 0.62619$$

$$= 2.50718$$

$\Delta^2 y_0$

$y_0 + \dots$

Using calculator

$$Y = \frac{(x - A)(x - B)(x - C)}{(D - A)(D - B)(D - C)} \times E$$

321.5 x_1 x_2 x_3

x_0

x_1

y_0

y_1

y_2

y_3

$D = x_1$ $A = x_0$ $B = x_2$ $C = x_3$

$= x_2$ x_0 x_1 x_3

x_3 x_0 x_1 x_2

sum all

$\frac{u^2 - 6u + 2}{3!}$ $\Delta^3 y_0$

$+$

$\frac{+24}{u^2 - 100u}$

$\Delta^5 y_0$

(Q) Using Lagrange's formula, find the form of function f(x) following table

	x_0	x_1	x_2	x_3
x	0	1	3	4
fx	-12	0	12	24
	y_0	y_1	y_2	y_3

∴ Using Lagrange formula

$$\therefore f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$f(x) = \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)} \times -12$$

$$+ \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)} \times 0$$

$$+ \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)} \times 12$$

$$+ \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)} \times 24$$

$$= (x-1)(x-3)(x-4) + 0 + 2x(x-1)(x-4) + 2x(x-1)(x-3)$$

$$= (x-1)(x^2-3x-4x+12 + 2x^2-8x+2x^2-6x)$$

$$= (x-1)(5x^2-21x+12)$$

$$= 5x^3-21x^2+12x-5x^2+21x-12$$

$$= 5x^3-26x^2+33x-12$$

$$\boxed{\therefore f(x) = 5x^3-26x^2+33x-12}$$

Ans/

Newton forward formula and its derivative

∴ Newton forward formula

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$\therefore u = \frac{x - x_0}{h}$$

$$\therefore du/dx = \frac{1}{h}(1-0) = 1/h$$

1st derivatives

$$dy/dx = 0 + \frac{1}{h}\Delta y_0 + \frac{2u-1}{2!} \times \frac{1}{h}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!\,h}\Delta^3 y_0$$

$$= \frac{1}{h}\left(\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0 + \cdots\right)$$

2nd derivatives

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \cdots\right)$$

3rd derivatives

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left(\Delta^3 y_0 + \cdots\right)$$

Q. Find 1st, 2nd, 3rd derivatives of the function tabulated below at $x = 1.5$

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.000	13.625	24.000	38.875	59.000

Sol Forward Difference Table

x	y	Δy_0 ($y_2 - y_1$)	$\Delta^2 y_0$	$\Delta^3 y_0$
1.5	3.375 y_1	3.625	3.000	0.750
2.0	7.000 y_2	6.625	3.750	0.750
2.5	13.625	10.375	4.500	0.750
3.0	24.000	14.875	5.250	
3.5	38.875	20.125		
4.0	59.000			

Here

$x = 1.5$

$x_0 = 1.5$ $\therefore u = \dfrac{x - x_0}{h} = 0$

$h = 0.5$

1st derivatives

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0\right]$$

$$= \frac{1}{0.5}\left[3.625 + \frac{0-1}{2!}(3.000) + \frac{0-0+2}{3!}(.750)\right]$$

$$= 4.750$$

2nd derivatives

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0\right]$$

$$= \frac{1}{.5^2}\left(3.000 + (0-1)\,.750\right)$$

$$= 9.000$$

3rd derivatives

$$\frac{d^3y}{dx^2} = \frac{1}{h^3}\Delta^3 y_0$$

$$= \frac{1}{.5^3} \times .750 = 6.000$$

∴ Thus at $x = 1.5$

$$\frac{dy}{dx} = 4.750 \quad \frac{d^2y}{dx^2} = 9.000$$

$$\frac{d^3y}{dx^3} = 6.000$$

Newton's forward Interpolation formula and its derivatives

Newton forward formula

$\cancel{f(x) = f(x_0 + uh)}$

$$y(x) = y(x_0 + uh) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 y_0 + \cdots$$

$$u = \frac{x - x_0}{h}$$

1st derivatives

$$y'(x) = \overset{y'}{\cancel{f}}(x_0 + uh)(h) = \Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{3!}\Delta^3 y_0$$

$$+ \frac{4u^3 - 18u^2 + 22u - 6}{4!}\Delta^4 y_0 + \frac{\cancel{5p^4} - \cancel{40}\; 5u^4 - 40u^3 + 105u^2 - 100u + 24}{5!}\Delta^5 y_0$$

$\cdots$

$$= y'(x_0+uh)$$

$$y'(x) = \frac{1}{h}\left\{\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{4!}\Delta^4 y_0 + \frac{5u^4-40u^3+105u^2-100u+24}{5!}\Delta^5 y_0 + \cdots\right\}$$

$\boxed{\text{2nd derivatives}}$

$$y''(x) = y''(x_0+uh)\cdot h$$

$$= \frac{1}{h}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{12u^2-36u+22}{4!}\Delta^4 y_0 + \frac{2u^3-12u^2+21u-10}{12}\Delta^5 y_0 + \cdots\right]$$

$$\therefore y''(x) = y''(x_0+uh)$$

$$= \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2-18u+11}{12}\Delta^4 y_0 + \frac{2u^3-12u^2+21u-10}{12}\Delta^5 y_0 + \cdots\right]$$

$$= y'(x_0+uh)$$

$$y'(x) = \frac{1}{h}\left\{ \Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{4!}\Delta^4 y_0 + \frac{5u^4-40u^3+105u^2-100u+24}{5!}\Delta^5 y_0 + \cdots \right\}$$

$\boxed{\text{2nd derivatives}}$

$$y''(x) = y''(x_0+uh)\cdot h$$

$$= \frac{1}{h}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{12u^2-36u+22}{4!}\Delta^4 y_0 + \frac{2u^3-12u^2+21u-10}{12}\Delta^5 y_0 + \cdots\right]$$

$$\therefore y''(x) = y''(x_0+uh)$$

$$= \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2-18u+11}{12}\Delta^4 y_0 + \frac{2u^3-12u^2+21u-10}{12}\Delta^5 y_0 + \cdots\right]$$

Find the first and 2nd, 3nd derivative at poin $x = 1.5$.

x	1.00	1.05	1.10	$1.\cancel{70}$ 15	1.20	1.25	1.30
$y=\sqrt{x}$	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017