Roof finding

① Bisection method :-

$f(a)f(b) < 0$

$\therefore$ appnoximate noot $x_1 = \frac{a+b}{2}$

$\therefore$ if $(f(x_1)f(b) < 0)$, $x_2 = \frac{x_1+b}{2}$

$\therefore$ if $(f(x_1)f(a) > 0)$, $x_2 = \frac{a+x_1}{2}$

② False position method

appnoximate noot, $x_1 = \frac{af(b) - f(a)\,b}{f(b)-f(a)}$

if $[f(x_1)f(b) < 0]$ $x_2 = \frac{x_1 f(b) - bf(x_1)}{f(b) - f(x_1)}$

$\therefore$ if $[f(a)f(x_1) < 0]$ $x_2 = \frac{af(x_1) - x_1 f(a)}{f(x_1) - f(a)}$

③ Newton Raphson

$\therefore$ appnoximate noot $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

n

$b = n$ not given

Numerical Integration

$h = \frac{b-a}{n}$

1. **Trapezoid rule** (n any)

$$\int_a^b f(x)dx = \frac{h}{2}\left(y_0 + 2(y_1+y_2+y_3+\dots+y_{n-1}) + y_n\right)$$

$$= \int_a^b y\,dx$$

2. **Simpson's $\frac{1}{3}$ rule** $= \frac{h}{3}\left(y_0 + 4(y_1+y_3+\dots+y_{n-1}) + 2(y_2+y_4+\dots+y_{n-2}) + y_n\right)$

(n even)

3. **Simpson's $\frac{3}{8}$ rule** $= \frac{3h}{8}\left(y_0 + 3(y_1+y_2+y_4+y_5+\dots+y_{n-1}+y_{n-2}) + 2(y_3+y_6+\dots+y_{n-3}) + y_n\right)$

(n is divisible by 3)

ODE

same for both

$$\frac{dy}{dx} = f(x) = f(x,y)$$

$\therefore f(x_0) = y_0$ (Given)

1. **Euler's Method:-**

$$\therefore y(x_n) = y_n = y_{n-1} + h f(x_{n-1}, y_{n-1})$$

$\therefore x_n = x_0 + nh$

$\therefore h = x_n - x_{n-1}$ or Given

2. **Runge-kutta Method:-**

$$y(x_n) = y_n = y_{n-1} + \frac{1}{2}(k_1 + k_2)$$

$\therefore k_1 = hf(x_{n-1}, y_{n-1})$

$\therefore k_2 = hf(x_{n-1} + h, y_{n-1} + k_1)$

$\therefore x_n = x_0 + nh$

$\Delta y_0 = y_2 - y_1$

Interpolation

$u = \frac{x - x_0}{h}$

Newton forward formula

$$y(x) = y_0 + \Delta y_0 u + \Delta^2 y_0 \frac{u(u-1)}{2!} + \Delta^3 y_0 \frac{u(u-1)(u-2)}{3!}$$

$$+ \Delta^4 y_0 \frac{u(u-1)(u-2)(u-3)}{4!} + \cdots$$

first derivatives,

$$y'(x) = \frac{1}{h}\left(\Delta y_0 + (2u-1)\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{3!} \Delta^3 y_0 + \frac{4u^3 - 18u^2 + 22u - 6}{4!} \Delta^4 y_0 + \cdots \right)$$

2nd derivatives

$$y''(x) = \frac{1}{h^2}\left(\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2 - 18u + 11}{12} \Delta^4 y_0 + \cdots \right)$$

3rd derivatives

$$y'''(x) = \frac{1}{h^3}\left(\Delta^3 y_0 + \frac{2u-3}{2} \Delta^4 y_0 + \cdots \right)$$

(*given)

Newton backward formula

$$\therefore y(x) = y_n + \Delta y_n u + \Delta^2 y_n \frac{u(u+1)}{2!} + \Delta^3 y_n \frac{u(u+1)(u+2)}{3!} + \Delta^4 y_n \frac{u(u+1)(u+2)(u+3)}{4!} + \cdots$$

$$\therefore u = \frac{x - x_n}{h}$$

Lagrange formula;

$$f(x) = y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

x given on not given

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$|a_1| > |b_1| + |c_1|$$
$$|b_2| > |a_2| + |c_2|$$
$$|c_3| > |a_3| + |b_2|$$

working rule

$$x = g_1(y, z)$$
$$y = g_1(x, z)$$
$$z = g_1(x, y)$$

Gauss Jaccobi :-

$$x_i = g_i(y_{i-1}, z_{i-1})$$
$$y_i = g_i(x_{i-1}, z_{i-1})$$
$$z_i = g_i(x_{i-1}, y_{i-1})$$

Gauss sedial :-

$$x_i = g_i(y_{i-1}, z_{i-1})$$
$$y_i = g_i(x_i, z_{i-1})$$
$$z_i = g_i(x_i, y_i)$$

Gaussian elimination method

covert the argument matrix into row echelon form

row operations only

$$\left[\begin{array}{ccc|c} 1 & A & B & C \\ 0 & 1 & D & E \\ 0 & 0 & 1 & F \end{array}\right]$$

$(x, y, z) = ?$

$\therefore$ then

$$x + yA + zB = C$$
$$y + zD = E$$
$$z = F$$

Gauss Jordan Elimination

row echelon form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & A \\ 0 & 1 & 0 & B \\ 0 & 0 & 1 & C \end{array}\right]$$

$\therefore x = A$

$\therefore y = B$

$\therefore z = C$