1) Compute the derivatives df/dx of the following function using chain rule.

$$f(z) = \log_e (1+z)$$

where, $z = x^T x$, $x \in \mathbb{R}^D$

<u>Sol^n:</u> Here,

$$f(z) = \log_e (1+z)$$
$$= \log_e (1+x^T x)$$
$$= \log_e (u)$$

Let,
$u = (1+z)$
$\rightarrow u = (1+x^T x)$
$\because z = x^T x$

$$\therefore \frac{du}{dx} = \frac{d}{dx}(1+x^T x)$$
$$= 0 + 2x$$
$$= 2x$$

Again, $$\frac{df}{du} = \frac{d}{du}(\log_e(u))$$
$$= \frac{1}{u} = \frac{1}{1+x^T x}$$

So, using chain rule, we get,

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = \frac{1}{1+x^T x} \cdot 2x$$

$$= \frac{2x}{1+x^Tx} \quad \text{(Ans)}$$

2) Compute denivatives df/dx of the following function using chain rule:

$$f(z) = e^{-z/2}$$

$$z = g(y) = y^T S^{-1} y$$

$$y = h(x) = x - \mu$$

Sol^n:

$$\frac{dz}{dy} = \frac{d(g(y))}{d(y)}$$

$$= \frac{d}{dy}(y^T S^{-1} y)$$

$$= 2S^{-1}y$$

$$\frac{df}{dz} = \frac{d}{dz}(e^{-z/2})$$

$$= -\frac{1}{2} e^{-z/2}$$

$$\frac{dy}{dx} = \frac{d(h(x))}{dx}$$

$$= \frac{d}{dx}(x - \mu)$$

$$= I \quad \text{(identity matrix)}$$

$\therefore$ Using chain rule, we get.

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{1}{2} e^{-z/2} \cdot 2 S^{-1} y \cdot I$$

$$= -e^{-z/2} S^{-1} y \cdot I \qquad \text{(Ans)}$$