SLE: System of Linear Equations.

Direct / Exact Method:

1) Gauss Jordan
2) Gaussian Elimination

Indirect / Iterative:

1) Gauss-Jacobi
2) Gauss-seidal

* Gauss-Jacobi Iterative Method:

$10x - 2y + z = 2 \dashrightarrow a_{11}x + a_{12}y + a_{13}z = b_1$

$-3x + 11y + 2z = 5 \dashrightarrow a_{21}x + a_{22}y + a_{23}z = b_2$

$x - y + 5z = 1 \dashrightarrow a_{31}x + a_{32}y + a_{33}z = b_3$

ignore sign

Application condition:

$$\left.\begin{array}{l} |a_{11}| > |a_{12}| + |a_{13}| \\ |a_{22}| > |a_{21}| + |a_{23}| \\ |a_{33}| > |a_{31}| + |a_{32}| \end{array}\right]$$

diagonally dominated

Solution:

Here, $x = (2 + 2y - z) / 10$

$y = (5 + 3x - 2z) / 11$

$z = (1 - x + y) / 5$

Step 1: Let $x = x_0 = 0,\ y = y_0 = 0,\ z = z_0 = 0$

$\therefore\ x_1 = (2+0-0)/10 = 0.2$

$y_1 = 5/11 = 0.4545$

$z_1 = 1/5 = 0.2$

Step 2: $x = x_1,\ y = y_1,\ z = z_1$

$x_2 = (2 + 2y_1 - z_1)/10$

$= (2 + 2 \times 0.4545 - 0.2)/10$

$= 0.2709$

$y_2 = (5 + 3x_1 - 2z_1)/11$

$= (5 + 3 \times 0.2 - 2 \times 0.2)/11$

$= 0.473$

$z_2 = (1 - x_1 + y_1)/5$

$= (1 - 0.2 + 0.4545)/5$

$= 0.2509$

Def: (an iterative method for determining the solutions for the system of linear equations, which is diagonally dominant.)

Step 3: $x = x_2 = 0.2709$

$y = y_2 = 0.473$

$z = z_2 = 0.2509$

$x_3 = (2 + 2y_2 - z_2)/10 = 0.269$

$y_3 = (5 + 3x_2 - 2z_2)/11 = 0.4828$

$z_3 = (1 - x_2 + y_2)/5 = 0.24$

Step 4: $x = x_3 = 0.269$, $y = y_3 = 0.4828$, $z = z_3 = 0.24$

$x_4 = (2 + 2y_3 - z_3)/10 = 0.2726$

$y_4 = (5 + 3x_3 - 2z_3)/11 = 0.4843$

$z_4 = (1 - x_3 + y_3)/5 = 0.24276$

Step 5: $x = x_4 = 0.2726$, $y = y_4 = 0.4843$, $z = z_4 = 0.24276$

$x_5 = (2 + 2y_4 - z_4)/10 = 0.2726$

$y_5 = (5 + 3x_4 - 2z_4)/11 = 0.48475$

$z_5 = (1 - x_4 + y_4)/5 = 0.24234$

Step 6: $x = x_5 = 0.2726$, $y = y_5 = 0.48475$

$z = z_5 = 0.24234$

$$x_6 = (2 + 2y_5 - z_5)/10 = 0.2727 \approx \frac{3}{11}$$

$$y_6 = (5 + 3x_5 - 2z_5)/11 = 0.4848 \approx 16/33$$

$$z_6 = (1 - x_5 + y_5)/5 = 0.2424 \approx 8/33$$

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First, $x = g_1(y, z)$

$y = g_2(x, z)$

$z = g_3(x, y)$

Then each step general form:

$$x_i = g_1(y_{i-1}, z_{i-1})$$

$$y_i = g_2(x_{i-1}, z_{i-1})$$

$$z_i = g_3(x_{i-1}, y_{i-1})$$

untill the $\boxed{\text{approx. values}}$ are found.

Gauss - Seidal Iterative method :

$$a_{11} x + a_{12} y + a_{13} z = b_1$$

$$a_{21} x + a_{22} y + a_{23} z = b_2$$

$$a_{31} x + a_{32} y + a_{33} z = b_3$$

Condition:

diagonally dominated

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$|a_{22}| > |a_{21}| + |a_{23}|$$

$$|a_{33}| > |a_{31}| + |a_{33}|$$

First,

$$x = g_1 (y, z)$$

$$y = g_2 (x, z)$$

$$z = g_3 (x, y)$$

Then each step's ~~st~~ general form :

$$x_i = g_1 (y_{i-1}, z_{i-1})$$

$$y_i = g_2 (x_i, z_{i-1})$$

$$z_i = g_3 (x_i, y_i)$$

Found in previous step (difference)

Ex:

$$10x - 2y + z = 2$$

$$-3x + 11y + 2z = 5$$

$$x - y + 5z = 1$$

Sol^n:

$$x = (2 + 2y - z)/10$$

$$y = (5 + 3x - 2z)/11$$

$$z = (1 - x + y)/5$$

Step 1: $\cancel{x}\ x_0 = 0,\ y_0 = 0,\ z_0 = 0$

$$x_1 = (2 + 2y_0 - z_0)/10 = 0.2$$

$$y_1 = (5 + 3x_1 - 2z_0)/11 = 0.509$$

$$z_1 = (1 - x_1 + y_1)/5 = 0.262$$

Step 2: $x_2 = (2 + 2y_1 - z_1)/10 = 0.2756$

$$y_2 = (5 + 3x_2 - 2z_1)/11 = 0.45825$$

$$z_2 = (1 - x_2 + y_2)/5 = 0.2365$$

Step 3: $x_3 = (2+2y_2 - z_2)/10 = .268$

$y_3 = (5 + 3x_3 - 2z_2)/11 = 0.48464$

$z_3 = (1 - x_3 + y_3)/5 = 0.2433$

Step 4: $x_4 = (2+2y_3 - z_3)/10 = 0.2726$

$y_4 = (5+3x_4 - 2z_3)/11 = 0.4847$

$z_4 = (1 - x_4 + y_4)/5 = 0.2424$

Step 5: $x_5 = (2+2y_4 - z_4)/10 = 0.2727$

$y_5 = (5+3x_5 - 2z_4)/11 = 0.4848$

$z_5 = (1 - x_5 + y_5)/5 = 0.2424$

$\therefore \ x = 0.2727 \approx 3/11$

$y = 0.4848 \approx 16/33$

$z = 0.2424 \approx 8/33$